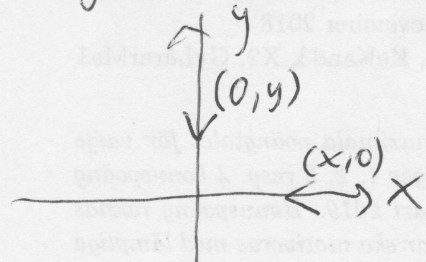
# Tenta 2012-05-24

**1.** Vi testar vad händer om vi närmar oss $(0,0)$ längs koordinataxlarna:




$$\lim_{(x,0)\to(0,0)} \frac{2x^2+0}{x^2+0} = \lim_{x\to 0} \frac{2x^2}{x^2} = 2$$

$$\lim_{(0,y)\to(0,0)} \frac{0+y^2}{0+y^2} = \lim_{y\to 0} \frac{y^2}{y^2} = 1$$

Olika resultat längs olika vägar $\Rightarrow$ gränsvärdet existerar INTE.

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**2** och **3** – identiska som på tentan 2011-12-12

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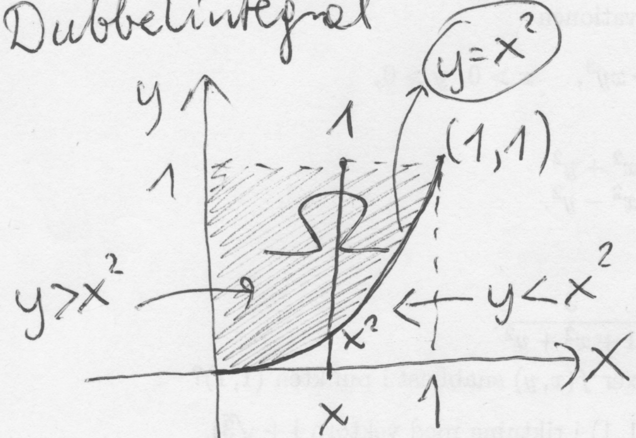
**4.** Dubbelintegral




$$\iint_{\Omega} \frac{x}{1+y^2}\,dx\,dy = \left[\begin{array}{c}y\text{-enkelt}\\ \text{område}\end{array}\right] = \int_0^1 \left( \int_{x^2}^1 \frac{x}{1+y^2}\,dy \right) dx =$$

$$= \int_0^1 \Big[ x \arctan y \Big]_{y=x^2}^{y=1} dx = \int_0^1 \left( -x\arctan x^2 + \frac{\pi}{4}x \right) dx =$$

$$\boxed{\begin{array}{c}\arctan 1 = \frac{\pi}{4} \\ \Uparrow \\ \tan\frac{\pi}{4} = 1\end{array}}$$

$$= -\frac{1}{2}\int_0^1 2x \cdot \arctan x^2\,dx + \left[ \frac{\pi x^2}{8} \right]_0^1 = \left[ \begin{array}{c}\text{sub.}\\ x^2 = t \Rightarrow 2x\,dx = dt\end{array} \right]$$

$$= -\frac{1}{2} \cdot \int_0^1 \arctan t\,dt + \frac{\pi}{8} = \circledast$$

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$$\int \arctan t\,dt = \left[ \begin{array}{l}\text{partiell int.}\\ f'=1,\ g=\arctan t\\ f=t,\ g'=\frac{1}{1+t^2}\end{array} \right] = t\arctan t - \int \frac{t}{1+t^2}\,dt =$$

$$= t\cdot\arctan t - \frac{1}{2}\ln(1+t^2) + C.$$

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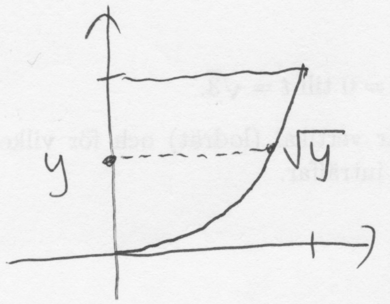
Tillbaka till dubbelint:

$$\circledast = -\frac{1}{2}\cdot\left[t\arctan t - \frac{1}{2}\ln(1+t^2)\right]_0^1 + \frac{\pi}{8} =$$

$$= -\frac{1}{2}\left(\arctan 1 - \frac{1}{2}\ln 2 - 0 + \frac{1}{2}\ln 1\right) + \frac{\pi}{8} =$$

$$= -\frac{1}{2}\cdot\left(\frac{\pi}{4} - \frac{1}{2}\ln 2\right) + \frac{\pi}{8} = \boxed{\frac{1}{4}\ln 2} \leftarrow \text{svar.}$$

