Project 4

Math512

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a) Estimate **Historical Volatility σ**

- CALCULATE RETURN $\frac{S_{t+1}-S_t}{S_t}$

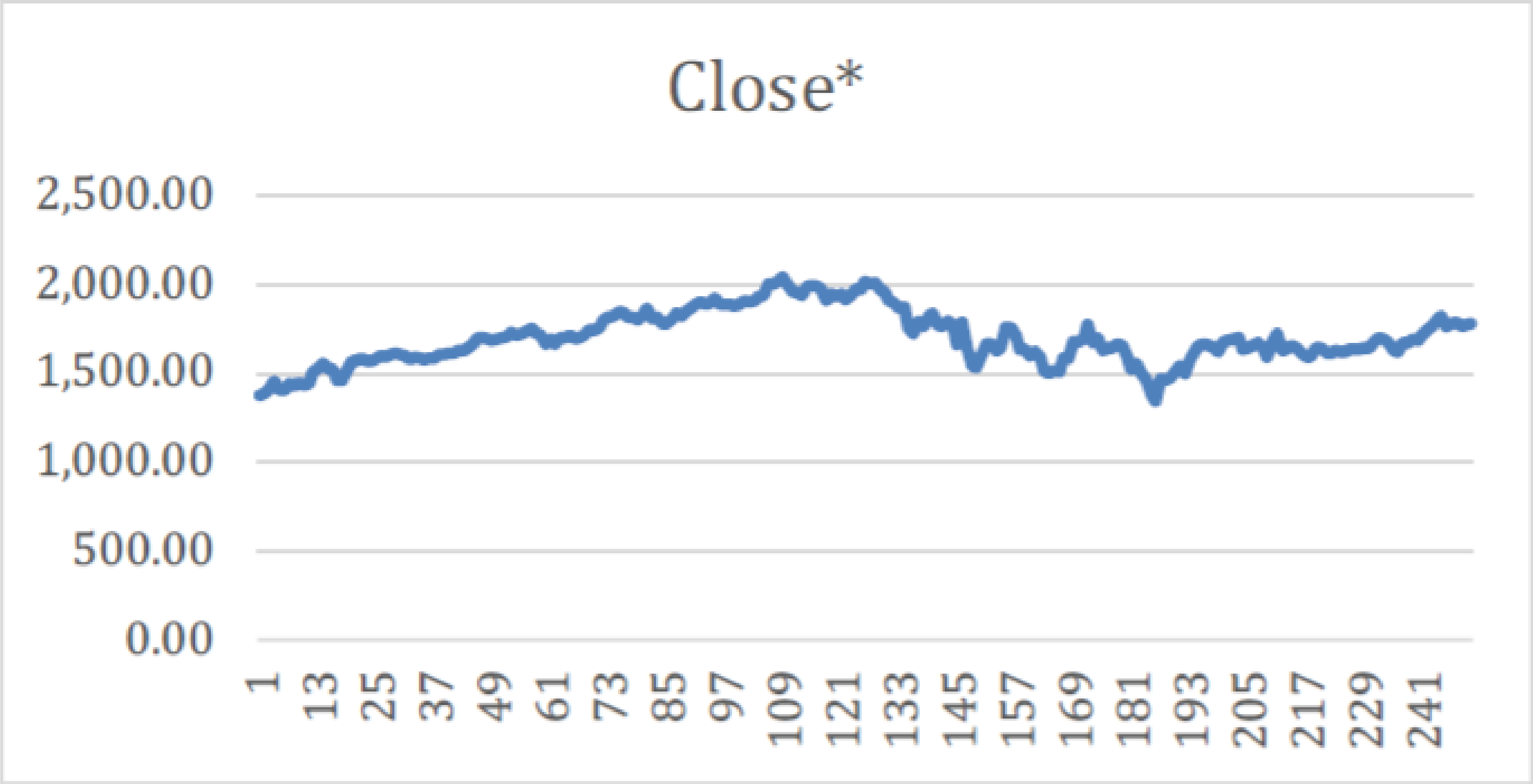
  VALUE OF STANDARD DEVIATION $S^2 = \frac{\Sigma(x_i-\bar{x})^2}{n-1}$.

