

# Early Chatter Detection using MaxEnt and SPRT

Yanqing Zhao, Kondo H. Adjallah, *Member, IEEE*, Alexandre Sava, Zhouhang Wang

**Abstract**— In high-speed milling process, the early detection of chatter contains two aspects, one is the fast detection of the initial symptom of the chatter, the other is to recognize the slight chatter embedded in the noisy signal. This paper presents an online cumulative chatter detection method for high-speed milling process based on once-per-revolution sampling, maximum entropy (MaxEnt) principle and sequential probability ratio test (SPRT). The method allows for coping with these two aspects of early chatter detection. This method has less computational complexity and is independent of the cutting conditions. The procedure consists of four steps. First, the prior knowledge of early chatter is determined. Secondly, once-per-revolution sampling data is sampled from the vibration signal. Thirdly, the MaxEnt principle is used to estimate the MaxEnt of the once-per-revolution sampling data as a chatter indicator. Finally, the SPRT cumulates the information of the estimated MaxEnt, and then detect the early chatter by using the prior knowledge. The proposed strategy is applied to a high-speed milling process, and two simulation experiments allowed to assess the effectiveness of the early chatter detection method.

## I. INTRODUCTION

Chatter originates from self-excited vibrations that arise in cutting processes, especially in high-speed milling. The occurrence of chatter not only degrades the quality of surfaces and causes noise but also wastes energy, impacts the life of the cutting tools and machine-tools, etc. So, detection and control of chatter are significant topics to improve surface quality, increase material removal rates and avoid damages in the machining process.

One of the main goals in machining process monitoring is early detection of chatter[1]. Here, the word ‘early’ has two meanings. On the one hand, early chatter detection should be completed at the beginning of a serious chatter. Before the complete development of serious chatter, there are no obvious chatter marks left on the work-piece [2]. Thus, the implementation of the initial chatter detection and control is crucial. It has important significance for ensuring surface quality

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and increasing the material removal rates, etc. On the other hand, the early chatter detection should also be achieved when the chatter is at a slight stage. The machining parameters are usually set nearly the boundary between the stable and unstable regions to increase the material removal rates. A slight variation of the system or/and machining parameters may lead to severe chatter. For example, the sub-critical bifurcation appears due to the slight variation of the machining parameters [3].

Actually, between chatter-free and complete chatter stage, there is a transient stage. The initial stage of serious chatter can be detected by identifying the transient stage. Yang et al. [4] presented an optimized variational mode decomposition and combined the approximate entropy and the sample entropy to detect the initial stage of serious chatter. The method was validated more sensitive than the empirical mode decomposition. Nonetheless, the computational cost is relatively expensive. Cao et al. [5] detected the initial stage of serious chatter by using the multi-feature fusion and self-organizing map neural network. Ding et al. [6] used the logistic regression and improved wavelet packet entropy to detect the early chatter. These two machine learning methods obtained satisfactory results. However, the learning model should be trained using the test data.

In term of the initial stage detection of serious chatter, the time of the signal processing should be as short as possible. One promising approach is based on the once-per-revolution sampling of the vibration/sound signal. This approach is derived from Poincaré sectioning technique which has been widely applied to instability research in a nonlinear system [7], [8]. The once-per-revolution sampling technique reduces the amount of the signal significantly compared to the traditional sampling approaches. Specifically, one sample is acquired during one spindle revolution synchronously. However, for the FFT method, tens of kilohertz sampling rates might be required to avoid aliasing which might occur in the signal translation process [9]. Less data usually need less computational cost for signal processing and data transmission. So, this technique could reduce the computational cost of the signal processing, and also can be used to the remote data transmission for the remote chatter detection, fault diagnosis, and system control. Using a once-per-revolution sampling technique, Schmitz et al.[9], [10] presented an online chatter detection method according to the different distribution of the once-per-revolution sampling data in chatter and chatter-free stage. Honeycutt and Schmitz [11], [12] used the once-per-revolution sampling data for the automatic stability identification and used sub-harmonic sampling data for identifying the type of chatter.

The methods mentioned above focus on the online detection of the developing or developed chatter. However, to our best knowledge, there are few published works to detect the slight

chatter from a noisy signal. Also, most of the feature extraction methods are time-consuming relatively, and they are not used timely in the high-speed or ultra-high-speed milling process. Aimed at filling these gaps, the authors have presented an online cumulative chatter detection strategy using a once-per-revolution sampling technique, the maximum entropy (MaxEnt) principle, and the sequential probability ratio test (SPRT). This strategy not only detects the initial stage of the serious chatter quickly but also identifies the slight chatter quickly from a noisy signal.

