[Draft] Sleepy Networks

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Abstract

We propose an extension of spatially extended spiking network models by allowing individual point-locations to emit chemicals and affect the properties of the network. We provide a network model example fulfilling rule 110 up to one error and sketch how this system attempts to correct this error without using backpropagation.

1 Chemical Spiking Neural Network (CSNN)

Definition 1 (Spatially extended Spiking Neural Network (SeSNN)) *Let $P \subset \mathbb{R}^d$. A Spatially extended Spiking Neural Network (SeSNN) $\mathcal{NN}$ within P is a Spiking Neural Network (SNN) with the following additional properties.*

1. *N Neurons ... (use some Spiking Neuron Model, for example:* Izkevitch Model (2003)*). The set of all neurons is called $\mathcal{N} = \mathcal{N}(\mathcal{NN}) = \{n_j : n_j$ is a neuron and $j \in \{0, ..., N-1\}\}$.*
2. *All N neurons have, as an attribute, an $\mathbb{R}^d$-valued position within P.*
3. *Synapses have distance dependent existence probabilities and delays.*

1.1 Location Groups and chemical Concentrations

Let $P = [0,1] \times [0,1] \times [0,1] \subset \mathbb{R}^3$ and $\mathcal{NN}$ be a SeSNN within P.

Partition P into $G \in \mathbb{N}_{>0}$ into equally sized cells. Each cell and the contained neurons define a **location group** g_i, $i \in \{0, ..., G-1\}$.

Define $\mathcal{LG} = \mathcal{LG}(\mathcal{NN}) = \{g_j : g_j$ is a location group and $j \in \{0, ..., G-1\}\}$ as the **set of all location groups**.

We say a neuron $n \in \mathcal{N} = \mathcal{N}(\mathcal{NN})$ belongs to $g_i \in \mathcal{LG}$ if n_j is positioned within the cell corresponding to g_i and denote it by $n \in g_i$. $\#g$ corresponds to the number of neurons in $g \in \mathcal{LG}$

The volume occupied by $g \in \mathcal{LG}$ is denoted by $Vol(g)$.

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Definition 2 (Self-Sustained Activity) *Let $R, T_I, T_b \in \mathbb{R}_{>0}$, and $N \in \mathbb{N}_{>0}$. We say a location group $g \in \mathcal{LG}$ is in a state of ((R, T_I, T_b-)**self-sustained activity** at time t if t belongs to a time interval of length T_I and there is a time interval of length T_b in which at least $k \in \mathbb{N}$ neurons $\frac{k}{\#g} \geq R$ would have been spiking without requiring external activation.*

Definition 3 (Cluster, R-Clusters) *A **cluster** is set of two or more location groups. Let C be a cluster. If the self-sustained activity of one or more location group belonging to $\mathcal{C}$ causes the self-sustained activity of all other groups in $\mathcal{C}$, we say $\mathcal{C}$ is a **R-Cluster**. An R-Cluster of which at least one location with at least one location group is in a state of self-sustained activity is said to be **activated**.*

R-Cluster activation notation: Let C be a cluster such that more than one cluster causes the self-sustained activity of the other groups. If $\{c_0, c_1, ..., c_i\}$ are clusters of C causing self-sustained activity in $\{c_{i+1}, ..., \}$ if their respective self-sustained activity time-intervals overlap (w.r.t. T_b), we denote this statement by:

$$C : \{c_0, c_1, ..., c_i\} \Rightarrow \{c_{i+1}, ..., \}.$$

Feedback effect may be loosely indicated using $\leftrightarrows$ w.r.t. their intensity or $\Leftrightarrow$ if

$$C : \{c_{i+1}, ..., \} \Rightarrow \{c_0, c_1, ..., c_i\}$$

if also true. If the inactivity (self-sustained activity) of one more location groups is necessary for (excludes) the self-sustained activity of one more other groups (not necessarily belonging to C) we denote this statement by:

$$C : \{\overline{c_0}, c_1, ..., c_i\} \Rightarrow \{c_{i+1}, ..., \}.$$

$$C : \{c_0, c_1, ..., c_i\} \Rightarrow \{c_{i+1}, ..., \overline{x}\}.$$

if c_0 or x are respectively the specifically non-activated group(s).

1.2 Fluid-Network (Bloodstream)

This Network is intended to approximate the circulation of blood within P. A Fluid network B is built similarly to a neural network (and graph) with delays, but with location groups (as nodes) instead of Neurons and vessels instead of synapses (as edges). Instead of transferring voltage spikes from neuron to neuron, the blood network transfers chemical concentration spikes.

