Intrahousehold Interaction and Married Family Labor Supply

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Abstract

Little is known about the role of household decision-making in the dramatic increase in labor supply of married women in the U.S. over the past half-century. In this paper, I use a bivariate probit and a Nash best-response game to look for changes in how married households supply labor over time in Census and PSID data. Consistent with past literature, I find in general that husbands prefer a household where one spouse specializes in home production and the other in market work, while wives prefer either both spouses work or both stay at home. Identification of these asymmetric effects comes from exclusion restrictions and sufficient variation in the characteristics of husbands and wives that push them to work in the market or at home. Next, I allow the interaction between spouses to depend on whether there are children in the home. I find that children increase complementarity of labor supply decisions for men and substitutability for women, but this has reversed over time for men. Lastly, I look at variation by spouse's labor income. Higher income husbands decrease the return to working for wives, but higher earning wives increase the return for husbands, consistent with a preference against being out-earned.

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Contents

1 Introduction

The employment decisions of married households are important drivers of overall employment trends, with the increase in married women's labor force participation accounting for most of the increase in women's labor force participation in recent decades (Blau & Kahn, 2007). Although much attention has been paid to household models of consumption, including unitary, collective, and cooperative game theoretic models, less work has focused on the mechanics of household labor supply decisions (Rode, 2011).

This paper follows the modeling innovation of Bjorn and Vuong (1984a), and later Peña (2011) and Kaya (2013), in treating the extensive margin of labor supply as a non-cooperative game between husband and wife. Outcomes are Nash equilibria, which are the solutions to mutual best response functions. One advantage of this framework is that no intrahousehold contracts are required to sustain the equilibria. By definition, no agent has the incentive to deviate, so no enforcement is required (Lundberg & Pollak, 1994). The non-cooperative game also has the advantage that non-pareto optimal outcomes are possible, which seems intuitively reasonable given the range of behaviors observed in households, including potentially limited communication between spouses.

Another advantage of the Nash solution is its simplicity. In collective models rich data on consumption is needed to pin down the pareto weights for each spouse (see for example (Blundell, Chiappori, & Meghir, 2005) and Cherchye, Rock, and Vermeulen (2012)). Likewise in bargaining models, a researcher must assume threat points either internal or external to the household to identify the model. In the non-cooperative game, one can estimate the impact of a spouse's decision without making assumptions beyond individual utility maximization.

I expand upon previous non-cooperative labor supply literature by allowing spousal interaction to vary by the presence of children in the household. Lundberg (1988) finds this to be an important source of heterogeneity in household labor supply. I then further allow a spouse's labor earnings to affect their partner's utility from entering the labor force. This is important because in a the baseline model, as in previous literature, utility from a spouse entering the labor force is not allowed to vary by the spouse's earnings. It is much more realistic to allow the possibility of a household budget constraint.

Results from a basic model similar to Bjorn and Vuong (1984b), Kaya (2013), and Kooreman (1994) allow the interaction between the spouses' decisions to be summarized in a single constant parameter. The basic model replicates the findings of (Bjorn & Vuong, 1984b), including the result that men want to work more if their wife is at home while women want to work more if their husband works. Kooreman (1994) found a positive impact for both spouses, but the asymmetric interaction result is consistent with their Stackelberg results, both under the assumption that the husband moves first and the wife moves first. Acosta-Pena (2011) find that husbands working is detrimental to the value to women of working in the context of Mexico. (Kaya, 2013) follows Kooreman (1994) in exploring various mechanisms, including bivariate probit, Nash, Stackleberg, and pareto optimality. In both the Nash outcome and the Stackleberg husband leader outcome, they also find opposite signs by gender using 2000 Census data, and focus on wage elasticities across education categories.

Estimates come from the Panel Study of Income Dynamics from 1970-2011, split into two time periods. The "early" period runs from 1970-1989, and the “late” period from 1990-2011. Estimates are also done using the decennial Census from 1970-2000 by year, and the 2011 3-year ACS. In the PSID in both periods men prefer to work more if their wives are at home, and women prefer to work more if their husbands are at work, implying that men may view leisure time as a complement to wife’s leisure time, and wives view it as a substitute. In the Census we see a change over time. In 1970 and 1980, men prefer to work more if their wife is home, in 1990 the interaction is insignificant, and in 2000 and 2011 men prefer to work if their wife works.[1]

I next allow the interaction parameter to vary by the presence of children either under the age of five or over the age of five in the household and find in general that children lead wives to want to work less if their husband works, and husbands to want to work more if their wives work. However, in later years, both husbands and wives prefer to stay at home more if they have children under 5 and their spouse works.

Lastly I introduce spouse’s labor income into the interaction term. I find that a higher earning husband decreases the return to working for wives, consistent with income pooling and diminishing utility from income. Surprisingly, I find that higher earning wives increases the return to working for men, and this effect has been increasing over time.

Unfortunately in this paper, as in past literature using cross-sectional data, spousal interaction cannot be separately disentangled from assortative matching or other marriage market selection effects. The asymmetric result by gender could in fact be confounded by asymmetric marriage preferences.

[1] Splitting the sample by education level (high school or more than high school) I see very little heterogeneity in the interaction, although if anything the lower education group appears to have a stronger interaction effect between the spouses.

Men who want to work may prefer to marry women who want to stay at home, while women who want to work may prefer to marry husbands who also work. The heterogeneity in results by income and children could also reflect to some degree heterogeneity in marriage preferences. It is important to keep this potential confounder in mind when interpreting the labor supply results as behavioral.

The paper is structured as follows: Section 2 reviews relevant household labor supply literature and entry game literature. Section 3 gives an overview of the PSID and Census datasets and results from bivariate probits. Section 4 specifies the model, its estimation, and results. Section 5 introduces the extended model in which the interaction between spouses depends on children or spouse's income. Section 6 concludes.

2 Literature Review

Unitary household labor supply models originated with Samuelson (1956) followed by Becker (1974). To allow for more individualistic behavior, Nash bargaining models were introduced by Manser and Brown (1980) and McElroy and Horney (1981). In bargaining models the position along the Pareto frontier is determined by the choice of the players' best outside option. The best outside option, or threat point, is usually assumed to be either the utility in case of divorce or the utility from the solution to a non-cooperative game (Donni & Chiappori, 2011). Manser and Brown (1980) used this approach to the marriage decision, while (McElroy & Horney, 1981) focused only on labor-leisure and consumption choices. Neither estimated their models empirically, rather laid out comparative statics.

These bargaining models were extended by Lundberg (1988) to allow for new threat points. Instead of facing the outside option of divorce, if divorce is costly household members may face the (inefficient) Nash solution for public goods provision. In this case gender norms may result in a focal point outcome. Chiappori (1988) introduced a collective labor supply model in which households maximize a joint utility function, and Pareto weights determine which Pareto-optimal outcome is achieved.

Other approaches to modeling the change in the labor supply of married women include dynamic lifetime utility models such as Heckman and MaCurdy (1980) and Attanasio, Low, and Sánchez-Marcos (2008) . These models have the advantage of being a more realistic reflection of the dynamic nature of labor supply decisions including experience accumulation and lifetime income, and the disadvantage of taking husbands' labor supply as exogenous. Eckstein and Lifshitz (2015) focuses in on the interaction between spouses in a dynamic context and identifies the fraction of households that behave collectively, simultaneous Nash, and Stackelberg leader Nash where the husband enters first.

The literature on non-cooperative game theoretic models of labor supply emerged from, and runs parallel to, the study of entry games in industrial organization. In the classic two firm entry game firms receive one payoff if they enter the market alone and a different payoff if they enter together. Generally, it is assumed that firms prefer to enter alone with no competition, meaning is the interaction term (θ below) is negative. In the household labor supply application, the direction of the interactions could be positive or negative, depending on whether couples prefer to specialize or if there are complementarities to leisure time.

This classic game is shown in Figure 1 below (de Paula, 2013).

This structure was first adapted to household labor supply by Bjorn

		Firm 2	
		Not Enter	Enter
Firm 1	Not Enter	0,0	0, ϵ_2
	Enter	ϵ_1,0	$\epsilon_1 + \theta_1$,$\epsilon_2 + \theta_2$

Figure 1: Classic Entry Game

and Vuong (1984b). The application is intuitive. Instead of firm profits depending on who enters a market, spousal utility depends on who decides to work. The work decision of one's spouse directly alters one's own decision to work. Bjorn and Vuong (1984b) solve and prove identification of the game and take it to data using the 1982 wave of the Panel Study of Income Dynamics. They find highly significant interaction terms, negative in the case of the husband and positive in the case of the wife.

The application of the two-player game to household labor supply was next taken up by Kooreman (1994), who applied it to Dutch data and later by Peña (2011), who applied it to Mexican data. Boca and Flinn (2012) bridge the gap between game-theoretic treatment of household labor supply and collective labor supply models by allowing spouses to play the Nash strategy if it individually dominates the pareto outcome, but what dominates depends on functional form assumptions.

The most similar works to this project are Kaya (2013) and Peña (2011). Kaya (2013) estimated non-cooperative games for various types of households according to relative education level of husband and wife and the presence of children. The focus in Kaya (2013) was labor supply elasticities of married women and cross elasticities with respect to the husbands' wage. Analysis was done using 1980 and 2000 U.S. Census data. Kaya (2013) estimated bivariate probit and Stackleberg models in addition to the simultaneous Nash and found the proportion of the households best modeled by each.

Bajari, Hong, and Ryan (2010), Tamer (2003), and Kline (2013) address identification in the two-player discrete non-cooperative game. Difficulty arises because for some parameter values the model has no prediction or multiple outcome predictions. In this case assumptions must be made as to the equilibrium selection mechanism. Identification in this paper follows closely with the methodology of Bjorn and Vuong (1984b), which sets a uniform distribution over regions of multiple equilibria and regions of no pure strategies equilibria. In addition some specifications assume that the husband always enters the labor market in these regions, as in the focal point suggested by Lundberg (1988)

3 Data

The datasets used in this paper are the Panel Study of Income Dynamics (PSID, n.d.) and the decennial Census and ACS (Ruggles, Genadek, Goeken, Grover, & Sobek, 2015). These datasets are useful because they collect data on both husbands and wives. In the Census husband and wife must be in the same household, and in the PSID the husband responds for both spouses.

The PSID spans a 40 year period with annual data from 1968-1996, and data every other year from 1997-2013. In this paper I take sample years from 1970-2011 to be consistent with a range of Census years from 1970-2011. The PSID sample is limited to married couples under the age of 65, leaving 13,563 households and 121,878 household-years[2]. The PSID does not

[2]An initial probability sample of 2,778 households was taken in 1968. In each subsequent year between 202 and 478 households enter the sample. Entering households result either from members of the original core sample or their children getting married. In

consistently identify unmarried partners, so these households are excluded. Sample excluded for missing data are 1,548 husbands and 972 wives missing education, 327 husbands and 281 wives missing race[3], and 3,821 husbands and 1,287 wives missing labor market experience[4]. The final sample consists of 11,095 households and 113,526 household-years before the end years are dropped.[5]

A binary employment variable is equal to one if the sample member reported positive work hours in the past year. The employment variable is equal to zero if the sample member reported zero working hours. Using this scheme for determining employment, 94% of husbands and 74% of wives in the PSID work. The trend in wives' employment over time is notable in Figure 3, moving from a mean of 51% in 1968 to a mean of 78% in 2011. Husbands have trended in the opposite direction, though less dramatically, moving from 98% employment in 1968 to 90% employment in 2011.

The Census data spans from 1970-2011. To be consistent with the PSID employment definition, I use "worked at all in the past 12 months" to define employment in the Census. This phrasing is slightly different from the PSID variable which is an indicator of working any hours in the past year, constructed from hours questions. In the Census data we see a similar trend

addition, a new sample of 250 immigrant households is added to the core sample in 1997 and their children are included as they marry. Each household is observed for an average of 9 years.

[3]Wife's race is not recorded until after 1985. Before that year I impute husband's race as wife's race

[4]Work experience, which is helpful to impute wages, was not asked on the PSID until 1974. Although many observations can be extrapolated before 1974, most observations for which experience is missing occur in the period 1968-1974, with up to 35% missing in 1968.

[5]All descriptive statistics are performed using Core and Core/Immigrant Longitudinal weights, inverse probability weights that create a nationally representative sample of non-immigrant households in the U.S. prior to 1997, and immigrant and non-immigrant households after 1997 (Gouskova, Heeringa, McGonagle, & Schoeni, 2008).

in individual work status, though leveling out around 75% participation for women in the Census as opposed to 80% in the PSID.

Looking more closely at this trend in terms of the joint labor supply decision of the household in Figure 5 below, one can see that almost the entirety of the movement in wives' labor supply has been a movement from single-earner households to dual-earner households. The rise in households in which the single-earner is the wife[6] since the 1990s is a very small contributing factor to the rise in wive's labor force participation by comparison.

Also notable is the fact that the bulk of the movement in household labor supply took place pre-1990, with the upward trend in dual earner households flattening out around 1990. Blau and Kahn (2007) identify a slow-down in the outward shift of married women's labor supply functions post-1990. Although it is not clear that a discontinuous change in labor supply functions occurred in 1990, it seems reasonable to break the dataset into two periods at this year, especially since 1990 is in the middle of the period 1970-2011 covered by the PSID. In this paper I will compare the period 1970-1989, during which the majority of the dramatic increase in dual-earner households occurred and the period 1990-2011, during which labor supply appears to be relatively stabilized.

In the PSID, missing years of labor income was imputed for workers using interpolation and extrapolation, exploiting the panel structure of the PSID. In both datasets labor income for non-workers was then imputed with a standard Heckman selection correction (Heckman, 1979) using non-labor income and children as identifying instruments. Imputed labor income from the Heckman model is then used for all observations, both workers and non-

[6]Many households with single earner wives are between the ages of 55 and 65, whereas households with single-earner husbands tend to be younger.

