ℹ Read and follow these instructions carefully to increase your chance of getting good marks.

This is an open book exam. You may use any material that was written prior to the start of the exam, but you must not communicate with anybody else about the exam while it is ongoing (i.e., before 14:00).

Send any questions during the exam by email to johannes.borgstrom@it.uu.se. Answers that are relevant for all students will appear in Piazza.

This exam contains several kinds of questions. Among them are multiple-choice questions, where only a single answer is correct (unless otherwise stated). These questions are followed by a motivation question, where you must give a sufficient motivation to why your chosen answer is the correct one. The multiple-choice question and its motivation count for one point each.

You can at most obtain 48 points on this exam. The preliminary grade limits are: 24p for grade 3, 34p for grade 4, 44p for grade 5.

Read the questions carefully, and watch out for negations (**not**, **except**, et c.).

Good luck!

1 What is the postorder traversal of the expression tree corresponding to ((3 * 1) + (4 - (11 / 5))) + 3?
Välj ett alternativ:

- 3 1 * 4 11 5 / - + 3 +
- 3 1 * 4 + 11 - 5 / 3 +
- No other alternative is correct.
- + + * 3 1 - 4 / 11 5 3
- + 3 + - / 5 11 4 * 1 3

Totalpoäng: 1

2 Motivate your answer to the previous question.

Skriv in ditt svar här

Totalpoäng: 1

3 Consider the following parts of a data-structure invariant for a general tree where each node has a key:

1. If the current node has k children, then they each have 0, 1, ..., k-1 children, in that order.
2. Each child of the current node (except for the leftmost) has a key that is greater than the key of the child to its left.
3. The key in the current node is less than or equal to any key in its subtrees.

Which combination of these corresponds to an invariant for an implementation of a binomial tree?

Välj ett alternativ:

- 1 and 3 hold at every node.
- 1, 2, and 3 hold at every node.
- 1 holds at every node.
- 1 holds at the root.
- 1 and 2 hold at every node.

Totalpoäng: 1

4 Motivate your answer to the previous question.

Skriv in ditt svar här

Totalpoäng: 1

5 How many nodes are there at least in a red-black tree of black height 3?
Välj ett alternativ:

- 4
- None of the above.
- 7
- 3
- 31

Totalpoäng: 1

6 Motivate your answer to the previous question.
Skriv in ditt svar här

Totalpoäng: 1

7 The worst-case time complexity of insertion in a red-black tree of height h is
Välj ett alternativ:

- O(2^h)
- O(h^2)
- O(h)
- O(1)
- O(log h)

Totalpoäng: 1

8 Motivate your answer to the previous question.
Skriv in ditt svar här

Totalpoäng: 1

9 Consider the following declaration, and assume that the function foo is pure.
foo :: Ord a => [a] -> [a]
Which statement below is **false**?
Välj ett alternativ:

- The type a must belong to the typeclass Ord.
- The implementation of foo can use overloaded definitions of the function ==.
- foo is a polymorphic function.
- The call 'foo []' must return the empty list.
- The implementation of foo can use overloaded definitions of the function signum.

Totalpoäng: 1

10 Motivate your answer to the previous question.
Skriv in ditt svar här

Totalpoäng: 1

11 Assume that a Table (as seen in Assignment 3) contains the entries

1 -> "PKD, 2 -> "2021".

What are the contents of the table if we perform the following operations?

exists 3, insert 1 "Easter", lookup 1, insert 2 "Break", delete 2

Välj ett alternativ:

- 1 -> "Easter"
- A runtime exception is thrown.
- 1 -> "PKD", 1 -> "Easter", 2 -> "2021"
- 1 -> "Easter", 2 -> "2021"
- 1 -> "PKD", 2 -> "2021"

Totalpoäng: 1

12 Motivate your answer to the previous question.

Skriv in ditt svar här

Totalpoäng: 1

13 Consider the following fragment of code

```
twoNumbers :: StdGen  -> (Int, Int)
twoNumbers gen =
  let (x, newGen) = random gen
      (y, newGen') = random gen
  in (x,y)
seed = 134342300
main :: IO ()
main = do
  let (x,y) = twoNumbers (mkStdGen seed)
  if x == y
    then putStrLn("The numbers are the same.")
    else putStrLn("The numbers are different.")
```

When running main which of the following statements are true?
Välj ett alternativ:

- If you change the value of seed then it is possible to get the message The numbers are different.
- When main is run it always prints The numbers are the same.
- When main is run it always prints The numbers are the different.
- When main is run sometimes it prints The numbers are the same and sometims it prints The numbers are different.

Totalpoäng: 1

14

Motivate your answer to the previous question.

Skriv in ditt svar här

Totalpoäng: 1

15 In the standard library for Haskell there is an array data type with type `IOArray Int a`. One of the following statements is true.

Välj ett alternativ:

- Arrays when used inside the IO Monad provide constant time access to elements.
- Arrays can be used either in pure code or within the IO Monad, but when used in pure code they are implemented as a binary search tree.
- Arrays can be used in pure code and provide constant time access to elements.
- Arrays cannot be used in pure code, and are implemented as a imperative red black trees.

