Dugga 2014-11-24

(I) Vi multiplicerar först ekv. (1) och (2) med $\frac{1}{2}$ och byter plats på dessa ekvationer. Då har vi

$$\left[\begin{array}{ccccc|c} 1 & 2 & 1 & 0 & 0 & 0 \\ 1 & 2 & 1 & 3 & 1 & 0 \\ 3 & 6 & 3 & c & 2 & 0 \\ 3 & 6 & 6 & 6 & 1 & 3 \end{array}\right] \begin{matrix} \boxed{-1} & \boxed{-3} \\ \leftarrow & \\ & \leftarrow \\ & \leftarrow \end{matrix} \quad \Leftrightarrow$$

$$\left[\begin{array}{ccccc|c} 1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & c & 2 & 0 \\ 0 & 0 & 3 & 6 & 1 & 3 \end{array}\right]$$

Vi studerar nu separat

$$\begin{cases} 3x_4 + x_5 = 0 \\ c x_4 + 2x_5 = 0 \\ 3x_3 + 6x_4 + x_5 = 3 \end{cases}$$

$$\left[\begin{array}{ccc|c} 3 & 6 & 1 & 3 \\ 0 & 3 & 1 & 0 \\ 0 & c & 2 & 0 \end{array}\right] \begin{matrix} \leftarrow & \\ \boxed{-2} & \boxed{-\frac{c}{3}} \\ & \leftarrow \end{matrix} \quad \Leftrightarrow$$

$$\left[\begin{array}{ccc|c} 3 & 0 & -1 & 3 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 2-\frac{c}{3} & 0 \end{array}\right] \Leftrightarrow \left[\begin{array}{ccc|c} 3 & 0 & -1 & 3 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & \frac{6-c}{3} & 0 \end{array}\right]$$

Om $c \neq 6 \Rightarrow \left[\begin{array}{ccc|c} 3 & 0 & -1 & 3 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{l} \leftarrow \\ \leftarrow \\ \textcircled{I} \ \textcircled{-1} \end{array}$

$\Leftrightarrow \left[\begin{array}{ccc|c} 3 & 0 & 0 & 3 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{l} \textcircled{\frac{1}{3}} \\ \textcircled{\frac{1}{3}} \\ \ \end{array} \Leftrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$

dvs ~~$x_3=1x$~~ $x_3 = 1 \quad x_4 = x_5 = 0$

Om $c = 6 \Rightarrow \left[\begin{array}{ccc|c} 3 & 0 & -1 & 3 \\ 0 & 3 & 1 & 0 \end{array}\right]$

Sätt $x_5 = t \Rightarrow x_3 = \frac{3+t}{3}, \ x_4 = -\frac{t}{3}$

Vi återvänder nu till $x_1 + 2x_2 + x_3 = 0$

$c \neq 6 \Rightarrow \quad x_1 + 2x_2 = -1$. Sätt $x_2 = s$

$\Rightarrow x_1 = -1 - 2s$

$c = 6 \Rightarrow \quad x_1 + 2x_2 = -\frac{3+t}{3}$. Sätt $x_2 = s$

$\Rightarrow x_1 = -\frac{3+t}{3} - 2s$

<u>Svar</u> ~~$c \neq 6$~~ $c = 6$ $\begin{cases} x_1 = -\frac{3+t}{3} - 2s \\ x_2 = s \\ x_3 = \frac{3+t}{3} \\ x_4 = -\frac{t}{3} \\ x_5 = t \end{cases}$ $\quad c \neq 6 \begin{cases} x_1 = -1-2s \\ x_2 = s \\ x_3 = 1 \\ x_4 = 0 \\ x_5 = 0 \end{cases}$

② $\det(A) = 1 \cdot \begin{vmatrix} 1 & 1 \\ a+1 & 2 \end{vmatrix} - a \begin{vmatrix} a & 1 \\ a+1 & 2 \end{vmatrix}$

$= 2 - a - 1 - a(2a - a - 1)$

$= 2 - a - 1 - 2a^2 + a^2 + a = 1 - a^2$

$\det(A) \neq 0$ då $a \neq \pm 1$

A inverterbar då $a \neq \pm 1$

A^{-1} kan bestämmas mha Jacobismetod!

Antag att $1-a^2 \neq 0 \Longrightarrow$

$$\left[\begin{array}{ccc|ccc} 1 & a & 0 & 1 & 0 & 0 \\ a & 1 & 1 & 0 & 1 & 0 \\ a+1 & a+1 & 2 & 0 & 0 & 1 \end{array}\right] \begin{array}{c} \circled{-a} \\ \leftarrow \\ \circled{\frac{1}{a+1}} \end{array} \iff$$

$$\left[\begin{array}{ccc|ccc} 1 & a & 0 & 1 & 0 & 0 \\ 0 & 1-a^2 & 1 & -a & 1 & 0 \\ 1 & 1 & \frac{2}{a+1} & 0 & 0 & \frac{1}{a+1} \end{array}\right] \begin{array}{c} \circled{-1} \\ \circled{\frac{1}{1-a^2}} \\ \leftarrow \end{array} \iff$$