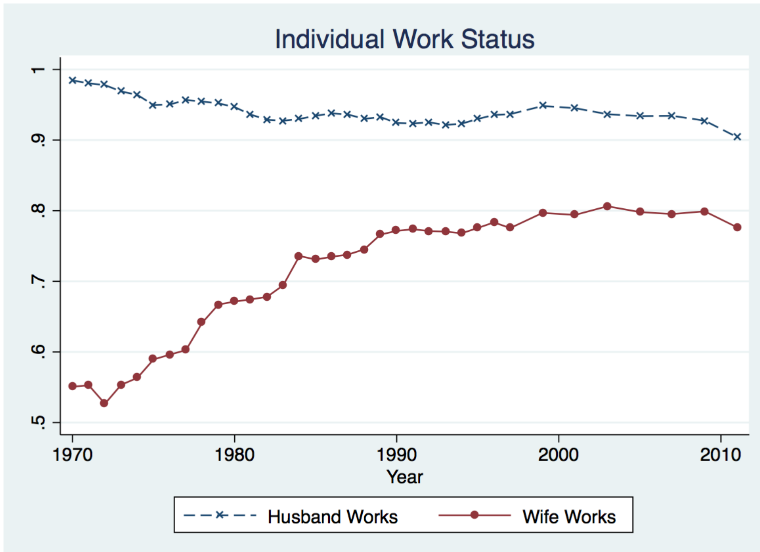

Figure 2: Individual Work Status in PSID Data

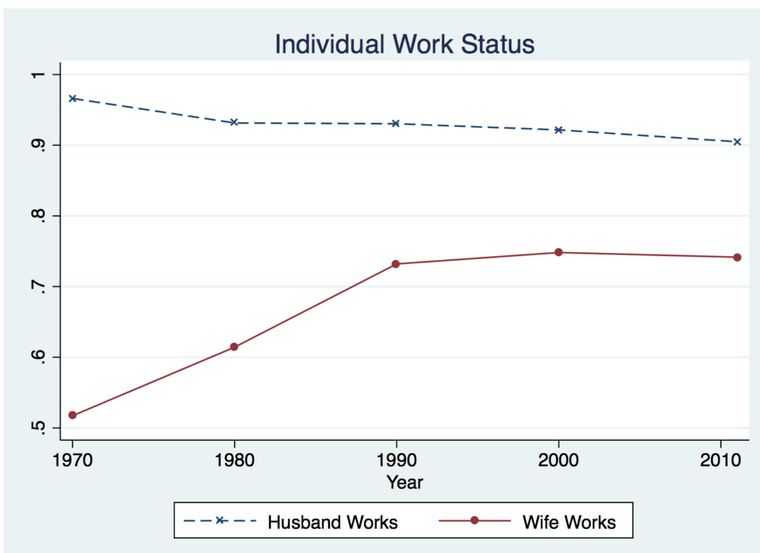

Figure 3: Individual Work Status in Census Data

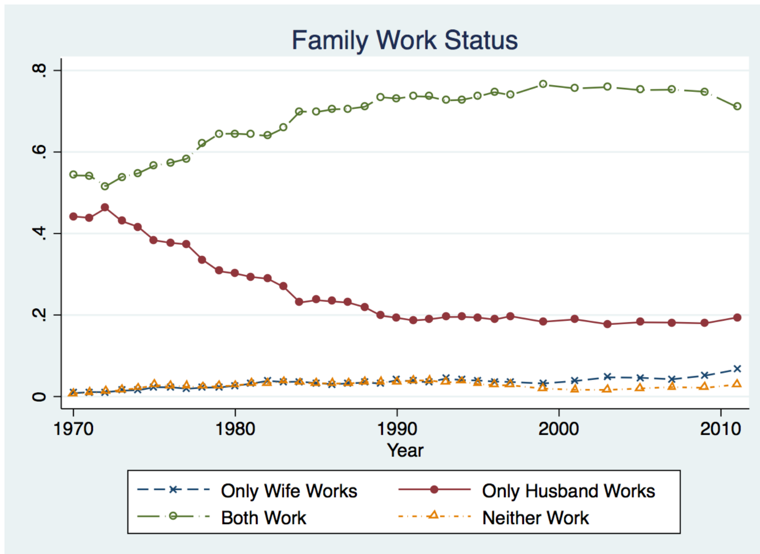

Figure 4: Family Work Status in PSID Data

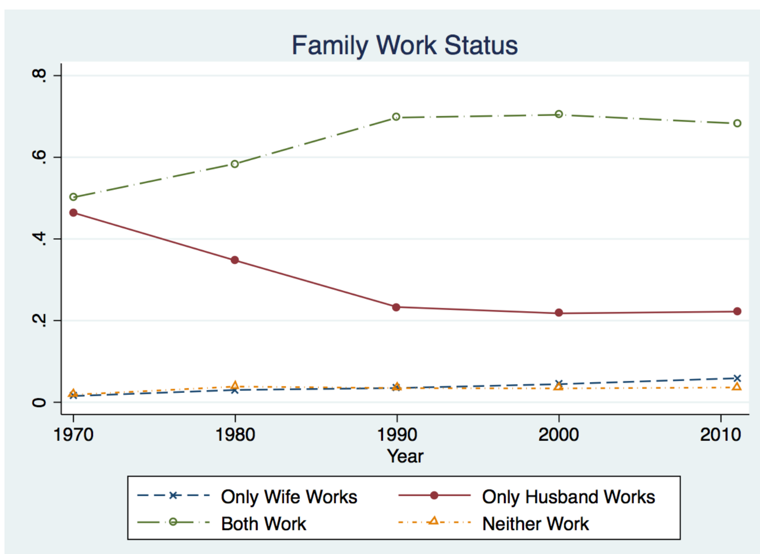

Figure 5: Family Work Status in Census Data

workers for consistency.[7] Imputed labor income values are truncated at 0 so all values are positive.[8] All income, labor and non-labor, is inflation adjusted using the Bureau of Labor Statistics CPI inflation calculator[9] and reported in year 2000 dollars.

Key variables are summarized below.

Table 1: PSID Summary Statistics

Variable	**Mean**	**Std. Dev.**	**Min.**	**Max.**
Husband Employment	0.94	0.24	0	1
Wife Employment	0.72	0.45	0	1
Number of Children	1.22	1.29	0	12
Indicator of Children Over 5	0.37	0.48	0	1
Indicator of Children Under 5	0.24	0.43	0	1
Husband Age	42.98	11.17	21	64
Wife Age	40.64	10.94	21	64
Husband Education	13.37	2.72	1	17
Wife Education	13.23	2.39	1	17
Non-Labor Income	14137.76	39638.77	-286967.22	2090004.25
Husband Labor Income	46124.72	17632.36	0	113551.59
Wife Labor Income	21040.56	9818.34	0	46982.77
Husband Black	0.07	0.25	0	1
Wife Black	0.07	0.25	0	1
Husband Hispanic	0.05	0.21	0	1
Wife Hispanic	0.05	0.21	0	1
Year	1989.05	11.41	1970	2011
N		100490		

In both datasets the most common arrangement is to have both spouses working, followed by having the husband at work and the wife at home. Surprisingly, the number of households in which only the wife works is similar to the number of households where neither spouse works in both datasets.

[7] For further discussion see Appendix

[8] The truncation affects around 3% of men and women in the PSID and .6% of men and women in the Census data.

[9] `http://www.bls.gov/data/inflation_calculator.htm`

Table 2: Census Summary Statistics

Variable	Mean	Std. Dev.	Min.	Max.
Husband Employment	0.93	0.25	0	1
Wife Employment	0.66	0.47	0	1
Number of Children	1.46	1.39	0	9
Children Over 5	0.45	0.5	0	1
Children Under 5	0.24	0.43	0	1
Husband Age	43.21	11.24	21	64
Wife Age	40.79	11.06	21	64
Husband Education	13.51	3.19	0	18
Wife Education	13.46	2.83	0	18
Non-Labor Income	7679.63	23677.97	-88729.49	777110.19
Husband Labor Income	49184.87	16471.6	0	86600.50
Wife Labor Income	27142.07	8233.03	0	48955.91
Husband Black	0.07	0.26	0	1
Wife Black	0.07	0.25	0	1
Husband Hispanic	0.08	0.26	0	1
Wife Hispanic	0.08	0.27	0	1
Year	1988.51	15.14	1970	2011
N		9379943		

Table 3: Employment Status in PSID

Husband Employment	Wife Employment		
	Not Working	Working	Total
	No.	No.	No.
Not Working	2,652.0	3,378.0	6,030.0
Working	24,533.0	70,172.0	94,705.0
Total	27,185.0	73,550.0	100,735.0

Table 4: Employment Status in Census

Husband Employment Status	Wife Employment Status		
	Not Working	Working	Total
	No.	No.	No.
Not Working	334,121.0	390,938.0	725,059.0
Working	2,487,707.0	6,167,402.0	8,655,109.0
Total	2,821,828.0	6,558,340.0	9,380,168.0

To get a sense of how household labor supply has changed over the past 40 years, one can observe the change in coefficients in a fixed effects[10] regression interacted with indicator variables for the time periods 1970-1989 and 1990-2011 in Table 6 below. The employment status of the wife is the dependent variable, and key independent variables are the wife's labor income, the husband's labor income, the husband's employment status, and the number of children in the household.[11] The fourth column of Figure 6 is the p-value from an F-test of equality between the coefficients in the two periods.

Table 5: Fixed Effects Regression: Wife Employment

	1968-1989	1990-2011	Difference	P-value
Husband Labor Income	-0.00258	-0.00011	0.00247	5.453e-10
Wife Labor Income	0.01806	0.01591	-0.00216	.00198812
Husband Employment	0.09046	0.06161	-0.02886	.10802195
Number of Children	-0.05249	-0.02767	0.02482	1.316e-08
Non-Labor Income	-0.00019	-0.00016	0.00004	.80027852
Wife Age	0.03895	0.00538	-0.03357	1.754e-15
Wife Age^2	-0.00050	-0.00011	0.00039	5.373e-14
Regional Unemployment	0.00705	-0.21307	-0.22012	.16128057
Constant	-0.21586	0.37906	0.59493	1.605e-15

Many coefficients are significantly different between the two periods, and the differences point to an increase in the preference for work in married women. Sensitivity to own income and to husband's income have gone down significantly, as found in (Blau & Kahn, 2007). The impact of husband employment status has gone down as well. The negative impact of children on

[10] A Hausman test rejects consistency of the random effects estimator with a $\chi^2(10)$ statistic of 794.63.

[11] Note that years of education and race are not included due to lack of time variation. Due to data quality issues, I imputed a fixed years of education for each observation. Results are very similar if spouse's income is also included as a control.

Table 6: Fixed Effects Regression: Husband Employment

	1968-1989	1990-2011	Difference	P-value
Husband Labor Income	0.00596	0.00847	0.00251	1.853e-16
Wife Labor Income	-0.00342	-0.00145	0.00197	.00005768
Wife Employment	0.01759	0.02823	0.01064	.06666399
Number of Children	-0.00765	-0.00802	-0.00037	.87924676
Non-Labor Income	-0.00075	-0.00026	0.00049	.00601598
Husband Age	-0.00411	-0.02773	-0.02362	6.146e-15
Husband Age^2	-0.00002	0.00027	0.00029	8.335e-15
Regional Unemployment	-0.24007	0.04811	0.28819	.00074994
Constant	0.99265	1.23409	0.24144	8.602e-07

labor supply of married women has become smaller in magnitude, significantly in the case of children under 5.

To jointly model the decision of husband and wife I turn first to a bivariate probit model. Because of coherency conditions (see eg. Heckman (1978)), we cannot allow both the husband's decision to impact the wife's decision and simultaneously allow the wife's decision to impact the husband's. The first bivariate probit table allows the husband's decision to influence the wife, while the second table allows the wife's decision to influence the husband. The bivariate probit regressions also allow husband and wife's labor supply equations to be correlated (see parameter "athrho" below).

Unlike the results of the fixed effect regression, the bivariate probit results are cross-sectional and therefore may be confounded by changes in who is selecting into marriage between the two periods and assortative mating. In fact, means tests reveal a very different pool of households in the two periods in observables. However, it is not within the scope of this paper to model selection into marriage separately. However, assortative matching in the bivariate probit will likely be soaked up in the correlation term between the spouses' employment equations, and in future work a marriage selection model could be used to learn more about this channel.

Table 7: Bivariate Probit With One-way Interactions in PSID Data

	(1) Husband Interaction		(2) Wife Interaction	
Husband Employment				
Wife Employment			-0.589***	(0.0661)
Husband Age	-15.16***	(0.893)	-15.40***	(0.846)
Husband Age^2	0.00139***	(0.000103)	0.00142***	(0.0000975)
Husband Education	-6.446***	(0.479)	-8.293***	(0.460)
Non-white	0.341***	(0.0264)	0.341***	(0.0253)
Non-Labor Income	-0.219***	(0.0280)	-0.234***	(0.0284)
Regional Unemployment	-0.0667	(0.416)	0.168	(0.398)
Husband Labor Income	4.379***	(0.0933)	4.195***	(0.0982)
Wife Labor Income	0.235*	(0.112)	1.884***	(0.202)
Constant	4.495***	(0.181)	4.841***	(0.169)
Wife Employment				
Husband Employment	0.749***	(0.0617)		
Wife Age	-0.823	(0.481)	-1.865***	(0.471)
Wife Age^2	-0.000145*	(0.0000586)	-0.0000590	(0.0000578)
Wife Education	-21.56***	(0.435)	-21.98***	(0.432)
Non-white	-0.0572**	(0.0183)	-0.0246	(0.0181)
Non-Labor Income	-0.0629***	(0.0190)	-0.0975***	(0.0188)
Regional Unemployment	2.528***	(0.269)	2.489***	(0.268)
Husband Labor Income	-0.582***	(0.0530)	-0.176***	(0.0466)
Wife Labor Income	10.85***	(0.111)	10.84***	(0.110)
Constant	1.284***	(0.124)	2.132***	(0.101)
athrho				
Constant	-0.213***	(0.0301)	0.524***	(0.0500)
Observations	100490		100490	

PSID data 1968-2011. Married sample ages 18-64. Robust standard errors in parentheses
$^{*}p < 0.05$, $^{**}p < 0.01$, $^{***}p < 0.001$

Table 8: Bivariate Probit With One-way Interaction for Women in Census

	1970		1980		1990		2000		2011	
Husband Employment										
Husband Age	15.27	(1.556)	4.771	(1.294)	2.385	(1.252)	4.087	(1.003)	0.540	(2.029)
Husband Age^2	-18.91	(1.229)	-12.25	(0.805)	-11.25	(0.826)	-12.77	(0.696)	-12.32	(1.157)
Husband Education	11.29	(3.518)	-8.069	(3.312)	-13.74	(3.016)	-10.93	(2.318)	-28.50	(5.270)
Non-white	-0.288	(0.0439)	-0.198	(0.0216)	-0.134	(0.0225)	-0.116	(0.0181)	0.0106	(0.0208)
Non-Labor Income	-0.602	(0.0438)	-0.959	(0.0261)	-0.609	(0.0242)	-0.408	(0.0159)	-0.519	(0.0180)
Regional Unemployment	-1.589	(2.197)	7.627	(0.732)	3.912	(1.153)	5.261	(1.686)	0.682	(0.719)
Husband Labor Income	-1.531	(0.809)	3.141	(0.746)	4.853	(0.684)	4.426	(0.536)	8.325	(1.200)
Wife Labor Income	0.586	(0.222)	0.975	(0.108)	1.175	(0.130)	0.663	(0.106)	0.237	(0.110)
Constant	-1.516	(0.539)	1.065	(0.488)	1.842	(0.456)	1.265	(0.365)	3.337	(0.793)
Wife Employment										
Husband Employment	0.948	(0.141)	0.653	(0.0553)	0.751	(0.0614)	1.042	(0.0507)	0.989	(0.0595)
Wife Age	1.703	(1.002)	3.420	(0.583)	5.356	(0.565)	8.063	(0.544)	9.236	(0.644)
Wife Age^2	-0.706	(0.880)	-4.709	(0.515)	-7.796	(0.534)	-9.022	(0.519)	-9.235	(0.595)
Wife Education	14.96	(2.920)	11.94	(1.697)	6.980	(1.448)	10.69	(1.347)	14.66	(1.652)
Non-white	0.231	(0.0326)	0.162	(0.0203)	0.0112	(0.0208)	-0.0562	(0.0191)	-0.0471	(0.0227)
Non-Labor Income	0.980	(0.232)	0.465	(0.0805)	0.488	(0.0667)	-0.0318	(0.0332)	-0.370	(0.0319)
Regional Unemployment	0.761	(1.031)	-0.800	(0.440)	2.608	(0.751)	-6.325	(1.135)	0.0485	(0.503)
Wife Labor Income	-3.357	(1.045)	-1.415	(0.610)	0.966	(0.536)	-0.393	(0.499)	-1.969	(0.605)
Husband Labor Income	-0.526	(0.0573)	-0.493	(0.0362)	-0.497	(0.0458)	-0.783	(0.0446)	-0.817	(0.0494)
Constant	-2.316	(0.380)	-1.816	(0.218)	-1.977	(0.196)	-2.697	(0.188)	-3.438	(0.225)
athrho										
Constant	-0.445	(0.0793)	-0.220	(0.0298)	-0.171	(0.0325)	-0.375	(0.0301)	-0.432	(0.0384)
Observations	33240		99558		106612		113226		117137	

Census data 1970-2011. Married sample ages 18-64. Robust standard errors in parentheses

