MAT 3 JM #6 4/10-16 2/

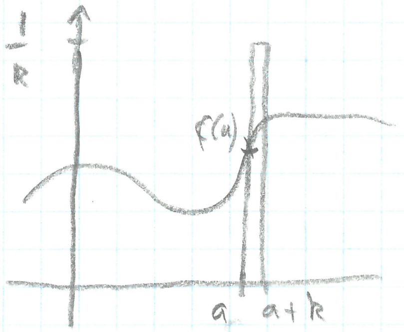
$$f_k(t-a) = \begin{cases} 1/k & a \le t \le a+k \\ 0 & \text{ellers} \end{cases}$$

DIRAC (p. 226)

$$\lim_{k\to 0} f_k(t-a) = \delta(t-a) = \begin{cases} \infty & t = a \\ 0 & \text{OTHERWISE} \end{cases} \quad \underline{\text{AND}} \quad \int_0^\infty \delta(t-a)\,dt = 1$$

SIFTING PROPERTY (IN GENERAL)

limits not important
$$\int_{-\infty}^{\infty} f(t)\,\delta(t-a)\,dt = f(a)$$



$$\int_{-\infty}^{\infty} f(t)\,\delta(t-a)\,dt = \lim_{k\to 0} \int_a^{a+k} f(t)\,f_k(t-a)\,dt$$

IN THE LIMIT, $f(t)$ ASSUMES THE (CONSTANT) VALUE $f(a)$, AND WE CAN SET IT OUTSIDE THE INTEGRAL. HENCE

$$\lim_{k\to 0} \int_a^{a+k} f(t)\,f_k(t-a)\,dt = f(a) \lim_{k\to 0} \int_a^{a+k} f_k(t-a)\,dt$$

AND SINCE BY DEF. $\int \delta(t-a)\,dt = 1$ WE HAVE

$$\int_{-\infty}^{\infty} f(t)\,\delta(t-a)\,dt = f(a)$$