The rest of the paper is organized as it follows. Section 2 introduces the MaxEnt principle and the SPRT briefly. Section 3 is devoted to showing the proposed online cumulative chatter detection strategy. In section 4, the proposed strategy is validated using two types of chatter signal. The results demonstrate that the proposed strategy can detect the initial stage of serious chatter and the slight chatter in a noisy signal effectively.

## II. RECALL OF THE MAXIMUM ENTROPY PRINCIPLE AND SEQUENTIAL PROBABILITY RATIO TEST

### A. Maximum Entropy Principle

In 1957, Jaynes introduced the maximum entropy (MaxEnt) principle based on the information entropy introduced by Shannon [13]. The MaxEnt principle indicates that the estimated probability density function (pdf) with MaxEnt is the optimized choice when insufficient information is used to determine system behavior. After the pdf estimation, the MaxEnt, which explains the characteristic of data uncertainty, can be calculated.

Considering a stochastic variable  $x$  which produces a set of data  $x=\{x_1, x_2, \dots, x_i, \dots, x_n\}$ , the pdf of  $x$  is defined as  $f(x)$ . Equation (1) gives the informational entropy quantifying the uncertainty:

$$H(f(x)) = - \int_{-\infty}^{+\infty} f(x) * \log(f(x)) dx . \quad (1)$$

Equation 2 expresses the constraints related to the moments of the stochastic variable  $x$ :

$$E(g_j(x)) = - \int_{-\infty}^{+\infty} g_j(x) * f(x) dx = \alpha_j, j = 0, 1, 2, \dots, m , \quad (2)$$

where  $g_j(x)$  is a specified function of  $x$ , and  $E(g_j(x))$  represents the statistical expectation of  $x$ . If  $j=0$  then the zero<sup>th</sup> moment  $E(g_0(x))$  is the total probability, if  $j=1$  then  $g_1(x)=x$ ,  $E(g_1(x))$  represents the first moment and so forth.

The MaxEnt principle aims to find  $f(x)$  without additional assumptions by maximizing the information entropy under the given constraints  $g_j(x)$ . The MaxEnt optimization problem can be solved using the Lagrange multipliers (see [13], [14] for detail).

### B. Sequential Probability Ratio Test

The sequential probability ratio test (SPRT) is a powerful method for hypotheses test (step by step) [15], [16]. This method was used, for instance, for detecting the change point or status in a wide range of fields, such as medicine [17], sewer network [18] and Chemical industry [19]. In this paper, the SPRT is used to detect the change of status corresponding to the occurrence of chatter. As classical hypothesis testing, SPRT also needs two hypotheses: the null hypothesis, denoted  $H_0$ , corresponding to

the normal mode and the alternative hypothesis, denoted  $H_1$ , corresponding to the abnormal mode. They are specified as follows:

$$H_0 : p = p_0 , \quad (3)$$

$$H_1 : p = p_1 . \quad (4)$$

The log-likelihood ratio (LLR), defined as the logarithm of the likelihood ratio of the above two hypotheses, is given by the formula:

$$\log \Lambda(x_i) = \log \left( \frac{p(x_i / H_1)}{p(x_i / H_0)} \right) . \quad (5)$$

When a new sample arrives, the cumulative sum of  $n$  LLRs is calculated as

$$S_n = S_{n-1} + \log \Lambda(x_n) = \log \left( \frac{p(x_1, x_2, \dots, x_n / H_1)}{p(x_1, x_2, \dots, x_n / H_0)} \right) . \quad (6)$$

Here,  $n=1, 2, 3, \dots$ , and  $S_0=0$ .

