Digital Signal Processing

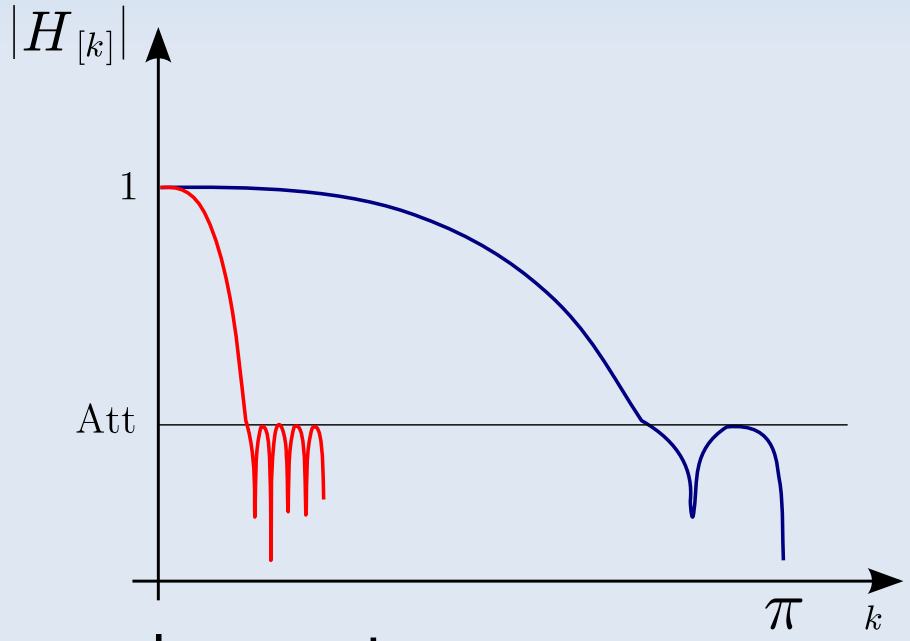
Multirate DSP

Bibliography:

- Lyons Ch 10
- Porat Ch. 12

Introduction

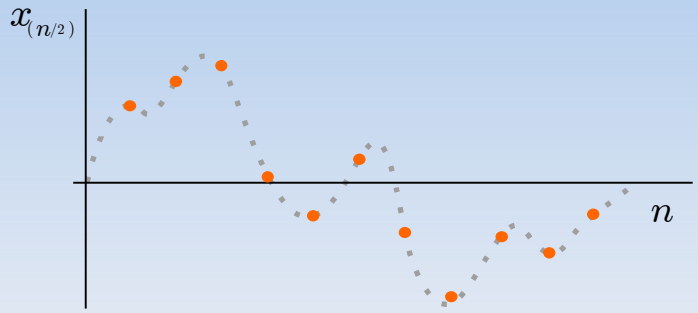
- Many situations motivates changing the sampling rate:
  - Computational cost optimization

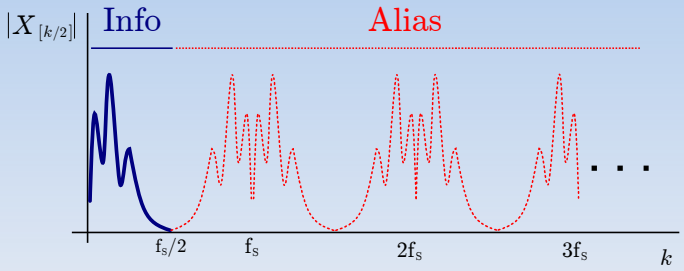


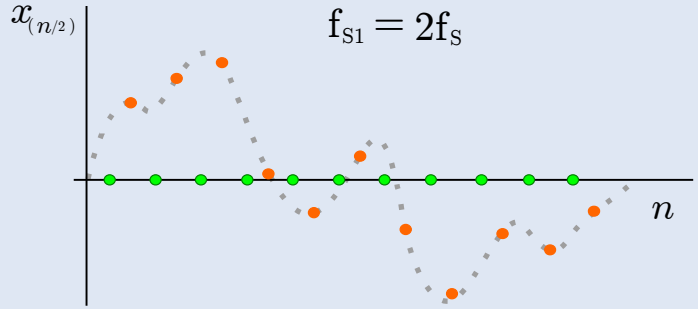
  - Application requirements

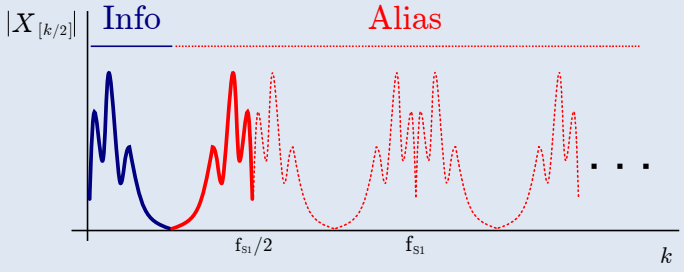
Introduction

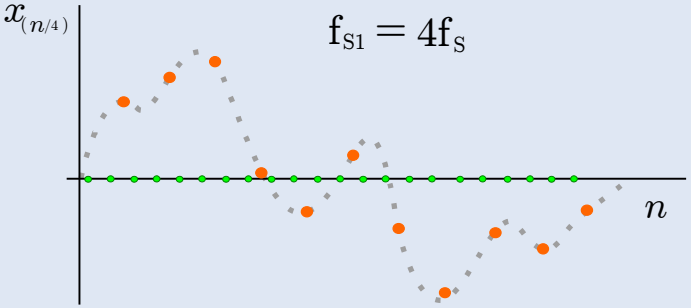
- Implications of adding samples:

