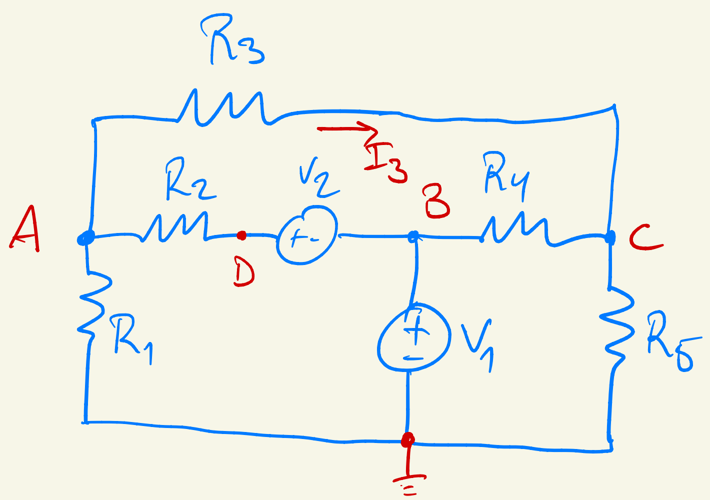
EXAM 1TE717 HT20, 2020-10-22

A1



Q: Find I_3.

Using the node voltage method:

$V_B = V_1 = 6V$; $V_D = V_2 + V_1 = 9V$

$V_A = ?$; $V_C = ?$ (2 equations)

KCL at A: $\frac{V_A}{R_1} + \frac{V_A - V_D}{R_2} + \frac{V_A - V_C}{R_3} = 0$

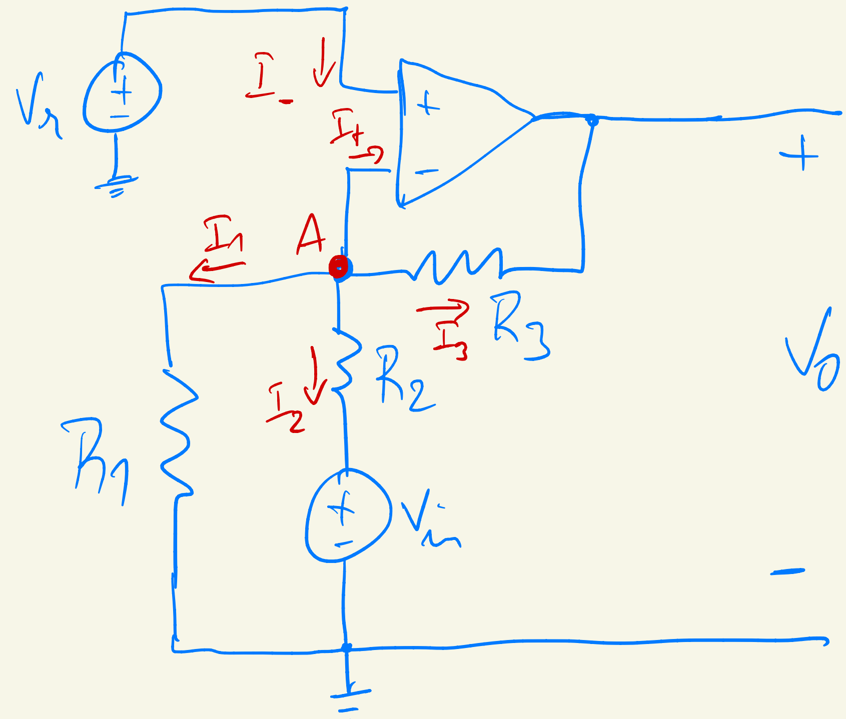
KCL at C: $\frac{V_C}{R_5} + \frac{V_C - V_B}{R_4} + \frac{V_C - V_A}{R_3} = 0$

$$(1) \begin{cases} \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) V_A - \frac{1}{R_3} V_C = \frac{9}{R_2} \\ -\frac{1}{R_3} V_A + \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right) V_C = \frac{6}{R_4} \end{cases}$$

(solve (1) for V_A & V_C)

$$I_3 = \frac{V_A - V_C}{R_3}$$ (with the direction depicted in the figure)

A2



- Find V_0 as a function of V_{in}, V_r
- Find I_3.