In the SPRT, there are two decision risk levels. The null hypothesis may be rejected when it is true (corresponding to the first decision risk level  $\alpha$ ). Of cause, the null hypothesis may be accepted when it is not true (called second decision risk level  $\beta$ ). These two decision risk levels can be preset according to the reliability requirement or risks acceptance, which allow setting the decision thresholds as it follows:

- ◆ If the  $S_n \leq a$ , accept  $H_0$  and stop the detection
- ◆ If the  $S_n \geq b$ , accept  $H_1$  and stop the detection
- ◆ If the  $a < S_n < b$ , continue the detection until either  $a$  or  $b$  is reached,

where  $a$  and  $b$  depend on the two decision risk levels ( $\alpha$  and  $\beta$ ),

$$a = \log \frac{\beta}{1-\alpha} , \quad (7)$$

$$b = \log \frac{1-\beta}{\alpha} . \quad (8)$$

For decision, the cumulative sum of LLRs is compared to both thresholds  $a$  and  $b$ . The process can be supervised in a recursive and real-time manner.

## III. CUMULATIVE ONLINE CHATTER DETECTION STRATEGY

In practice, due to the influence of the noise from the system (sensors noise, in-homogeneous material, etc.), the boundary between chatter-free and early chatter stage is fuzzy. The standard deviation of data used in [9], [11] may fail to detect early chatter in this situation.

The MaxEnt principle evaluates the pdf of data without additional assumptions under incomplete constraints. So, the MaxEnt calculated according to pdf reflects characteristics of the data objectively. The SPRT is a cumulative test strategy and often used to detect the change point in a noisy signal. The combination of these two methods can allow detecting incipient chatter. The flowchart of the proposed chatter detection strategy is depicted in Fig. 1.

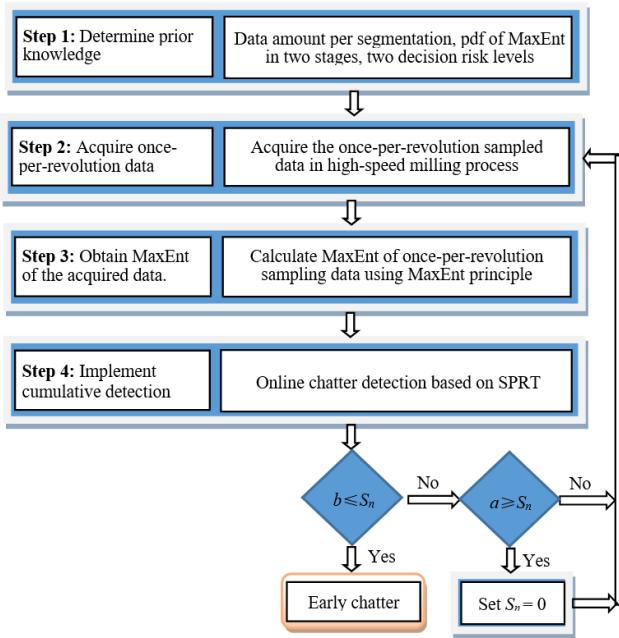


Figure 1. Flowchart of the proposed early chatter detection method.

### Step 1: Determine prior knowledge

Before detecting the chatter, some prior knowledge (data amount per segmentation for estimating MaxEnt, the pdf of MaxEnt in chatter-free and early chatter stage, two decision risk levels) need requiring. The data amount per segmentation impacts the accuracy of estimated pdf using MaxEnt principle and the sensitivity of the early chatter detection. The higher the sensitivity, the shorter the detection time and the smaller the amount of data processed in one segmentation. However, the small amount of data will lead to poor estimation accuracy of the MaxEnt. Considering the two conflicting aspects and their trade-off, we recommend a value of data amount per segmentation of 20 to 30.

The setting of MaxEnt pdf corresponding to the early chatter and chatter-free stage is a crucial step for detecting the early chatter. It is set generally according to expert's knowledge or a large number of experimental studies. After setting, it is necessary to verify if the MaxEnt pdf meets the assumption.

Two decision risk levels for SPRT relate directly to the chatter detection risks or reliability. In the case of certain samples, the two decision risk levels cannot be reduced synchronously. The increase of samples amount can reduce the two decision risks synchronously. However, the increase of samples amount would lead to an increase in the detection delay accordingly. The two decision risk levels are set according to the reliability requirements or risks acceptance. Traditionally, the two decision risk levels are set to 0.05 or 0.01 in practice. In this paper, we set both to 0.01.

### Step 2: Acquire once-per-revolution sampling data

Acquire the data through a once-per-revolution sampling of vibration signal in real time and separate the acquired data into some segmentation according to the setting data amount per segmentation.

Step 3: Obtain the MaxEnt of the acquired data per segmentation

Evaluate the pdf of one observed segmentation data using the MaxEnt principle and calculate the MaxEnt according to the evaluated pdf as chatter indicator.