B has an entry surface B_{in} and exit surface B_{out} at the border ∂P of P. Weights are inter-dependent. The sum of weights between two layers is constant, i.e a vessel contraction leads to the expansion of all other vessels in the respective layer. The corresponding expansion/contraction is applied to all previous and following layers. Through B_{in} there is a constant (*TODO: this may change*) inflow of liquid B_l. Through B_{out} there is an equivalent outflow of liquid B_l.

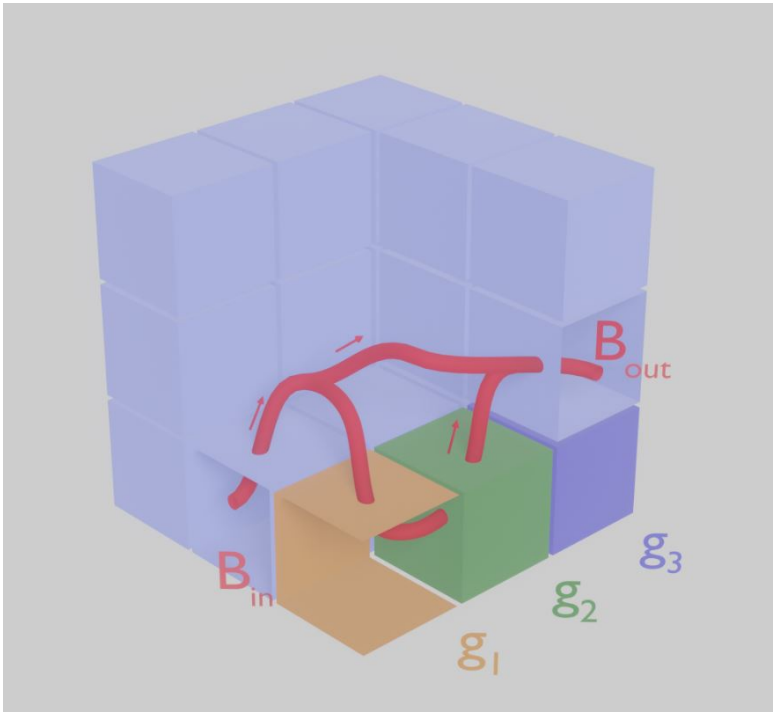

Figure 1: Fluid-Network Example [TODO: Resize diameters]

Definition 4 (Attributes) *Let X, Y and Z be sets such that $X \subset Y$. An (Y, Z)-valued-attribute (or Z-valued-attribute z) of X is an Z-valued map from Y to Z.*

We assume that P can be separated in three non-overlapping subsets: Neurons (Points), denoted by P_N, Synapse(edges without endpoints), denoted by P_S, fluid-network vessels (if present), denoted by P_F and P minus the previous sets, denoted by P_R.

Definition 5 (set of chemicals, chemical concentration, chemicals,) *Let C be a non-empty set of $\mathbb{R}_{\geq 0}$-valued attributes of P. C may be called a **set of chemicals** of P if for each attribute $c \in C$ and for each of the following (non-overlapping) subsets of P, P_N, P_S, P_F, and P_R, there exists a time(-step) dependent 'diffusion' function $f_c(p, c(p, t), t, ...)$*

$$f : (\mathbb{R}^3, \mathbb{R}_{\geq 0}, \mathbb{R}_{\geq 0}, ...) \to \mathbb{R}_{\geq 0},$$

such that

$$c(p, t_{i+1}, ...) = f_c(p, c(p, t_i), t_i, ...)$$

and f_c fulfills one of the following properties

1. [*some mathematical property: diffusion w.r.t a subset of P (including flow for blood vessels)*],

2. *... .*

If C is a set of chemicals in P, the elements of C are called ***chemicals****. A value $c(p,t)$ is called* ***chemical concentration*** *(of the chemical c in the point p at time t). The diffusion functions are denoted by $d_{c,N}$, $d_{c,S}$, $d_{c,F}$ and $d_{c,R}$ (w.r.t. P_N, P_S, P_F, and P_R).*

Definition 6 (gas-emission, production (points, neurons)) *If P has at least one chemical attribute c and $p \in P$ can increase the concentration of c in it's neighborhood we say that p is a* ***gas-emitting point*** *and that p* ***produces*** *c. If $p \in P_N$ we call the corresponding neuron a* ***gas-emitting neuron****.*