  VOLATILITY $\sqrt{S^2}$ = **35.586%**

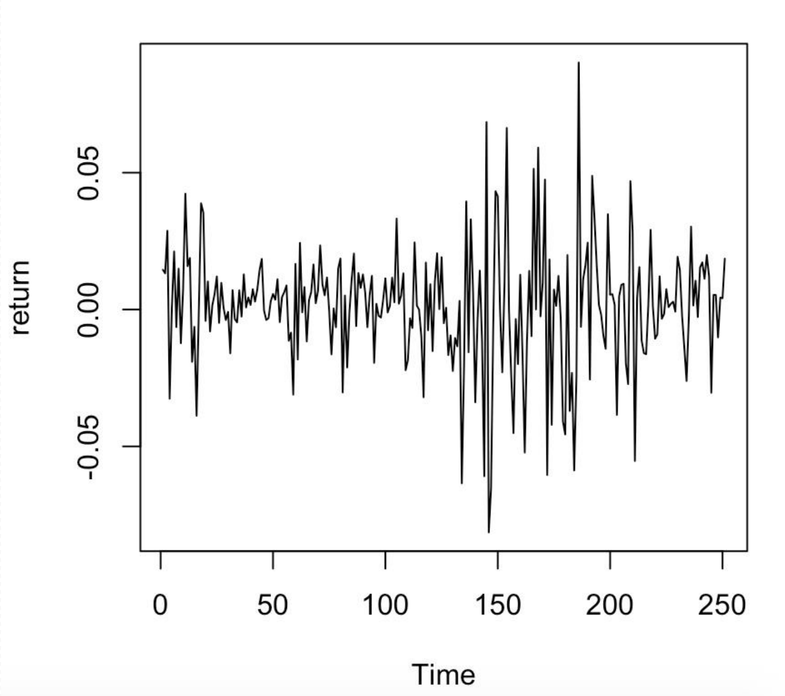
- CALCULATE LOG RETURN Ln $\left(\frac{S_{t+1}}{S_t}\right)$

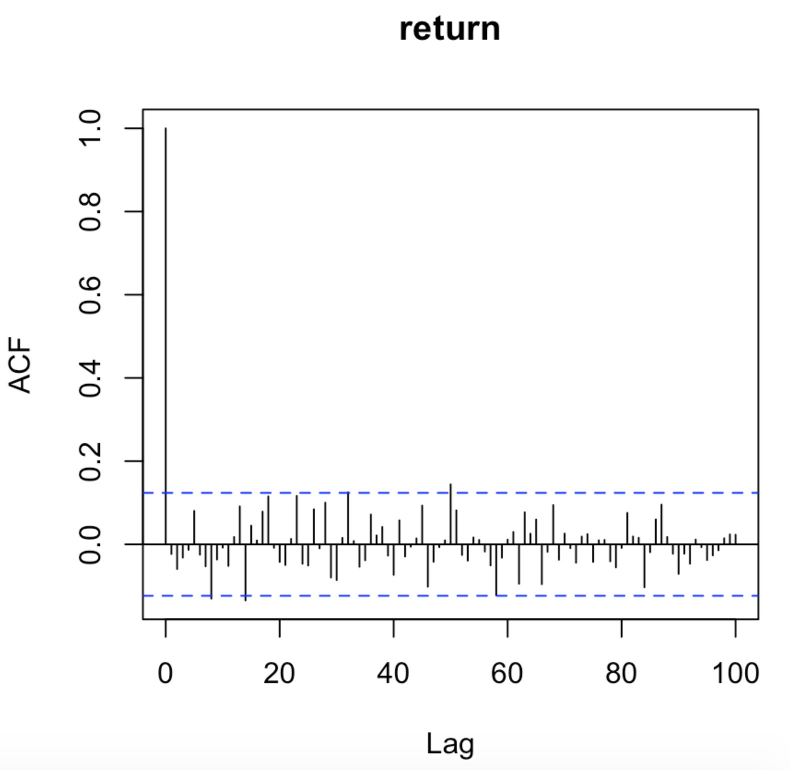
  HISTORICAL VOLATILITY $\sqrt{S^2}$ = **35.5889%**