### Step 4: Cumulative hypothesis test

Calculate the two critical values  $a$  and  $b$  using the two decision risk levels according to (7) and (8), and then conduct the SPRT through calculating the sum of LLRs of the evaluated entropy. Compare the calculating sum of LLRs with the two critical values ( $a$  and  $b$ ), if the sum of LLRs is less than or equal the critical value  $a$ , the cutting state is stable. There is no reason to stop the detection procedure. So, we present a new stopping rule to detect chatter. To avoid the influence of cumulative results of the chatter-free stage on the later test, we reset the sum of LLRs to zero for the later test when it is less than or equals  $a$ . Then, the test procedure returns to step 2 and continue to monitor the milling process. If the sum of LLRs is more than or equals the critical value  $b$ , it means that the early chatter has appeared and been detected successfully. Thus, the control procedure is triggered to change the control parameters, for example, to change the spindle speed, or the test procedure is stopped for removing the assignable cause. If the sum of LLRs locates in range of the two critical values ( $a$  and  $b$ ), the test procedure return to step 2 and continue to monitor the milling process.

## IV. NUMERICAL VALIDATION AND DISCUSSIONS

Two simulation experiments were conducted to evaluate the proposed chatter detection method. In one simulation experiment, a chatter simulation signal used in [20] is chosen and improved by adding harmonic components, a noise component and by modifying the frequencies. In the other experiment, a dynamic model of a milling process with bearing clearance studied by [21] is chosen to generate the once-per-revolution displacement data.

### A. Simulation Validation I

Refer to the knowledge of chatter in the high-speed milling process, a simulation vibration signal from [20] is chosen and improved through adding the harmonic components, a noise component and modifying the frequencies. The improved vibration signal composed of seven components are

$$y = 4\sin(800\pi t) + 1.5\sin(1600\pi t) + 4\sin(2400\pi t) + A(t)(1 + 0.3\sin(1995\pi t))\cos(8991\pi t) + \sin(4800\pi t) + 0.6\sin(7200\pi t) + w(t) \quad (9)$$

$$A(t) = 5t \quad (10)$$

The first and second component is the sinusoidal signal and represents the rotating frequency and its second harmonic respectively. The third component is also the sinusoidal signal and represents the tooth passing frequency and the third harmonic of the rotating frequency. Here implies that the cutting tool has three flutes. The fourth component is designed to simulate the chatter mode. Here, the amplitude  $A(t)$  is a time-varying function and represents the chatter variation trend with time. The fifth and sixth components are also harmonics of rotating frequency and tooth passing frequency. The noise emerged in process and measurement is the seventh component. The signal to noise ratio (SNR) is assumed as 10. Here, the simulated signal without the chatter component is seen as the chatter-free signal.

For this signal, we set the ratio of the sampling frequency and the spindle rotating frequency to 250. In other words, there are 250 sampling points in each spindle revolution period. Thus, the once-per-revolution data are acquired with a constant interval of 250 sampling points synchronously. The acquired once-per-revolution data can indicate the mode of chatter [9]. In practice, the noise emerges in milling and measurement process and impacts the character of the once-per-revolution data. The time domain signals with the noise in the chatter-free and chatter stage are shown in Fig. 2(a) and (b), respectively. The red points represent the once-per-revolution sampling data. Under the noisy background, we can see that it is difficult to detect the initial chatter according to the distribution of the once-per-revolution sampling data.

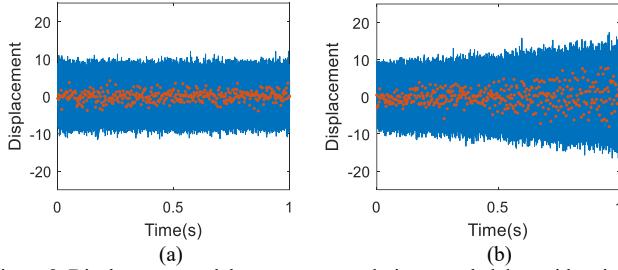


Figure 2. Displacement and the once-per-revolution sampled data with noise, (a): chatter-free; (b): chatter

After the once-per-revolution sampling of displacement signal, the MaxEnt principle is performed on the sampled data to calculate the corresponding MaxEnt. The obtained MaxEnt is used to hypothesis test. The prior probability distribution of MaxEnt in both chatter-free and early chatter stage needs to be set for the SPRT. The normal distribution is a very common and important probability distribution in practice and statistics theory. So, we assume the once-per-revolution sampling data and the MaxEnt in chatter-free and early chatter stage follow the normal distribution.