Definition 7 (gland, production (glands)) *An R-Cluster whose activation directly causes the increase of chemical concentrations at particular points in the blood stream is called a* ***gland****. If a gland increases the chemical concentrations of (say) chemicals C_1, C_2 and C_3 at a rate x, we speak of a $C_1C_2C_3$-**gland*** *and say that this glands* ***produces*** *C_1, C_2 and C_3 at rate x.*

Definition 8 (chemically sensitive neurons) *Let x be a Neuron with a set of attributes A_x positioned in $p \in P$, and C is a set of chemicals in P. The subset of attributes of x that are chemical concentrations at point p is denoted by $C_x \subset C$. If a change in concentration of an element of C_x causes a change in another element of A_x, the neuron is called chemically sensitive.*

1.3 Definition

A Chemical Spiking Neural Network (CSNN) within $P \subset \mathbb{R}^3$ is a SeSNN with the following properties.

1. One or more Neurons of the SeSNN are chemically sensitive.

2. One or more points in P are gas-emitting.

2 Example

2.1 Rule 100

Consider the following tuples:

$$p_0 = (0,0,0), \quad p_1 = (0,0,1), \quad p_2 = (0,1,0), \quad p_3 = (0,1,1),$$
$$p_4 = (1,0,0), \quad p_5 = (1,0,1), \quad p_6 = (0,1,1), \quad p_7 = (1,1,1).$$

Set $P = \{p_0, p_1, ..., p_7\}$ and define the map $r_{110} : P \to \{0,1\}$ by:

$$r_{110} : p_0, p_4, p_7 \mapsto 0,$$
$$r_{110} : p_1, p_2, p_3, p_5, p_6 \mapsto 1.$$

Figure 2: Rule 110 (highlighted: $p_1 = (0,0,1)$). The first row corresponds to the inputs, the second one to the outputs (i.e $r_{110}(p_1) = 1$). [*Image source: Wikipedia*]

Definition 9 (Rule 110) *Let $\mathcal{NN}$ be a CSNN containing a sufficient number of R-Clusters such that $\mathcal{I} = \{i_0, i_1, i_2, i_3\}$, $\mathcal{O} = \{o_0, o_1\}$ and $\mathcal{R} = \{r \in \mathcal{LG}(\mathcal{NN}), r \notin \mathcal{I} \cup \mathcal{C} \cup \mathcal{O}\}$ are clusters of distinct location groups $\subset \mathcal{LG} = \mathcal{LG}(\mathcal{NN})$.*

We say that $\mathcal{NN}$ ***fulfills the rule 110*** *if the following statements are true:*

$$A_0 = \mathcal{I} \cup \mathcal{O} : \{\overline{i_0}, \overline{i_1}, \overline{i_2}, i_3\} \Rightarrow \{o_0, \overline{o_1}\},$$
$$A_1 = \mathcal{I} \cup \mathcal{O} : \{\overline{i_0}, \overline{i_1}, i_2, i_3\} \Rightarrow \{\overline{o_0}, o_1\},$$
$$A_2 = \mathcal{I} \cup \mathcal{O} : \{\overline{i_0}, i_1, \overline{i_2}, i_3\} \Rightarrow \{\overline{o_0}, o_1\},$$
$$A_3 = \mathcal{I} \cup \mathcal{O} : \{\overline{i_0}, i_1, i_2, i_3\} \Rightarrow \{\overline{o_0}, o_1\},$$
$$A_4 = \mathcal{I} \cup \mathcal{O} : \{i_0, \overline{i_1}, \overline{i_2}, i_3\} \Rightarrow \{o_0, \overline{o_1}\},$$
$$A_5 = \mathcal{I} \cup \mathcal{O} : \{\overline{i_0}, i_1, \overline{i_2}, i_3\} \Rightarrow \{\overline{o_0}, o_1\},$$
$$A_6 = \mathcal{I} \cup \mathcal{O} : \{\overline{i_0}, \overline{i_1}, i_2, i_3\} \Rightarrow \{\overline{o_0}, o_1\},$$
$$A_7 = \mathcal{I} \cup \mathcal{O} : \{i_0, i_1, i_2, i_3\} \Rightarrow \{o_0, \overline{o_1}\}.$$

2.2 Network

From now on let $\mathcal{NN}$ be a CSSN such that the parameters R, T_I and T_b defining self-sustained activity are constant and containing a sufficient number of R-clusters such that all statements of rule 110 except A_1 are fulfilled. Instead the

following is true:

$$\overline{A_1} = \mathcal{I} \cup \mathcal{O} : \{\overline{i_0}, \overline{i_1}, i_2, i_3\} \Rightarrow \{o_0, \overline{o_1}\};$$

and the weights of $\mathcal{NN}$ can be modified such that $A1$ is true.

