# 3D Geometry Regularization

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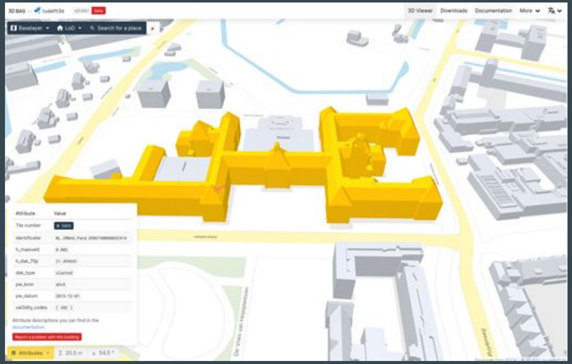
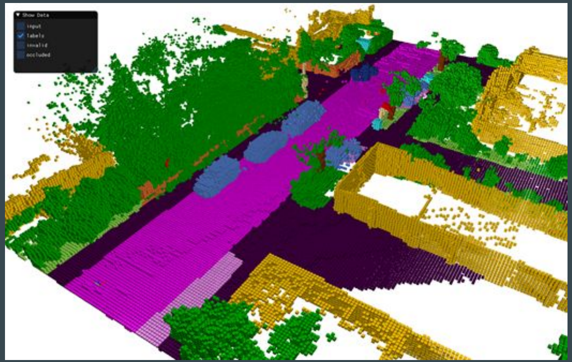
# Background

- Modelling 3D urban objects is crucial for many applications



Urban modelling

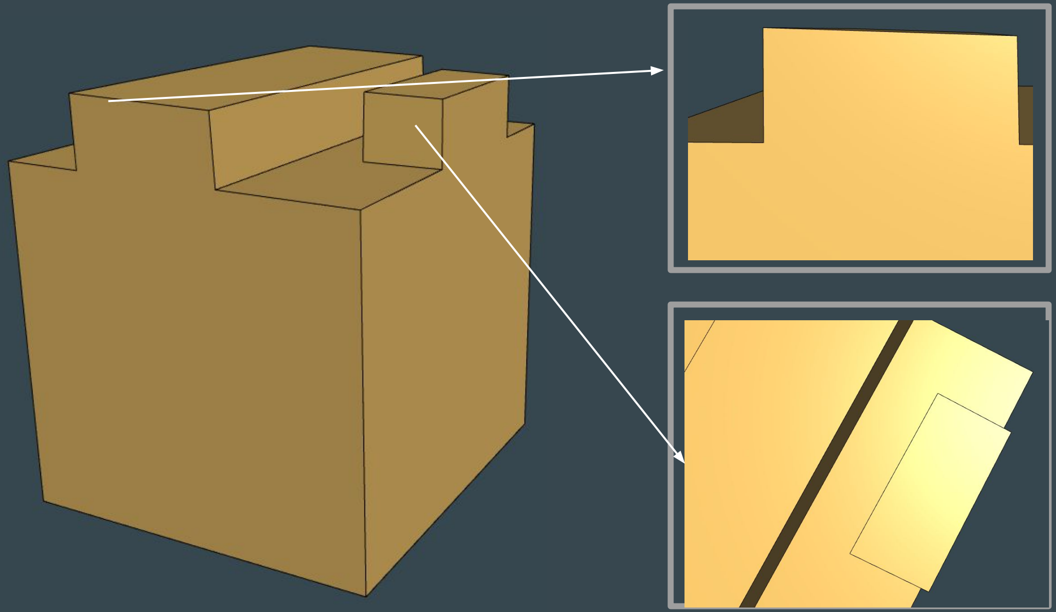
City management



Environment monitoring

# Background

- 3D object modelling suffers from data incompleteness, occlusion, outliers ……
- The output model usually has noisy geometry



# Problem Statement

Given a model with n vertices:

$$P = (x_1, y_1, z_1, x_2, y_2, z_2, ..., x_n, y_n, z_n)$$

We aim to find new vertices $X \subseteq R^{3n}$ which minimize the vertex change $\|X - P\|_2^2$ while fulfilling the geometrical constraint:

- Near-orthogonal edge pairs should be orthogonal

# Formulating Edge Pairs

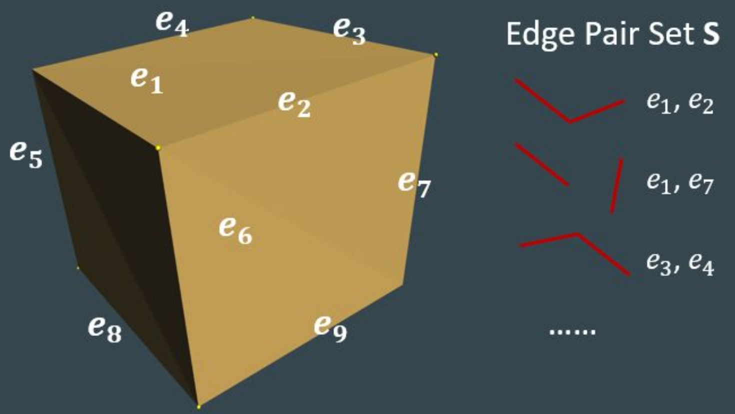
- Search for near-orthogonal edge pairs and store in a set S

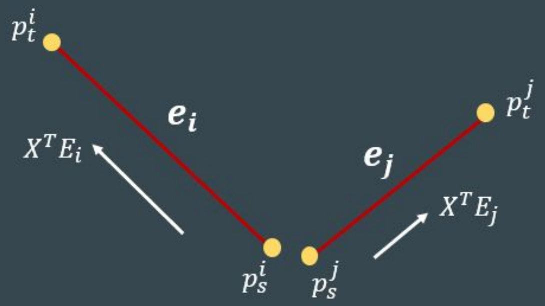
- Dot product of edge i and j is:

$$(X^T E_i) * (X^T E_j) = X^T E_i E_j^T X$$

  - E is a 3n by 3 matrix indicating the edge connection of the vertices, i.e.,

$$E = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ \dots & -1 & 0 \\ +1 & \dots & -1 \\ 0 & +1 & \dots \\ 0 & 0 & +1 \end{bmatrix}$$



# Formulating Optimization

- True Problem

$$\begin{aligned} &min \quad \|X - P\|_2^2 \\ &s.t. \quad X^T E_i {E_j}^T X = 0, \forall e_i \perp e_j \end{aligned}$$

- Approximated Problem

$$min \, \|X - P\|_2^2 + \lambda \sum_{i \perp j} \left| X^T E_i {E_j}^T X \right|$$

  - We use the subgradient descent method to solve this approximated problem

# Implementation details

Tools that we used:

- Modern C++ (C++11)
- Easy3D

https://github.com/Mirmix/3D_shape_regularization

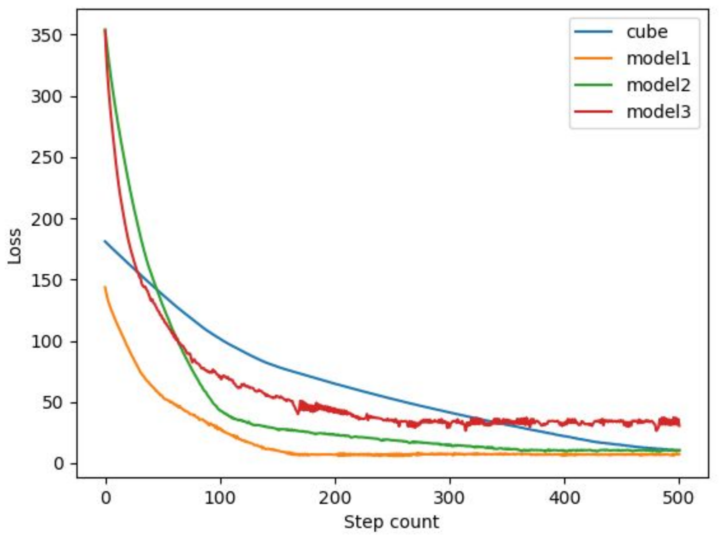
# Subgradient Descent Optimization

- Penalize L2 norm of X deviation from input P
- Penalize L1 norm of  orthogonality residuals