KCL at A: $$\frac{V_A}{R_1} + \frac{V_A - V_{in}}{R_2} + \frac{V_A - V_0}{R_3} + \underbrace{\cancel{I_+}}_{=0 \text{ (ideal OpAmp)}} = 0$$

$$V_A = \underline{V_- = V_+} = V_r$$

(ideal OpAmp in negative feedback)

$$\Rightarrow \frac{V_0}{R_3} = -\frac{V_{in}}{R_2} + \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) V_r \ (=)$$

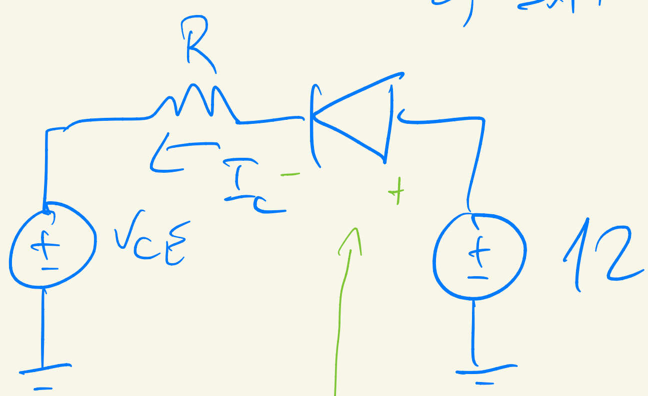
$$V_o = -\frac{R_3}{R_2} V_{in} + \left(1 + \frac{R_3(R_1+R_2)}{R_1 R_2}\right) V_r$$

$$\therefore V_0 = -0.25\, V_{in} + 1.75\, V_r$$

$$I_3 = \frac{V_A - V_0}{R_3} = \frac{V_r - V_0}{R_3} = \frac{1 - (-1.25 + 1.75)}{100} = \frac{0.5}{100} = 5 \text{ mA}$$