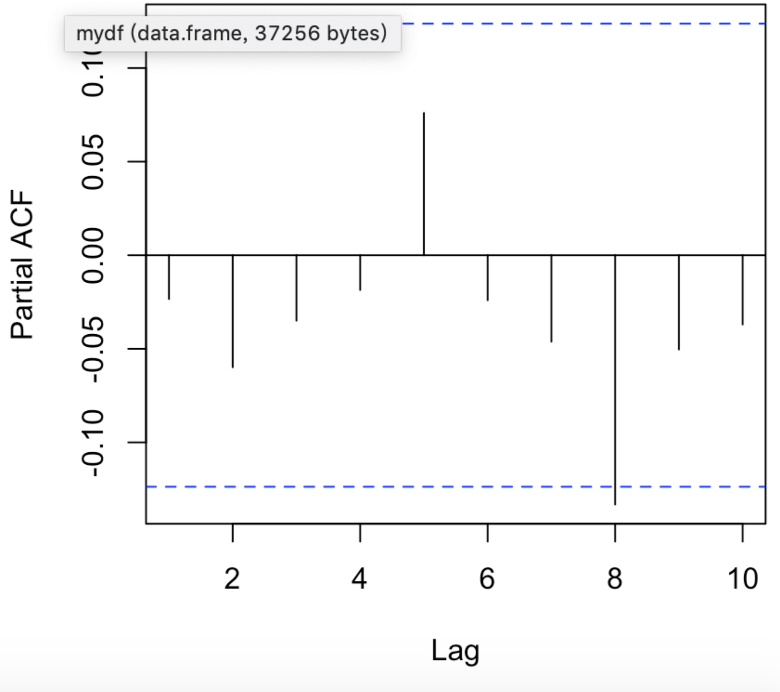
AMAN Closing Prices



Time Series Analysis







Log-return plot

Looks like WN

ADF test p-value: 0.01
--> stationary

ARCH and GARCH Model

- ARCH model (autoregressive conditional heteroskedasticity) - equation for variance σ_t^2:

$$\sigma_t^2 = \omega + \alpha_1 u_{t-1}^2 + \ldots \alpha_q u_{t-q}^2$$

- Constraints on parameters:
  - variance has to be positive:

$$\omega > 0, \alpha_1, \ldots, \alpha_{q-1} \geq 0, \alpha_q > 0$$

  - stationarity:

$$\alpha_1 + \ldots + \alpha_q < 1$$

We tried ARCH(1), ARCH(2), ARCH(3) and ARCH(4), but the residuals are not white noise;

AIC and BIC test:
we got ARCH(5)