$$\left[\begin{array}{ccc|ccc} 1 & a & 0 & 1 & 0 & 0 \\ 0 & 1 & \frac{1}{1-a^2} & -\frac{a}{1-a^2} & \frac{1}{1-a^2} & 0 \\ 0 & 1-a & \frac{2}{a+1} & -1 & 0 & \frac{1}{a+1} \end{array}\right] \begin{array}{c} \leftarrow \\ \circled{-a} \\ \circled{\frac{1}{1-a}} \end{array} \iff$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{a}{1-a^2} & \left(1 + \frac{a^2}{1-a^2}\right) & -\frac{a}{1-a^2} & 0 \\ 0 & 1 & \frac{1}{1-a^2} & -\frac{a}{1-a^2} & \frac{1}{1-a^2} & 0 \\ 0 & 1 & \frac{2}{1-a^2} & -\frac{1}{1-a} & 0 & \frac{1}{1-a^2} \end{array}\right] \begin{array}{c} \\ \circled{-1} \\ \leftarrow \end{array}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{-a}{1-a^2} & \frac{1}{1-a^2} & \frac{-a}{1-a^2} & 0 \\ 0 & 1 & \frac{1}{1-a^2} & -\frac{a}{1-a^2} & \frac{1}{1-a^2} & 0 \\ 0 & 0 & \frac{1}{1-a^2} & \left(\frac{-1}{1-a} + \frac{a}{1-a^2}\right) & \frac{-1}{1-a^2} & \frac{1}{1-a^2} \end{array}\right]$$

$$\Leftrightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{a}{1-a^2} & \frac{1}{1-a^2} & \frac{-a}{1-a^2} & 0 \\ 0 & 1 & \frac{1}{1-a^2} & -\frac{a}{1-a^2} & \frac{1}{1-a^2} & 0 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{array}\right] \leftarrow \boxed{\frac{a}{1-a^2}}$$

$$\Leftrightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1-a}{1-a^2} & -\frac{2a}{1-a^2} & \frac{a}{1-a^2} \\ 0 & 1 & \frac{1}{1-a^2} & -\frac{a}{1-a^2} & \frac{1}{1-a^2} & 0 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{array}\right] \leftarrow \boxed{-\frac{1}{1-a^2}}$$

$$\Leftrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \\ 0 & 0 & 1 & \end{array}\right. \underbrace{\left.\begin{array}{ccc} \frac{1-a}{1-a^2} & -\frac{2a}{1-a^2} & \frac{a}{1-a^2} \\ \frac{1-a}{1-a^2} & \frac{2}{1-a^2} & -\frac{1}{1-a^2} \\ -1 & -1 & 1 \end{array}\right]}_{A^{-1}}$$

Svar A inverterbar da $a \neq \pm 1$

$$A^{-1} = \frac{1}{1-a^2}\begin{bmatrix} 1-a & -2a & a \\ 1-a & 2 & -1 \\ a^2-1 & a^2-1 & 1-a^2 \end{bmatrix}$$

③ $B + X^{-1} = C \iff X^{-1} = C - B$

Om $C-B$ är inverterbar $\Rightarrow$

$$X = (X^{-1})^{-1} = (C-B)^{-1}$$

$$C-B = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Jacobismetod:

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \leftarrow \\ (-2) \\ \end{array} \iff$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & -2 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \leftarrow \\ \leftarrow \\ (1) \quad (-2) \end{array} \iff$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \underbrace{\begin{matrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{matrix}}_{(C-B)^{-1}} \\ 0 & 0 & 1 & \end{array}\right]$$

Svar $X = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

④ $$\begin{vmatrix} x & 2 & x & 2x \\ x & x & x-1 & 2x \\ x-1 & x & x-2 & 1 \\ x & x & x-3 & 2x \end{vmatrix} \begin{matrix} (-1) \\ \leftarrow \\ \\ \leftarrow \end{matrix} =$$

$$= \begin{vmatrix} x & 2 & x & 2x \\ 0 & x-2 & -1 & 0 \\ x-1 & x & x-2 & 1 \\ 0 & x-2 & -3 & 0 \end{vmatrix} = \left\{ \begin{matrix} \text{utveckling} \\ \text{längs} \\ \text{4:e kolumnen} \end{matrix} \right\}$$

$$= -2x \begin{vmatrix} 0 & x-2 & -1 \\ x-1 & x & x-2 \\ 0 & x-2 & -3 \end{vmatrix} - 1 \cdot \begin{vmatrix} x & 2 & x \\ 0 & x-2 & -1 \\ 0 & x-2 & -3 \end{vmatrix}$$

$$= 2x(x-1) \underbrace{\begin{vmatrix} x-2 & -1 \\ x-2 & -3 \end{vmatrix}}_{4-2x} - x \underbrace{\begin{vmatrix} x-2 & -1 \\ x-2 & -3 \end{vmatrix}}_{4-2x}$$

$$= (4-2x)(2x^2-2x-x) =$$

$$= (4-2x)(2x-3)\cdot x \quad .$$

Ekvationen blir således

$$(4-2x)(2x-3)x = 0$$

<u>Svar</u> $x = 2$, $x = \frac{3}{2}$, $x = 0$.