$^{*}p < 0.05, ^{**}p < 0.01, ^{***}p < 0.001$

Table 9: Bivariate Probit With One-way Interaction for Men in Census

	1970		1980		1990		2000		2011	
Husband Employment										
Wife Employment	-0.437	(0.193)	-0.0137	(0.218)	0.313	(0.136)	0.504	(0.132)	0.916	(0.135)
Husband Age	14.71	(1.655)	4.141	(1.429)	1.206	(1.324)	1.872	(1.148)	-3.721	(2.298)
Husband Age^2	-18.44	(1.352)	-11.91	(0.897)	-10.12	(0.928)	-10.81	(0.868)	-8.590	(1.442)
Husband Education	10.78	(3.640)	-9.412	(3.739)	-15.53	(3.140)	-14.63	(2.542)	-35.61	(5.723)
Non-white	-0.249	(0.0452)	-0.187	(0.0222)	-0.130	(0.0228)	-0.0864	(0.0201)	0.0626	(0.0225)
Non-Labor Income	-0.477	(0.0388)	-0.867	(0.0266)	-0.512	(0.0219)	-0.338	(0.0150)	-0.414	(0.0194)
Regional Unemployment	-2.411	(2.160)	7.540	(0.741)	3.568	(1.170)	5.291	(1.700)	0.360	(0.701)
Husband Labor Income	-1.581	(0.841)	3.422	(0.855)	5.241	(0.715)	5.267	(0.588)	9.952	(1.299)
Wife Labor Income	1.023	(0.264)	1.028	(0.248)	0.826	(0.211)	0.0746	(0.189)	-0.673	(0.168)
Constant	-1.197	(0.568)	1.303	(0.526)	2.074	(0.472)	1.702	(0.396)	4.132	(0.854)
Wife Employment										
Wife Age	2.521	(1.034)	3.919	(0.581)	6.023	(0.563)	9.301	(0.549)	9.968	(0.670)
Wife Age^2	-1.906	(0.892)	-5.836	(0.508)	-9.321	(0.518)	-11.55	(0.507)	-11.31	(0.597)
Wife Education	15.76	(3.038)	11.55	(1.698)	6.263	(1.456)	9.663	(1.410)	11.37	(1.759)
Non-white	0.221	(0.0338)	0.143	(0.0204)	-0.0132	(0.0209)	-0.0912	(0.0197)	-0.0906	(0.0231)
Non-Labor Income	1.033	(0.256)	0.442	(0.110)	0.406	(0.0745)	-0.0648	(0.0339)	-0.445	(0.0343)
Regional Unemployment	0.660	(1.031)	-0.145	(0.437)	3.005	(0.749)	-5.559	(1.142)	0.209	(0.507)
Wife Labor Income	-3.527	(1.090)	-1.143	(0.611)	1.405	(0.540)	0.230	(0.523)	-0.541	(0.644)
Husband Labor Income	-0.498	(0.0575)	-0.423	(0.0360)	-0.369	(0.0452)	-0.607	(0.0448)	-0.714	(0.0498)
Constant	-1.585	(0.389)	-1.297	(0.214)	-1.387	(0.192)	-1.935	(0.193)	-2.462	(0.240)
athrho										
Constant	0.292	(0.128)	0.118	(0.137)	0.0168	(0.0823)	-0.131	(0.0785)	-0.455	(0.0913)
Observations	33240		99558		106612		113226		117137	

Census data 1970-2011. Married sample ages 18-64. Robust standard errors in parentheses

$^{*}p < 0.05, ^{**}p < 0.01, ^{***}p < 0.001$

Note that for scaling purposes, years of education and age are divided by 10, and income was measured in \$100,000s.

For women the interaction parameter (measuring the influence of the husband's labor supply) is large and grows in magnitude when imputed wage offer is added. Other controls include variables that could be expected to impact own labor supply such as education and presence of children. For men the interaction parameter negative and significant as more controls are added.

Estimating the bivariate probit on all years of the Census pooled, I find similarly strong and significant positive impact of husband working on wife's utility from work. However, estimating on each wave of the Census individually, while there is no systematic change in the sign or magnitude of the interaction parameters for women, for men the estimates go from negative and significant in 1970 and 1980, to positive and significant in 1990, 2000, and 2011. This could indicate that for husbands, leisure time has become more of a complement than a substitute for wife's leisure.

The negative impact of children on women's labor supply has decreased over Census waves, while the overall intercept for women has gotten somewhat more negative. The impact of non-labor income for women has moved from positive to negative.

The limitation of the bivariate probit model is that it only allows one interaction term. The non-cooperative game framework outlined below will allow for identification of interaction parameters for both the husband and wife simultaneously. I impose that husband and wife must play mutual best response functions and use the structure of the Nash equilibrium to estimate interaction parameters. Similar to the bivariate probit model, propensity to work is allowed to vary by a constant term and other covariates. Later, I will also allow the interaction parameter to vary by child status.

4 Basic Model

4.1 General Form

There are two agents, a husband and a wife. All husbands and all wives have the same payoff function given observed covariates, except for individual unobserved heterogeneity in the overall taste for work.

The husband and wife independently choose between work and home, and each spouse maximizes their own payoff given the choice of their spouse. The husband's choice changes the payoff structure for the wife and vice versa. Mutual best response functions determine the Nash equilibrium.

Let $y_1 = \{0, 1\}$ denote the decision of spouse 1 (husband) and $y_2 = \{0, 1\}$ denote the decision of spouse 2 (wife), where 0 indicates staying home and 1 indicates working. There are four possible outcomes:

1. $(y_1 = 1, y_2 = 1) \implies$ both work
2. $(y_1 = 1, y_2 = 0) \implies$ husband works, wife home
3. $(y_1 = 0, y_2 = 1) \implies$ husband home, wife works
4. $(y_1 = 0, y_2 = 0) \implies$ both home

Utility when not working is normalized to zero, regardless of the spouse's work decision. Utility when working is dependent on observable characteristics X with coefficients β. To account for heterogeneity and make the model empirically tractable, the individual heterogeneity in work payoff, ϵ, will be assumed normally distributed.

To model the relationship between the spouses' decisions there is a constant interaction term Δ, adopting the notation of Tamer (2003). This term is the difference in utility between working when the spouse also works, and working when the spouse is at home. Thus payoffs are as follows:

Spouse 1:

$$U_1(y_1, y_2) = (X_1\beta_1 + \epsilon_1) * y_1 + \Delta_1 * y_1 y_2$$

Spouse 2:

$$U_2(y_1, y_2) = (X_2\beta_2 + \epsilon_2) * y_1 + \Delta_2 * y_1 y_2$$

These characteristics in X are own education, race, age, children under the age of five, children over the age of five, non-labor income, regional unemployment rate, and a constant term. This differs slightly from the specification of Bjorn and Vuong (1984b), which takes number of children younger and older than age fourteen instead of age five. I consider age five a more appropriate cut-off due to the availability of child care during the day in the form of public school for children over the age of five.

Unlike the reduced form regression, spouse's characteristics enter into own payoff only through the interaction parameter and the spouse's propensity to work.

		Wife	
		0 (Home)	1 (Work)
Husband	0 (Home)	0,0	0, $X_2\beta_2 + \epsilon_2$
	1 (Work)	$X_1\beta_1 + \epsilon_1$, 0	$X_1\beta_1 + \Delta_1 + \epsilon_1$, $X_2\beta_2 + \Delta_2 + \epsilon_2$

4.2 Identification

Identification follows from three normalizations. First, the heterogeneity terms ϵ_1 and ϵ_2 are normally distributed with mean zero, setting location. Second, a normalization must be made to set scale. For simplicity I assume that the variance of ϵ_1 is the same as the variance of ϵ_2 and equal to one, though this assumption could be relaxed later, and a correlation term added as well.[12]

Lastly, the payoff when not working is normalized to zero. The empirical content of the model comes from comparing the payoffs of working to not working for each individual. If, as in Bjorn and Vuong (1984b), the not working payoffs are made to depend on covariates, only differences in coefficients between working and not working are identified.

Decision rules are as follows:

$$
\begin{aligned}
(y_1 = 1 | y_2 = 0) \quad \text{if} \quad & X_1\beta_1 + \epsilon_1 && > 0 \\
& \epsilon_1 && > -X_1\beta_1 \\
(y_1 = 1 | y_2 = 1) \quad \text{if} \quad & X_1\beta_1 + \Delta_1 + \epsilon_1 && > 0 \\
& \epsilon_1 && > -X_1\beta_1 - \Delta_1 \\
(y_2 = 1 | y_1 = 0) \quad \text{if} \quad & X_2\beta_2 + \epsilon_2 && > 0 \\
& \epsilon_2 && > -X_2\beta_2 \\
(y_2 = 1 | y_1 = 1) \quad \text{if} \quad & X_2\beta_2 + \Delta_2 + \epsilon_2 && > 0 \\
& \epsilon_2 && > -X_2\beta_2 - \Delta_2
\end{aligned}
$$

[12] Normalizing the variance of heterogeneity is better than normalizing one of the coefficients on the covariates because the coefficients represent how the relative value of working v.s. not working depends on that covariate, and it is therefore difficult to interpret a normalization to one.

Identification can then be shown identically to Bjorn and Vuong (1984b), by writing out the Hessian of the likelihood function as a product of full rank matrices.[13] Intuitively, identification comes from the exclusion of some variables from each payoff function, and sufficient variation in such variables. As certain covariates push the husband to be more or less likely to work, we observe how the wife responds.

4.3 Estimation

If a single Nash equilibrium existed for each combination of ϵ_{1i} and ϵ_{2i}, then the likelihood function could be written simply as follows using the probabilities taken from the normal distribution. I assume the heterogeneity terms ϵ_1 and ϵ_2 are independently distributed in order to reduce computation time, but theoretically they could be bivariate normal with a correlation parameter to be estimated as in Bjorn and Vuong (1984b) and Kaya (2013).

$$\mathcal{L}_i = P(1,1)^{\mathbb{I}(y_{1i}=1,y_{2i}=1)} P(1,0)^{\mathbb{I}(y_{1i}=1,y_{2i}=0)} P(0,1)^{\mathbb{I}(y_{1i}=0,y_{2i}=1)} P(0,0)^{\mathbb{I}(y_{1i}=0,y_{2i}=0)}$$

However, in several cases there are regions of values of ϵ_1 and ϵ_2 (see solution matrix in Figures 6 and 7 below) where given values of ϵ_1 and ϵ_2 produce multiple pure strategy Nash equilibria, and hence are often referred to as regions of multiplicity. Cutoff values (W_1, W_2, H_1, H_2) depend only on the sign and magnitude of the interaction parameters Δ_1 and Δ_2 in the basic model.

[13]The proof for the basic model is not elaborated here, but Bjorn and Vuong (1984b) may be referred to. The proof for the basic model is also subsumed by the proof for the extended model which is in Appendix A.

$$
\begin{aligned}
H_1 &= -X_1\beta_1 \\
H_2 &= -X_1\beta_1 - \Delta_1 \\
W_1 &= -X_2\beta_2 \\
W_2 &= -X_2\beta_2 - \Delta_2
\end{aligned}
$$

The following conditions determine game structure:

$$
\begin{aligned}
H_1 &\gtrless H_2 \\
W_1 &\gtrless W_2
\end{aligned}
$$

In the first crosshatch figure illustrated below, interaction parameters are negative so only one spouse will want to enter in general, and the region of multiplicity is a region where either spouse could enter and have a stable Nash outcome. By contrast in the second crosshatch figure, interaction parameters are positive, and the region of multiplicity is an area of heterogeneity where either both entering or neither entering would be stable.

4.4 Equilibrium Areas

I follow Bjorn and Vuong (1984b) and assume that pure strategy equilibria in these regions of multiplicity are chosen with exogenous probability which is then included in the likelihood. This likelihood is either uniform, or, when it aids computation, that the man always enters if there are mul-

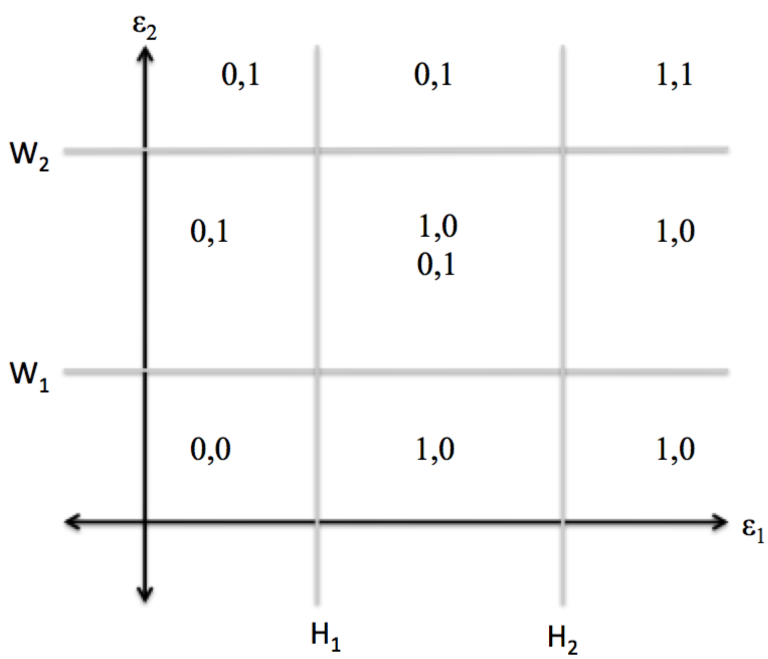

Figure 6: Generally in this construction, the spouses would like to take the opposite action from one another and will have an equilibrium with one working and one home.

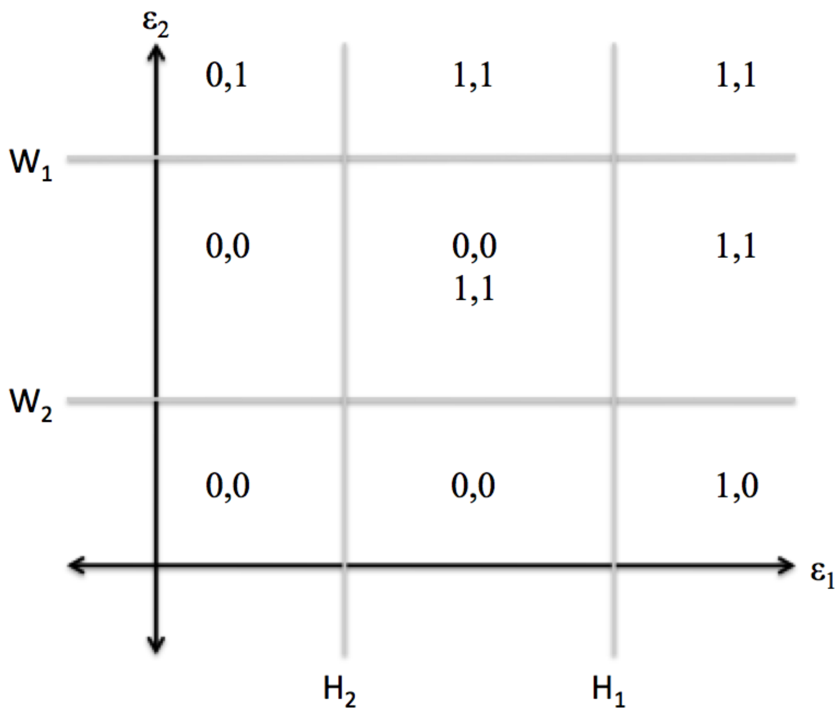

Figure 7: Generally in this construction, spouses would like to take the same action, and will generally have an equilibrium with both working or both at home.

tiple equilibria. Future extensions would allow these probability weights to depend on covariates using a logit or probit model as suggested by Bjorn and Vuong (1984b), or use partial identification tools to set identify parameters as in Beresteanu, Molchanov, and Molinari (2011). The existence of a region of multiplicity in the model, even without the imposition of exogenous equilibrium selection probabilities, is not an insurmountable barrier to identification or estimation, as shown in Tamer (2003) and Kline (2013).

Another challenge to estimation is the existence of parameter values and values of ϵ_1 and ϵ_2 for which there are no pure strategy Nash equilibrium (see solution matrix figures 8 and 9 below). In the basic model, this case arises when the signs of Δ are mismatched. In this case, spouses do not agree on whether they should take the same or opposite action, for example if one spouse views leisure as a complement but for the other spouse it is a substitute. In most estimation results this is the case: the husband views leisure as a complement and prefers to take the same action as the wife, while the wife views leisure as a substitute and prefers to take the opposite action. Leisure could be complementary if it is for example watching a movie together, whereas if it is changing diapers that would be substitutable.

In these cases of opposite-signed interaction parameters, there is a unique Nash equilibrium in mixed strategies (Bjorn & Vuong, 1984b). The mixed strategies would be a function of all parameters in the model. Although theoretically feasible, I continue to select each outcome with equal probability, or to assume that the man always enters, as in the case of multiple pure strategy equilibria.

