Intermediate Exam A in Signals and Transforms (1TE746)

November 17, 2020

Instructions

General:
- Use a separate A4-page for each question and number the pages;
- Always write down your intermediate steps;
- Clearly mark your final answers (e.g., using double underlines or a frame);
- Always provide the correct unit, missing units may lead to deductions.

Allowed Aids:
- The course's formula sheet;
- Template for Bode diagrams;
- One (1) A4 sheet with your own, hand-written notes (front and back);
- Calculator;
- **Old exams, exercises, lecture notes, and other aids are not allowed!**

Grading

Pass (3): To pass the exam, you need to obtain at least 50 % of the points in the questions related to a specific intended learning outcome. The questions associated to the four learning outcomes examined in this exam are as follows:
- Learning outcome 1: 1a)–b), (2p+3p)
- Learning outcome 2: 1c)–3a) (2p+2p)
- Learning outcome 3: 3b), 4a) (2p+2p)
- Learning outcome 4: 2a)–b), 4b) (1p+2p+2p)

Higher Grades: Given the criterion for passing above is fulfilled (i.e., 50 % of the questions related to each learning outcome), the final grade is calculated according to the total number of points. The grading scale is as follows:
- Less than 60 %: Fail or 3 (depending on the criterion for passing).
- 60 % to 80 %: 4.
- 80 % to < 100 %: 5.

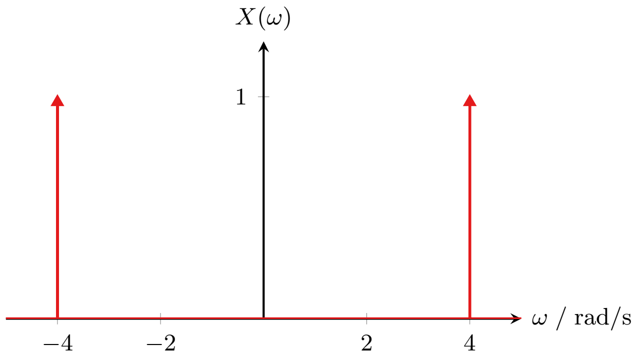
Questions

1. The continuous time Fourier transform and unitary Laplace transforms are given by

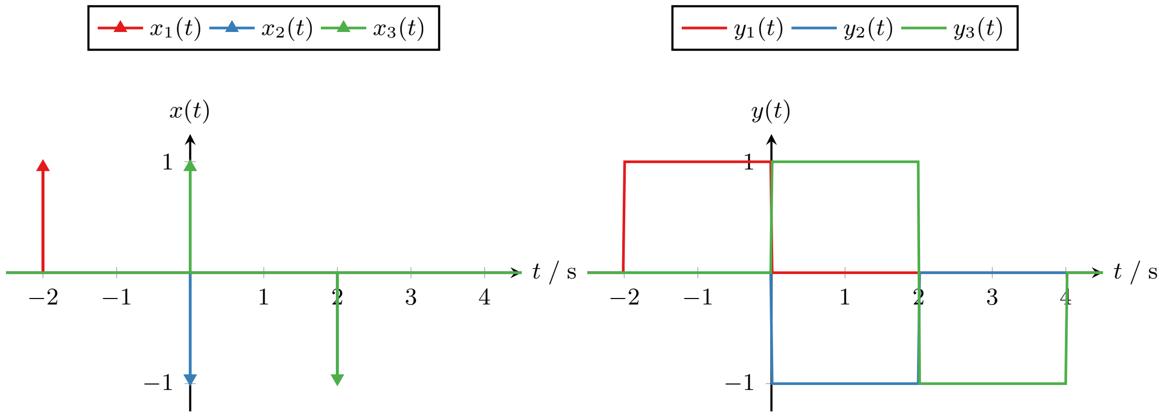
$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-\mathrm{j}\omega t}\,\mathrm{d}t \quad \text{and} \quad X(s) = \int_{0}^{\infty} x(t)e^{-st}\,\mathrm{d}t,$$

respectively.

   a) (2p) The continuous time Fourier series can be seen as a decomposition of T_0-periodic signals into harmonic components of frequencies $\omega = n\frac{2\pi}{T_0}$. What is a similar interpretation of the Fourier transform shown above?
   b) (3p) What are the three requirements for the Fourier and Laplace transforms to be equivalent?
   c) (2p) Consider the spectrum of the signal $x(t)$ shown in the Figure below. Sketch the spectrum of the signal $z(t) = \frac{2}{\pi}\,\mathrm{sinc}\left(\frac{2t}{\pi}\right)x(t)$. Recall to mark important frequencies, amplitudes, etc.



2. The figure below shows three input-output signal pairs $x_1(t) \mapsto y_1(t)$, $x_2(t) \mapsto y_2(t)$, and $x_3(t) \mapsto y_3(t)$ for an unknown dynamic system.



   a) (1p) Is the system a linear, time-invariant system?
   b) (2p) Determine the system's impulse response $h(t)$.
   c) (4p) Determine the system's output signal $y(t)$ if the input signal is $x(t) = 2\cos\left(\frac{\pi}{2}t\right)$.

3. The transfer function of a linear, time-invariant system is given by

$$H(s) = \frac{1}{(s+2)(s+3)}.$$

   a) (2p) Determine the system's impulse response $h(t)$.

b) (2p) Determine the system's frequency response $H(\omega)$. Expand all parentheses as much as possible.
c) (4p) Sketch the system's Bode plot (both magnitude and phase). Recall to mark important properties such as corner frequencies, asymptotes, etc.

4. Consider a linear, time-invariant system described by the differential equation

$$\frac{\mathrm{d}^2y(t)}{\mathrm{d}t^2}+6\frac{\mathrm{d}y(t)}{\mathrm{d}t}+13y(t)=\frac{\mathrm{d}x(t)}{\mathrm{d}t}+2x(t).$$

Assume that all initial conditions are zero.

a) (2p) Determine the system's transfer function $H(s)$.
b) (2p) Determine the system's poles and zeros and sketch the pole-zero map. Is the system stable?
c) (4p) Determine the output signal if the input is $x(t)=u(t)$.