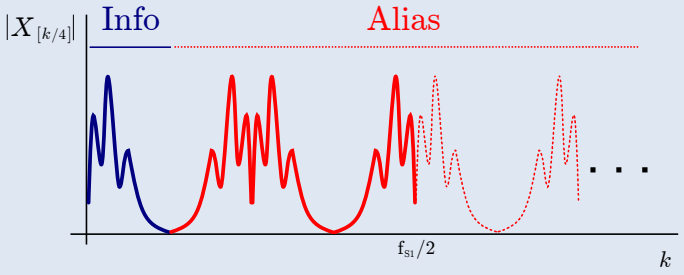






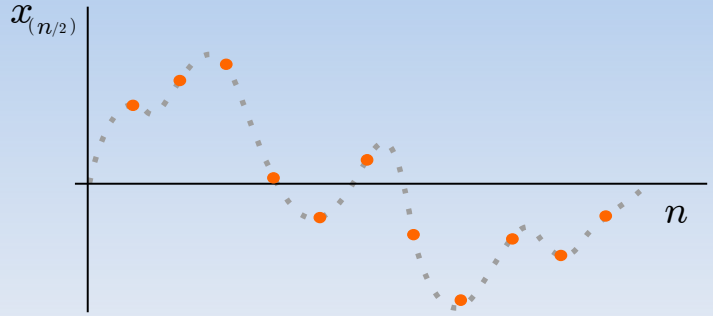


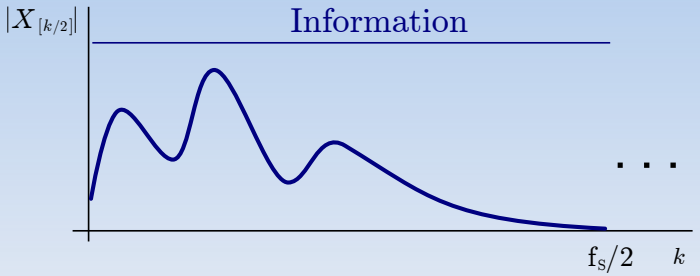


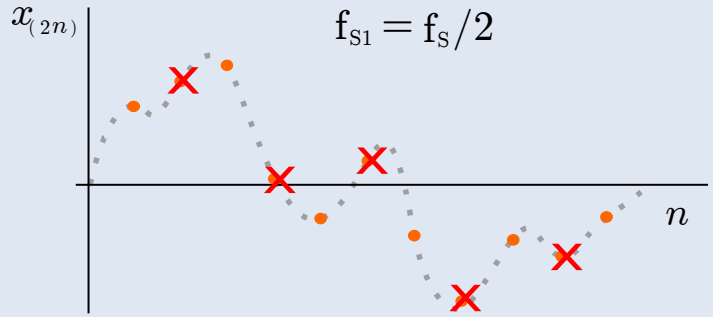


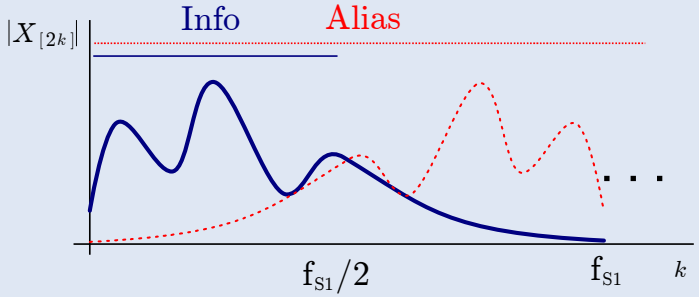
Introduction

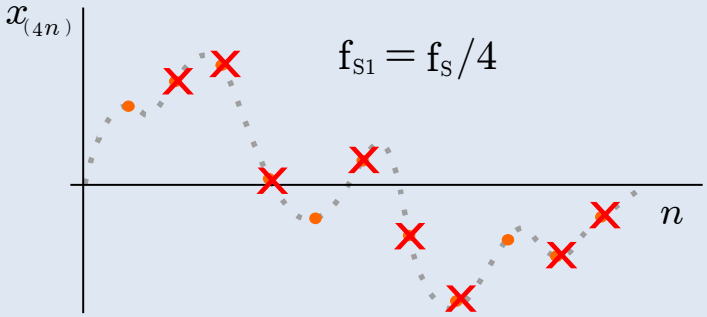
- Implications of discarding samples:

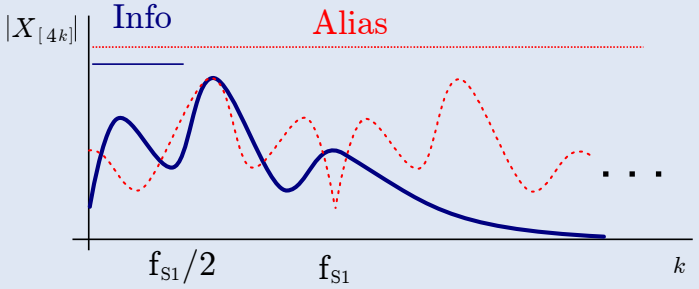






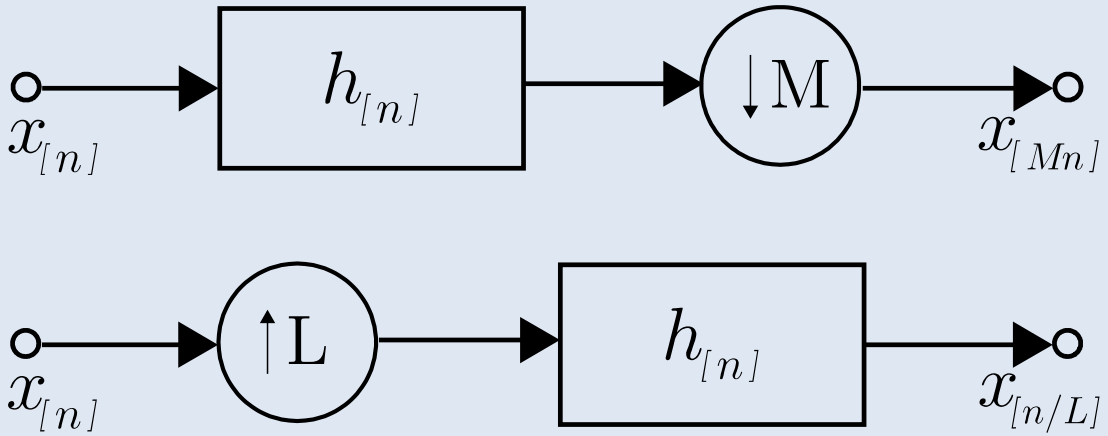




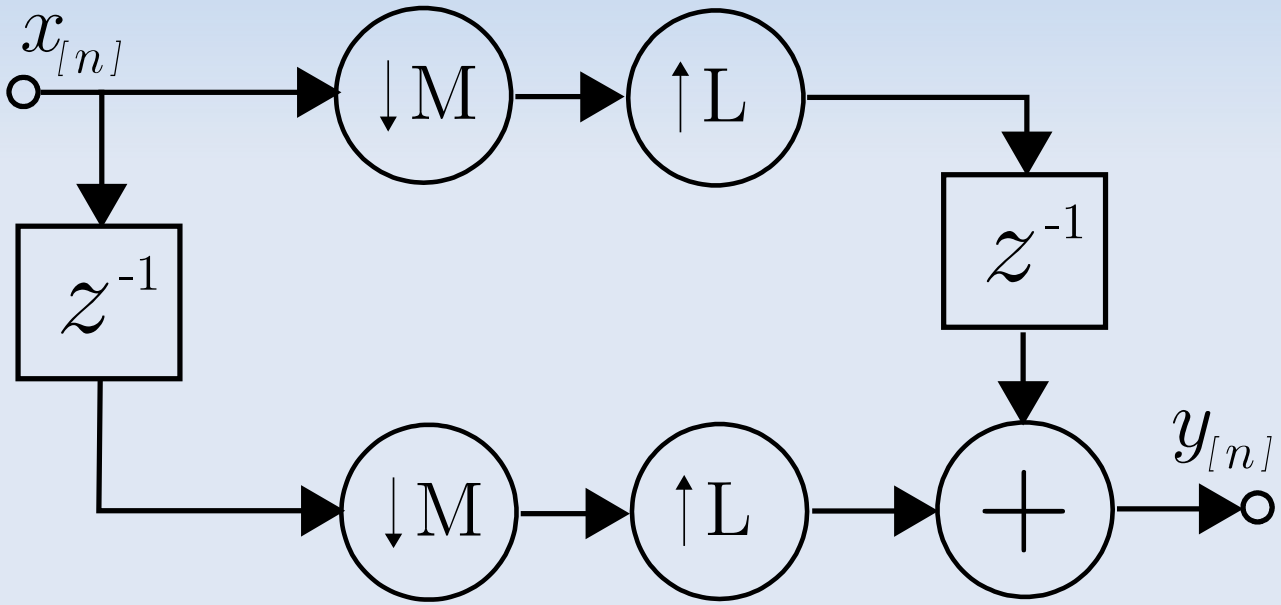


Introduction

- Decimation by M means discarding M-1 of M samples.
- Interpolation by L means adding L-1 zeros
- What should h[n] be for each case ?

