Problem 1: basic questions (6/30)

Answer only *'true'* or *'false'*. Each correct answer gives 1 point, each wrong answer gives -1 point. Minimum total points for Part A and B is 0 , respectively.

Part A

Note: Write 'skip' if your total home assignment score ≥ 8

i) The following system is observable

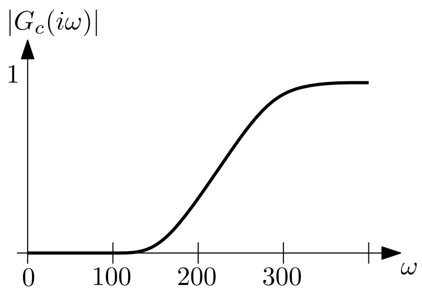
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

ii) The system

$$G(s) = \frac{(s+2)(s+1)}{(s^2+2s-3)(s+5)}$$

is input-output stable.

iii) Consider a closed-loop system $G_c(s)$ with a frequency response as illustrated in the figure. We use a reference signal $r(t) = A\sin(\omega t)$, where $\omega \leq 100$.



Then the output $y(t)$ is zero.

(3 p)

Part B

Note: Write 'skip' if your total home assignment score ≥ 12

i) It is possible to reduce both sensitivity functions $|S(i\omega)|$ and $|T(i\omega)|$ to 0 at every frequency ω.

ii) Given an observable system, the poles of a closed-loop system can never be placed arbitrarily.

iii) Consider a system with transfer function

$$G(s) = \frac{s+1}{s^2+6s+8}.$$

Suppose the same system is described in state-space that is a minimal realization. Then the eigenvalues of A equal -2 and -5.

(3 p)

Problem 2 (6/30)

a) Consider a mechanical system with input $u(t)$ and output $y(t)$ that can be written as

$$Y(s) = G(s)U(s) = \frac{1}{s+a}U(s).$$

We apply feedback controller to output to follow a reference signal $r(t)$. Specifically a P-controller $F(s) = K$ is used with K set to 5.

We now are interested to know for which pole locations of $G(s)$ is the closed-loop system stable: For which a will $G_c(s)$ be stable?

(3 p)

b) Consider the following configuration of the system above

$$Y(s) = G(s)U(s) = \frac{1}{s+3}U(s),$$

which we control using a feedback controller $F(s) = 1 + \frac{K_i}{s}$.

Let the reference signal $r(t)$ be a step. That is,

$$r(t) = \begin{cases} r_0, & t \geq 0 \\ 0 & \text{otherwise} \end{cases}.$$

Set $K_i = 1$ and calculate the stationary error of the controller: $e(t) = r(t) - y(t)$ as $t \to \infty$.

(3 p)

Problem 3 (6/30)

a) Consider a controlling a thermal process with an output modeled as

$$Y(s) = G(s)U(s),$$

where the process model can be written as

$$G(s) = \frac{s+1}{s^2 + 6s + 11}.$$

Assuming the states can be obtained, design a stable closed-loop system from r to y with poles located at -4 and -2.

(4 p)

b) To prepare for the design of a state observer,

$$\dot{\hat{x}} = A\hat{x} + Bu + K(y - C\hat{x}),$$

write the system above on observable canonical form.

(2 p)

Problem 4 (6/30)

(a) Taking a course in computer controlled system, a student has finished the second home assignment. The assignment included a task to generate Bode diagrams along with step responses of both the open loop systems and closed loop systems with unit feedback $F(s) = 1$, that is Bode plots of $G(s)$ and step responses for $G(s)$ and $G_c(s) = \frac{G(s)}{1+G(s)}$. On route to hand in the assignment, the student realizes that the order of the systems are all mixed up in the three figures. Help create a key to pair the Bode diagramd with the step responses.

The Bode diagrams are shown in Figure 1, the open-loop step responses ($G(s)$) are shown in Figure 2 and the closed-loop step responses ($G_c(s)$) are shown in Figure 3

Remember to motivate your answer.

(I) Pair the Bode diagrams A-D in Figure 1 with the open-loop step responses 1-4 in Figure 2.