Möjligen blir det enklare om vi ser $\Omega$ som ett x-enkelt område:




$$\iint_\Omega \frac{x}{1+y^2}\,dx\,dy = \int_0^1\left(\int_0^{\sqrt{y}}\frac{x}{1+y^2}\,dx\right)dy =$$

$$= \frac{1}{2}\int_0^1\left[\frac{x^2}{1+y^2}\right]_{x=0}^{x=\sqrt{y}} dy =$$

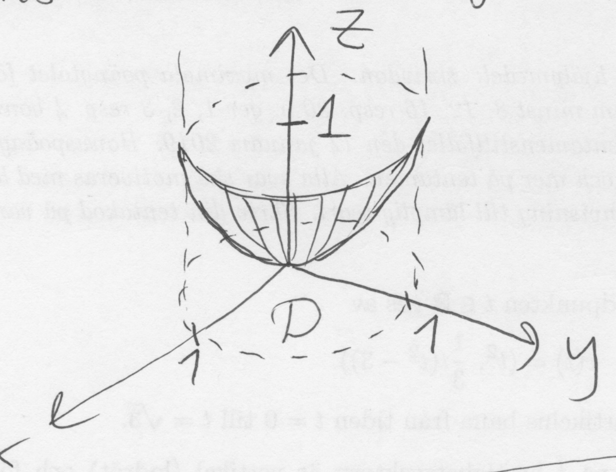
$$= \frac{1}{2}\cdot\int_0^1 \frac{y}{1+y^2}\,dy = \left[\begin{array}{l}\text{subst. } 1+y^2 = t\\ 2y\,dy = dt\end{array}\right] =$$

$$= \frac{1}{4}\cdot\int_1^2 \frac{dt}{t} = \frac{1}{4}\cdot\left[\ln t\right]_1^2 =$$

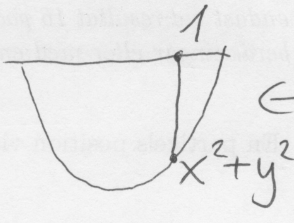
$$= \frac{1}{4}(\ln 2 - \ln 1) = \boxed{\frac{1}{4}\ln 2}$$

5. $\displaystyle\iiint_{\Omega} z\sqrt{x^2+y^2}\,dx\,dy\,dz = \circledast$

$\Omega$ är z-enkelt område inuti paraboloiden.
Cylindriskt koordinatbyte fungerar bäst.




$D: x^2+y^2 \leq 1$




← integr. gränser på z

$$\circledast = \iint_D \left( \int_{x^2+y^2}^{1} z\sqrt{x^2+y^2}\,dz \right) dx\,dy =$$

$$= \iint_D \underbrace{\left[ \frac{z^2}{2}\sqrt{x^2+y^2} \right]_{x^2+y^2}^{1}}_{\text{gränserna för } z} dx\,dy = \frac{1}{2}\cdot \iint_D \Big( \underbrace{-(x^2+y^2)^2\sqrt{x^2+y^2}}_{z=x^2+y^2} + \underbrace{\sqrt{x^2+y^2}}_{z=1} \Big)\,dx\,dy$$

insättning

$$= \left[ \begin{array}{l} \text{polärt koord.} \\ x = r\cos\theta \\ y = r\sin\theta \\ dx\,dy = r\,dr\,d\theta \\ x^2+y^2 = r^2 \\ 0 \leq r \leq 1,\ 0 \leq \theta \leq 2\pi \end{array} \right] = \frac{1}{2}\cdot \int_0^{2\pi}\int_0^1 \left(-r^4\cdot r + r\right) r\,dr\,d\theta =$$

integranden beror ej på $\theta$

$$= \pi \cdot \int_0^1 (-r^6 + r^2)\,dr = \pi \cdot \left[ -\frac{r^7}{7} + \frac{r^3}{3} \right]_0^1 =$$

$$= \pi \cdot \left( -\frac{1}{7} + \frac{1}{3} \right) = \pi \cdot \frac{-3+7}{21} = \boxed{\frac{4\pi}{21}}.$$

(6.) Alla antaganden i Greens sats (förutom kurvens orientering) är uppfyllda.

Därför är $\oint P\,dx + Q\,dy = -\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dxdy =$

$$\boxed{\begin{aligned} P(x,y) &= xy^2 \Rightarrow \frac{\partial P}{\partial y} = 2xy \\ Q(x,y) &= -yx^2 \Rightarrow \frac{\partial Q}{\partial x} = -2xy \end{aligned}}$$

$$= -\iint_D (-2xy - 2xy)\,dxdy = 4 \cdot \iint_D xy\,dxdy =$$

$D$: $x^2+y^2 \leq 1$

$$= \left[\begin{aligned} &\text{polärt koordinatbyte} \\ &x = r\cos\theta \\ &y = r\sin\theta \\ &dxdy = r\,dr\,d\theta, \\ &0 \leq r \leq 1,\ 0 \leq \theta \leq 2\pi \end{aligned}\right] = 4 \cdot \int_0^{2\pi}\int_0^1 r^2 \cos\theta \sin\theta\, r\,dr\,d\theta =$$

$$= 4 \cdot \underbrace{\int_0^{2\pi} \cos\theta \sin\theta\,d\theta}_{\boxed{\begin{aligned}&\text{subst} \\ &\sin\theta = t \\ &\cos\theta\,d\theta = dt\end{aligned}}} \int_0^1 r^3\,dr = 4 \cdot \underbrace{\left[\frac{\sin^2\theta}{2}\right]_0^{2\pi}}_{=0} \cdot \left[\frac{r^4}{4}\right]_0^1 = 0$$

$\boxed{\text{Svar: } 0!}$

(8.) – Se lösning till BÖ9 (i filen Lektion16-fyra-valda-uppgifter.

(7.) Räknas på samma sätt som upp. 7 i 2011-12-12, fast cylindern är liggande längs x-axeln.