The likelihood function depends on the signs of Δ_1 and Δ_2, thus there are four possible likelihood functions that must be maximized over. Given this and uniform probability weights assigned over regions of multiplicity or

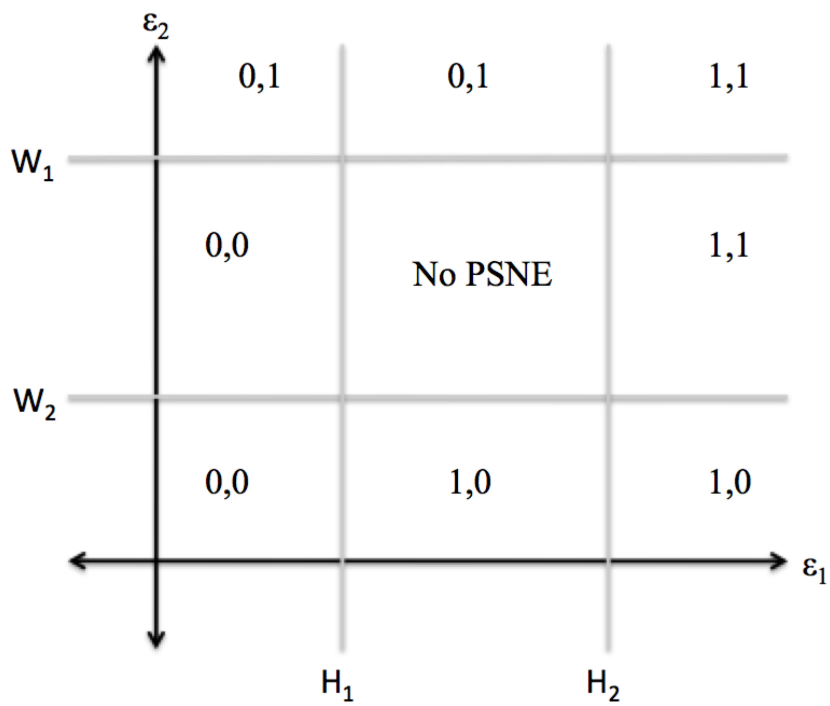

Figure 8: In this construction, the husband generally wants to take the opposite action while the wife would like to take the same action, resulting in a region of no equilibrium in pure strategies.

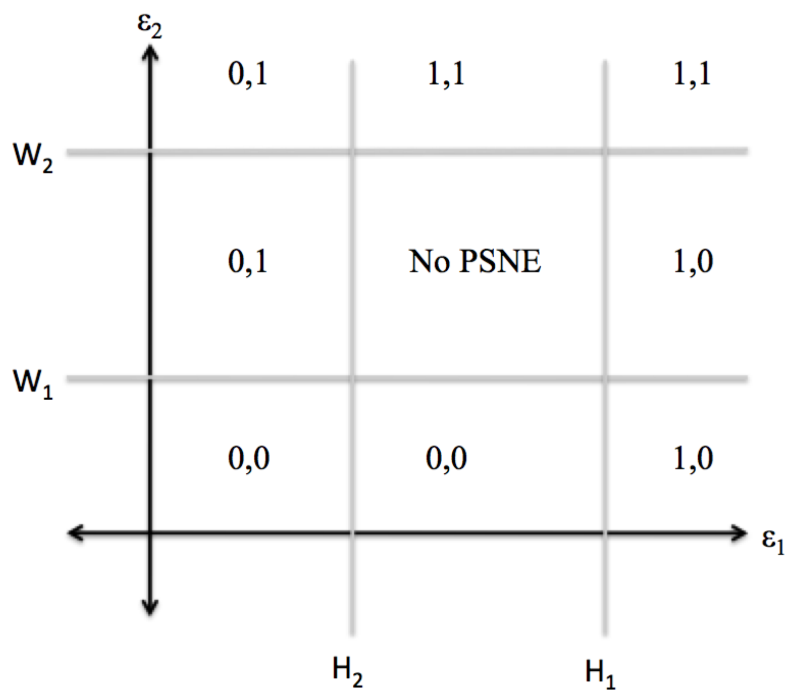

Figure 9: In this construction, the husband generally wants to take the same action while the wife would like to take the opposite action, resulting in a region of no pure strategy equilibrium.

of no pure strategy Nash equilibrium, the likelihoods are as follows:

Let $H_1 = -X_2\beta_2$, $H_2 = -X_1\beta_1 - \Delta_1$, $W_1 = -X_2\beta_2$, and $W_2 = -X_2\beta_2 - \Delta_2$. Let $P00$, $P01$, $P10$, and $P11$ represent the sample moments where $P00 = P(y_1 = 0, y_2 = 0)$ etc.

If $\mathbf{\Delta_1 \leqslant 0, \Delta_2 \leqslant 0}$

$\mathcal{L} = (\Phi(H_1) * \Phi(W_1))^{P00}(\Phi(-H_1) * \Phi(W_2) - .5 * (\Phi(H_2) - \Phi(H_1)) * (\Phi(W_2) - \Phi(W_1)))^{P10}$

$(\Phi(H_2) * \Phi(-W_1) - .5 * (\Phi(H_2) - \Phi(H_1)) * (\Phi(W_2) - \Phi(W_1)))^{P10}(\Phi(-H_2) * \Phi(-W_2))^{P11}$

If $\mathbf{\Delta_1 > 0, \Delta_2 > 0}$

$\mathcal{L} = (\Phi(H_1) * \Phi(W_1) - .5 * (\Phi(H_1) - \Phi(H_2)) * (\Phi(W_1) - \Phi(W_2)))^{P00}(\Phi(-H_1) * \Phi(W_2))^{P10}$

$(\Phi(H_2) * \Phi(-W_1))^{P01}(\Phi(-H_2) * \Phi(-W_2) - .5 * (\Phi(H_1) - \Phi(H_2)) * (\Phi(W_1) - \Phi(W_2)))^{P11}$

If $\mathbf{\Delta_1 \leqslant 0, \Delta_2 > 0}$

$\mathcal{L} = (\Phi(H_1) * \Phi(W_1) + .25 * (\Phi(H_2) - \Phi(H_1)) * (\Phi(W_1) - \Phi(W_2)))^{P00}$

$(\Phi(-H_1) * \Phi(W_2) + .25 * (\Phi(H_2) - \Phi(H_1)) * (\Phi(W_1) - \Phi(W_2)))^{P10}$

$(\Phi(H_2) * \Phi(-W_1) + .25 * (\Phi(H_2) - \Phi(H_1)) * (\Phi(W_1) - \Phi(W_2)))^{P01}$

$(\Phi(-H_2) * \Phi(-W_2) + .25 * (\Phi(H_2) - \Phi(H_1)) * (\Phi(W_1) - \Phi(W_2)))^{P11}$

If $\mathbf{\Delta_1 > 0, \Delta_2 \leqslant 0}$

$\mathcal{L} = (\Phi(H_1) * \Phi(W_1) + .25 * (\Phi(H_1) - \Phi(H_2)) * (\Phi(W_2) - \Phi(W_1)))^{P00}$

$(\Phi(-H_1) * \Phi(W_2) + .25 * (\Phi(H_1) - \Phi(H_2)) * (\Phi(W_2) - \Phi(W_1)))^{P10}$

$(\Phi(H_2) * \Phi(-W_1) + .25 * (\Phi(H_1) - \Phi(H_2)) * (\Phi(W_2) - \Phi(W_1)))^{P01}$

$(\Phi(-H_2) * \Phi(-W_2) + .25 * (\Phi(H_1) - \Phi(H_2)) * (\Phi(W_2) - \Phi(W_1)))^{P11}$

4.5 Results of Basic Model

The results of the basic model in both the PSID and all waves Census indicate that men prefer to take the same action as their spouse whereas women prefer to take the opposite action. These results are consistent with Bjorn and Vuong (1984b) and some specifications of Kooreman (1994). The positive interaction term for women matches the bivariate probit results above in both the PSID and Census. The negative interaction term for men matches the bivariate probit results in the PSID and the earlier Census years.[14]

The other covariates in the basic model are chosen following Bjorn and Vuong (1984b). The signs of the coefficients on the other covariates (shown below) mostly match those of Bjorn and Vuong (1984b) in both the PSID and Census. Differences include that I find a negative coefficient on age for men, they find a positive coefficient on non-labor income for men, and they find the impact of children to be negative for both men and women whereas I find it to be positive for men.

The result that men want to work more when their wives are at home, while women want to work more when their husbands are at work is somewhat puzzling. In the absence of other covariates this would imply that for many values of the heterogeneity term ϵ there would be no pure strategies Nash equilibrium. If the husband enters, the wife wants to enter, but if the wife enters then husband wants to leave, but if the husband leaves then wife wants to leave and so on.

These asymmetric interaction coefficients by gender may be consistent

[14]Note that the bivariate probit results in the Census for men where I do not include imputed wage offer as a covariate are uniformly positive which does not match the model estimates.

with cultural norms. For example if there is a stigma against husbands staying at home when their wives work, that could explain the result that women want to work more if their husband works. Bertrand, Kamenica, and Pan (2015) find evidence that husbands do not wish to be out-earned by their wives. Similarly social norms could dictate that more of wives' leisure time is spent engaging in household production, which would be a substitute good for husband's leisure time. If leisure time is a substitute good we would expect a negative interaction term, which is want I find for husbands. On the other hand if leisure time is not used for household production we might expect complementarity between the leisure time of husband and wife, which is what I find for wives.[15]

5 Extended Models

The basic model does not allow for the interaction between spouses to depend on any observed characteristics. In the next sections I relax these assumptions, first by allowing the spousal interaction to vary by whether there are children under 5 or over 5 or no children in the household, then on spouse's income.

In the baseline model, more children in the household causes husbands to want to work more and wives to work less. When looking at how children affects the interaction between spouses I look not at the effect of an additional child, but rather whether there are any children under 5 or over 5 in the household, since the need for childcare may change discretely at these points. I expect that the presence of children, especially under age 5, will

[15] These results could be confounded by marriage market preferences: perhaps career-oriented men prefer to marry stay at home wives while career-oriented women prefer to marry career-oriented husbands, all else equal.

cause leisure to be more of a substitute good between spouses as there are more household tasks needed, and therefore that the coefficients on children in the interaction will be negative. In other words I expect children to make it more valuable to have one spouse at home if the other one works.

Lastly, I allow the interaction terms to depend on spouse's income. I expect that it would be easier to stay at home if a spouse is working and high earning due to relaxation of the budget constraint. In addition, if there is diminishing marginal utility form consumption, you should get less utility from your spouse's income if you both work than if only your spouse worked. Therefore I expect the coefficients on spouse's income in the interaction terms to be negative: the higher earning your spouse is predicted to be, the less likely you are to enter the labor market.[16] However, if labor supply affects how spouses pool income, the coefficients on spouse's income in the interaction terms could in fact be positive. I could get more utility from my spouse's income when I work, if working allows me to share more of my spouse's income.

The extended models are detailed below. In both of these models, in addition to the constant interaction terms Δ_1 and Δ_2 from the baseline model, we have covariates Z (indicators of children or spouse's income) and coefficients δ_1 and δ_2.

[16]I do not assume a utility function with diminishing returns to consumption because identification then relies on the specific choice of functional form, which also makes the estimator computationally unstable.

Spouse 1:

$$
U_1(y_1, y_2) = \begin{cases} = 0 & & : (y_1 = 0, y_2 = 0) \\ = X_1\beta_1 & +\epsilon_{1i} & : (y_1 = 1, y_2 = 0) \\ = 0 & & : (y_1 = 0, y_2 = 1) \\ = X_1\beta_1 + Z_1\delta_1 + & \epsilon_{1i} & : (y_1 = 1, y_2 = 1) \end{cases}
$$

Spouse 2:

$$
U_2(y_1, y_2) = \begin{cases} = 0 & & : (y_1 = 0, y_2 = 0) \\ = X_2\beta_2 + & \epsilon_{2i} & : (y_1 = 0, y_2 = 1) \\ = 0 & & : (y_1 = 1, y_2 = 0) \\ = X_2\beta_2 + Z_2\delta_2 & +\epsilon_{2i} & : (y_1 = 1, y_2 = 1) \end{cases}
$$

Solving for ϵ_{1i} and ϵ_{2i} yields cut-off points similar to the basic model above, but now dependent on more than just the sign of Δ_1 and Δ_2. It is now possible for individuals with the same parameters to face different game structures depending on their covariates Z.

In the extended specifications, in order to gain stability for the estimator, I assume that the husband enters in regions of multiple equilibria or no equilibria rather than assume a uniform probability. The full proof of identification is in Appendix A.

5.1 Heterogeneity by Child Status

The presence of children in a household might make spouses' leisure time more substitutable by increasing hours spent in household production, especially for children under 5 years old. To assess whether the interaction terms vary by the presence of children in the home, I parameterize the interaction term into a constant, an indicator of children under age 5, and an indicator of children over age 5.

Table 10: Bjorn and Vuong Replication in Census data

	Parameters and SEs				
Parameter	1970	1980	1990	2000	2011
Δ_1 (interaction)	-0.5418	-0.7915	-0.8531	-0.6363	-0.5791
	0.1557	0.0894	0.1283	0.0728	0.0484
Δ_2 (interaction)	0.8754	1.2906	1.4696	1.1393	0.9913
	0.2115	0.1125	0.1460	0.0802	0.0526
$\beta_{0,1}$ (constant)	2.8115	2.3618	2.3990	2.4829	2.4456
	0.1633	0.0972	0.1281	0.0948	0.0808
$\beta_{0,2}$ (constant)	-0.9235	-0.8960	-1.2090	-0.7094	-0.9089
	0.2175	0.1136	0.1518	0.0935	0.0741
σ_1 (variance of error)	1.0000	1.0000	1.0000	1.0000	1.0000
	0.0000	0.0000	0.0000	0.0000	0.0000
σ_2 (variance of error)	1.0000	1.0000	1.0000	1.0000	1.0000
	0.0000	0.0000	0.0000	0.0000	0.0000
age1	-2.7369	-3.7630	-4.1311	-4.1615	-4.2092
	0.1676	0.0968	0.1187	0.0794	0.0704
yrsch1	6.7318	8.1426	10.0842	9.4973	8.5064
	0.4573	0.2286	0.2840	0.2267	0.1999
racebv1	-0.1827	-0.2609	-0.1148	-0.1853	-0.1234
	0.0424	0.0199	0.0201	0.0167	0.0153
nonlabor1	-0.6387	-1.0314	-0.7527	-0.4662	-0.5316
	0.0631	0.0352	0.0342	0.0183	0.0163
nchild1	0.4531	0.4517	0.2337	0.4827	0.7095
	0.1039	0.0550	0.0604	0.0564	0.0551
unemp1	-6.0494	6.3993	5.2582	1.3045	1.4932
	2.0889	0.7639	1.0765	1.4752	0.5692
age2	-0.6529	-1.3563	-1.3823	-0.7991	-0.6186
	0.0780	0.0601	0.0869	0.0651	0.0544
yrsch2	4.5648	6.0047	7.8455	7.5377	7.5714
	0.2836	0.1812	0.2158	0.1808	0.1651
racebv2	0.2990	0.2573	0.1415	0.0201	-0.0025
	0.0236	0.0145	0.0145	0.0122	0.0114
nonlabor2	0.3005	0.6675	0.6345	-0.0675	-0.3642
	0.1133	0.0845	0.0800	0.0341	0.0285
nchild2	-1.3594	-1.4121	-1.3661	-1.1881	-1.2763
	0.0477	0.0365	0.0435	0.0388	0.0380
unemp2	0.4662	-1.3733	2.7884	-5.4425	0.5473
	1.0561	0.4640	0.7116	1.0661	0.4192

Note: Standard Errors Below

Table 11: Bjorn and Vuong Replication in PSID data

	Parameters and SEs
Parameter	PSID 1970-2011
Δ_1 (interaction)	-0.6263
	0.0609
Δ_2 (interaction)	1.0223
	0.0702
$\beta_{0,1}$ (constant)	5.6565
	0.1490
$\beta_{0,2}$ (constant)	0.9451
	0.1090
σ_1 (variance of error)	1.0000
	0.0000
σ_2 (variance of error)	1.0000
	0.0000
age1	-17.3042
	0.6998
agesquare1	15.6835
	0.8102
yrsch1	-9.7037
	0.3799
racebv1	0.3515
	0.0199
nonlabor1	-0.2857
	0.0151
nchild1	-0.2719
	0.0603
unemp1	-1.5011
	0.3602
heckincwage1	4.4675
	0.0749
heckincwage2	1.9312
	0.1440
age2	0.7764
	0.4231
agesquare2	-4.2150
	0.5212
yrsch2	-20.2385
	0.3579
racebv2	0.0811
	0.0132
nonlabor2	-0.0401
	0.0151
nchild2	-1.1014
	0.0390
unemp2	1.6801
	0.2326
heckincwage2	10.7539
	0.0973
heckincwage1	-0.6804
	0.0505