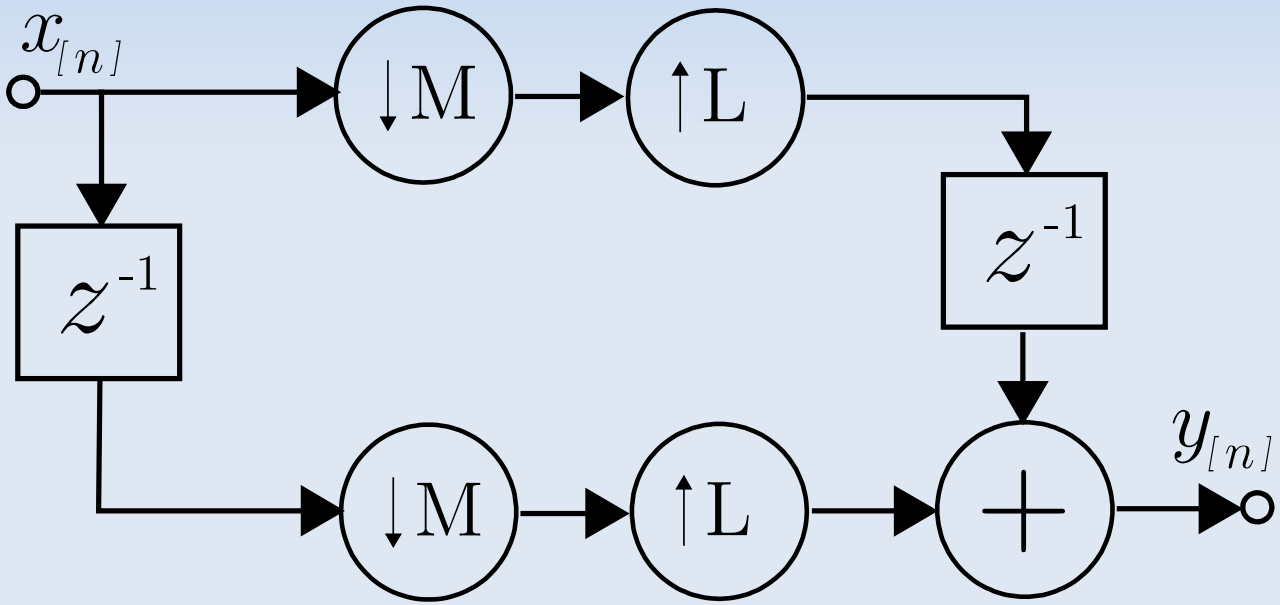


More about up/downsampling



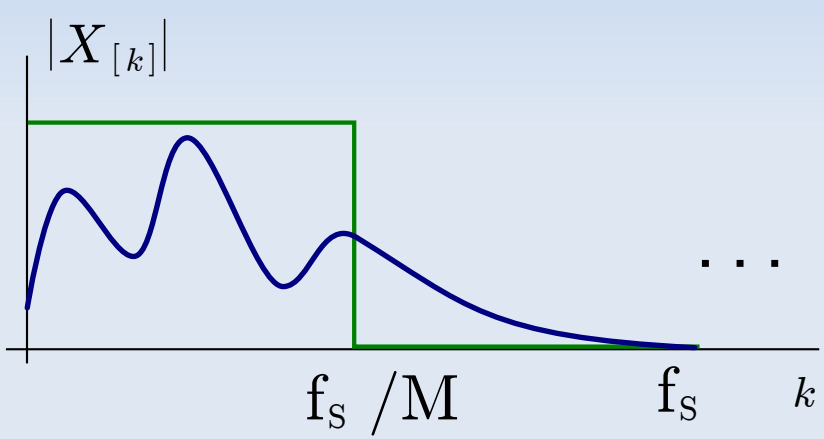
More about up/downsampling

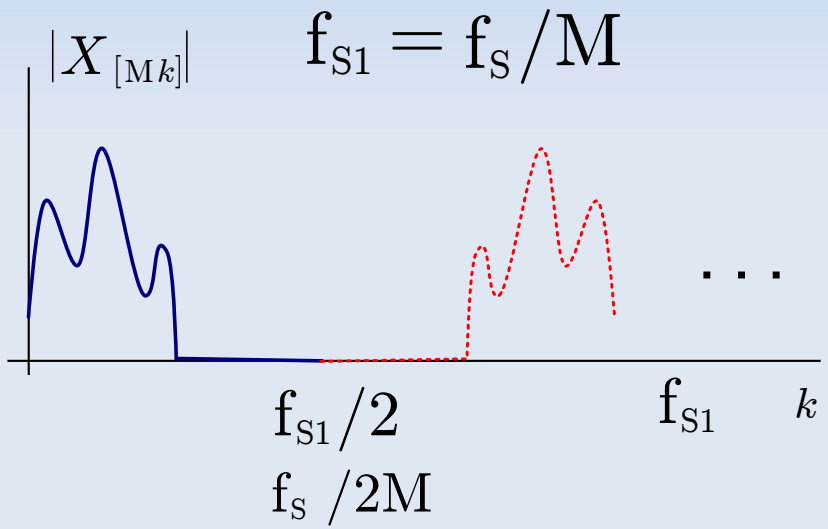
- Up/downsampling are not time invariant ops.

