

- i** Read and follow these instructions carefully to increase your chance of getting good marks.

This is an open book exam. You may use any material that was written prior to the start of the exam, but you must not communicate with anybody else about the exam while it is ongoing (i.e., before 14:00). The use of a Haskell interpreter or compiler is recommended.

Send any questions during the exam by email to [justin.pearson@it.uu.se](mailto:justin.pearson@it.uu.se). Answers that are relevant for all students will appear in [Piazza](#). If you don't have access, email Justin. We recommend that you check the questions and answers before handing in.

This exam contains several kinds of questions. Among them are multiple-choice questions, where only a single answer is correct (unless otherwise stated). These questions are followed by a motivation question, where you must give a sufficient motivation to why your chosen answer is the correct one. The multiple-choice question and its motivation count for one point each.

You can at most obtain 48 points on this exam. The preliminary grade limits are: 24p for grade 3, 34p for grade 4, 44p for grade 5.

Read the questions carefully, and watch out for negations (**not**, **except**, et c.).

Good Luck

- 1 What is the preorder traversal of the expression tree corresponding to  $((3 * 1) + (4 - (11 / 5))) + 3$ ?  
**Välj ett alternativ:**

- ☐ 3 1 \* 4 11 5 / - + 3 +
- ☐ + + \* 3 1 - 4 / 11 5 3
- ☐ 3 1 \* 4 + 11 - 5 / 3 +
- ☐ + 3 + - / 5 11 4 \* 1 3
- ☐ No other alternative is correct.

Totalpoäng: 1

2 Motivate your answer to the preceding question.

**Skriv in ditt svar här**

Totalpoäng: 1

3 Consider the following parts of a data-structure invariant for a binary tree where each node has a key of type Int.

1. If the current node is coloured red, then none of its children are coloured red.
2. The key of the current node is less than all keys in the tree rooted at its right child.
3. The key of the current node is less than all keys in the tree rooted at its left child.
4. The key of the current node is greater than all keys in the tree rooted at its left child.

Which combination of these corresponds to an invariant for an implementation of a binary search tree?

**Välj ett alternativ:**

- ☐ 2 and 4 hold at the root
- ☐ 2 and 3 hold at the root
- ☐ 2 and 4 hold at every node
- ☐ 2 and 3 hold at every node
- ☐ 1, 2, and 4 hold at every node

Totalpoäng: 1

4 Motivate your answer to the preceding question.

**Skriv in ditt svar här**

Totalpoäng: 1

5 How many nodes are there at most in a red-black tree of black height 3?

**Välj ett alternativ:**

- ☐ 7
- ☐ 31
- ☐ 15
- ☐ None of the other alternatives is correct.
- ☐ 3

Totalpoäng: 1

6 Motivate your answer to the preceding question.

**Skriv in ditt svar här**

Totalpoäng: 1

7 The worst-case time complexity of insertion into a binary search tree of height  $h$  is

**Välj ett alternativ:**

- ☐  $O(1)$
- ☐  $O(\log h)$
- ☐  $O(h)$
- ☐  $O(h^2)$
- ☐  $O(2^h)$

Totalpoäng: 1

8 Motivate your answer to the preceding question.

**Skriv in ditt svar här**

Totalpoäng: 1

9 Consider the following declaration, and assume that the function `foo` is pure.

`foo :: Integral a => [a] -> [a]`

Which alternative below is **false**?

**Välj ett alternativ:**

- ☐ The call `foo []` must return the empty list `[]`
- ☐ The type `a` must belong to the typeclass `Ord`
- ☐ `foo` is a polymorphic function
- ☐ The implementation of `foo` can use overloaded definitions of the function `abs`
- ☐ The implementation of `foo` can use overloaded definitions of the function `div`

Totalpoäng: 1

10 Motivate your answer to the preceding question.

**Skriv in ditt svar här**

Totalpoäng: 1

**11** Consider the following operations.

$\text{init} :: \text{Ast } a$

$\text{f1} :: \text{Eq } a \Rightarrow \text{Ast } a \rightarrow a \rightarrow \text{Ast } a$

$\text{f2} :: \text{Ast } a \rightarrow a \rightarrow \text{Bool}$

$\text{f3} :: \text{Ast } a \rightarrow \text{Ast } a$

$\text{f4} :: \text{Ast } a \rightarrow \text{Ast } a \rightarrow \text{Ast } a$

Which AST are these operations from?

**Välj ett alternativ:**

- ☐ None of the other alternatives is correct.
- ☐ PriorityQueue
- ☐ Set
- ☐ Table
- ☐ Queue or Stack

Totalpoäng: 1

**12** Motivate your answer to the preceding question.

**Skriv in ditt svar här**

Totalpoäng: 1

**13** Consider the following fragment of code

```
twoNumbers :: StdGen -> (Int, Int)
twoNumbers gen =
  let (x, newGen) = random gen
      (y, newGen') = random newGen
  in (x,y)
seed = 134342300
main :: IO ()
main = do
  let (x,y) = twoNumbers (mkStdGen seed)
  if x == y
    then putStrLn("The numbers are the same.")
    else putStrLn("The numbers are different.")
```

When running the code which of the following statements are true?

.

**Välj ett alternativ:**

- ☐ When main is run sometimes it prints The numbers are the same and sometimes it prints The numbers are different
- ☐ If you change the value of seed then it is possible to get the message The numbers are different.
- ☐ When main is run it always prints The numbers are the same.
- ☐ When main is run it always prints The numbers are the different.

Totalpoäng: 1

**14** Motivate your answer to the previous question

**Skriv in ditt svar här**

Totalpoäng: 1

15

Which of the following is *not* true about native/truly mutable arrays (the ones in `Data.Array.IO`) in Haskell?