Note: Standard Errors Below

Table 12: Bjorn and Vuong Replication in Census data

	Parameters and SEs				
Parameter	1970	1980	1990	2000	2011
Δ_1 (interaction)	-0.8549	-1.0955	-1.0016	-0.8456	-0.5998
	0.1507	0.0697	0.0813	0.0704	0.0503
Δ_2 (interaction)	1.2298	1.5879	1.5402	1.2945	0.9627
	0.1957	0.0854	0.0860	0.0708	0.0537
$\beta_{0,1}$ (constant)	0.9940	0.8566	1.3110	1.3273	1.9054
	0.7271	0.4228	0.3312	0.2944	0.3706
$\beta_{0,2}$ (constant)	-3.2768	-3.2211	-3.2327	-3.2880	-3.5889
	0.4053	0.2210	0.2020	0.1909	0.1885
σ_1 (variance of error)	1.0000	1.0000	1.0000	1.0000	1.0000
	0.0000	0.0000	0.0000	0.0000	0.0000
σ_2 (variance of error)	1.0000	1.0000	1.0000	1.0000	1.0000
	0.0000	0.0000	0.0000	0.0000	0.0000
age1	9.1759	7.3589	6.2278	5.8241	4.7547
	2.1135	1.1810	0.9685	0.8519	1.0110
agesquare1	-14.9389	-15.1753	-15.5571	-14.5929	-14.8676
	1.5915	0.8391	0.7542	0.6731	0.6932
yrsch1	0.4611	-2.9122	-9.0090	-7.2952	-16.9270
	4.9151	2.8733	2.1593	1.8375	2.4222
racebv1	-0.1580	-0.2305	-0.0865	-0.1543	-0.0754
	0.0474	0.0223	0.0211	0.0174	0.0160
nonlabor1	-0.7604	-1.1547	-0.7861	-0.4931	-0.5235
	0.0678	0.0356	0.0305	0.0190	0.0168
nchild1	0.0052	-0.0595	-0.2042	0.1001	0.2630
	0.1100	0.0604	0.0635	0.0590	0.0564
unemp1	-10.1210	6.6463	5.6101	1.1584	1.2096
	2.2112	0.8008	1.1032	1.6618	0.5785
heckincwage1	1.0299	2.0194	3.7804	3.4648	5.6559
	1.1174	0.6487	0.4961	0.4285	0.5582
heckincwage2	1.6881	2.0186	2.1289	1.6924	1.0347
	0.2426	0.1224	0.1289	0.1176	0.1045
age2	7.9754	8.2102	9.7642	11.8902	12.5044
	1.0151	0.5829	0.5556	0.5185	0.5200
agesquare2	-9.1358	-10.2885	-12.8825	-13.9729	-14.0709
	0.9344	0.5422	0.5337	0.5040	0.4911
yrsch2	12.4861	12.6643	8.0607	11.1202	11.8793
	2.8545	1.5988	1.3967	1.2691	1.3107
racebv2	0.3124	0.2934	0.1059	0.0222	-0.0179
	0.0336	0.0212	0.0209	0.0194	0.0193
nonlabor2	0.4172	0.8558	0.7702	-0.0056	-0.3182
	0.1156	0.0863	0.0765	0.0350	0.0289
nchild2	-1.6212	-1.7800	-1.7582	-1.6114	-1.6767
	0.0511	0.0372	0.0407	0.0394	0.0388
unemp2	2.1252	-1.3964	3.0928	-6.1213	0.9623
	1.0686	0.4655	0.7181	1.0910	0.4219
heckincwage2	-2.2467	-1.9461	0.3610	-0.8169	-0.7634
	1.0262	0.5795	0.5179	0.4773	0.4885
heckincwage1	-0.7362	-0.6644	-0.7494	-0.8599	-0.9862
	0.0601	0.0389	0.0458	0.0439	0.0415

Note: Standard Errors Below

Table 13: Bjorn and Vuong Replication by Child Status in PSID data

	Parameters and SEs
Parameter	PSID 1970-2011
Δ_1 (interaction)	0.1040
	0.0574
under51	0.1211
	0.0285
over51	0.1826
	0.0237
Δ_2 (interaction)	0.1781
	0.0661
under52	-0.5499
	0.0179
over52	-0.0327
	0.0151
$\beta_{0,1}$ (constant)	2.7039
	0.0730
$\beta_{0,2}$ (constant)	-0.0839
	0.0784
σ_1 (variance of error)	1.0000
	0.0000
σ_2 (variance of error)	1.0000
	0.0000
age1	-3.7137
	0.0853
yrsch1	6.2732
	0.2641
racebv1	-0.2998
	0.0161
nonlabor1	-0.2674
	0.0139
nchild1	-0.0001
	0.0618
unemp1	-4.1945
	0.3312
age2	-2.0483
	0.0626
yrsch2	11.9127
	0.2080
racebv2	0.2096
	0.0109
nonlabor2	-0.1520
	0.0134
nchild2	-0.8291
	0.0435
unemp2	0.8840
	0.2161

Note: Standard Errors Below

Table 14: Bjorn and Vuong Replication by Child Status in PSID data

	Parameters and SEs
Parameter	PSID 1970-2011
Δ_1 (interaction)	-0.4015
	0.0434
Δ_2 (interaction)	0.7887
	0.0422
$\beta_{0,1}$ (constant)	5.5377
	0.1445
$\beta_{0,2}$ (constant)	1.2172
	0.0949
σ_1 (variance of error)	1.0000
	0.0000
σ_2 (variance of error)	1.0000
	0.0000
age1	-17.6637
	0.6914
agesquare1	16.3580
	0.7996
yrsch1	-9.6708
	0.3802
racebv1	0.3498
	0.0197
nonlabor1	-0.2805
	0.0149
nchild1	-0.3397
	0.0648
unemp1	-1.5946
	0.3548
heckincwage1	4.4923
	0.0743
heckincwage2	1.4679
	0.1292
age2	0.9503
	0.4135
agesquare2	-4.9777
	0.5135
yrsch2	-20.0588
	0.3485
racebv2	0.0944
	0.0129
nonlabor2	-0.0656
	0.0143
nchild2	-0.6801
	0.0472
unemp2	1.5627
	0.2287
heckincwage2	10.6549
	0.0930
heckincwage1	-0.4692
	0.0405
anychild1	0.1157
	0.0252
anychild2	-0.2372
	0.0156

Note: Standard Errors Below

The impact of children under 5 for wives is consistent with a story of increasing the substitutability of leisure between husband and wife. Wives without any children in the household continue to prefer to work if their husbands work in all years except 1970, but if children under 5 are present wives prefer to work more if their husband stays at home in all Census years except the most recent, 2000 and 2011. Interestingly, the impact of children over 5 has changed from negative in 1970 and 1980 to positive in more recent years, implying perhaps leisure time has become more complementary over time for wives with school-aged children. By contrast, for husbands the impact of children has become more negative over time, meaning that leisure maybe be more substitutable in recent years for husbands.

The positive coefficient for husbands for the children under five interaction in the PSID and early Census years implies that husbands with children at home prefer to work more if their spouse works, whereas wives prefer to work less if their spouse works. Children might increase the opportunity cost of working and thereby not spending time with children, while also increasing the need for labor income. These competing explanations may explain why the coefficient for men is shifting over time, with the time explanation gaining prominence with more negative coefficients in later years. Men spending more time with children could also explain the switch in sign for wives with children over 5 over this period.

The pattern for husbands could also results from a cultural norm against stay-at-home dads being eroded, or the lower average labor income among wives ($23,901 vs. husbands' $41,953) could mean that husbands are required to work to support children even when the wife works, but as women's earnings have gone up over time this is less true.

A disadvantage of this model is that spouses who stay at home are assumed to be indifferent to whether their spouse works. In reality the spouses decision to work should make staying at home much more attractive given that household income is likely shared. To address this in the next section I include spouse's income into the interaction effect. In this case the coefficient on spouse's income represents how much more utility I get from my spouse's earnings if I work as opposed to staying at home and we could expect it to be positive or negative. This parameter could be positive if entry into the labor force increases bargaining power, or negative if breadwinners share more of their income with a stay-at-home spouse or there is diminishing marginal utility to income.

5.2 Heterogeneity by Spouse's Income

Unlike in the child status specification, the signs of the intercepts of the interaction term (Δ_1 and Δ_2) are uniformly similar to the baseline Bjorn and Vuong (1984b) replication.[17] The coefficients on spouse's income, which I expected to be negative for both husband and wife, are in fact negative for the wife but positive for the husband. This implies that husbands may be getting more utility from their wife's labor income if they also work.

[17] In the following specifications I also control for own income. Results are similar without this control in the Census, but in the PSID the coefficient on husban's income in the interaction also becomes positive.

Table 15: Bjorn and Vuong Replication by Child Status in Census data

	Parameters and SEs				
Parameter	1970	1980	1990	2000	2011
Δ_1 (interaction)	1.3025	-0.0675	0.0966	-0.0341	-0.1829
	0.0665	0.0470	0.0421	0.0397	0.0393
under51	0.4642	0.2218	0.1376	-0.0289	-0.2104
	0.0629	0.0348	0.0313	0.0292	0.0284
over51	0.3721	0.1498	0.0900	-0.0328	-0.0588
	0.0436	0.0253	0.0226	0.0207	0.0198
Δ_2 (interaction)	-1.6700	0.3711	0.2971	0.4688	0.5697
	0.0860	0.0509	0.0469	0.0434	0.0413
under52	-1.1694	-0.7286	-0.5162	-0.4642	-0.4225
	0.0530	0.0174	0.0182	0.0179	0.0183
over52	-0.2826	-0.0641	0.0603	0.0983	0.1162
	0.0321	0.0144	0.0149	0.0144	0.0141
$\beta_{0,1}$ (constant)	0.8413	1.6954	1.6032	2.0083	2.2333
	0.1318	0.0720	0.0779	0.0793	0.0756
$\beta_{0,2}$ (constant)	2.2864	0.5238	0.2991	0.3009	-0.1699
	0.1053	0.0618	0.0654	0.0672	0.0667
σ_1 (variance of error)	1.0000	1.0000	1.0000	1.0000	1.0000
	0.0000	0.0000	0.0000	0.0000	0.0000
σ_2 (variance of error)	1.0000	1.0000	1.0000	1.0000	1.0000
	0.0000	0.0000	0.0000	0.0000	0.0000
age1	-1.2706	-3.1158	-3.3498	-3.8619	-4.2397
	0.1556	0.0784	0.0757	0.0729	0.0738
yrsch1	4.3023	7.3522	8.6378	8.7012	8.2744
	0.3845	0.2046	0.2114	0.2046	0.1991
racebv1	-0.2099	-0.2775	-0.1228	-0.1649	-0.1121
	0.0366	0.0192	0.0193	0.0162	0.0150
nonlabor1	0.0851	-0.8645	-0.5844	-0.4000	-0.4828
	0.0569	0.0291	0.0252	0.0164	0.0155
nchild1	0.1950	0.4249	0.2830	0.6337	0.9099
	0.0883	0.0597	0.0674	0.0657	0.0674
unemp1	-5.3262	6.4166	4.4226	1.7739	1.2391
	1.8336	0.7311	1.0269	1.5241	0.5610
age2	-3.6136	-3.0085	-2.9717	-2.1286	-1.7647
	0.1475	0.0491	0.0544	0.0542	0.0534
yrsch2	9.0216	7.1952	9.4864	8.6546	8.4352
	0.4893	0.1689	0.1854	0.1690	0.1628
racebv2	0.1547	0.2310	0.1214	0.0065	-0.0116
	0.0360	0.0134	0.0137	0.0118	0.0113
nonlabor2	-0.0638	0.3648	0.3914	-0.0848	-0.3863
	0.1366	0.0735	0.0632	0.0329	0.0278
nchild2	-0.8520	-0.8784	-1.0016	-0.9575	-1.1645
	0.0824	0.0435	0.0494	0.0492	0.0509
unemp2	-2.9424	-0.2130	3.8063	-4.4078	0.9880
	1.5734	0.4487	0.6867	1.0570	0.4149

Note: Standard Errors Below

Table 16: Extended Model by Spouse's Income in PSID data

	Parameters and SEs
Parameter	PSID 1970-2011
Δ_1 (interaction)	-0.3964
	0.0493
heckincwage2	1.0027
	0.1381
Δ_2 (interaction)	0.6278
	0.0396
heckincwage1	-0.2949
	0.0352
$\beta_{0,1}$ (constant)	2.8643
	0.0622
$\beta_{0,2}$ (constant)	1.8942
	0.0592
σ_1 (variance of error)	1.0000
	0.0000
σ_2 (variance of error)	1.0000
	0.0000
age1	-3.5548
	0.0699
yrsch1	-5.7981
	0.3267
racebv1	0.2127
	0.0183
nonlabor1	-0.2692
	0.0147
nchild1	-0.5282
	0.0575
unemp1	-1.8762
	0.3504
heckincwage1	3.5461
	0.0558
age2	-2.8325
	0.0522
yrsch2	-20.6917
	0.3384
racebv2	0.1245
	0.0124
nonlabor2	-0.0688
	0.0143
nchild2	-1.0044
	0.0362
unemp2	1.5230
	0.2281
heckincwage2	10.7968
	0.0912

Note: Standard Errors Below

Table 17: Bjorn and Vuong Replication by Income in PSID data

	Parameters and SEs
Parameter	PSID 1970-2011
Δ_1 (interaction)	-0.5332
	0.0569
Δ_2 (interaction)	0.7112
	0.0404
$\beta_{0,1}$ (constant)	5.6214
	0.1484
$\beta_{0,2}$ (constant)	1.2438
	0.0944
σ_1 (variance of error)	1.0000
	0.0000
σ_2 (variance of error)	1.0000
	0.0000
age1	-17.6232
	0.6981
agesquare1	16.1340
	0.8053
yrsch1	-9.9304
	0.3870
racebv1	0.3606
	0.0200
nonlabor1	-0.2850
	0.0150
nchild1	-0.2931
	0.0608
unemp1	-1.5635
	0.3577
heckincwage1	4.4604
	0.0757
heckincwage2	1.6698
	0.1675
age2	0.4580
	0.4115
agesquare2	-4.0522
	0.5092
yrsch2	-20.0218
	0.3480
racebv2	0.0920
	0.0129
nonlabor2	-0.0570
	0.0143
nchild2	-1.1010
	0.0385
unemp2	1.5295
	0.2283
heckincwage2	10.6390
	0.0932
heckincwage1	-0.9195
	0.1232
heckincwage2	0.2580
	0.1807
heckincwage1	0.4322
	0.1200

Note: Standard Errors Below

For husbands in the PSID if their wives earned more an additional \$39,000 per year, the sign of the interaction term would flip and husbands would actually prefer to work more if their wives worked. This puzzling results could indicate that at low levels of wife income husbands are happy to stay at home, but at higher levels would prefer to also work. For wives in the PSID, the interaction with husband income is signed as expected: as husbands earn more wives get less utility from working if their husbands also work. This is consistent with income sharing and/or diminishing marginal utility from consumption.

Another interpretation is that wives who stay at home are receiving more utility from their spouse's income that husbands who stay at home. Surprisingly, the coefficient on wife's income for husbands has been going up over time, implying that men are wanting more and more to work as their wife's income increases. Either working allows men more access to their wife's income, or men are working more in order to avoid being outearned by their wives as in Bertrand et al. (2015), especially as women's earnings are going up and maybe more likely to surpass those of their husbands in more recent years.

6 Conclusion

I build on the literature on intra-household interaction and labor supply. I assume non-cooperative Nash outcomes in order to estimate how labor supply functions have changed over time, including the impact of husband's decision on wife and vice versa. The non-cooperative strategy has the advantage of allowing non-pareto, self-enforcing, outcomes, and is superior to a bivariate probit model in that it allows for spouses decisions to simultaneously affect one another.

