Dynamic Programming
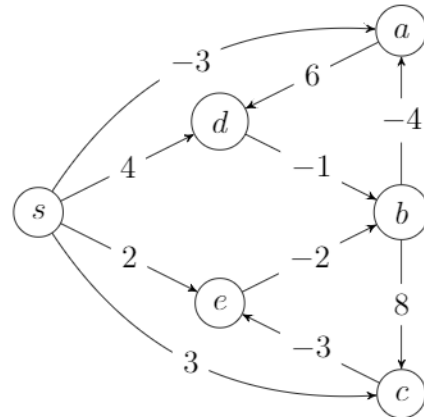
Consider a weighted directed graph G with a set V of n vertices, a set E of m edges, and an edge-weight function $w \colon E \to \mathbb{R}$, such as in the figure to the right.

Consider the following recurrence, parameterised by $\langle \alpha, \beta_1, \ldots, \beta_4, \gamma \rangle$, for a quantity $T[i, v]$:

$$
T[i, v] = \begin{cases}
0 & \text{if } \alpha \wedge (v = s) \\
\beta_1 & \text{if } \alpha \wedge (v \neq s) \\
\gamma \left(T[i-1, \beta_2], \ \min_{(x,v) \in E} (T[i-1, \beta_3] + \beta_4) \right) & \text{if } \neg\alpha
\end{cases}
$$

Question 1: If $T[n-1, v]$ is returned by a correct algorithm for computing the weight of a shortest path in G from s to v, then $T[i, v]$, with $0 \leq i < n$ and $v \in V$, denotes the weight of a shortest path from s to v with how many edges?

| A | $< i$ | | B | $\leq i$ | | C | $= i$ | | D | $\geq i$ | | E | $> i$ |

Question 2: For the graph given above, what is the sum $T[3, a] + T[3, b]$?

| A | -8 | | B | -6 | | C | -5 | | D | -4 | | E | -3 |

Question 3: What is the logical condition α?

| A | $i = 0$ | | B | $i = n - 1$ | | C | $i = m - 1$ | | D | $i = m$ | | E | $i \cdot v = 0$ |

Question 4: What is the numeric expression β_1?

| A | $-\infty$ | | B | -1 | | C | 0 | | D | $+1$ | | E | $+\infty$ |

Question 5: What is the index expression β_2?

| A | s | B | $v-1$ | C | v | D | $v+1$ | E | x |

Question 6: What is the index expression β_3?

| A | s | B | $v-1$ | C | v | D | $v+1$ | E | x |

Question 7: What is the numeric expression β_4?

| A | $w(s,v)$ | B | $w(s,x)$ | C | $w(v,x)$ | D | $w(x,v)$ | E | 1 |

Question 8: What is the two-argument operator γ (written in prefix form above)?

| A | $+$ | B | Σ | C | Π | D | w | E | min |