

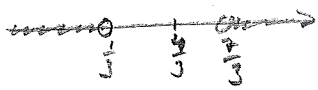
$$1. \sin \frac{8\pi}{3} = \frac{\sqrt{3}}{2}$$

$$2. \frac{4a^2 - b^2}{b^2 - 2ab} = \frac{(2a+b)(2a-b)}{-b(2a-b)}$$

$$= -\frac{2a+b}{b}$$

$$3. |4-3x| > 3$$

$$3|x - \frac{4}{3}| > 3$$

$$|x - \frac{4}{3}| > 1$$


$$x < -\frac{1}{3}, x > \frac{7}{3}$$

$$4. \frac{a-3i}{2+i} = \frac{(a-3i)(2-i)}{4+1}$$

$$= \frac{2a - a^2 - 6i + 3i^2}{5}$$

$$-i(a+6) = 0$$

$$a = -6$$

$$5. \sum_{k=1}^{50} (2k+1) = 3+5+\dots+101$$

$$S_{50} = \frac{50}{2} (3+101) = 25 \cdot 104 = 2600$$

$$6. 27^{-4/3} = \left(\frac{1}{27^{1/3}}\right)^4 = \frac{1}{3^4} = \frac{1}{81}$$

$$7. \log_5 0.04 = \log_5 \frac{4}{100} = \log_5 \frac{1}{25} =$$

$$= \log_5 5^{-2} = -2$$

$$8. 2kx^2 - 4kx + 1 = 0$$

$$\Delta = 16k^2 - 4 \cdot 2k \cdot 1$$

$$= 16k^2 - 8k$$

$$\Delta = 0 \quad 16k^2 - 8k = 0$$

$$8k(2k-1) = 0$$

$$k = \frac{1}{2} \quad (k \neq 0)$$

$$9. x^2 - 4x + 3 + y^2 - 7y = 0$$

$$(x-2)^2 - 4 + 3 + \left(y - \frac{7}{2}\right)^2 - \frac{49}{4} = 0$$

$$(x-2)^2 + \left(y - \frac{7}{2}\right)^2 = \frac{53}{4}$$

$$(x-2)^2 + \left(y - \frac{7}{2}\right)^2 = \left(\frac{\sqrt{53}}{2}\right)^2$$

Medelpunkt: $(2, \frac{7}{2})$

Radie: $\frac{\sqrt{53}}{2}$

$$10. \left(2x + \frac{1}{x^2}\right)^{12} = \sum_{k=0}^{12} \binom{12}{k} (2x)^{12-k} \left(x^{-2}\right)^k$$

$$12-k-2k=0$$

$$12-3k=0, k=4 \quad \therefore \binom{12}{4} \cdot 2^8$$

11. Tot. antal sätt: $5!$

Antal sätt med C bredvid B: $2 \cdot 4!$

$$\therefore 5! - 2 \cdot 4! = 120 - 2 \cdot 24 = 72$$

(eller $3 \cdot 4! = 3 \cdot 24 = 72$)

$$12. \sin\left(3x + \frac{2\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

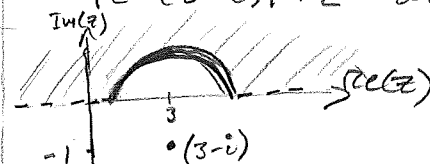
$$3x + \frac{2\pi}{3} = \frac{4\pi}{3} + n \cdot 2\pi \quad \text{eller} \quad 3x + \frac{2\pi}{3} = \frac{5\pi}{3} + n \cdot 2\pi$$

$$3x = \frac{2\pi}{3} + n \cdot 2\pi \quad \text{eller} \quad 3x = \pi + n \cdot 2\pi$$

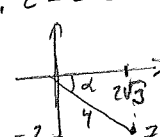
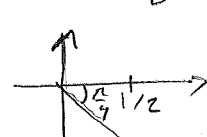
$$x = \frac{2\pi}{9} + n \cdot \frac{2\pi}{3} \quad \text{eller} \quad x = \frac{\pi}{3} + n \cdot \frac{2\pi}{3}$$

13. $|z-3+i| \geq 2$ och $\text{Im}(z) > 0$

$|z-(3-i)| \geq 2$ och $\text{Im}(z) > 0$



14. $z = 2\sqrt{3} - 2i$ $w = \frac{1}{2} - \frac{1}{2}i$

$$\sin \alpha = \frac{2}{4} = \frac{1}{2}$$

$$\alpha = \frac{\pi}{6}$$

$$\arg z = \frac{11\pi}{6}$$

$$\arg w = \frac{7\pi}{4}$$

(acceptera $-\frac{\pi}{4}$)