To examine the interaction between husband and wife I first run a fixed effect regression of female labor supply, where husband's employment is an explanatory variable. I find that husband's labor supply actually increases wife's labor supply, which is consistent with complementarity of leisure time between spouses, or of wives wanting to avoid out-earning their spouses.

I then run cross-sectional bivariate probit regressions pooling the PSID years, and by waves of the Census. I continue to find a strong positive effect of husband labor supply on wife labor supply. The impact of wife's labor supply on husband is found to be negative, but this result is less robust. This result could be confounded by a preference on the part of men to marry stay-at-home wives.

The complementarity of leisure, or preference for dual earner households, is found to increase differentially across households with and without children. Husbands who have children are more likely to work if their wives work, and wives who have children are less likely to work if their husbands work. This could indicate that for women children in the household increases the substitutability of leisure. However, these estimates have been changing over time.

Finally I examine the impact of spouse's income level on labor supply. I expected that having a higher income spouse would decrease the return to working due to diminishing marginal utility from consumption in the household. I find that although this is the case for wives, for husbands having a higher earning wife actually increases labor supply. This effect has been increasing over time. Perhaps with the closing of the gender wage gap the risk that a wife might out-earn her husband has gone up, driving up with it husbands' positive labor supply response to wife's labor supply.

7 Appendix

8 Identification Proof of Model

Recall from above that the empirical content of the extended model can be summarized in the following four expressions:

$$
\begin{aligned}
H_2 =& log(.5inc + (1-\lambda_2)w_2) - (X_1\beta_1 + Z_1\delta_1 + log(.5inc + w_1)) \\
H_1 =& log(.5inc) - (X_1\beta_1 + log(.5inc + \lambda_1 w_1)) \\
W_2 =& log(.5inc + (1-\lambda_1)w_1) - (X_2\beta_2 + Z_2\delta_2 + log(.5inc + w_2)) \\
W_1 =& log(.5inc) - (X_2\beta_2 + log(.5inc + \lambda_2 w_2))
\end{aligned}
$$

$$
\begin{aligned}
H_2 =& - X_1\beta_1 - Z_1\delta_1 \\
H_1 =& - X_1\beta_1 \\
W_2 =& - X_2\beta_2 - Z_2\delta_2 \\
W_1 =& - X_2\beta_2
\end{aligned}
$$

For each structure of the model, which may vary by individual covariates, the probabilities are as follows. Subscripts for each individual i are omitted for simplicity.

If $\mathbf{H_2} \geqslant \mathbf{H_1}, \mathbf{W_2} \geqslant \mathbf{W_1}$

$P00 = \Phi(H_1) * \Phi(W_1)$

$P10 = \Phi(-H_1) * \Phi(W_2) - .5 * (\Phi(H_2) - \Phi(H_1)) * (\Phi(W_2) - \Phi(W_1))$

$P01 = \Phi(H_2) * \Phi(-W_1) - .5 * (\Phi(H_2) - \Phi(H_1)) * (\Phi(W_2) - \Phi(W_1))$

$P11 = \Phi(-H_2) * \Phi(-W_2)$

If $\mathbf{H_2} < \mathbf{H_1}, \mathbf{W_2} < \mathbf{W_1}$

$P00 = \Phi(H_1) * \Phi(W_1) - .5 * (\Phi(H_1) - \Phi(H_2)) * (\Phi(W_1) - \Phi(W_2))$

$P10 = \Phi(-H_1) * \Phi(W_2)$

$P01 = (\Phi(H_2) * \Phi(-W_1)$

$P11 = \Phi(-H_2) * \Phi(-W_2) - .5 * (\Phi(H_1) - \Phi(H_2)) * (\Phi(W_1) - \Phi(W_2))$

If $\mathbf{H_2} \geqslant \mathbf{H_1}, \mathbf{W_2} < \mathbf{W_1}$

$P00 = \Phi(H_1) * \Phi(W_1) + .25 * (\Phi(H_2) - \Phi(H_1)) * (\Phi(W_1) - \Phi(W_2))$

$P10 = \Phi(-H_1) * \Phi(W_2) + .25 * (\Phi(H_2) - \Phi(H_1)) * (\Phi(W_1) - \Phi(W_2))$

$P01 = \Phi(H_2) * \Phi(-W_1) + .25 * (\Phi(H_2) - \Phi(H_1)) * (\Phi(W_1) - \Phi(W_2))$

$P11 = \Phi(-H_2) * \Phi(-W_2) + .25 * (\Phi(H_2) - \Phi(H_1)) * (\Phi(W_1) - \Phi(W_2))$

If $\mathbf{H_2} < \mathbf{H_1}, \mathbf{W_2} \geqslant \mathbf{W_1}$

$P00 = \Phi(H_1) * \Phi(W_1) + .25 * (\Phi(H_1) - \Phi(H_2)) * (\Phi(W_2) - \Phi(W_1))$

$P10 = \Phi(-H_1) * \Phi(W_2) + .25 * (\Phi(H_1) - \Phi(H_2)) * (\Phi(W_2) - \Phi(W_1))$

$P01 = \Phi(H_2) * \Phi(-W_1) + .25 * (\Phi(H_1) - \Phi(H_2)) * (\Phi(W_2) - \Phi(W_1))$

$P11 = \Phi(-H_2) * \Phi(-W_2) + .25 * (\Phi(H_1) - \Phi(H_2)) * (\Phi(W_2) - \Phi(W_1))$

And the log-likelihood for the full sample:

$$\mathcal{L} = \sum_i log(P00_i)((y_{1i} = 0)(y_{2i} = 0)) + log(P11_i)((y_{1i} = 1)(y_{2i} = 1)) +$$
$$log(P10_i)((y_{1i} = 1)(y_{2i} = 0)) + log(P01_i)((y_{1i} = 0)(y_{2i} = 1))$$

Showing identification of the model is equivalent to showing that the expectation of the Hessian is invertible, which through the information matrix equality is equivalent to showing that the outer product of gradients of the log-likelihood is invertible. Let us call this matrix B as in (Bjorn & Vuong, 1984b). Let θ be a $1 \times K$ vector of all parameters in the model. X_{1i}, X_{2i}, Z_{1i}, and Z_{2i} are column vectors of covariates of individual i of length corresponding to β_1, β_2, δ_1 and δ_2 respectively.

$$\theta = \quad [\beta_1 \quad \beta_2 \quad \delta_1 \quad \delta_2 \quad \rho]$$

The expected value of the Hessian is then

$$E\left(\frac{\partial^2 \mathcal{L}}{\partial\theta\partial\theta'}\right) = E\left(\frac{\partial \mathcal{L}}{\partial\theta'} \cdot \frac{\partial \mathcal{L}}{\partial\theta}\right) = B$$

For instance, the derivative with respect to the first element of θ can be written as follows. Note that no probability can be equal to zero.

$$\frac{\partial \mathcal{L}}{\partial\theta_1} = \frac{(y_1 = 0)(y_2 = 0)}{P00}\frac{\partial P00}{\partial\theta_1} + \frac{(y_1 = 1)(y_2 = 0)}{P10}\frac{\partial P10}{\partial\theta_1}$$
$$+ \frac{(y_1 = 0)(y_2 = 1)}{P01}\frac{\partial P01}{\partial\theta_1} + \frac{(y_1 = 1)(y_2 = 1)}{P11}\frac{\partial P11}{\partial\theta_1}$$

Only one event can occur, so for example $((y_1 = 0)(y_2 = 0)) * ((y_1 = 1)(y_2 = 0))$ must equal zero. Therefore multiplying $\frac{\partial \mathcal{L}}{\partial \theta}$ and $\frac{\partial \mathcal{L}}{\partial \theta'}$, all elements except squared elements are zero and we are left with

$$\left(\frac{\partial \mathcal{L}}{\partial \theta_1}\right)^2 = \frac{((y_1 = 0)(y_2 = 0))^2}{P00^2}\left(\frac{\partial P00}{\partial \theta_1}\right)^2 + \frac{((y_1 = 1)(y_2 = 0))^2}{P10^2}\left(\frac{\partial P10}{\partial \theta_1}\right)^2 + \frac{((y_1 = 0)(y_2 = 1))^2}{P01^2}\left(\frac{\partial P01}{\partial \theta_1}\right)^2 + \frac{((y_1 = 1)(y_2 = 1))^2}{P11^2}\left(\frac{\partial P11}{\partial \theta_1}\right)^2$$

The expectation of $((y_1 = 0)(y_2 = 0))^2 = (y_1 = 0)(y_2 = 0)$ is $P00$ etc. leading to cancellation. So the first element of B, for instance, will be

$$\frac{\partial \mathcal{L}}{\partial \theta_1} = \frac{1}{P00}\left(\frac{\partial P00}{\partial \theta_1}\right)^2 + \frac{1}{P10}\left(\frac{\partial P10}{\partial \theta_1}\right)^2 + \frac{1}{P01}\left(\frac{\partial P01}{\partial \theta_1}\right)^2 + \frac{1}{P11}\left(\frac{\partial P11}{\partial \theta_1}\right)^2$$

Now let

$$A_i = \begin{bmatrix} \frac{\partial P00_i}{\partial \theta'} & \frac{\partial P10_i}{\partial \theta'} & \frac{\partial P01_i}{\partial \theta'} & \frac{\partial P11_i}{\partial \theta'} \end{bmatrix}$$

be a $K \times 4$ matrix of partial derivatives, and D_i be a 4×4 diagonal matrix of moments $\frac{1}{P00_i}$, $\frac{1}{P10_i}$, $\frac{1}{P01_i}$ and $\frac{1}{P11_i}$. Then observation i's contribution to the expected gradient can be written as

$$B_i = A_i \;\; D_i \;\; A_i'$$

Since the likelihood $\mathcal{L}$ is the sum of the likelihood of each observation i, B is also a sum over the product of the gradient of each individual i. Thus as in Bjorn and Vuong (1984b), B can also be written as a product of two matrices. Let N be the number of observations. Let A be a $K \times 4N$ block matrix where the i_{th} block is A_i, and D be the $4N \times 4N$ block diagonal matrix whose i_{th} block is D_i. Then

$$B = A \;\; D \;\; A'$$

Since D is diagonal with non-zero diagonal entries (by assumption) and therefore invertible, we must only show the A is full rank. This must be done for each of the four structures corresponding to $H_2 \gtrless H_1$ and $W_2 \gtrless W_1$.

Let us adopt the following notation, common to all cases, as in Bjorn and Vuong (1984b).

$$\begin{aligned}
e_i^1 &= \phi(H2)\Phi(-W2) & e_i^2 &= \phi(W2)\Phi(-H2) \\
f_i^1 &= \phi(H1)\Phi(W1) & f_i^2 &= \phi(W1)\Phi(H1) \\
g_i^1 &= \phi(H1)\Phi(W2) & g_i^2 &= \phi(W2)\Phi(-H1) \\
h_i^1 &= \phi(H2)\Phi(-W1) & h_i^2 &= \phi(W1)\Phi(H2) \\
a_i^1 &= \tfrac{-w_1}{.5inc+\lambda_1 w_1} & a_i^2 &= \tfrac{-w_1}{.5inc+(1-\lambda_1)w_1} \\
b_i^1 &= \tfrac{-w_2}{.5inc+(1-\lambda_2)w_2} & b_i^2 &= \tfrac{-w_2}{.5inc+\lambda_2 w_2}
\end{aligned}$$

$$s_i(x) = \left(\int_{-\infty}^{x} e^{\left(-\frac{t^2}{2\sigma^2}\right)} \left(-\frac{t^2}{\sqrt{2\pi}\sigma^4} - \frac{1}{\sqrt{2\pi}\sigma^2}\right) dt\right)$$

8.1 Case 1: $H_2 \geqslant H_1, W_2 \geqslant W_1$

$P00 = \Phi(H_1) * \Phi(W_1)$

$P10 = \Phi(-H_1) * \Phi(W_2) - .5 * (\Phi(H_2)\Phi(W_2) - \Phi(H_2)\Phi(W_1) - \Phi(H_1)\Phi(W_2) + \Phi(H_1)\Phi(W_1))$

$P01 = \Phi(H_2) * \Phi(-W_1) - .5 * (\Phi(H_2)\Phi(W_2) - \Phi(H_2)\Phi(W_1) - \Phi(H_1)\Phi(W_2) + \Phi(H_1)\Phi(W_1))$

$P11 = \Phi(-H_2) * \Phi(-W_2)$

$$\frac{\partial P00_i}{\partial \beta_1} = f_i^1 \frac{\partial H1_i}{\partial \beta_1} = -f_i^1 X_{1i} \qquad \frac{\partial P00_i}{\partial \beta_2} = f_i^2 \frac{\partial W1_i}{\partial \beta_2} = -f_i^2 X_{2i}$$

$$\frac{\partial P00_i}{\partial \delta_1} = 0 \qquad \frac{\partial P00_i}{\partial \delta_2} = 0$$

$$\frac{\partial P00_i}{\partial \lambda_1} = f_i^1 \frac{\partial H1_i}{\partial \lambda_1} = \frac{-f_i^1 w_1}{.5inc+\lambda_1 w_1} = f_i^1 a_i^1 \qquad \frac{\partial P00_i}{\partial \lambda_2} = f_i^2 \frac{\partial W1_i}{\partial \lambda_2} = \frac{-f_i^2 w_2}{.5inc+\lambda_2 w_2} = f_i^2 b_i^2$$

$$\frac{\partial P00_i}{\partial \sigma} = s_i(H1_i)\Phi(W1_i) + s_i(W1_i)\Phi(H1_i)$$