After assuming the distribution both of the sampled once-per-revolution data and the MaxEnt, it is necessary to check if they meet the assumption of normal distribution. The normal distribution hypothesis testing should be conducted on them. Lilliefors test and Jarque-Bera test are two commonly normality test methods to test if the data come from a normally distributed population. We test the sampled once-per-revolution data and the MaxEnt in two stages using these two test methods. The results of the Lilliefors test and Jarque-Bera test prove that they follow the normal distribution with a 5% significance level. The distribution of the MaxEnt in two stages are shown in Fig. 3. The distributions in two stages are closed, the mean of the MaxEnt distribution in early chatter stage is a little bigger than that in the chatter-free stage.

The two decision risk levels are set to 0.01 and SPRT is used to test the calculated MaxEnt. Fig. 4 shows the results of chatter detection. The red and green horizontal dashed lines represent the upper and lower thresholds respectively. When the sum of LLRs lies below the lower threshold, it means the milling process belongs to the chatter-free stage. Here, the preceding sum of LLRs is set to zero and continue to detect the early chatter. When the sum of LLRs crosses the upper threshold, it means the chatter has occurred. When the sum of LLRs is between the up and down threshold, no decision can be made, and the chatter detection procedure needs to continue. For the

chatter-free data shown in Fig. 4(a), the lower threshold is reached at the 0.25s for the first time, the cutting state is recognized as the chatter-free. The detection procedure needs to continue to monitor the cutting state. Due to the test result of SPRT is based on the cumulated sum of the likelihood ratio, to reduce the influence of accumulation result on the later detection, the sum of LLRs  $S_{14}$  is reset to zero for calculating the later sum of LLRs. At the 0.875 s, the lower threshold is reached again. The cutting state is still chatter-free, and then the sum of LLRs  $S_{14}$  is set to zero again. The detection procedure continues to monitor the cutting state until the upper threshold is reached. For the chatter data plotted in Fig. 4(b), the upper threshold is crossed at the 0.375s. It means the early chatter is recognized successfully.

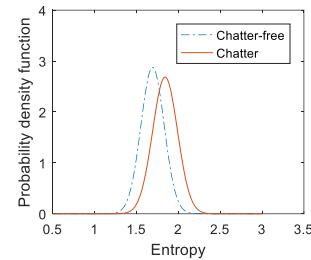


Figure 3. Threshold variant of the MaxEnt in early chatter and chatter-free stage.

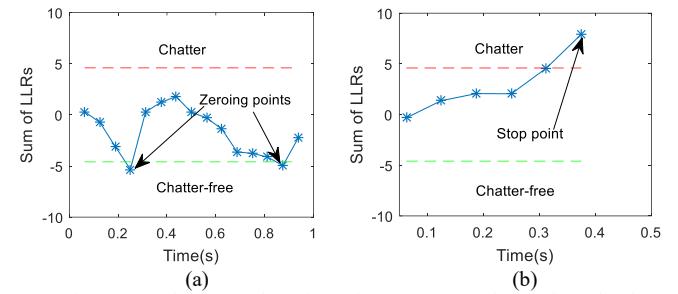


Figure 4. Early chatter detection using SPRT. (a): chatter-free; (b):chatter.

## B. Simulation Validation II

This part uses the high-speed milling simulation model in [21] to generate the vibration signal. The generated signal is used to validate the proposed method for detecting the slight chatter state. In this simulation model, the cutting tool is seen as an equivalent 2-degree-of-freedom (DOF) dynamic system which is represented by the spring-mass-damper system. The work-piece is seen as rigid. The effect of the bearing clearance fault on the dynamic system is represented by the variable stiffness which has been studied in [22].