(acceptera $-\frac{\pi}{6}$)

$$\arg\left(\frac{z}{w}\right) = \arg z - \arg w = \frac{11\pi}{6} - \frac{7\pi}{4}$$

$$= \frac{\pi}{12}$$

15. $\log_3 x + \log_x \frac{1}{27} = 2$ $y = -1$

$$\log_3 x + \log_x 3^{-3} = 2$$

$$\log_3 x - 3 \log_x 3 = 2$$

$$\log_3 x - 3 \frac{\log_3 3}{\log_3 x} = 2$$

$$(\log_3 x)^2 - 3 \cdot 1 = 2 \log_3 x$$

$$y^2 - 2y - 3 = 0 \quad (y = \log_3 x)$$

$$(y+1)(y-3) = 0$$

$$y = -1 \quad \text{eller} \quad y = 3$$

$\log_3 x = -1 \quad \left| \quad x = 3^{-1} = \frac{1}{3} \right.$

$\log_3 x = 3 \quad \left| \quad x = 3^3 = 27 \right.$

16. $x=i$ är en rot

$\Rightarrow x=-i$ är en rot

$$(x+i)(x-i) = x^2 + 1$$

$$\begin{array}{r} x^2 - 5x + 6 \\ x^4 - 5x^3 + 7x^2 - 5x + 6 \quad | x^2 + 1 \\ \hline x^4 + x^2 \\ \hline -5x^3 + 6x^2 - 5x + 6 \\ -5x^3 - 5x \\ \hline 6x^2 + 6 \\ 6x^2 + 6 \\ \hline 0 \end{array}$$

$$\therefore x^4 - 5x^3 + 7x^2 - 5x + 6 =$$

$$= (x^2 + 1)(x^2 - 5x + 6)$$

$$= (x^2 + 1)(x-3)(x-2)$$

Övriga rötter: $x = -i$
 $x = i$
 $x = 2$

17. $z^3 = -2 + 2i$, $z = r(\cos\theta + i\sin\theta)$

$$r^3(\cos 3\theta + i\sin 3\theta) = \sqrt{8}(\cos \frac{3\pi}{4} + i\sin \frac{3\pi}{4})$$

$$r^3 = 8^{1/2}$$

$$3\theta = \frac{3\pi}{4} + n \cdot 2\pi$$

$$r = ((2^3)^{1/2})^{1/3} = \sqrt{2}$$

$$\theta = \frac{\pi}{4} + n \cdot \frac{2\pi}{3}$$

$$n=0: \theta = \frac{\pi}{4}$$

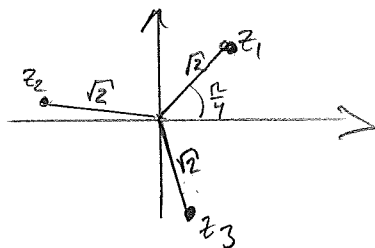
$$\therefore z_1 = \sqrt{2}(\cos \frac{\pi}{4} + i\sin \frac{\pi}{4})$$

$$n=1: \theta = \frac{11\pi}{12}$$

$$z_2 = \sqrt{2}(\cos \frac{11\pi}{12} + i\sin \frac{11\pi}{12})$$

$$n=2: \theta = \frac{19\pi}{12}$$

$$z_3 = \sqrt{2}(\cos \frac{19\pi}{12} + i\sin \frac{19\pi}{12})$$



18. $P_n: \sum_{k=1}^n k \cdot k! = (n+1)! - 1$, $n \in \mathbb{Z}^+$

$$P_1: \text{V.L.} = 1 \cdot 1! = 1$$

$$\text{H.L.} = (1+1)! - 1 = 1$$

$\therefore P_1$ är sant

Antag att P_r är sant, $r \in \mathbb{Z}^+$, dvs

$$1 \cdot 1! + 2 \cdot 2! + \dots + r \cdot r! = (r+1)! - 1 \quad (*)$$

$$P_{r+1}:$$

$$1 \cdot 1! + 2 \cdot 2! + \dots + r \cdot r! + (r+1) \cdot (r+1)! =$$

$$= (\text{enligt } (*)) = (r+1)! - 1 + (r+1)(r+1)! =$$

$$= (r+1)!(1 + (r+1)) - 1 =$$

$$= (r+2)(r+1)! - 1 = (r+2)! - 1$$

P_{r+1} är sant om P_r är sant, P_1 är sant, sålunda är P_n sant för alla $n \in \mathbb{Z}^+$ enligt induktionsaxiomet.