# Intermediate Exam B in Signals and Transforms (1TE746)

December 10, 2020

## Instructions

**General:**

- Use a separate A4-page for each question and number the pages;
- Always write down your intermediate steps, missing intermediate steps may lead to deductions;
- Clearly mark your final answers (e.g., using double underlines or a frame);
- Always provide the correct unit, missing units may lead to deductions.

**Allowed Aids:**

- The course's formula sheet;
- Template for Bode diagrams;
- One (1) A4 sheet with your own, hand-written notes (front and back);
- Calculator;
- **Old exams, exercises, lecture notes, and other aids not listed above are not allowed!**

## Grading

**Pass (3):** To pass the exam, you need to obtain at least 50 % of the points in the questions related to a specific intended learning outcome (ILO). The questions associated to the five ILOs examined in this exam are as follows (ILOs 1 & 2 and ILOs 3 & 4 are counted together in this exam):

| Learning Outcome(s) | Questions | Points |
|---|---|---|
| 1–2: Transform Methods | 1c), 3a)–b) | 2p + 3p + 2p = 7p |
| 3–4: LTI Systems | 2a)–c) | 1p + 3p + 2p = 6p |
| 6: Sampling & Reconstruction | 1a)–b) | 2p + 2p = 4p |

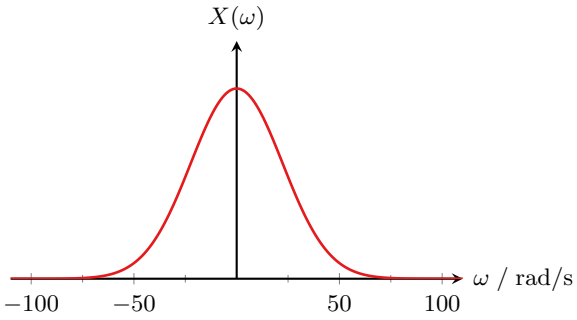
*Questions not listed above only count towards higher grades (see below).*

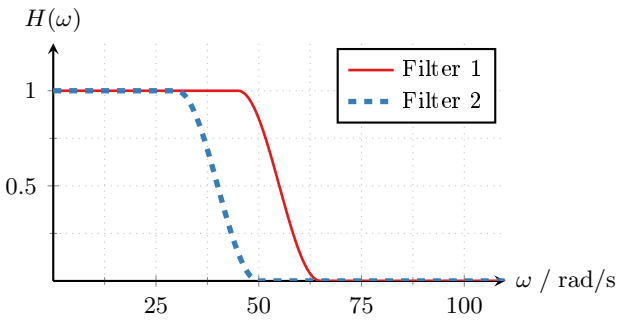
**Higher Grades:** Given the criterion for passing above is fulfilled (i.e., 50 % of the questions related to each learning outcome), the final grade is calculated according to the total number of points from all questions. The grading scale is as follows:

- Less than 60 %: Fail or 3 (depending on the criterion for passing).
- 60 % to 80 % (15 to 19.5 points): 4.
- 80 % to $< 100$ % (20 to 25 points): 5.

## Questions

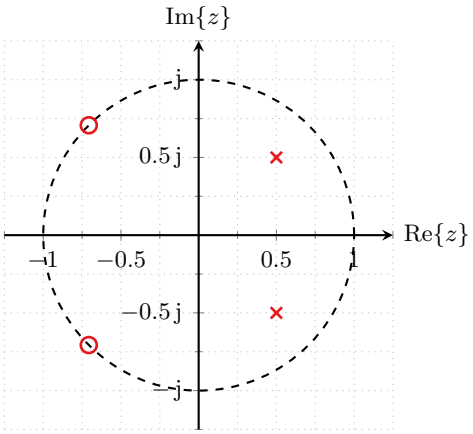
1. The continuous time signal $x(t)$ with spectrum shown below (left) is to be sampled by an ideal analog to digital converter.




   a) (2p) Assume that the sampling frequency has been chosen to $\omega_s = 100\,\text{rad/s}$. Is this choice of sampling frequency adequate? Motivate your answer.
   b) (2p) You decide that it might be a good idea to also include an analog prefilter before the analog to digital converter. The figure above (right) shows the frequency responses of two candidates for such prefilters. Which of the two filters would you prefer and why?
   c) (2p) Sketch the spectrum $X(\Omega)$ of the sampled signal $x[k]$ for the interval $-3\pi \leq \Omega \leq 3\pi$, assuming that the sampling frequency is as in a) but without the prefilter from b). Recall to properly label your axes, add ticks, etc.

2. Consider the discrete time LTI system with the pole-zero map shown below. Furthermore, it is known that the system has DC gain 1, that is, $H(\Omega = 0) = H(z = 1) = 1$ and that there are no multiple poles or zeros.




   a) (1p) Is the system stable? Motivate your answer.
   b) (3p) Determine the system's transfer function $H(z)$ on negative form (in terms of $z^{-1}$). Expand as much as possible until only a single fraction of numerator and denominator polynomials is left.
   *Hint: Do not forget the gain term in the pole-zero form.*
   c) (2p) What is the system's difference equation (with $y[k]$ as the most recent term)?
   d) (4p) Determine the output signal $y[k]$ if the input to the system is

$$x[k] = \cos\left(\frac{\pi}{2}k\right) + \cos\left(\frac{3\pi}{4}k\right).$$

3. Assume that you are asked to perform a spectral analysis of a signal $x(t)$ that is 2 s long, that is, you know that $x(t) = 0$ *outside* the interval $0\,\text{s} \leq t \leq 2\,\text{s}$. The sampling frequency of your measurement equipment is fixed to $f_s = 100\,\text{Hz}$.

   a) (3p) Assume that you want to:
      i. Obtain an error-free approximation of the signal's spectrum $X(f)$ using the DFT.
      ii. Obtain a spectral resolution of $\Delta f = 0.25\,\text{Hz}$.

      What is the minimum number of samples $K$ you need to measure from the signal to fulfill *both* conditions above at the same time? Is there a shortcut to achieve the above requirements given this particular setting?

   b) (2p) How would your result from a) be affected if you used a window different from the rectangular window? Would this improve your approximation of $X(f)$ or not?

   c) (4p) Assume that the signal $x(t)$ is passed through an LTI system with frequency response $H(\omega)$ and yields the output $y(t)$. The output $y(t)$ is also assumed to be time-limited to the interval $0\,\text{s} \leq t \leq 2\,\text{s}$. Explain how you can estimate the frequency response $H(\omega)$ using the DFTs of the input and output signals.