(2 p)

(II) Pair the Bode diagrams A-D in Figure 1 with the closed-loop step responses I-IV in Figure 3.

(2 p)

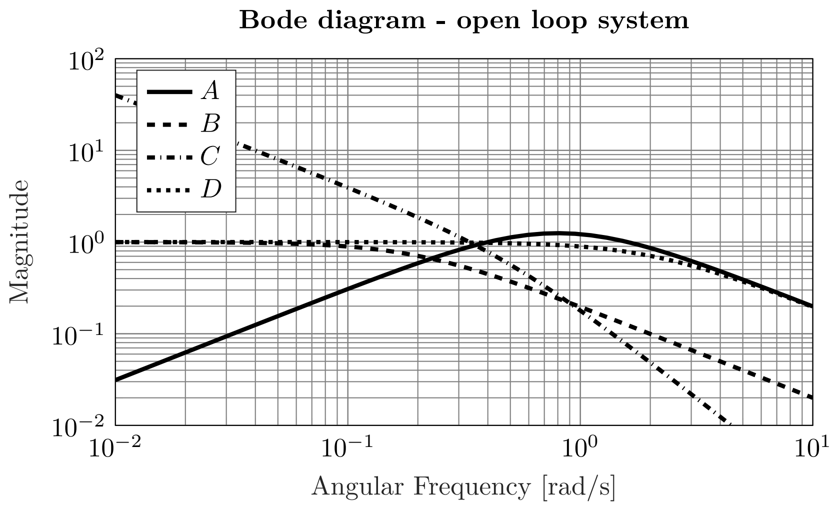


Figure 1: Bode diagrams of the open loop systems.

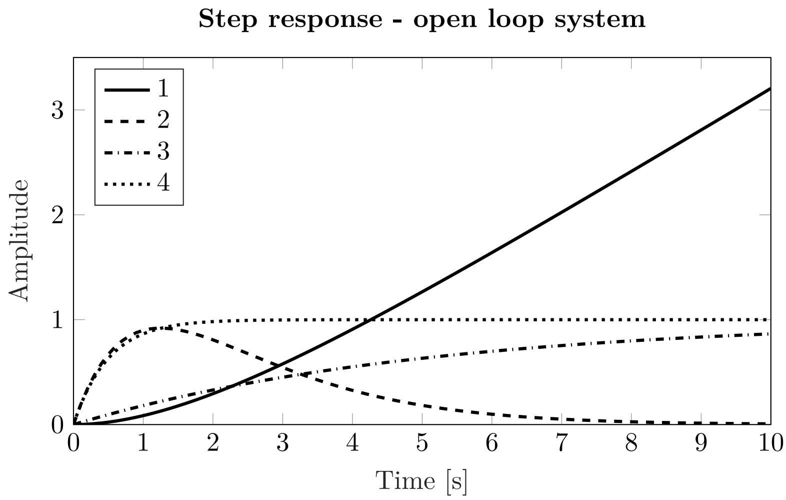


Figure 2: Step responses for the open loop systems.

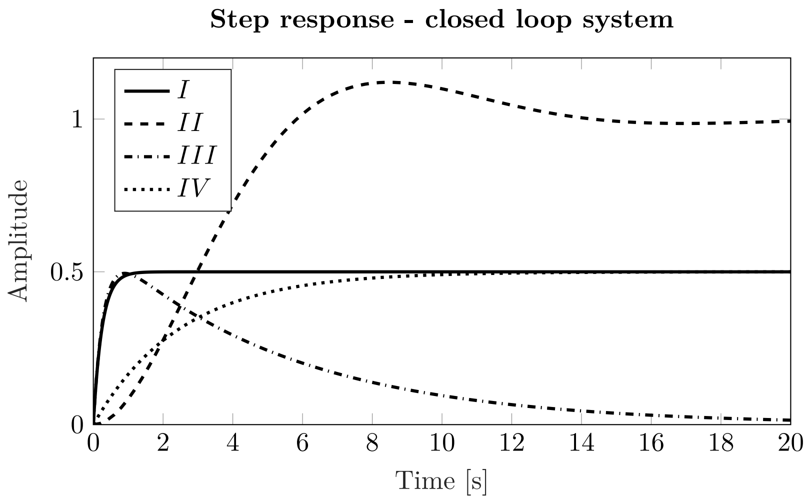


Figure 3: Step responses for the closed loop systems using standard feedback with $F(s) = 1$.