$$
\begin{aligned}
\frac{\partial P10_i}{\partial \beta_1} &= g_i^1 \frac{\partial -H1_i}{\partial \beta_1} - .5((\phi(H_2) - e_i^1)\frac{\partial H2_i}{\partial \beta_1} - (\phi(H2) - h_i^1)\frac{\partial H2_i}{\partial \beta_1} - g_i^1 \frac{\partial H1_i}{\partial \beta_1} + f_i^1 \frac{\partial H1_i}{\partial \beta_1}) \\
&= g_i^1 \frac{\partial -H1_i}{\partial \beta_1} - .5(-e_i^1 \frac{\partial H2_i}{\partial \beta_1} + h_i^1 \frac{\partial H2_i}{\partial \beta_1} - g_i^1 \frac{\partial H1_i}{\partial \beta_1} + f_i^1 \frac{\partial H1_i}{\partial \beta_1}) \\
&= g_i^1 X_{1i} - .5(e_i^1 X_{1i} - h_i^1 X_{1i} + g_i^1 X_{1i} - f_i^1 X_{1i}) \\
&= (g_i^1 - .5(e_i^1 - h_i^1 + g_i^1 - f_i^1))X_{1i} \\
\frac{\partial P10_i}{\partial \beta_2} &= g_i^2 \frac{\partial W2_i}{\partial \beta_2} - .5((\phi(W2_i) - e_i^2)\frac{\partial W2_i}{\partial \beta_2} - h_i^2 \frac{\partial W1_i}{\partial \beta_2} - (\phi(W2_i) - g_i^2)\frac{\partial W2_i}{\partial \beta_2} + f_i^2 \frac{\partial W1_i}{\partial \beta_2}) \\
&= g_i^2 \frac{\partial W2_i}{\partial \beta_2} - .5(-e_i^2 \frac{\partial W2_i}{\partial \beta_2} - h_i^2 \frac{\partial W1_i}{\partial \beta_2} + g_i^2 \frac{\partial W2_i}{\partial \beta_2} + f_i^2 \frac{\partial W1_i}{\partial \beta_2}) \\
&= (g_i^2 - .5(-e_i^2 - h_i^2 + g_i^2 + f_i^2))(-X_{2i}) \\
&= (-g_i^2 - .5(e_i^2 + h_i^2 - g_i^2 - f_i^2))X_{2i} \\
\frac{\partial P10_i}{\partial \delta_1} &= -.5((\phi(H2_i) - e_i^1)\frac{\partial H2_i}{\partial \delta_1} - (\phi(H2_i) - h_i^1)\frac{\partial H2_i}{\partial \delta_1}) \\
&= -.5(-e_i^1 \frac{\partial H2_i}{\partial \delta_1} + h_i^1 \frac{\partial H2_i}{\partial \delta_1}) \\
&= -.5(-e_i^1 + h_i^1)(-Z_{1i}) \\
&= -.5(e_i^1 - h_i^1)Z_{1i} \\
\frac{\partial P10_i}{\partial \delta_2} &= g_i^2 \frac{\partial W2_i}{\partial \delta_2} - .5((\phi(W2_i) - e_i^2)\frac{\partial W2_i}{\partial \delta_2} - (\phi(W2_i) - g_i^2)\frac{\partial W2_i}{\partial \delta_2}) \\
&= (g_i^2 - .5(-e_i^2 + g_i^2))(-Z_{2i}) \\
&= (-g_i^2 - .5(e_i^2 - g_i^2))Z_{2i} \\
&= (-.5(e_i^2 + g_i^2))Z_{2i} \\
\frac{\partial P10_i}{\partial \lambda_1} &= g_i^1 \frac{\partial -H1_i}{\partial \lambda_1} + g_i^2 \frac{\partial W2_i}{\partial \lambda_1} - .5((\phi(W2_i) - e_i^2)\frac{\partial W2_i}{\partial \lambda_1} - (g_i^1 \frac{\partial H1_i}{\partial \lambda_1} + (\phi(W2_i) - g_i^2)\frac{\partial W2_i}{\partial \lambda_1}) + f_i^1 \frac{\partial H1_i}{\partial \lambda_1}) \\
&= g_i^1 \frac{\partial -H1_i}{\partial \lambda_1} + g_i^2 \frac{\partial W2_i}{\partial \lambda_1} - .5(-e_i^2 \frac{\partial W2_i}{\partial \lambda_1} - g_i^1 \frac{\partial H1_i}{\partial \lambda_1} + g_i^2 \frac{\partial W2_i}{\partial \lambda_1} + f_i^1 \frac{\partial H1_i}{\partial \lambda_1}) \\
&= -g_i^1 a_i^1 + g_i^2 a_i^2 - .5(-e_i^2 a_i^2 - g_i^1 a_i^1 + g_i^2 a_i^2 + f_i^1 a_i^1) \\
&= -.5 g_i^1 a_i^1 + .5 g_i^2 a_i^2 + .5 e_i^2 a_i^2 - .5 f_i^1 a_i^1 \\
&= -.5 a_i^1 (g_i^1 + f_i^1) + .5 a_i^2 (g_i^2 + e_i^2) \\
\frac{\partial P10_i}{\partial \lambda_2} &= -.5((\phi(H2_i) - e_i^1)\frac{\partial H2_i}{\partial \lambda_2} - ((\phi(H2_i) - h_i^1)\frac{\partial H2_i}{\partial \lambda_2} + h_i^2 \frac{\partial W1_i}{\partial \lambda_2}) + f_i^2 \frac{\partial W1_i}{\partial \lambda_2}) \\
&= -.5(-e_i^1 \frac{\partial H2_i}{\partial \lambda_2} + h_i^1 \frac{\partial H2_i}{\partial \lambda_2} - h_i^2 \frac{\partial W1_i}{\partial \lambda_2} + f_i^2 \frac{\partial W1_i}{\partial \lambda_2}) \\
&= -.5(-e_i^1 b_i^1 + h_i^1 b_i^1 - h_i^2 b_i^2 + f_i^2 b_i^2) \\
&= .5 b_i^1 (e_i^1 - h_i^1) + .5 b_i^2 (h_i^2 - f_i^2) \\
\frac{\partial P10_i}{\partial \sigma} &= s_i(-H1_i)\Phi(W2_i) + s_i(W2_i)\Phi(-H1_i) - .5 * (s_i(H2_i)\Phi(W2_i) + s_i(W2_i)\Phi(H2_i) \\
&\quad - s_i(H2_i)\Phi(W1_i) - s_i(W1_i)\Phi(H2_i) - s_i(H1_i)\Phi(W2_i) - s_i(W2_i)\Phi(H1_i) \\
&\quad + s_i(H1_i)\Phi(W1_i) + s_i(W1_i)\Phi(H1_i))
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial P01_i}{\partial \beta_1} &= h_i^1 \frac{\partial H2_i}{\partial \beta_1} - .5((\phi(H2_i) - e_i^1) \frac{\partial H2_i}{\partial \beta_1} - (\phi(H2_i) - h_i^1) \frac{\partial H2_i}{\partial \beta_1} - g_i^1 \frac{\partial H1_i}{\partial \beta_1} + f_i^1 \frac{\partial H1_i}{\partial \beta_1}) \\
&= h_i^1 \frac{\partial H2_i}{\partial \beta_1} - .5(-e_i^1 \frac{\partial H2_i}{\partial \beta_1} + h_i^1 \frac{\partial H2_i}{\partial \beta_1} - g_i^1 \frac{\partial H1_i}{\partial \beta_1} + f_i^1 \frac{\partial H1_i}{\partial \beta_1}) \\
&= -h_i^1 X_{1i} - .5(e_i^1 X_{1i} - h_i^1 X_{1i} + g_i^1 X_{1i} - f_i^1 X_{1i}) \\
&= (-h_i^1 - .5(e_i^1 - h_i^1 + g_i^1 - f_i^1)) X_{1i} \\
\frac{\partial P01_i}{\partial \beta_2} &= h_i^2 \frac{\partial - W1_i}{\partial \beta_2} - .5((\phi(W2_i) - e_i^2) \frac{\partial W2_i}{\partial \beta_2} - h_i^2 \frac{\partial W1_i}{\partial \beta_2} - (\phi(W2_i) - g_i^2) \frac{\partial W2_i}{\partial \beta_2} + f_i^2 \frac{\partial W1_i}{\partial \beta_2}) \\
&= h_i^2 \frac{\partial - W1_i}{\partial \beta_2} - .5(-e_i^2 \frac{\partial W2_i}{\partial \beta_2} - h_i^2 \frac{\partial W1_i}{\partial \beta_2} + g_i^2 \frac{\partial W2_i}{\partial \beta_2} + f_i^2 \frac{\partial W1_i}{\partial \beta_2}) \\
&= h_i^2 X_{2i} - .5(e_i^2 X_{2i} + h_i^2 X_{2i} - g_i^2 X_{2i} - f_i^2 X_{2i}) \\
&= (h_i^2 - .5(e_i^2 + h_i^2 - g_i^2 - f_i^2)) X_{2i} \\
\frac{\partial P01_i}{\partial \delta_1} &= h_i^1 \frac{\partial H2_i}{\partial \delta_1} - .5((\phi(H2_i) - e_i^1) \frac{\partial H2_i}{\partial \delta_1} - (\phi(H2_i) - h_i^1) \frac{\partial H2_i}{\partial \delta_1}) \\
&= h_i^1 \frac{\partial H2_i}{\partial \delta_1} - .5(-e_i^1 \frac{\partial H2_i}{\partial \delta_1} + h_i^1 \frac{\partial H2_i}{\partial \delta_1}) \\
&= -h_i^1 Z_{1i} - .5(e_i^1 Z_{1i} - h_i^1 Z_{1i}) \\
&= (-h_i^1 - .5(e_i^1 - h_i^1)) Z_{1i} \\
\frac{\partial P01_i}{\partial \delta_2} &= -.5((\phi(W2_i) - e_i^2) \frac{\partial W2_i}{\partial \delta_2} - (\phi(W2_i) - g_i^2) \frac{\partial W2_i}{\partial \delta_2}) \\
&= -.5(-e_i^2 \frac{\partial W2_i}{\partial \delta_2} + g_i^2 \frac{\partial W2_i}{\partial \delta_2}) \\
&= -.5(e_i^2 Z_{2i} - g_i^2 Z_{2i}) \\
&= -.5(e_i^2 - g_i^2) Z_{2i} \\
\frac{\partial P01_i}{\partial \lambda_1} &= -.5((\phi(W2_i) - e_i^2) \frac{\partial W2_i}{\partial \lambda_1} - g_i^1 \frac{\partial H1_i}{\partial \lambda_1} - (\phi(W2_i) - g_i^2) \frac{\partial W2_i}{\partial \lambda_1} + f_i^1 \frac{\partial H1_i}{\partial \lambda_1}) \\
&= -.5(-e_i^2 \frac{\partial W2_i}{\partial \lambda_1} - g_i^1 \frac{\partial H1_i}{\partial \lambda_1} + g_i^2 \frac{\partial W2_i}{\partial \lambda_1} + f_i^1 \frac{\partial H1_i}{\partial \lambda_1}) \\
&= -.5(-e_i^2 a_i^2 - g_i^1 a_i^1 + g_i^2 a_i^2 + f_i^1 a_i^1) \\
&= -.5 a_i^1 (f_i^1 - g_i^1) - .5 a_i^2 (-e_i^2 + g_i^2) \\
\frac{\partial P01_i}{\partial \lambda_2} &= h_i^1 \frac{\partial H2_i}{\partial \lambda_2} - h_i^2 \frac{\partial W1_i}{\partial \lambda_2} - .5((\phi(H2_i) - e_i^1) \frac{\partial H2_i}{\partial \lambda_2} - (\phi(H2_i) - h_i^1) \frac{\partial H2_i}{\partial \lambda_2} - h_i^2 \frac{\partial W1_i}{\partial \lambda_2} + f_i^2 \frac{\partial W1_i}{\partial \lambda_2}) \\
&= h_i^1 \frac{\partial H2_i}{\partial \lambda_2} - h_i^2 \frac{\partial W1_i}{\partial \lambda_2} - .5(-e_i^1 \frac{\partial H2_i}{\partial \lambda_2} + h_i^1 \frac{\partial H2_i}{\partial \lambda_2} - h_i^2 \frac{\partial W1_i}{\partial \lambda_2} + f_i^2 \frac{\partial W1_i}{\partial \lambda_2}) \\
&= h_i^1 b_i^1 - h_i^2 b_i^2 - .5(-e_i^1 b_i^1 + h_i^1 b_i^1 - h_i^2 b_i^2 + f_i^2 b_i^2) \\
&= h_i^1 b_i^1 - h_i^2 b_i^2 + .5 e_i^1 b_i^1 - .5 h_i^1 b_i^1 + .5 h_i^2 b_i^2 - .5 f_i^2 b_i^2 \\
&= (h_i^1 + .5 e_i^1 - .5 h_i^1) b_i^1 + (.5 h_i^2 - .5 f_i^2 - h_i^2) b_i^2 \\
&= .5 b_i^1 (e_i^1 + h_i^1) - .5 b_i^2 (h_i^2 + f_i^2) \\
\frac{\partial P01_i}{\partial \sigma} &= s_i(H2_i)\Phi(W1_i) + s_i(W1_i)\Phi(H2_i) - .5 * (s_i(H2_i)\Phi(W2_i) + s_i(W2_i)\Phi(H2_i) \\
&\quad - s_i(H2_i)\Phi(W1_i) - s_i(W1_i)\Phi(H2_i) - s_i(H1_i)\Phi(W2_i) - s_i(W2_i)\Phi(H1_i) \\
&\quad + s_i(H1_i)\Phi(W1_i) + s_i(W1_i)\Phi(H1_i))
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial P11_i}{\partial \beta_1} &= e_i^1 \frac{\partial H2_i}{\partial \beta_1} = e_i^1 X_{1i} & \frac{\partial P11_i}{\partial \beta_2} &= e_i^2 \frac{\partial W2_i}{\partial \beta_2} = e_i^2 X_{2i} \\
\frac{\partial P11_i}{\partial \delta_1} &= e_i^1 \frac{\partial H2_i}{\partial \delta_1} = e_i^1 Z_{1i} & \frac{\partial P11_i}{\partial \delta_2} &= e_i^2 \frac{\partial W2_i}{\partial \delta_2} = e_i^2 Z_{2i} \\
\frac{\partial P11_i}{\partial \lambda_1} &= e_i^2 \frac{\partial -W2_i}{\partial \lambda_1} = -e_i^2 a_i^2 & \frac{\partial P11_i}{\partial \lambda_2} &= e_i^1 \frac{\partial -H2_i}{\partial \lambda_2} = -e_i^1 b_i^1 \\
\frac{\partial P11_i}{\partial \sigma} &= s_i(-H2_i)\Phi(-W2_i) + s_i(-W2_i)\Phi(-H2_i)
\end{aligned}
$$

Therefore the i_{th} element of matrix A may be written as follows:

$$
A = \left| \begin{array}{cccc}
-f_i^1 X_{1i} & (g_i^1 - .5(e_i^1 - h_i^1 + g_i^1 - f_i^1))X_{1i} & (-h_i^1 - .5(e_i^1 - h_i^1 + g_i^1 - f_i^1))X_{1i} & e_i^1 X_{1i} \\
-f_i^2 X_{2i} & (-g_i^2 - .5(e_i^2 + h_i^2 - g_i^2 - f_i^2))X_{2i} & (h_i^2 - .5(e_i^2 + h_i^2 - g_i^2 - f_i^2))X_{2i} & e_i^2 X_{2i} \\
0 & -.5(e_i^1 - h_i^1)Z_{1i} & (-h_i^1 - .5(e_i^1 - h_i^1))Z_{1i} & e_i^1 Z_{1i} \\
0 & -.5(e_i^2 + g_i^2))Z_{2i} & -.5(e_i^2 - g_i^2)Z_{2i} & e_i^2 Z_{2i} \\
f_i^1 a_i^1 & -.5a_i^1(g_i^1 + f_i^1) + .5a_i^2(g_i^2 + e_i^2) & -.5a_i^1(f_i^1 - g_i^1) - .5a_i^2(-e_i^2 + g_i^2) & -e_i^2 a_i^2 \\
f_i^2 b_i^2 & .5b_i^1(e_i^1 - h_i^1) + .5b_i^2(h_i^2 - f_i^2) & .5b_i^1(e_i^1 + h_i^1) - .5b_i^2(h_i^2 + f_i^2) & -e_i^1 b_i^1 \\
\frac{\partial P00_i}{\partial \sigma} & \frac{\partial P10_i}{\partial \sigma} & \frac{\partial P01_i}{\partial \sigma} & \frac{\partial P11_i}{\partial \sigma}
\end{array} \right| .
$$

A_i is of dimension $K \times 4$, where K is the number of parameters and 4 is the number of probabilities. Since A is stacked horizontally with a matrix A_i for each observation in the sample, A is of dimension $K \times 4N$, where N is sample size.

Let us take first only the first six rows of A, ignoring the seventh row, which is derivatives with respect to sigma. Call this $K - 1 \times 4N$ matrix A_1. We want to show that A_1 has rank $K - 1$. To do so we first perform column operations to simplify A_{1i}.