The 2-DOF milling dynamic system is excited by two variable cutting forces in the feed ( $x$ -) direction, and crossfeed ( $y$ -) direction and the dynamic governing equations are written as follow:

$$\begin{cases} m_x \ddot{x} + c_x \dot{x} + k_{x1} + \delta(D-\gamma)k_{x2}(D-\gamma)\cos(\phi) = F_x(t) \\ m_y \ddot{y} + c_y \dot{y} + k_{y1} + \delta(D-\gamma)k_{y2}(D-\gamma)\cos(\phi) = F_y(t) \end{cases}, \quad (11)$$

where  $x$ ,  $\dot{x}$ ,  $\ddot{x}$ ,  $y$ ,  $\dot{y}$  and  $\ddot{y}$  are the displacement, velocity, and acceleration of the cutting tool head in  $x$ -direction and  $y$ -direction, respectively. The  $m_x$ ,  $c_x$ ,  $m_y$  and  $c_y$  are the model mass, model damping in  $x$ - and  $y$ -direction, respectively. The  $k_{x1} + \delta(D-\gamma)k_{x2}$  and  $k_{y1} + \delta(D-\gamma)k_{y2}$  are the model stiffness caused by bearing clearance in  $x$ - and  $y$ -direction, respectively.

Here,  $D = \sqrt{x^2 + y^2}$ ,  $\cos(\phi) = x/D$ ,  $\sin(\phi) = y/D$ , and  $\gamma$  is the radial clearance of the bearing, and  $\delta(\cdot)$  is a switch function with  $\delta(\Delta)=1$ , if the  $\Delta \geq 0$ , and  $\delta(\Delta)=0$ , if the  $\Delta < 0$ . The model parameters used in this work are listed in Tab I.  $F_x(t)$  and  $F_y(t)$  are the variable cutting force at time  $t$  in  $x$ - and  $y$ -directions. The two variable cutting forces are modelled based on the regeneration effect [23], [11].

TABLE I. MODEL PARAMETERS OF THE CUTTING TOOL HEAD

	Mass(kg)	Stiffness( $k_1+k_2$ )	Damping ratio (%)
x	$2.75 \times 10^{-2}$	$8.79 \times 10^5$	1.39
y	$2.96 \times 10^{-2}$	$9.71 \times 10^5$	1.38

The machining parameters are shown in Tab II, respectively. The carbide square end cutting tool with a diameter 12mm and helix angle  $30^\circ$  is chosen. The displacements of the cutting tool are calculated using the time domain simulation approach [24]. The displacement with SNR 10 is obtained by adding the noise to the pure signal to simulate the noisy circumstance. According to the spindle speed and sampling frequency, the once-per-revolution sampled data of displacement in the  $x$ -direction are acquired with a constant interval of 200 sampling points synchronously. Because there are 200 sampling points in each spindle revolution. In the application process, we recommend using a capacitive sensor to record the displacement of the cutting tool (see [25] for the detail of the experimental setup and calibration).

TABLE II. MACHINING PARAMETERS

Spindle speed (rpm)	Axial depth (mm)	Radial depth (mm)	Feed per tooth (mm/tooth)
7000	3	0.5	0.1

Fig. 5(a) shows the once-per-revolution data without bearing clearance and with bearing clearance  $\gamma = 5\mu m$  which represents the chatter-free signal and the early chatter signal respectively. One can see in Fig. 5 (a) that the two data series are close. It is difficult to detect the chatter using standard deviation which was used as an indicator of chatter detection in [10], [11].

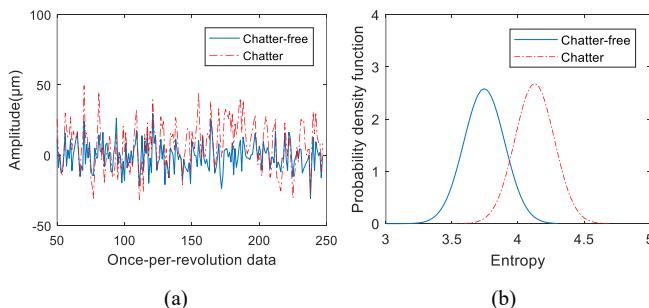


Figure 5. Once-per-revolution data plot and MaxEnt pdf in the chatter-free and early chatter stage.

In this example, we freely choose the MaxEnt pdf shown in Fig. 5(b) as prior MaxEnt pdf. The once-per-revolution sampling data and MaxEnt in chatter and the chatter-free stage pass the Lilliefors test and Jarque-Bera test with 5% significance level.