(b) The Bode diagram for an industrial process is shown in Figure 4.

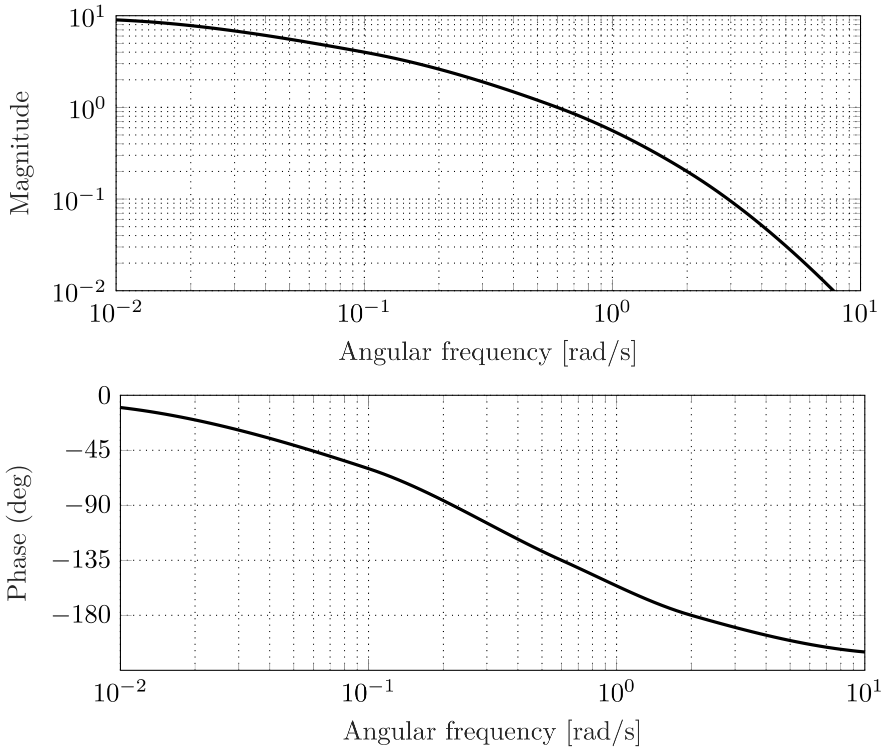


Figure 4: Bode diagram for an industrial process.

(I) What is the crossover frequency[1] ω_c and phase margin[2] φ_m for the process?

(1 p)

(II) Using a P-controller $F(s) = K$, what is the largest possible gain K that gives a stable closed loop system?

(1 p)

[1]sv: skärfrekvens
[2]sv: fasmarginal

Problem 5 (6/30)

A minesweeper vessel uses hydrophones in order to detect underwater mines. In particular, the hydrophones are arranged in an array in order to cover a larger terrain. The transfer function from the drive shaft which is connected to the vessel to the hydrophone array can be given as a second order system:

$$G(s) = \frac{1}{Js^2 + k_d s}$$

where J is the moment of inertia of the array and k_d represents the viscous force of the water. The transfer function above can be rewritten as a state-space representation by using e.g. the controller canonical form:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} -\frac{k_d}{J} & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & \frac{1}{J} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

where u is the input from the drive shaft and y is the output to the array.

We want to control the array with a computer.

a) Discretize the state-space system above with a sampling time T

(3 p)

b) Assume that $J = 2\ \mathrm{Nm\,s^2\,rad^{-1}}$, $k_d = 8\ \mathrm{Nm\,s\,rad^{-1}}$ and $T = \frac{1}{4}$ s. By using state feedback on the discrete-time system, compute a suitable matrix $\mathbf{L}$ so that the system error will decay as e^{-t}, i.e. the poles in the continuous-time system are at -1.

Hint: Substitute in b) before doing anything. Answers in rounded decimals are fine.

(3 p)