**Välj ett alternativ**

- ☐ Arrays can only be used inside the IO monad.
- ☐ Any element of an array can be written in  $O(1)$  time.
- ☐ Arrays are implemented internally using a binary search tree.
- ☐ Any element of an array can be read in  $O(1)$  time.

Totalpoäng: 1

16 Motivate your answer to the previous question.

**Skriv in ditt svar här**

Totalpoäng: 1

17

Consider a table with 30 slots that uses chaining to resolve conflicts. What is the *maximal* possible chain length (worst case) of a single slot when the load factor is 2:

**Välj ett alternativ**

- ☐ 60
- ☐ 1
- ☐ 20
- ☐ None of the above the load factor can never be larger than 1

Totalpoäng: 1

**18** Motivate your answer to the previous question..

**Skriv in ditt svar här**

Totalpoäng: 1

**19** Consider a hash table with 10 slots using the hash function  $h(k) = k \bmod 10$  with linear probing, and the following integers

After inserting the following items into the hash table 121 231 42 43 44 78 77 21

Which item will be in slot 6 (the slots are numbers 0,1,2, . . . , 9).

.

**Välj ett alternativ:**

- ☐ 21
- ☐ 121
- ☐ 42
- ☐ Nothing

Totalpoäng: 1

**20**

Motivate your answer to the previous question.

**Skriv in ditt svar här**

Totalpoäng: 1



**21** Given a directed graph with nodes A,B,C,D,E,F,G and directed edges

A		
B		E, G
C		I
D		E, I
E		F
F		
G		
H		
I		A, F, G

Which of the following options is *not* a topological sort of the graph

**Välj ett alternativ:**

- ☐ C,D,B,I,E,A,F,G
- ☐ D,I,A,B,E,C,F,G
- ☐ B,D,E,C,I,G,A,F
- ☐ B,C,D,E,I,A,F,G

Totalpoäng: 1

**22** Motivate your answer to the previous question.

**Skriv in ditt svar här**

Totalpoäng: 1

**23** Given a graph with nodes A,B,C,D,E,F,G and (non directed) edges

A		B C
B		A C D
C		A B D E
D		B C E G
E		C D F
F		E
G		D

which of the following is *not* a breadth first search (BFS) order.

**Välj ett alternativ:**

- ☐ D,E,F,G,C,B, A
- ☐ G,D,E,C,B,F,A
- ☐ C,E,D,B,A,F,G
- ☐ D,G,E,C,B,F,A

Totalpoäng: 1

**24** Motivate your answer to the previous question. In particular say how you would repair the incorrect search path to make it BFS.

**Skriv in ditt svar här**

Totalpoäng: 1

**25** Consider a binary tree where each node contains a key of type `Int`.

Write down the datatype, and a recursive function that yields both the smallest and the greatest element in the tree. Document your code.

Make sure that each leaf is visited exactly once, and argue why this is true.

**Skriv in ditt svar här**

1	
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Totalpoäng: 4

**26** Consider a heap implemented as a binary tree satisfying the min-heap invariant.

Write down the invariant.

Given such a heap with more than one element, is it always true that all leaves are less than or equal to all children of the root?

Motivate your answer.

**Skriv in ditt svar här**

Totalpoäng: 4

- 27** Recall the ADT Stack with the functions push, pop, and peek.  
Implement a Stack where the type Stack a of a stack containing elements of type a is

data Stack a = Empty | Stack a (Stack a)

Document the code.

**Skriv in ditt svar här (4p)**

1	
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Totalpoäng: 4

**28** Consider the following function that is supposed to remove duplicate entries in a list of integers.

```
remove_duplicates :: [Int] -> [Int]
remove_duplicates [] = []
remove_duplicates (x : xs) = if x `elem` xs
                             then xs
                             else ( x : remove_duplicates xs)
```

The function is not correct. Your job is to work out why and correct it.

- Write 3 test cases that show when the function is not performing correctly.
- Provide a corrected version of the above function.

**Skriv in ditt svar här (4p)**

1	
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Totalpoäng: 4

- 29** The following piece of code is an attempt to do the classic FizzBuzz puzzle. If a number is divisible by 3 then you should Fizz if the number is divisible by 5 then you should print Buzz and if the number is divisible by both 3 and 5 then print FizzBuzz. The two helper functions

```
fizz n = if (n `mod` 3) == 0 then "Fizz" else ""  
buzz n = if (n `mod` 5) == 0 then "Buzz" else ""
```

are correct, but the following function program does not work. The idea is you start at n and count down until you get to 0.

```
main n =  
  if n == 0  
  then  
    putStrLn "That's all folks."  
  else  
    str1 <- (fizz n)  
    str2 <- (buzz n)  
    putStrLn ((show n) ++ " is " ++ str1 ++ str2)  
    main (n - 1)
```

1. Explain why the code is incorrect. This does not mean copying and pasting Haskell's error message, but you need to explain what the programmer has misunderstood.
2. Rewrite the code using the do notation.
3. Rewrite the code using >>= and >> instead of using do blocks.

**Write your answer here (4p)**

1	
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Totalpoäng: 4

**30** Consider the following graph on the nodes A,B,C,D,E,F,G,H,I,J with directed edges

A		B C
B		E H
C		D I
D		B
E		G F
F		G
H		
I		J
J		I

1. List all the strongly connected components of the graph.
2. Then apply ( Kosaraju) algorithm to compute strongly connected components from the slides. You should explain each step in the algorithm.

**Skriv in ditt svar här (4p)**

Totalpoäng: 4