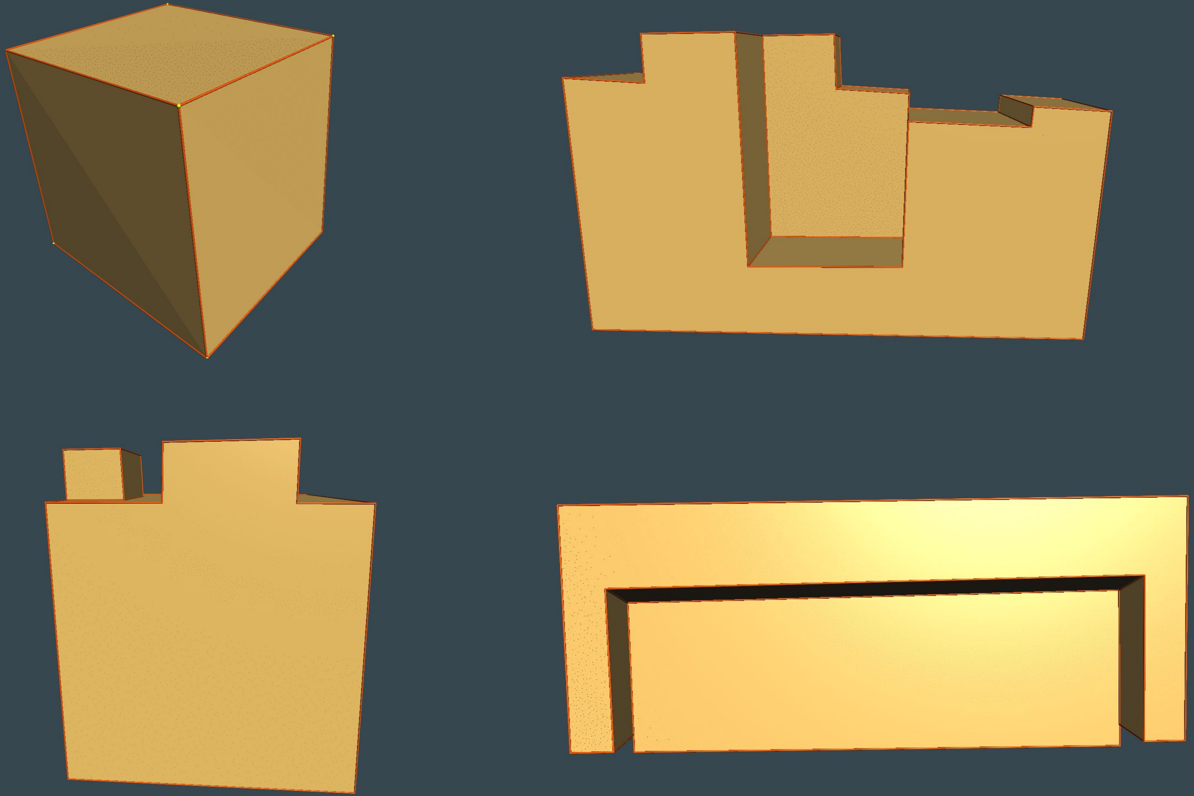
$$\mathcal{L} = (X - P)^T (X - P) + \lambda \sum_{i \perp j} |X^T E_i E_j^T X|$$

$$\frac{\partial \mathcal{L}}{\partial X} = 2(X - P) + \lambda \sum_{i \perp j} \pm \left( E_i E_j^T + E_j E_i^T \right) X$$

$$X_{k+1} = X_k - \eta \frac{\partial \mathcal{L}}{\partial X}$$



# Subgradient Descent Results



# Thank you! Questions?