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Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-5.028436 -4.930117 -5.029936 -4.988870
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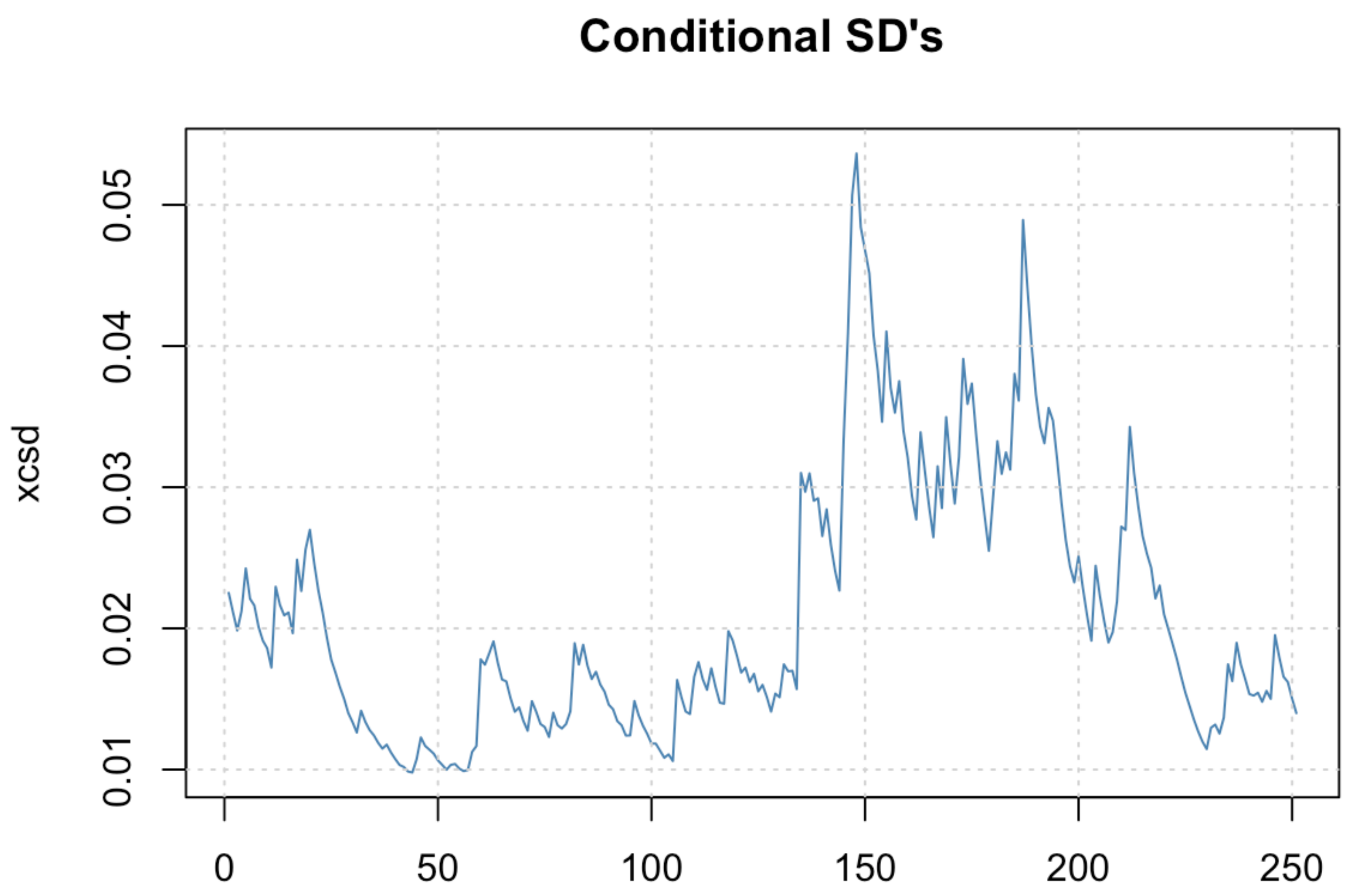
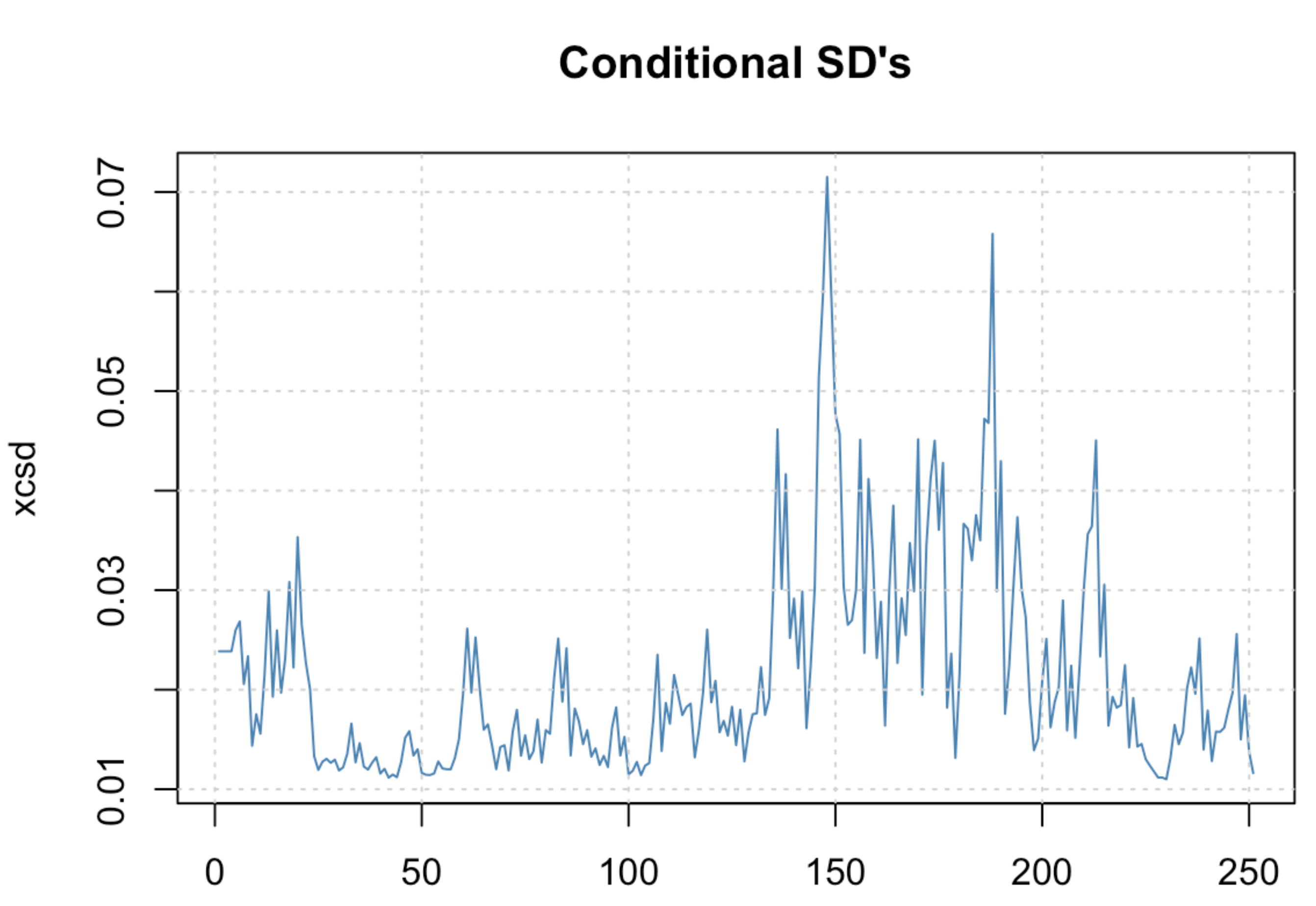
ARCH(5) Model



