1. $\sin\frac{8\pi}{3} = \frac{\sqrt{3}}{2}$

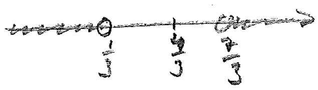
2. $\frac{4a^2-b^2}{b^2-2ab} = \frac{(2a+b)(2a-b)}{-b(2a-b)}$

$= -\frac{2a+b}{b}$

3. $|4-3x| > 3$

$3|x-\frac{4}{3}| > 3$

$|x-\frac{4}{3}| > 1$



$x < \frac{1}{3}, \ x > \frac{7}{3}$

4. $\frac{a-3i}{2+i} = \frac{(a-3i)(2-i)}{4+1}$

$= \frac{2a-ai-6i+3i^2}{5}$

$\frac{-i(a+6)}{5} = 0$

$a = -6$

5. $\sum_{k=1}^{50}(2k+1) = 3+5+\ldots+101$

$S_{50} = \frac{50}{2}(3+101) = 25\cdot 104 = 2600$

6. $27^{-4/3} = \left(\frac{1}{27^{1/3}}\right)^4 = \frac{1}{3^4} = \frac{1}{81}$

7. $\log_5 0{,}04 = \log_5 \frac{4}{100} = \log_5 \frac{1}{25} =$

$= \log_5 5^{-2} = -2$

8. $2kx^2 - 4kx + 1 = 0$

$\Delta = 16k^2 - 4\cdot 2k\cdot 1$

$= 16k^2 - 8k$

$\Delta = 0 \quad 16k^2 - 8k = 0$

$8k(2k-1) = 0$

$\underline{k = \frac{1}{2}} \quad (k \neq 0)$

9. $x^2 - 4x + 3 + y^2 - 7y = 0$

$(x-2)^2 - 4 + 3 + (y-\frac{7}{2})^2 - \frac{49}{4} = 0$

$(x-2)^2 + (y-\frac{7}{2})^2 = \frac{53}{4}$

$(x-2)^2 + (y-\frac{7}{2})^2 = \left(\frac{\sqrt{53}}{2}\right)^2$

Medelpunkt: $(2, \frac{7}{2})$

Radie $= \frac{\sqrt{53}}{2}$

10. $(2x + \frac{1}{x^2})^{12} = \sum_{k=0}^{12}\binom{12}{k}(2x)^{12-k}\cdot(x^{-2})^k$

$12 - k - 2k = 0$

$12 - 3k = 0, \ k = 4 \quad \therefore \ \underline{\binom{12}{4}\cdot 2^8}$

11. Tot. antal sätt: $5!$

Antal sätt med C bredvid B: $2\cdot 4!$

$\therefore \ 5! - 2\cdot 4! = 120 - 2\cdot 24 = \underline{72}$

(eller $3\cdot 4! = 3\cdot 24 = \underline{72}$)

12. $\sin(3x + \frac{2\pi}{3}) = -\frac{\sqrt{3}}{2}$

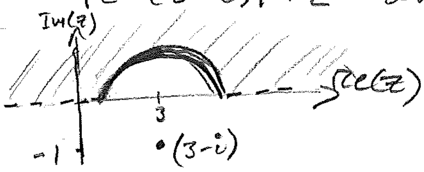
$3x + \frac{2\pi}{3} = \frac{4\pi}{3} + n\cdot 2\pi$ eller $3x + \frac{2\pi}{3} = \frac{5\pi}{3} + n\cdot 2\pi$

$3x = \frac{2\pi}{3} + n\cdot 2\pi$ eller $3x = \pi + n\cdot 2\pi$

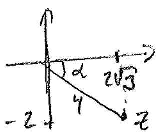
$\underline{x = \frac{2\pi}{9} + n\cdot\frac{2\pi}{3}}$ eller $\underline{x = \frac{\pi}{3} + n\cdot\frac{2\pi}{3}}$

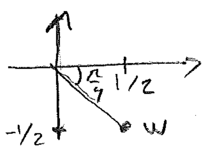
13. $|z - 3 + i| \geq 2$ och $\mathrm{Im}(z) > 0$