Use the MaxEnt principle to estimate the MaxEnt of once-per-revolution sampling data. The SPRT is carried out using the estimated MaxEnt. Here, the two decision risk levels are set to 0.01. Fig.6 (a) and (b) illustrate the chatter detection results of once-per-revolution data in chatter-free and early chatter stage, respectively. For the chatter-free stage, the sum of

LLRs fluctuates around the lower threshold. That's the result of the resetting of the sum of LLRs to zero when the lower threshold is reached. Due to much-zeroing points, we don't show them in Fig. 6 (a). For the early chatter data plotted in Fig. 6(b), the early chatter stage is quickly detected just after three tests.

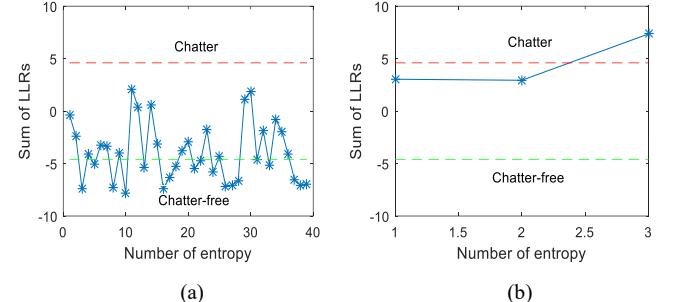


Figure 6. Chatter detection using SPRT, (a):chatter-free; (b):early chatter.

From the results of two simulation experiments, one can conclude on the effectiveness of the proposed cumulative method for detecting two types of early chatter.

Due to the property of the once-per-revolution sampling, the proposed method is independent of the cutting conditions as it is based on the comb filters [26]. Moreover, our method has much less computational complexity than that based on comb filters. Because the once-per-revolution sampling technique significantly reduce the amount of sampling data, and less data generally need less computational cost for signal processing.

To illustrate the low computational cost of the proposed MaxEnt, the displacement signal, with 25 spindle revolution periods used in simulation I, is chosen to calculate the proposed MaxEnt, approximate entropy(ApEn) [27], sample entropy (SampEn) [28], fuzzy entropy [29]. These entropies are calculated on the desktop computer with CPU of Intel Core i5-6600 3.3GHz. The computational costs of the abovementioned four entropies are about 0.0195s, 0.7417s, 0.7629s, and 1.0973s, respectively. It is clear that the computational cost of the proposed MaxEnt is much lower than ones of other three entropies. Thus, the proposed MaxEnt is more suitable than the other three entropies for the high-speed milling process. Because the period from chatter onset to mature chatter is very short, less than 0.1s [7].

In engineering filed, the instability frequently appears in many dynamic systems, such as the rotor-bearings system [30], the pulse energy conversion systems [31], [32]. According to the literature [31], [32], the boundary of the stable and unstable areas is a region instead of a line. In this region, the bifurcation phenomenon has intermittent property. In other words, the alternation of stable and unstable phenomenon occurs. The ratio between the duration of the unstable phenomenon and the all-time (unstable duration and stable duration) can be used to indicate the instability. When the ratio reaches the value 1, it means the unstable phenomenon occupies in all the period. So, the dynamic process has entered the unstable region completely and vice versa. Our method, while aimed at this phenomenon, seems to apply more successfully to detect the instability in pulse energy conversion systems.

## V. CONCLUSION

For the two aspects of early chatter detection, a novel cumulative strategy based on the MaxEnt and SPRT was presented in this paper. The two simulation experiments were conducted to validate the effectiveness of the proposed method. Two types of early chatter were detected successfully online using the proposed early chatter detection method. The experimental results also showed that the proposed method is low-cost and independent early chatter detection strategy. Beyond this study, the proposed method could be used to detect the instability of the rotor-bearings systems and pulse energy conversion systems.

The noise emerged in once-per-revolution sampling data impacts the pdf of the MaxEnt in chatter-free and early chatter stage. Then, the early chatter detection accuracy will be reduced when the noise level varies. A dynamic threshold variant may deal with this problem.

There are also two limitations of the presented method which need to be improved to detect the early chatter in the high-speed milling process. For the first limitation, at the beginning of the milling process, the initial transients impact the distribution of once-per-revolution data and then reduce the chatter detection performance. The second limitation is that the presented method can be applied to the milling process using the single flute cutting tool. However, it fails to detect some period- $n$  bifurcations for the cutting tool with multiple flutes.

In the future, the improved version of the proposed method with the dynamic threshold variant will be presented to adapt to the variable noisy background.

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