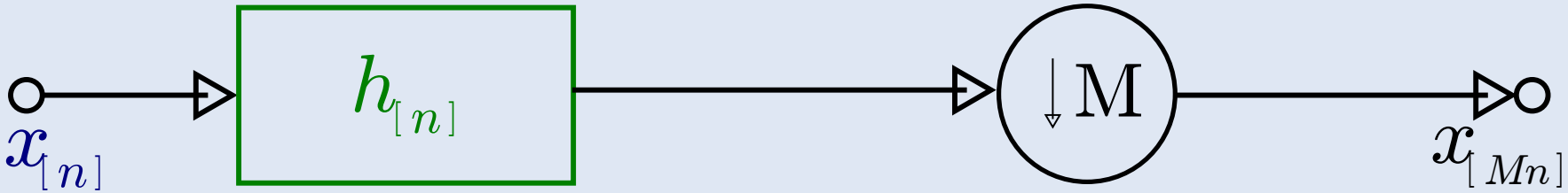


Antialiasing filter

- When decimating, alias must be eliminated previous to decimation

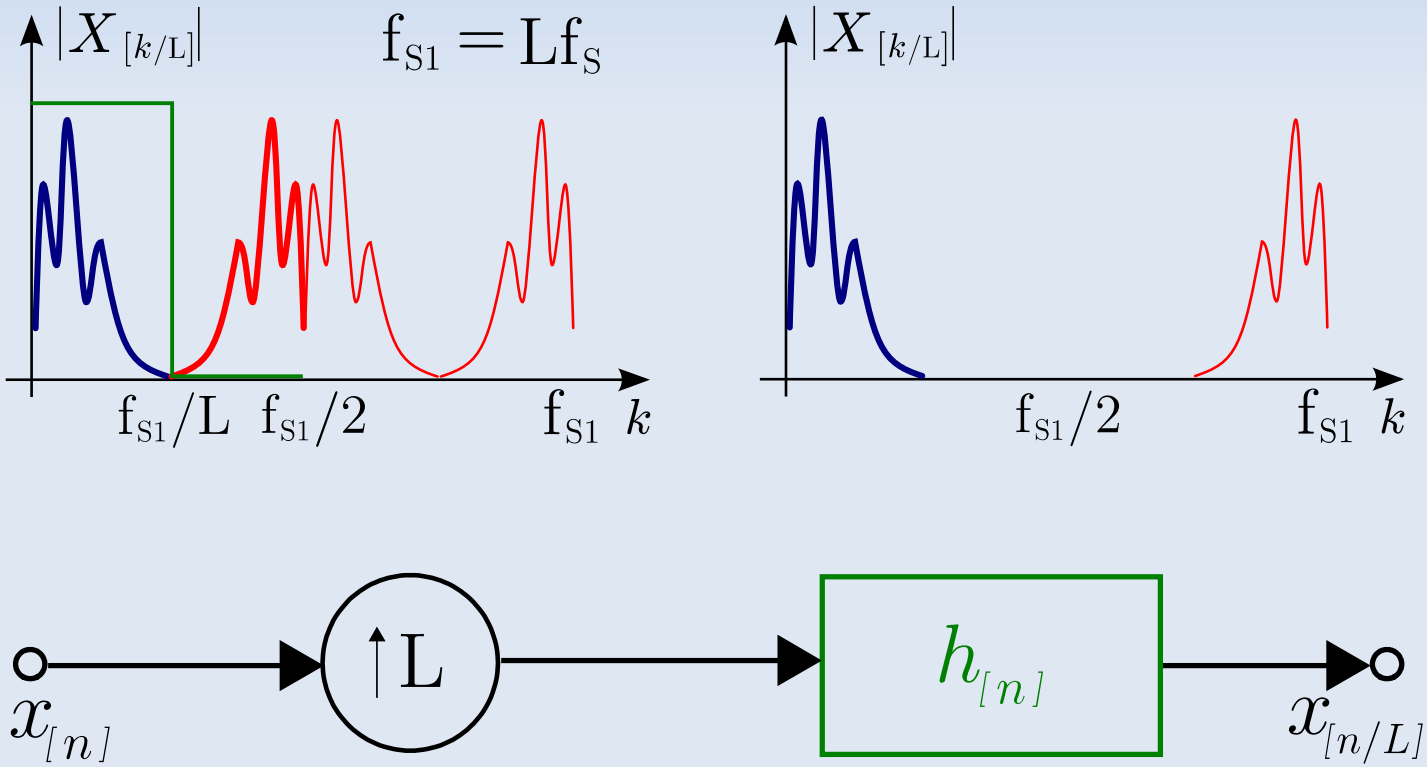






Antialiasing filter

- When interpolating, first add zeros and then filter alias



Sampling rate conversion

- Sampling rate conversion can be achieved via interpolation or decimation

$$f_2 = k \cdot f_S$$

- When k is not an integer, can be aproximated

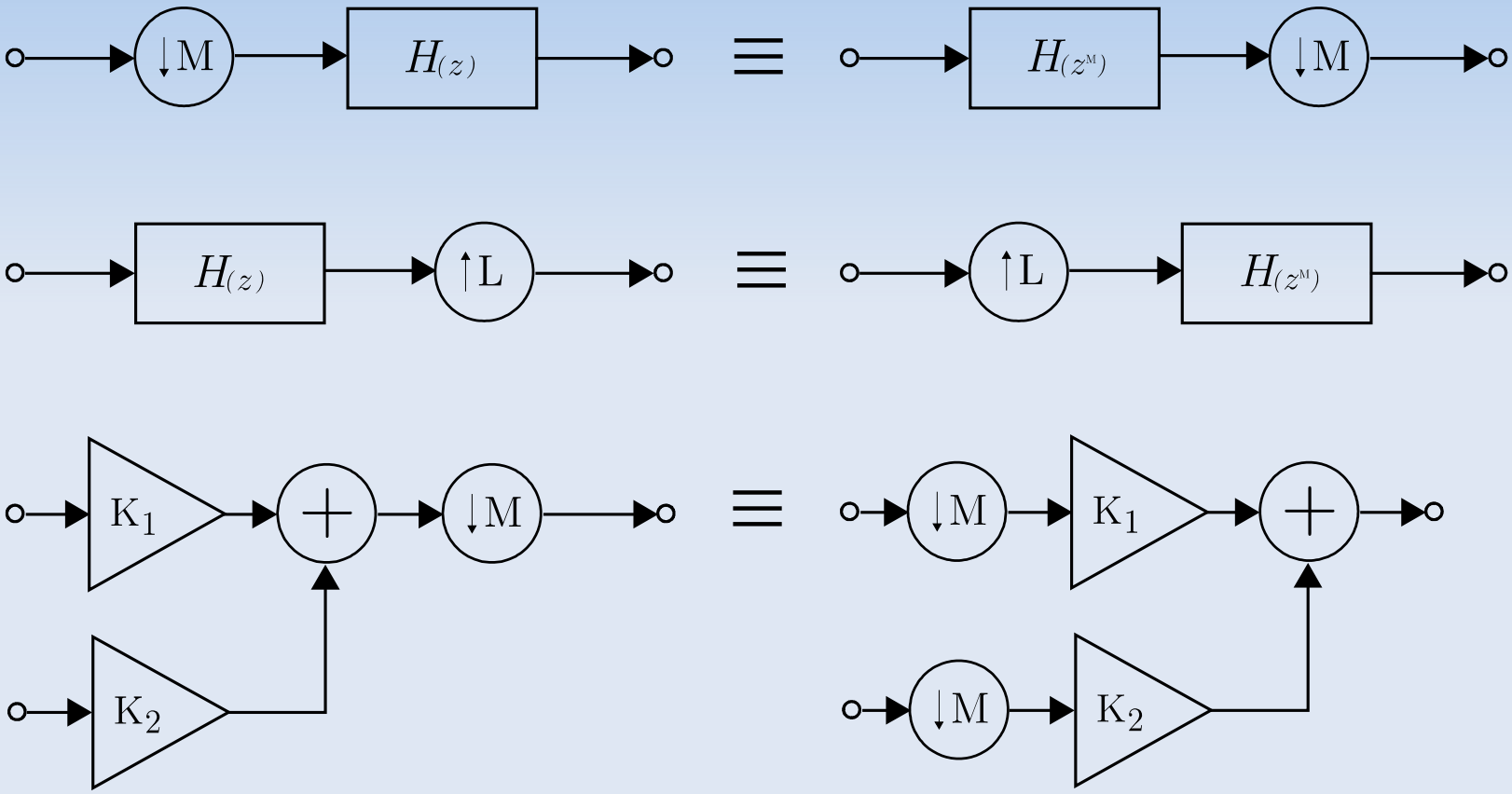
$$f_2 = L/M \cdot f_S$$

Sampling rate conversion

- For large M or L sampling rate changes, try the factorization of M and L