A_3

Switch : $V_{CE} = V_{CE,SAT} = 0.2$ ✓



$V_D = 1.5V$ ✓

KVL : $12 - V_D - R_D I_c - R I_c - V_{CE} = 0 \iff$

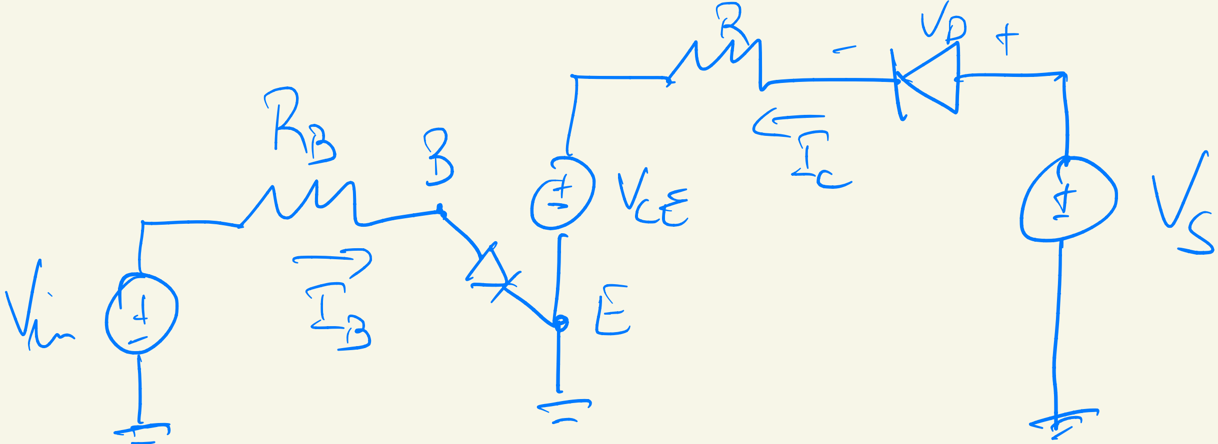
$12 - 1.5 - 0.2 = (R + R_D) I_c \iff$

$I_c = \frac{10.3}{R + R_D}$; $I_c \leq 20 \, mA \iff$

$\frac{10.3}{R+20} \leq 20 \cdot 10^{-3} \iff$

$R + 20 \geq \frac{10.3}{20 \cdot 10^{-3}} = 515 \, \Omega$

$R \geq 495 \, \Omega$



choose R_B so that $I_B \in \left[\frac{I_c}{\beta}, 1mA\right]$

$$I_B = \frac{V_{in} - V_B}{R_B} = \frac{5 - 0.6}{R_B} = \frac{4.4}{R_B}$$

$$I_B \le 1mA \iff \frac{4.4}{R_B} \le 10^{-3} \iff R_B \ge 4.4\,k\Omega$$

$$I_B \ge \frac{I_c}{\beta} \iff \beta I_B \ge \frac{10.3}{R+R_D}$$ (I_c when $V_{CE} = V_{CE,SAT}$)

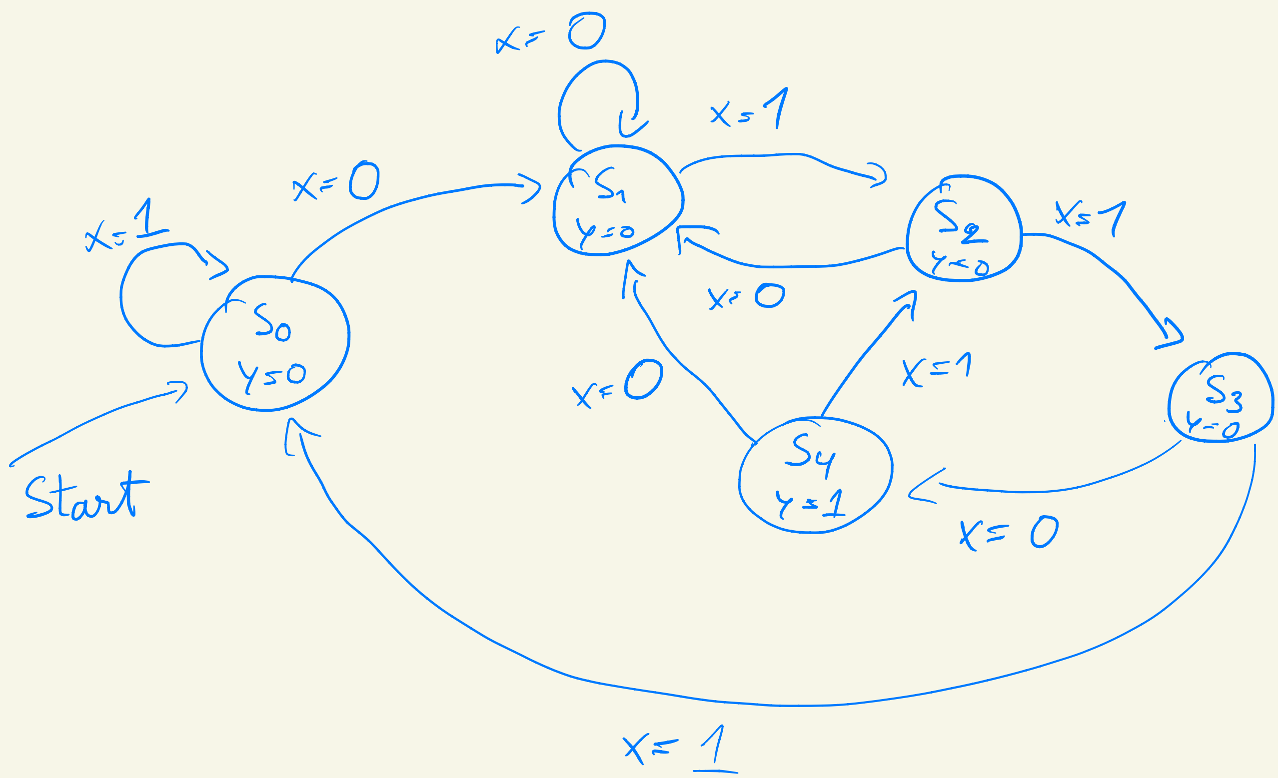
$$\iff \frac{\beta\, 4.4}{R_B} \ge \frac{10.3}{R+R_D} \iff R_B \le \frac{\beta \cdot 4.4(R+R_D)}{10.3}$$