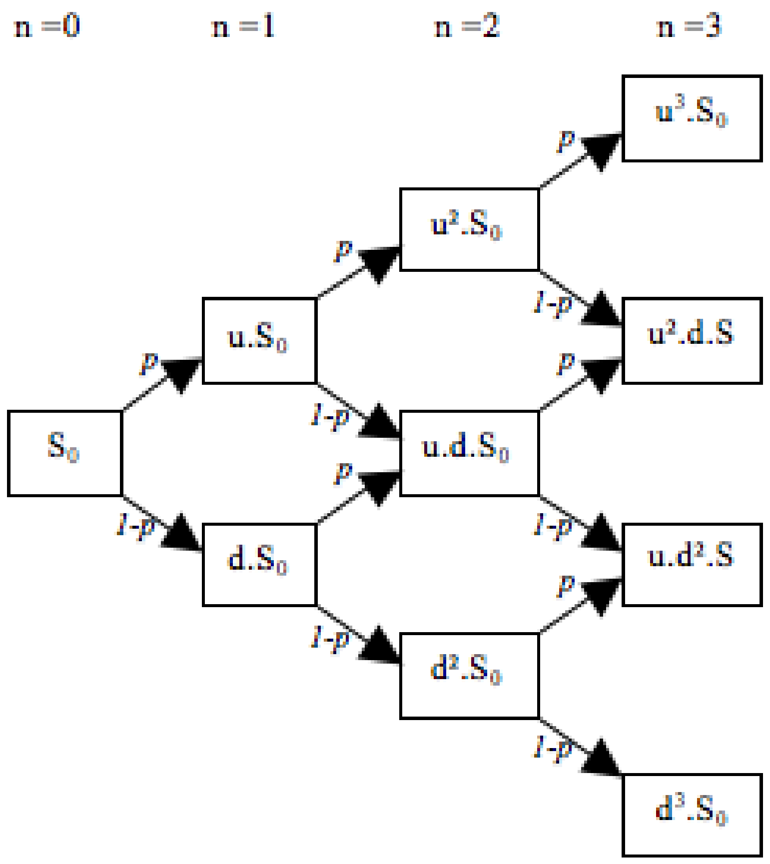
Implied volatility forecast:
0.03056744

GRCH(1,1) Model

$$\sigma_n^2 = \gamma v_L + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$$



Implied volatility forecast: 0.03452849



$$u = e^{\sigma\sqrt{t}} \quad d = e^{-\sigma\sqrt{t}} = \frac{1}{u}$$

σ: Stock Volatility

b) **American Call Option Price** - Binomial Tree Approach

Maturity Date: July 1st (from April 1st , T = 0.25)