- This is equivalent to perform decimation/interpolation in several stages.

Noble identities



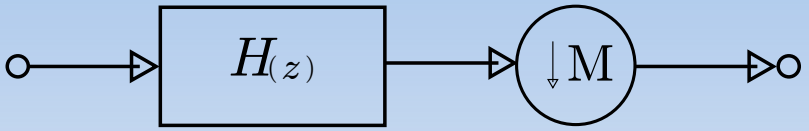
Computational improvements

- Efficient decimation motivation:
  - Save computations posteriorly discarded

- Decimate, then filter

Computational improvements

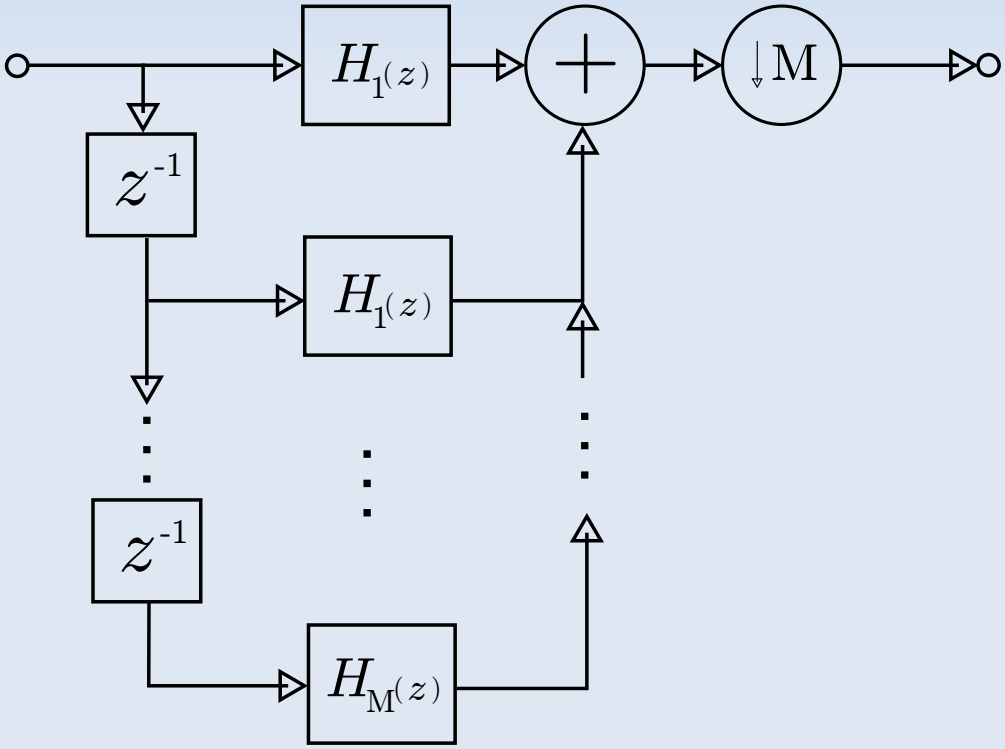
- Efficient decimation



$$H(z) = \sum_{l=0}^{N-1} z^{-l} E_l(z^N)$$

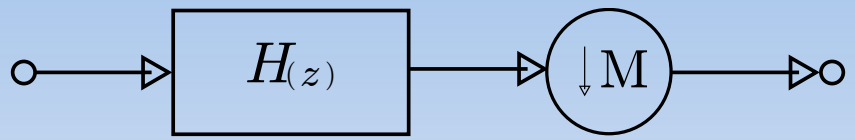
$$E_l(z) = \sum_{n=-\infty}^{\infty} e_l(n) z^{-n},\ l = 0, 1, \ldots, N-1,$$

