Exam in Automatic Control II
Reglerteknik II 5hp (1RT495)

Date: January 27, 2024
Time: 8:00 – 13:00
Venue: Bergsbrunnagatan 15: Sal 2

Responsible teacher: Sérgio Pequito

Aiding material: Calculator, mathematical handbooks, textbooks by Glad & Ljung (Reglerteori/Control theory & Reglerteknik). Laplace table and the automatic control glossary between Swedish and English. Additional handwritten notes in the textbooks are allowed.

Preliminary grades: $\geq$ 43p for grade 5, $\geq$ 33p for grade 4, and $\geq$ 23p for grade 3.

Use separate sheets for each problem, i.e., <u>only one problem per sheet.</u> Write your exam code on every sheet.

Important: Your solutions should be well motivated unless else is stated in the problem formulation. Vague or lacking motivation may lead to a reduced number of points.

Good luck!

Please use English in your solutions!

Problem 1 Consider the following discrete-time state-space representation of a single-input single-output system:

$$\begin{pmatrix} x_1[k+1] \\ x_2[k+1] \end{pmatrix} = \begin{pmatrix} -0.1 & 0.8 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_1[k] \\ x_2[k] \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u[k], \tag{1}$$

$$y[k] = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} x_1[k] \\ x_2[k] \end{pmatrix} + u[k]. \tag{2}$$

(a) Is the state-space representation (1)-(2) in an observable canonical form, controllable canonical form, or both? Explain your answer. **(2p)**

(b) Provide a state-space representation associated with the same system (1)-(2) that is not in controllable neither observable canonical form. **(2p)**

(c) Provide the block diagram representation associated with the state-space representation in (1)-(2). **(3p)**

(d) Is the system associated with (1)-(2) asymptotically stable? Explain your answer. **(2p)**

(e) Determine the state feedback control $u = -Kx$ that makes the poles of the closed-loop associated with (1)-(2) to be at $-0.5i$ and $+0.5i$. **(3p)**

Please use English in your solutions!

Problem 2 Consider the following state space representation

$$\dot{x}(t) = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t), \tag{3}$$

$$y(t) = \begin{bmatrix} 1 & -1 \end{bmatrix} x(t). \tag{4}$$

(a) Determine the transfer function associated with (3)-(4). **(2p)**

(b) Can the system in (3)-(4) be a minimum realization? Explain your answer. **(2p)**

(c) Compute the zero-order hold (Z.O.H.) realization (3)-(4) with a sampling step $h > 0$. **(4p)**

Please use English in your solutions!

Problem 3 Consider the following discrete-time state-space representation of a single-input single-output system of a continuous-time system:

$$\begin{pmatrix} qx_1 \\ qx_2 \end{pmatrix} = \begin{pmatrix} h & 0 \\ 1 & e^{-h} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u, \tag{5}$$

$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + u. \tag{6}$$

(a) State the values of $h > 0$ that make the system (5)-(6) controllable. Explain your answer. **(2p)**

(b) State the values of $h > 0$ that make the system (5)-(6) observable. Explain your answer. **(2p)**

(c) If the Z.O.H. realization of a system (not necessarily (5)-(6)) is a minimum realization, then the continuous-time system that gave rise to it is a minimum realization. Please state the validity of the implication and explain your reasoning. **(2p)**

Please use English in your solutions!

Problem 4 Consider the following dynamical system

$$\begin{aligned} x(k+1) &= \begin{bmatrix} 0 & 0 \\ 0 & -0.9 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} v_1(k), \\ y(k) &= \begin{bmatrix} 1 & -1 \end{bmatrix} x(k) + v_2(k), \end{aligned} \tag{7}$$

where the disturbance and sensor noise are uncorrelated, and they are independent and identically distributed over time with unitary intensity.

(a) What is the steady state variance of the stationary stochastic process $x(k)$? **(3p)**

(b) Describe the Kalman filter (or estimator) associated with the dynamics in (7), and compute the optimal gain for such estimator. **(5p)**

(c) Determine the intensity of the innovation $\nu(k) = y(k) - \hat{y}(k|k-1)$ noise. **(2p)**

(d) What is the spectral density $\Phi_z(\omega)$ associated with the reference $z(k) = \begin{bmatrix} 1 & -1 \end{bmatrix} x(k)$? **(3p)**

Please use English in your solutions!

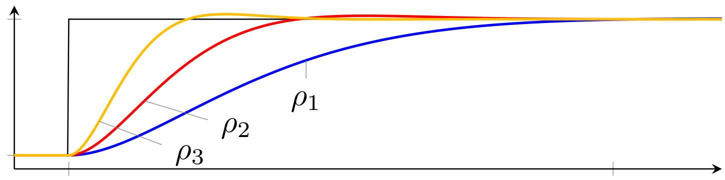
Problem 5 Consider the following objective associated with a continuous-time deterministic linear quadratic optimal control problem

$$V = E[z^2 + \rho_i u^2], \tag{8}$$

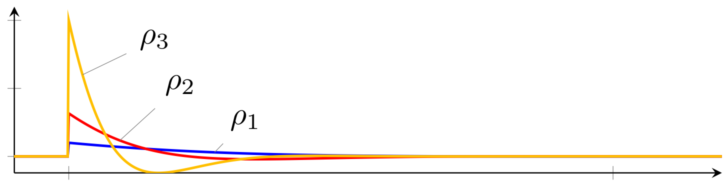
for three different scenarios for the values of ρ_i, with $i = 1, 2, 3$.

Additionally, consider the following reference outputs and inputs associated with the three different scenarios.

- The reference outputs, z:



- The inputs, u:



(a) What is the relation between ρ_1, ρ_2 and ρ_3, i.e., which one has a higher value? Explain your reasoning. **(3p)**

(b) Suppose that the dynamics of the linear quadratic regulator in (8) is as follows:

$$\begin{aligned}\dot{x}(t) &= \begin{bmatrix} 0 & 0 \\ 0 & -0.9 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t), \\ z(t) &= \begin{bmatrix} 1 & -1 \end{bmatrix} x(t). \end{aligned} \tag{9}$$

Compute the steady state optimal control solution for (8), when $\rho_i = 1$, and where the solution to the corresponding algebraic Riccati equation is given by

$$S = \begin{bmatrix} 1 & 0 \\ 0 & 1.7578 \end{bmatrix}.$$

(3p)

(c) Suppose that there is some uncertainty in both dynamics and the measured output associated with the reference output in (9). Specifically, consider the following state-space representation of the previous system in the standard state-space form

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} 0 & 0 \\ 0 & -0.9 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t) + v_1(t), \\ z(t) &= \begin{bmatrix} 1 & -1 \end{bmatrix} x(t), \\ y(t) &= \begin{bmatrix} 1 & -1 \end{bmatrix} x(t) + v_2(t), \end{aligned} \tag{10}$$

where the disturbance and sensor noises are correlated and have intensities R_1, R_2, and $R_{1,2}$, respectively.

What would be the optimal control strategy you would choose to attain the objective in (8)? Explain the different steps and the values to be taken by the known parameters from the statement and equations that would be invoked. It is important to notice that there is no need to compute the numerical results for those equations. **(5p)**