Number of Steps: N = 2,000

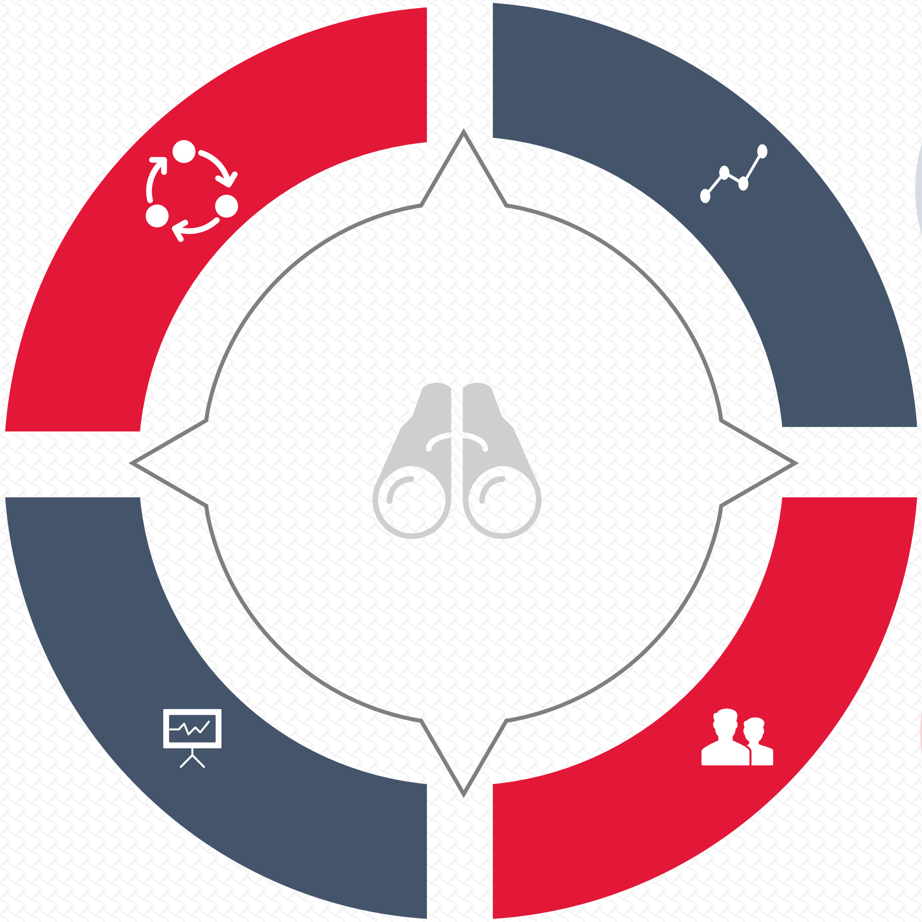
$r_0 = 0.03, S_0 = \$1640, K = \$1750$

Price of American Call Option: **$77.3925**

c) **Convergence Rate** of Binomial Tree Approach

Binomial Model provide a discrete time approximation to the continuous process underlying the Black-Scholes model

The Binomial model value converges on the Black-Scholes formula value as the number of steps increases.



Plot Error against n, the graph has a clear sawtooth pattern and periodic humps

The rate of convergence of Binomial Tree approach is 1 (linear convergence).

d) **European Call Option Price** - Binomial Tree Approach

$$r_0 = 0.03, S_0 = \$1640, K = \$1750$$

Price of European Call Option: **$77.3925**

SAME as part b)!
Reasoning: The option has no dividend, so the buyers of this American call option will not exercise until the maturity date.

e)

European Call Option Price	$77.4034
Corresponding Put Option Price	$174.3275

Vega

334.23612

Impact of a change in *volatility*

$$\nu = S\phi(d1)\sqrt{t}$$

$$where: \phi(d1) = \frac{e^{-\frac{d1^2}{2}}}{\sqrt{2\pi}};$$

$$d1 = \frac{ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

Delta

0.40761

Impact of a change in *stock price*

$$Call\ delta = e^{-qt} * N(d_1)$$

Gamma

0.0013987

Impact of a change of *Delta*

$$Gamma = \frac{e^{-qt}}{S_0\ \sigma\ \sqrt{t}} * \frac{1}{\sqrt{2\pi}} * e^{\frac{-d_1{}^2}{2}}$$

f) European Call Option Price – **Simple Monte Carlo Method**

Method	European Call Option Price
Simple Monte Carlo	$77.9116
Black-Scholes	$77.4034
Binomial Tree	$77.3925

When N = 1500, the value derived from the Binomial Tree method is more accurate than the one derived from the Simple Monte Carlo method

g) European Call Option Price – **Monte Carlo with Antithetic Variates**

European Call Option price: **$77.9056**

Standard deviation: 158.875

h) European Call Option Price – **Monte Carlo with Control Variates**

European Call Option price: **$77.8381**

Standard deviation: 108.117

h)

Method	European Call Option Price	Standard Deviation
Simple Monte Carlo	$77.9116	161.326
Antithetic Variates	$77.9056	158.875
Control Variates	$77.8381	135.117

- Pricewise, no big discrepancy
- Variance: Simple Monte Carlo > Control Variates > Antithetic Variates

h) Convergence of Monte Carlo Method

Method	# of time steps =10, paths=1000	100	1000	10,000
Simple Monte Carlo	$87.80 (abs err = 9.8884) Sd = 176.88	76.77 (1.1416) 158.40	79.25 (1.3384) 165.49	77.37 (0.5416) 165.326
Antithetic Variates	$77.53 (0.3816) Sd = 162.8674	73.9655 (3.9461) 155.798	78.23 (0.3184) 160.77	77.91 (0.0016) 156.875
Control Variates	$72.92 (4.9916) Sd = 148.35	74.47 (3.4416) 149.36	77.43 (0.4816) 144.85	77.25 (0.6616) 143.117