$$e_l(n) \triangleq h(Nn + l). \qquad (l\text{th subphase filter}).$$



Computational improvements

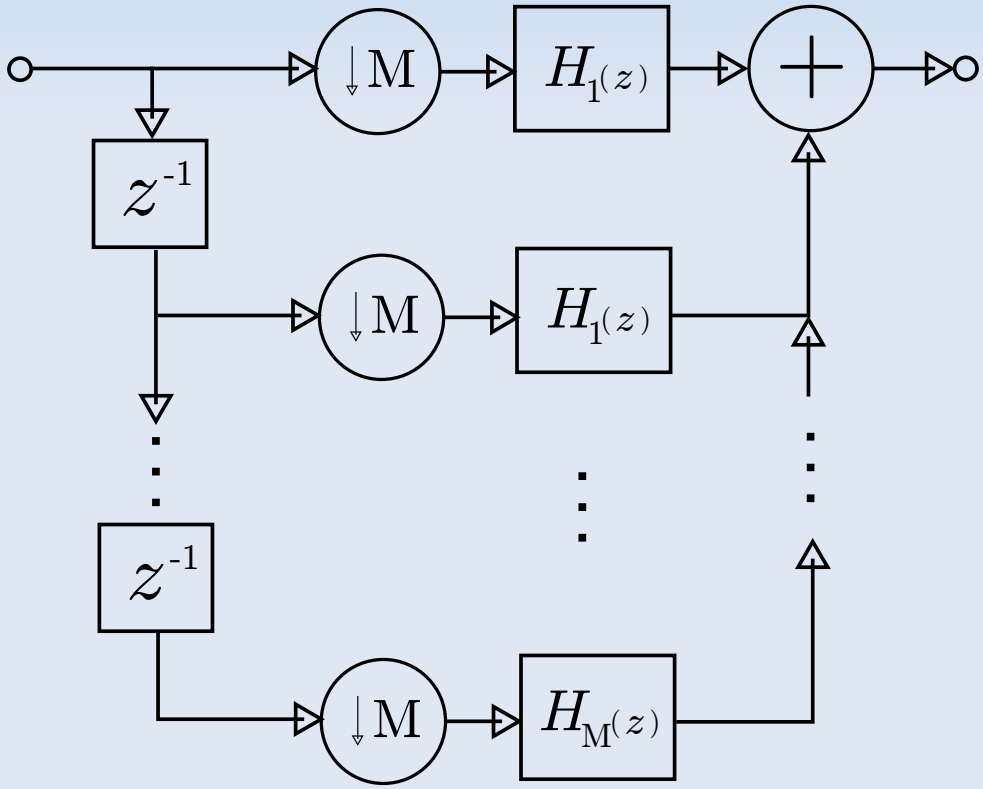
- Efficient decimation



$$H(z) = \sum_{l=0}^{N-1} z^{-l} E_l(z^N)$$

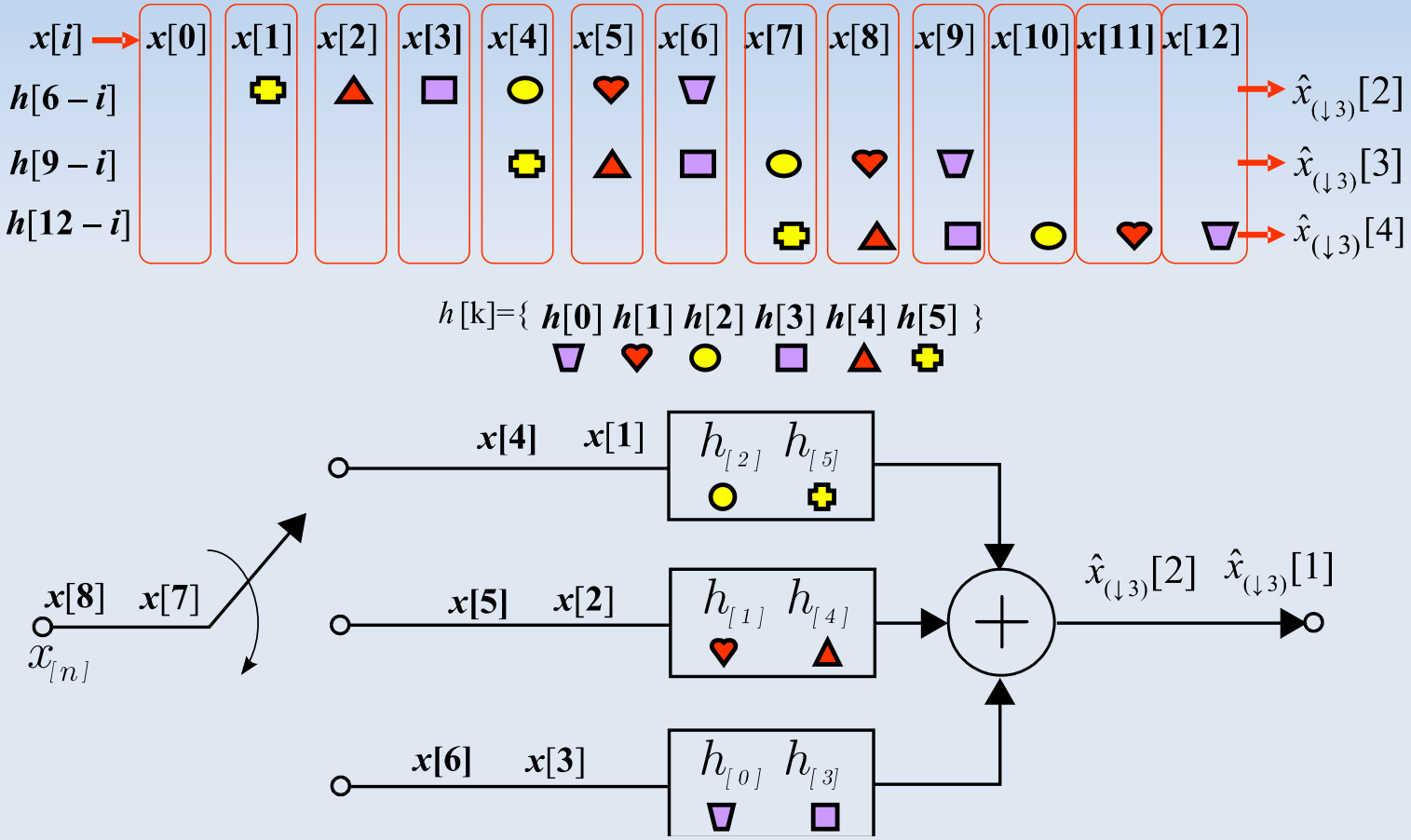
$$E_l(z) = \sum_{n=-\infty}^{\infty} e_l(n) z^{-n}, \; l = 0, 1, \ldots, N-1,$$