$|z - (3-i)| \geq 2$ och $\mathrm{Im}(z) > 0$



14. $z = 2\sqrt{3} - 2i \qquad w = \frac{1}{2} - \frac{1}{2}i$





$\sin\alpha = \frac{2}{4} = \frac{1}{2}$

$\alpha = \frac{\pi}{6}$

$\arg z = \frac{11\pi}{6}$

(acceptera $-\frac{\pi}{6}$)

$\arg w = \frac{7\pi}{4}$

(acceptera $-\frac{\pi}{4}$)

$\arg\left(\frac{z}{w}\right) = \arg z - \arg w = \frac{11\pi}{6} - \frac{7\pi}{4}$

$= \frac{\pi}{12}$

15. $\log_3 x + \log_x \frac{1}{27} = 2$

$\log_3 x + \log_x 3^{-3} = 2$

$\log_3 x - 3\log_x 3 = 2$

$\log_3 x - 3\frac{\log_3 3}{\log_3 x} = 2$

$(\log_3 x)^2 - 3\cdot 1 = 2\log_3 x$

$y^2 - 2y - 3 = 0 \quad (y = \log_3 x)$

$(y+1)(y-3) = 0$

$y = -1$ eller $y = 3$

$\underline{y = -1}$

$\log_3 x = -1 \ \Big| \ \underline{x = 3^{-1} = \frac{1}{3}}$

eller

$\underline{y = 3}$

$\log_3 x = 3 \ \Big| \ \underline{x = 3^3 = 27}$

16. $x = i$ är en rot

$\Rightarrow x = -i$ är en rot

$(x+i)(x-i) = x^2+1$

$$
\begin{array}{l}
\quad x^2-5x+6 \\
\hline
x^4-5x^3+7x^2-5x+6 \;\big|\; x^2+1 \\
x^4+x^2 \\
\hline
\quad -5x^3+6x^2-5x+6 \\
\quad -5x^3-5x \\
\hline
\qquad 6x^2+6 \\
\qquad 6x^2+6 \\
\hline
\qquad\quad 0
\end{array}
$$

$\therefore\; x^4-5x^3+7x^2-5x+6 =$

$= (x^2+1)(x^2-5x+6)$

$= (x^2+1)(x-3)(x-2)$

Övriga rötter: $x = -i$

$x = 3$

$x = 2$

18. $P_n: \sum_{k=1}^{n} k\cdot k! = (n+1)!-1$, $n \in \mathbb{Z}^+$

$\underline{P_1}$: V.L. $= 1\cdot 1! = 1$

H.L. $= (1+1)!-1 = 1$

$\therefore\; P_1$ är sant

Antag att P_r är sant, $r \in \mathbb{Z}^+$, dvs

$1\cdot 1! + 2\cdot 2! + \ldots + r\cdot r! = (r+1)!-1$ $(*)$

$\underline{P_{r+1}}$:

$1\cdot 1! + 2\cdot 2! + \ldots + r\cdot r! + (r+1)\cdot(r+1)! =$

$=$ (enligt $(*)$) $= (r+1)!-1+(r+1)(r+1)! =$

$= (r+1)!\,(1+(r+1))-1 =$

$= (r+2)(r+1)!-1 = (r+2)!-1$

P_{k+1} är sant om P_k är sant, P_1 är sant. Sålunda är P_n sant för alla $n \in \mathbb{Z}^+$ enligt induktionsaxiomet.

17. $z^3 = -2+2i$, $z = r(\cos\theta + i\sin\theta)$

$r^3(\cos 3\theta + i\sin 3\theta) = \sqrt{8}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

$r^3 = 8^{1/2}$

$r = ((2^3)^{1/2})^{1/3} = \sqrt{2}$

$3\theta = \frac{3\pi}{4} + n\cdot 2\pi$

$\theta = \frac{\pi}{4} + n\cdot\frac{2\pi}{3}$

$n=0: \theta = \frac{\pi}{4}$

$n=1: \theta = \frac{11\pi}{12}$

$n=2: \theta = \frac{19\pi}{12}$

$\therefore\; z_1 = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$z_2 = \sqrt{2}\left(\cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}\right)$

$z_3 = \sqrt{2}\left(\cos\frac{19\pi}{12} + i\sin\frac{19\pi}{12}\right)$