Method	# of time paths =10, steps=1000	100	1000	10,000
Simple Monte Carlo	$6.54 (71.3716 Sd = 19.63	94.37 (16.4584) 187.83	78.51 (0.5984) 160.05	77.92 (0.0084) 161.326
Antithetic Variates	$114.83 (36.9184) Sd = 178.21	80.01 (2.0984) 158.04	77.32 (0.5916) 158.30	77.82 (0.0916) 158.875
Control Variates	$170.62 (92.7084) Sd = 213.6	98.13 (20.2104) 175.32	77.51 (0.4016) 131.08	77.97 (0.0584) 128.117

i) Estimate **Implied Volatility**

K = $1750

AMZN Call Option Price on April 3rd : $158.7

Stock price: S_0 = $1820.7

Maturity Date: July-19-2019

Time until Maturity: T = 110/365 = 0.301

Backward Solving B-S Formula

Implied Volatility = **0.2832146**

Different from the Implied Volatility on Yahoo Finance: 0.3021

Reasoning: Estimate r = 0.0068

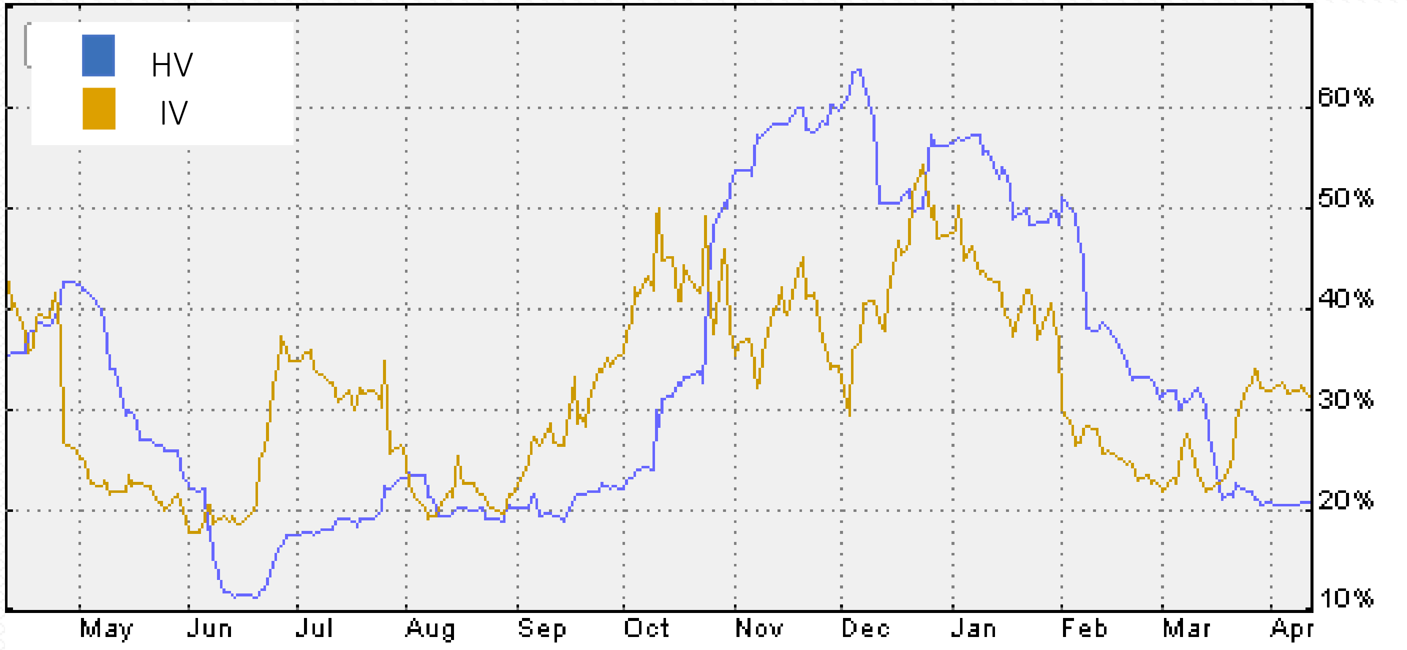
For T=$\frac{110}{365}$, Annual r = 0.0226

FED r = 0.025

I) **Historical Volatility (HV)** vs. **Implied Volatility (IV)**

HV: annualized standard deviation of past stock price movement

IV: derived from an option's price and shows what the market implies about the stock's volatility in the future




THANK YOU