Totalpoäng: 1

16 Motivate your answer to the previous question.

Skriv in ditt svar här

Totalpoäng: 1

17 Consider a hash table with 20 slots that uses chaining to resolve conflicts. What is the *maximal* possible chain length (worst case) of a single slot when the load factor is 1.5

Välj ett alternativ:

- 30
- 1
- 64
- None of the above the load factor can never be larger than 1
- 20

Totalpoäng: 1

18 Motivate your answer to the previous question.

Skriv in ditt svar här

Totalpoäng: 1

19 Consider a hash table with 10 slots using the hash function h(k) = k mod 10 with linear probing, and and the following 10 integers

After insert the following 10 items into the hash table 1 11 3 33 8 9 7 88 99 100

Which item will be in slot 5

Välj ett alternativ:

- 88
- 11
- 1
- Nothing

Totalpoäng: 1

20 Motivate your answer to the previous question.

Skriv in ditt svar här

Totalpoäng: 1

21 Given a graph with nodes A,B,C,D,E,F,G,H,I,J and directed edges

```
A | B
B | C, H
C | D
F | G
G | B
H | I
```

Nodes not listed have no outgoing edges.
Which of the following options is *not* a topological sort of the graph
Välj ett alternativ:

- E J F G A B C H I D
- E J A B F G H C D I
- F G A B H I C D E J
- A F G B C D J E H I

Totalpoäng: 1

22 Motivate your answer to the previous question.
Skriv in ditt svar här

Totalpoäng: 1

23 Given a graph with nodes A,B,C,D,E,F,G and (non directed) edges

```
A | B C
B | A D
C | A D
D | B C E F
E | D G
F | G D
G | E F
```

which of the following is **not** a breadth first search (BFS) order.

Välj ett alternativ:

- D B C E F A G
- D E F B C A G
- A B C D E F G
- D B A C E F G

Totalpoäng: 1

24 Motivate your answer to the previous question.
Skriv in ditt svar här

Totalpoäng: 1

25 Recall the definition of arithmetic expressions over integers (with plus and minus only):
data Expr = Lit Integer | Add Expr Expr | Sub Expr Expr deriving (Eq, Show)

a) Write a function that given an expression e, returns the number of literals 2 in e. (1p)
b) write a function that uses the associativity of addition (t1 + (t2 + t3) = (t1 + t2) + t3) to put all sums in a given expression into right associate form (t1 + (t2 + (... + (tn)...))). (3p)
Skriv in ditt svar här

```
1
```

Totalpoäng: 4

26 Consider a path from the root to a leaf in a binary search tree.
Is it always true that any node to the left of the path is less than every node on the path?
Give your answer, and a proof why it is true. (4p)

Totalpoäng: 4

27 Some algorithms need an ADT that can behave both as a queue and as a stack. Define the interface for a module that implements such a data structure, where we can add and remove elements at both ends. Include the types of functions, but not their documentation nor their implementation.

Skriv in ditt svar här (2p)

```
1
```

Totalpoäng: 2

28 Some algorithms need an ADT that can behave both as a queue and as a stack, where we can add and remove elements at both ends.

What concrete Haskell datatype would you use to implement the above datatype? Motivate your answer.

Skriv in ditt svar här (2p)

Totalpoäng: 2

29 Consider the following function. The function is to return the maximum element in non-empty set. If the list is empty the function returns 0.

```
maxelem :: [Int] -> Int
maxelem [] = 0
maxelem [x] = x
maxelem (x:xs) =
  let smaller = maxelem [a | a <- xs,  a < x]
      bigger =  maxelem [a | a <- xs,  a > x]
  in if smaller < bigger then bigger else smaller
```

The function is not correct. Your job is to work out why it is not correct and then:

- Write 3 test cases that show when the function is not performing correctly.
- Provide a corrected version of the above function.

You should make it clear with comments were the test cases are. If possible then you should use the HUnit syntax.

Skriv in ditt svar här

```
1
```

Totalpoäng: 4

30 Consider the following pieace of code:

```
main = putStrLn "Please type two strings:" >>
        getLine >>= (\x1 -> getLine >>=
                       (\x2 ->
                          if x1 == x2
                          then putStrLn "Equal"
                          else putStrLn "NotEqual"
                       )
                     )
```

Write an equivlent piece of code using the `do` notation. You must also explain why your code is equivelent to the above fragement. You can put your explanations as comments in the code.

.

Skriv in ditt svar här

```
1
```

Totalpoäng: 4

31 Consider the following graph on the nodes A,B,C,D,E,F with directed edges

```
A | B
B | A C
C | D E
D | F
E | F
```

List all the strongly connected components of the graph.

Skriv in ditt svar här

Then apply (Kosaraju) algorithm to compute strongly connected components from the slides. You should explain each step in the algorithm.

Skriv in ditt svar här

Totalpoäng: 4