1. Subtract column 2 from column 3.

2. Add half of columns 1, 3 and 4 to column 2.

$$A_{1i} = \left| \begin{array}{rcrr} -f_i^1 X_{1i} & 0 & (-g_i^1 - h_i^1) X_{1i} & e_i^1 X_{1i} \\ -f_i^2 X_{2i}^2 & 0 & (g_i^2 + h_i^2) X_{2i} & e_i^2 X_{2i} \\ 0 & 0 & -h_i^1 Z_{1i} & e_i^1 Z_{1i} \\ 0 & 0 & g_i^2 Z_{2i} & e_i^2 Z_{2i} \\ f_i^1 a_i^1 & 0 & g_i^1 a_i^1 - g_i^2 a_i^2 & -e_i^2 a_i^2 \\ f_i^2 b_i^2 & 0 & h_i^1 b_i^1 - h_i^2 b_i^2 & -e_i^1 b_i^1 \end{array} \right| .$$

Note that the second column is now equal to zeros. This is to be expected since $P00 + P01 + P10 + P11 = 1$, not all four gradients can carry new information. Column two is therefore omitted. To be consistent with Bjorn and Vuong (1984b) let us now look at the transpose of A_{1i}, A'_{1i} which is $3 \times K - 1$. Note A'_1 is $3N \times K - 1$.

$$A'_{1i} = \left| \begin{array}{rrrrrr} -f_i^1 X'_{1i} & -f_i^2 X'_{2i} & 0 & 0 & f_i^1 a_i^1 & f_i^2 b_i^2 \\ (-g_i^1 - h_i^1) X'_{1i} & (g_i^2 + h_i^2) X'_{2i} & -h_i^1 Z'_{1i} & g_i^2 Z'_{2i} & g_i^1 a_i^1 - g_i^2 a_i^2 & h_i^1 b_i^1 - h_i^2 b_i^2 \\ e_i^1 X'_{1i} & e_i^2 X'_{2i} & e_i^1 Z'_{1i} & e_i^2 Z'_{2i} & -e_i^2 a_i^2 & -e_i^1 b_i^1 \end{array} \right| .$$

To show that A'_1 is full rank, we can write A'_1 as the product of four matrices, as in Bjorn and Vuong (1984b). First let us switch second and third columns of A'_1 and move the last column to be the third column.

$$A'_{1i} = \left| \begin{array}{rrrrrr} -f_i^1 X'_{1i} & 0 & f_i^2 b_i^2 & -f_i^2 X'_{2i} & 0 & f_i^1 a_i^1 \\ (-g_i^1 - h_i^1) X'_{1i} & -h_i^1 Z'_{1i} & h_i^1 b_i^1 - h_i^2 b_i^2 & (g_i^2 + h_i^2) X'_{2i} & g_i^2 Z'_{2i} & g_i^1 a_i^1 - g_i^2 a_i^2 \\ e_i^1 X'_{1i} & e_i^1 Z'_{1i} & -e_i^1 b_i^1 & e_i^2 X'_{2i} & e_i^2 Z'_{2i} & -e_i^2 a_i^2 \end{array} \right| .$$

Define $\bar{W}_1$ and $\bar{W}_2$ as stacked matrices of length $3N$ whose i_{th} elements are

$$\bar{W_{1i}} = \begin{vmatrix} 0 & 0 & b_i^2 \\ X'_{1i} & 0 & 0 \\ X'_{1i} & Z'_{1i} & -b_i^1 \end{vmatrix}$$

$$\bar{W_{2i}} = \begin{vmatrix} 0 & 0 & a_i^1 \\ X'_{2i} & 0 & 0 \\ X'_{2i} & Z'_{2i} & -a_i^2 \end{vmatrix}$$

Let D_h and D_w $(3N \times 3N)$ be block diagonal matrices with i_{th} elements D_{hi} and D_{wi}

$$D_{hi} = \begin{vmatrix} f_i^2 & f_i^1 & 0 \\ -h_i^2 & -g_i^1 & -h_i^1 \\ 0 & 0 & e_i^1 \end{vmatrix}$$

$$D_{wi} = \begin{vmatrix} f_i^1 & -f_i^2 & 0 \\ g_i^1 & h_i^2 & g_i^2 \\ 0 & 0 & e_i^2 \end{vmatrix}$$

We now can write A' as two matrix products appended:

$$A'_1 = D_h\bar{W}_1 | D_w\bar{W}_2$$

Lemma 1 D_{hi} can be reduced to an upper triangular matrix with non-zero elements in its diagonal as long as $g_i^1 \neq f_i^1 * \frac{h_i^2}{f_i^2}$, $f_i^2 \neq 0$ and $e_i^2 \neq 0$.

Proof. Multiply the first row of D_{hi} by $\frac{h_i^2}{f_i^2}$ and add to the second row. The matrix is now

$$D_{hi} = \begin{vmatrix} f_i^2 & f_i^1 & 0 \\ -h_i^2 + h_i^2 & -g_i^1 + f_i^1 * \frac{h_i^2}{f_i^2} & -h_i^1 \\ 0 & 0 & e_i^1 \end{vmatrix}$$

Lemma 2 D_{wi} can be made into an upper triangular matrix with non-zero diagonal entries as long as $h_i^2 \neq -f_i^2 \frac{g_i^1}{f_i^1}$, $f_i^1 \neq 0$ and $e_i^1 \neq 0$.

Proof. Multiplying the first row of D_{wi} by $\frac{g_i^1}{f_i^1}$ and subtracting it from the second row yields

$$D_{wi} = \begin{vmatrix} f_i^1 & -f_i^2 & 0 \\ g_i^1 - g_i^1 & h_i^2 + f_i^2 \frac{g_i^1}{f_i^1} & g_i^2 \\ 0 & 0 & e_i^2 \end{vmatrix}$$

Thus both D_{hi} and D_{wi} are reducible to triangular matrices with non-zero diagonal elements under certain minimal conditions. Therefore D_{hi} and D_{wi} are generally invertible. By extension, D_h and D_w, the block diagonal matrices composed of D_{hi} and D_{wi}, are generally invertible.

According to Bjorn and Vuong (1984b) the rank of A_1' can equivalently be seen by observing the matrix

$$\tilde{A_1}' = \bar{W}_1 | D_h^{-1} D_w \bar{W}_2$$

By assumption the data matrices W_1 and W_2 are full column rank. $D_h^{-1} D_w$ the product of two invertible matrices and therefore invertible, with rank $3N$. Thus $D_h^{-1} D_w \bar{W}_2$ the rank of $\bar{W}_2$. As in Bjorn and Vuong (1984b), it would take a particular combination of parameters and data for $\bar{W}_1$ and

$D_h^{-1} D_w \bar{W}_2$ to be collinear. Therefore A_1' is generally the rank of $\bar{W}_1$ plus the rank of $\bar{W}_2$ which is $K-1$. Thus A_1 is generally full column rank.

9 Wage Imputation

Labor income is imputed for those who have wage and hours data but no labor income data as wage $\times$ hours. This affects 530 husbands and 807 wives. This leaves missing labor income for approximately 12% of husbands and 32% of wives. Of husbands missing labor income data, 50% work, whereas among wives missing labor income, only 10% work. We are missing labor income data for 98% of wives who don't work and 95% of husbands who don't work. Since the employment decision is the focus of the model, and it is expected that labor income is a critical component of this decision, it is clear that missing labor income data must be filled in somehow.

Exploiting the panel structure, labor income can be interpolated and extrapolated for individuals from observations in different years. If labor income is observed in even in one year for an individual, this value is extrapolated to all other years the individual is observed. After doing this, labor income is missing for just 3% of husbands and 7% of wives. We are now missing labor income for 20% of both wives and husbands who don't work.

Now a Heckman selection regression can be used to impute labor income for the remaining observations. The selection and income regressions are as follows, done separately for husband and wife:

$$\text{Work} = X\beta + Z\delta + u_1$$

$$\text{Labor income} = X\beta' + \lambda\beta'_{lambda} + u_2$$

Where X is age, age squared, education, experience, experience squared, black and hispanic, Z is instruments, number of children and non-labor income, and λ is the selection correction term estimated in the first regression. Instruments in the first stage are children, non-labor income, spouse's age and age squared, spouse's experience and experience squared. An F-test of these instruments rejects that they are jointly equal to zero in the first stage probit except for the case of wives in the second period, where the statistic is 8.53 with p-value of 0.2015. In all cases a Wald test rejects that the selection and income equations are independent.

Heckman Regression for Husband: Period 1

	Husband Labor Income	Husband Employment Status	athrho	l
Husband Age	4,440.264	0.208		
	(1,463.847)**	(0.056)**		
Husband Age Squared	-65.804	-0.003		
	(20.417)**	(0.001)**		
Husband Experience	1,112.621	-0.011		
	(603.549)	(0.031)		
Husband Experience Squared	-0.365	0.001		
	(15.407)	(0.001)		
Husband Education	3,449.632			
	(426.520)**			
Husband Black	-8,714.999	-0.455		
	(2,625.572)**	(0.126)**		
Husband Hispanic	-8,900.824	-0.003		
	(3,157.720)**	(0.270)		
Number of Children		-0.058		
		(0.027)*		
Non-Labor Income		-0.000		
		(0.000)		
Wife Age		0.021		
		(0.047)		
Wife Age Squared		-0.000		
		(0.001)		
Wife Experience		0.000		
		(0.016)		
Wife Experience Squared		-0.000		
		(0.000)		
Constant	-87,745.157	-2.132	2.231	
	(22,817.285)**	(0.695)**	(0.432)**	(
N	10,135			

* $p < 0.05$; ** $p < 0.01$

Heckman Regression for Wife: Period 1

	Wife Labor Income	Wife Employment Status	athrho	lnsigma
Wife Age	-36.823	0.060		
	(136.019)	(0.019)**		
Wife Age Squared	-0.560	-0.001		
	(1.769)	(0.000)**		
Wife Experience	926.525	0.140		
	(84.404)**	(0.006)**		
Wife Experience Squared	-14.607	-0.002		
	(2.416)**	(0.000)**		
Wife Education	1,760.256			
	(116.097)**			
Wife Black	-238.840	0.066		
	(731.755)	(0.062)		
Wife Hispanic	-497.639	0.147		
	(968.811)	(0.083)		
Number of Children		-0.146		
		(0.013)**		
Non-Labor Income		0.000		
		(0.000)		
Husband Age		0.030		
		(0.023)		
Husband Age Squared		-0.000		
		(0.000)		
Husband Experience		-0.038		
		(0.009)**		
Husband Experience Squared		0.001		
		(0.000)**		
Constant	-8,417.921	-0.963	-0.321	9.541
	(2,957.312)**	(0.283)**	(0.041)**	(0.019)**
N	59,778			

* $p < 0.05$; ** $p < 0.01$

Heckman Regression for Husband: Period 2

	Husband Labor Income	Husband Employment Status	athrho	l
Husband Age	10,045.745	0.128		
	(965.149)**	(0.015)**		
Husband Age Squared	-138.854	-0.002		
	(12.538)**	(0.000)**		
Husband Experience	-293.646	0.001		
	(456.703)	(0.007)		
Husband Experience Squared	55.158	0.001		
	(9.418)**	(0.000)**		
Husband Education	4,582.338			
	(372.049)**			
Husband Black	-20,437.319	-0.292		
	(2,053.710)**	(0.040)**		
Husband Hispanic	-8,747.847	-0.170		
	(2,874.690)**	(0.045)**		
Number of Children		0.001		
		(0.004)		
Non-Labor Income		0.000		
		(0.000)		
Wife Age		-0.012		
		(0.009)		
Wife Age Squared		0.000		
		(0.000)		
Wife Experience		0.005		
		(0.004)		
Wife Experience Squared		-0.000		
		(0.000)		
Constant	-200,075.095	-1.353	4.002	
	(17,056.407)**	(0.217)**	(0.512)**	(
N	45,467			

* $p < 0.05$; ** $p < 0.01$

Heckman Regression for Wife: Period 2

	Wife Labor Income	Wife Employment Status	athrho	lnsigma
Wife Age	-636.590	-0.024		
	(225.064)**	(0.009)**		
Wife Age Squared	-6.907	-0.000		
	(2.820)*	(0.000)*		
Wife Experience	3,405.181	0.128		
	(184.717)**	(0.007)**		
Wife Experience Squared	-35.921	-0.001		
	(4.641)**	(0.000)**		
Wife Education	154.930			
	(47.082)**			
Wife Black	-2,975.142	-0.122		
	(1,096.934)**	(0.040)**		
Wife Hispanic	-6,380.750	-0.249		
	(1,223.011)**	(0.045)**		
Number of Children		-0.001		
		(0.001)		
Non-Labor Income		-0.000		
		(0.000)		
Husband Age		-0.001		
		(0.002)		
Husband Age Squared		-0.000		
		(0.000)		
Husband Experience		0.001		
		(0.001)		
Husband Experience Squared		-0.000		
		(0.000)		
Constant	13,345.969	0.636	5.084	10.194
	(4,023.407)**	(0.160)**	(0.292)**	(0.053)**
N	45,435			

* $p < 0.05$; ** $p < 0.01$

Income cannot be imputed using this Heckman specification for 5% of husbands and wives due to missing values of work experience. Now we are missing labor income data on 5% of wives who do not work and 7% of husbands who do not work. If the predicted income is less than zero, it is censored at zero. This affects 2093 of husbands and 87 wives.

If Heckman predicted income is imputed only for non-workers who were missing wages even after interpolation and extrapolation, then income is missing for only 2% of wives and husbands. Standard errors of imputed income are 36 and 15 for husbands and wives, vs. 160 and 366 for non-imputed income.

The maximum values for imputed labor income and actual labor income differ dramatically as well. The maximum values for imputed wife and husband labor income are $48,260 and $89,082 respectively, whereas actual maximum values are $2,884,600 and $4,168,000. Labor-income was top-coded at $99,999 until 1983, $999,999 until 1993, and $9,999,999 thereafter. To ameliorate the impact of the top-coding and outliers, and to reduce scaling problems in the likelihood function[18], I censor labor income and non-labor income at $500,000 in the basic model.

1.5

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[18] The normal cumulative density function in Matlab is identically equal to one for values above nine. For this reason censoring income at $500,000 and dividing by $100,000 is key to estimation in the basic model, though not in the extended model due to the use of logs.

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Table 18: Extended Model by Spouse's Income in Census data

	Parameters and SEs				
Parameter	1970	1980	1990	2000	2011
Δ_1 (interaction)	-0.4953	-0.5817	-0.3544	-0.8019	-1.1327
	0.1710	0.0680	0.0684	0.0683	0.0717
heckincwage2	0.6587	1.3589	1.1052	1.7893	2.0262
	0.3330	0.1623	0.1563	0.1484	0.1471
Δ_2 (interaction)	0.8490	0.7986	0.7342	1.0372	1.3182
	0.1506	0.0498	0.0530	0.0481	0.0451
heckincwage1	-0.6583	-0.4955	-0.4780	-0.6181	-0.8090
	0.0581	0.0365	0.0414	0.0399	0.0397
$\beta_{0,1}$ (constant)	7.3189	7.1905	6.7514	6.5506	9.6025
	0.5557	0.2630	0.2318	0.2014	0.2387
$\beta_{0,2}$ (constant)	0.3766	1.2029	1.7848	1.0769	0.5760
	0.2557	0.1221	0.1197	0.1171	0.1134
σ_1 (variance of error)	1.0000	1.0000	1.0000	1.0000	1.0000
	0.0000	0.0000	0.0000	0.0000	0.0000
σ_2 (variance of error)	1.0000	1.0000	1.0000	1.0000	1.0000
	0.0000	0.0000	0.0000	0.0000	0.0000
age1	-10.9542	-12.8128	-12.5812	-11.8793	-17.2337
	0.9682	0.4625	0.4023	0.3461	0.4098
yrsch1	-35.5224	-40.0917	-36.8828	-30.8391	-58.5885
	4.8146	2.2941	1.9881	1.7062	2.0336
racebv1	-0.0372	-0.1360	-0.0250	-0.1093	-0.0128
	0.0462	0.0206	0.0199	0.0168	0.0161
nonlabor1	-0.5995	-0.9259	-0.6328	-0.4381	-0.5131
	0.0551	0.0291	0.0254	0.0166	0.0158
nchild1	0.2946	0.2478	0.1447	0.3227	0.3427
	0.1073	0.0560	0.0596	0.0562	0.0565
unemp1	-7.6339	6.6406	4.5485	-0.9874	-0.5182
	2.0842	0.7362	1.0268	1.4086	0.5853
heckincwage1	9.5287	10.6833	10.4121	9.1383	15.3617
	1.0891	0.5186	0.4545	0.3965	0.4710
age2	-2.1750	-3.6949	-4.8194	-3.0865	-2.7406
	0.2494	0.1419	0.1361	0.1285	0.1243
yrsch2	-8.9702	-11.9263	-18.1719	-11.6294	-13.5659
	1.9633	1.1311	1.0555	0.9828	1.0110
racebv2	0.1256	0.0200	-0.1891	-0.2537	-0.3037
	0.0285	0.0170	0.0175	0.0162	0.0163
nonlabor2	0.3114	0.4637	0.4081	-0.0947	-0.3810
	0.1113	0.0728	0.0639	0.0328	0.0278
nchild2	-1.4328	-1.4518	-1.4290	-1.2455	-1.3566
	0.0480	0.0334	0.0373	0.0361	0.0366
unemp2	0.3508	-0.6092	3.3085	-6.5195	-0.1588
	1.0374	0.4465	0.6838	1.0521	0.4092
heckincwage2	5.5836	7.1966	10.4220	7.9649	8.7347
	0.7063	0.4083	0.3896	0.3678	0.3769

Note: Standard Errors Below