2.2.1 Input-Clusters

Each input cluster contains one signal neuron that will emit spikes at different rates, depending if the cluster is active or not. All 8 frequencies are distinct.

2.2.2 Chemicals and Glands

$\{W, T, S, F, R, L_1, L_2, NL_1, NL_2\}$ is the set of chemical of $\mathcal{NN}$. $\mathcal{R} \subset \mathcal{LG}$ contains four glands:

1. fear-gland (F-gland)
2. reward-gland (R-gland)
3. wake-gland (WT-gland)
4. sleep-gland (S-gland)
5. learn-gland (L_1L_2-gland)

Each gland has two modes:

1. active (producing the corresponding chemicals at a (time-limited) fixed rate)
2. inactive

2.2.3 Frequency-Network

The input-clusters, output-clusters and the F-Gland are connected in such a way that the addition of all input-spike-signals and output signals are relayed to a specific neuron(s) of the F-Gland (receiver) and a frequency decoder: If the signal contains a frequency, a corresponding neurons 'activates'. A number of neurons of the F-Gland function as an emitter. The F-gland can recognize if the frequencies of the emitter match those of the receiver. Such a **signal-match**

2.2.4 Network-States

Depending on the chemical concentrations in P, at any point in time, we attribute one of the following 'states' to $\mathcal{NN}$

1. awake-state; sub-states:
   (a) interpreting-state (only the WT-gland is active),
   (b) rewarded-state (only the WT-gland and R-gland are active),

(c) fear-state (only the WT-gland and F-gland are active),

2. asleep; sub-states:

   (a) dream-state (only S-gland in active),

   (b) fear-state (only the WT-gland and F-gland are active),

   (c) learning-state (only the WT-gland and F-gland are active),

The only possible transitions are $1.a \to 1.b$, $1.a \to 1.b$, $1.a \to 2.a$ (falling-asleep-state), $1.b \to 1.a$, $1.c \to 1.a$, $2.a \to 1.a$(waking-up-state), $2.a \to 2.b$, $2.a \to 2.c$, $2.b \to 2.a$ and $2.c \to 2.a$. We also denote *wake* and *sleep* for a state

The S-gland will activate and the WT-gland will deactivate if T exceeds a threshold in a point $p \in P$. In the *interpreting-state*, the R-gland will activate if triggered from outside. If the R-concentration exceeds a threshold in a point $p \in P$, the WT-gland stops producing T. ... [TODO: chemical-state relationships]

2.2.5 Memory formation

[TODO: precise Definitions]

L-T-S-Memory: Activation pattern change caused by long-term changes in synaptic weights or new synapses

S-T-S-Memory: Activation pattern change caused by short-term changes in synaptic weights or new short-lived synapses

S-T-C-Memory: Activation pattern change caused by changes in chemical concentrations

... [TODO:]

2.2.6 Correcting the Error

References

[1] Izkevitch *[...]* 2003.

[2] Authors *Shot-Term Memory* 20xx.

[3] .. *Rule 110. ..*

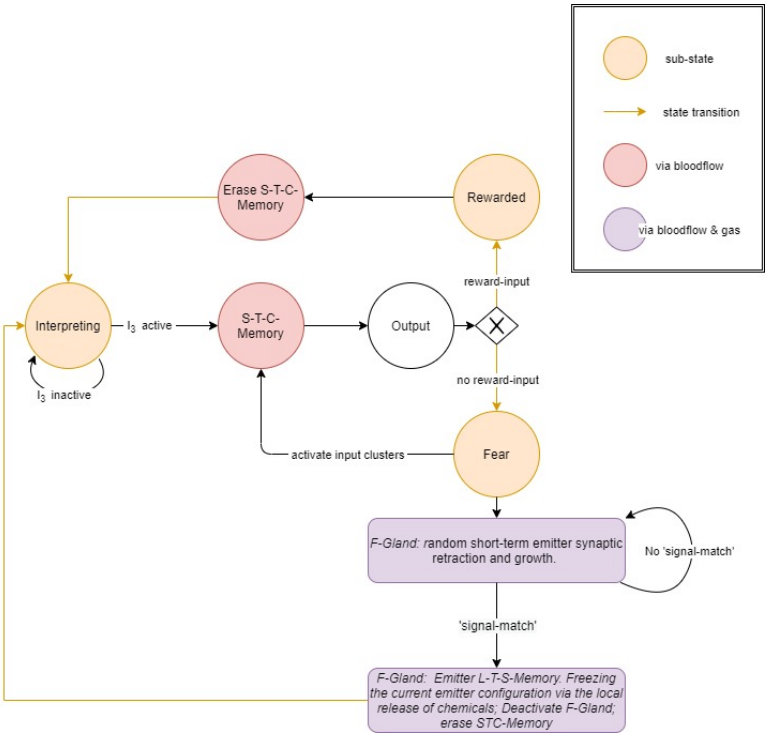

Figure 3: Wake.