$$e_l(n) \triangleq h(Nn + l). \qquad (l\text{th subphase filter}).$$



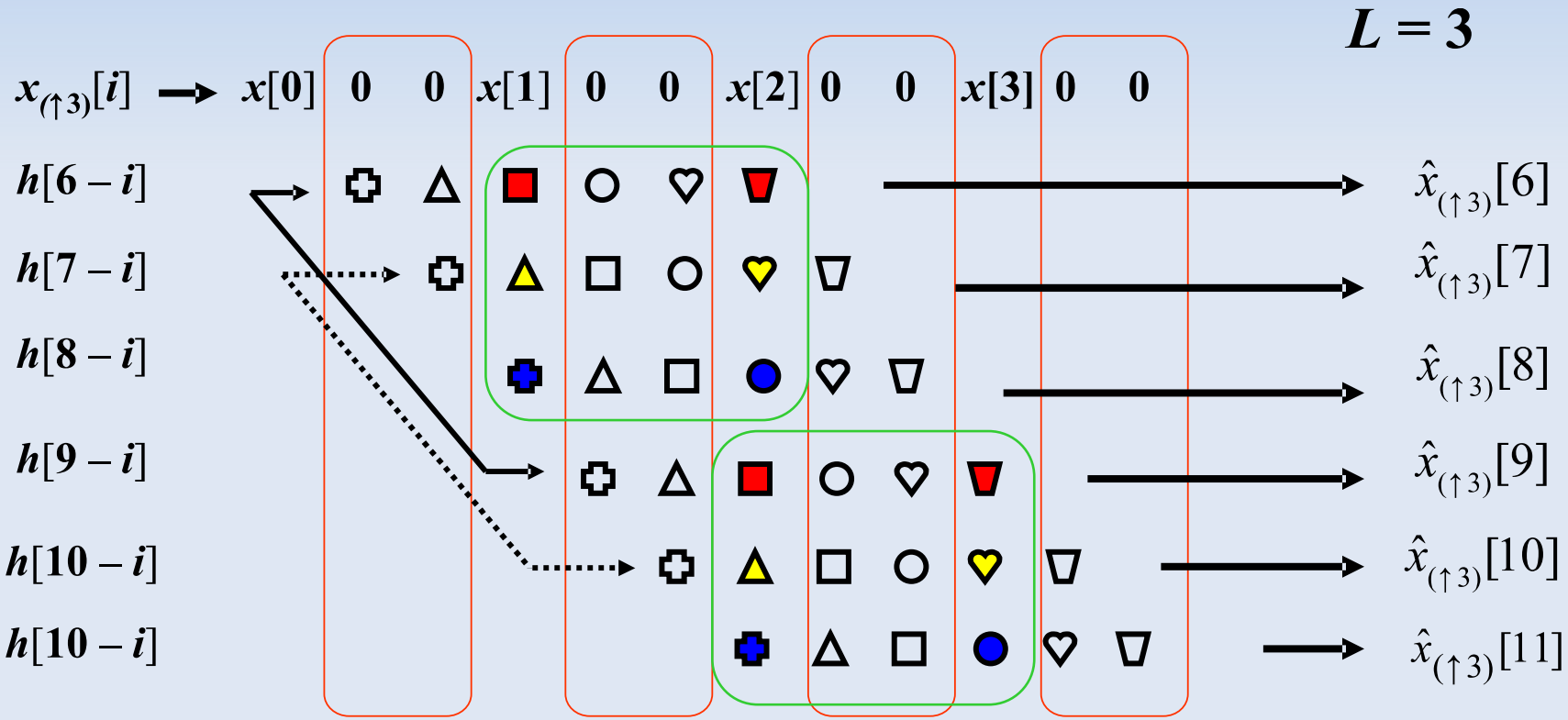
Computational improvements

- Following the initial example for M = 3



Computational improvements

- Efficient interpolation motivation:
  - Save computations when mult. by zeros



Computational improvements

- Following the interp. example