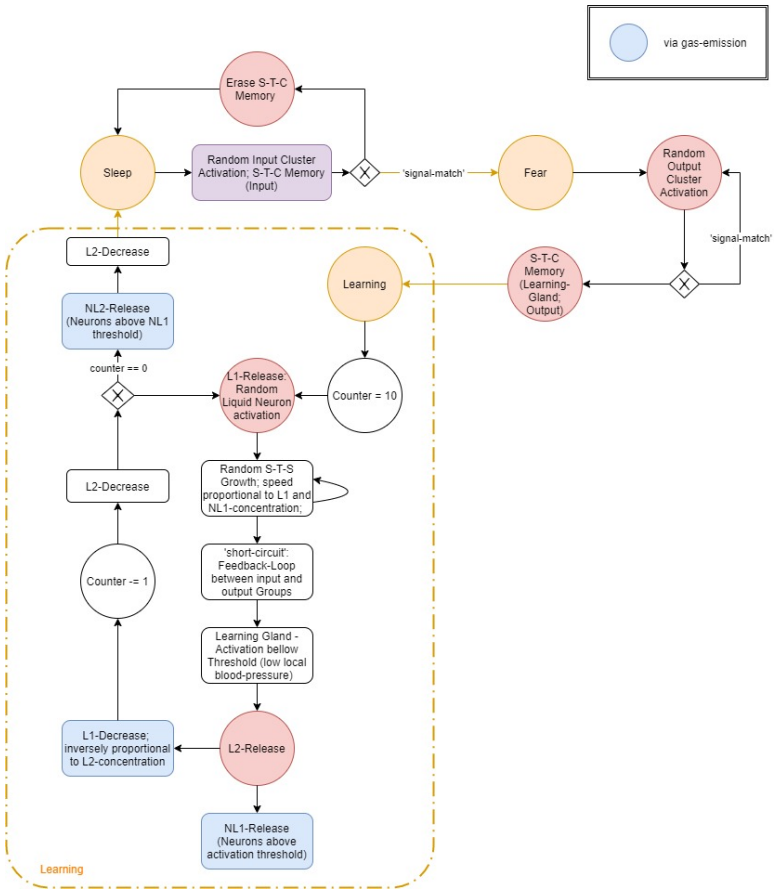

Figure 4: Sleep.

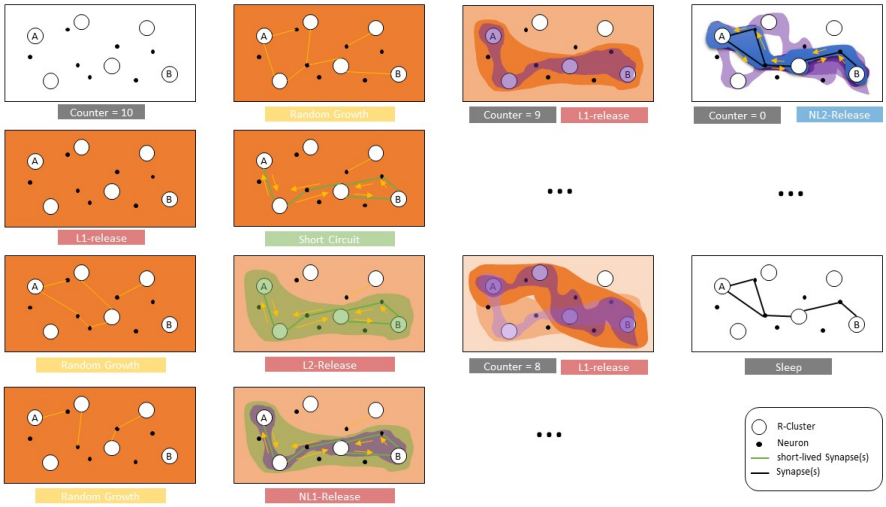

Figure 5: Learning (2D).