Supposing $R \approx 500\,\Omega$; $R_B \le \frac{300 \times 4.4 \times 500}{10.3} \iff$

$$R_B \le \frac{6.6 \times 10^5}{10.3} \simeq 64\,k\Omega$$

$\therefore R_B = 5\,k\Omega$ would do.

A4. a)



State coding | output (for Moore circuit)

	q_2	q_1	q_0	y
S_0	0	0	0	0
S_1	0	0	1	0
S_2	0	1	0	0
S_3	0	1	1	0
S_4	1	0	0	1

Transition Truth Table

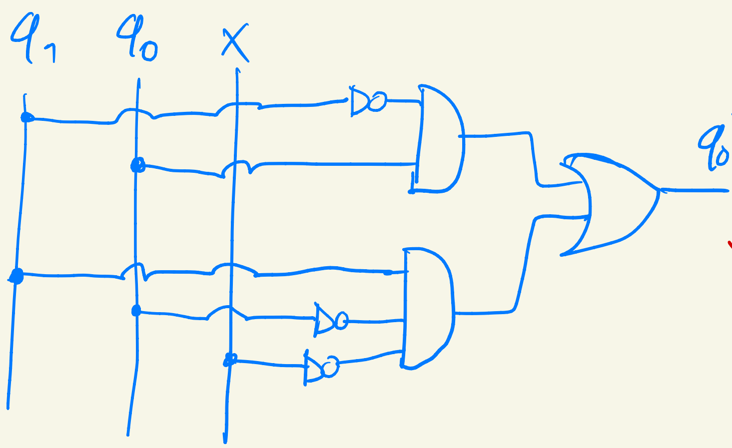
q_2	q_1	q_0	x_i	q_2^+	q_1^+	q_0^+
0	0	0	0	0	0	1
0	0	0	1	0	0	0
0	0	1	0	0	0	1
0	0	1	1	0	1	0
0	1	0	0	0	0	1
0	1	0	1	0	1	1
0	1	1	0	1	0	0
0	1	1	1	0	0	0
1	0	0	0	0	0	1
1	0	0	1	0	1	0
–	–	–	–			

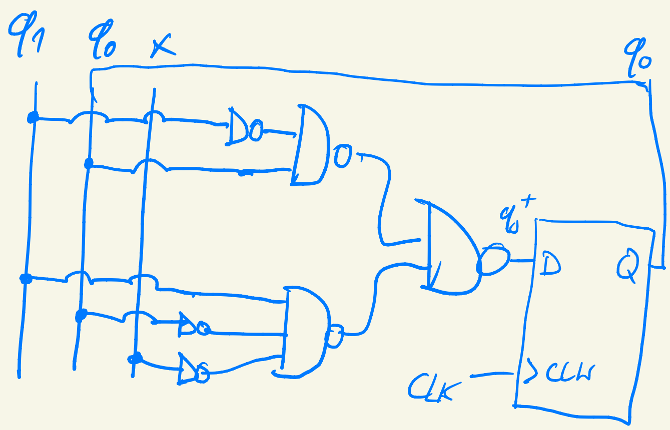
3 flip-flops are needed

A4 b)

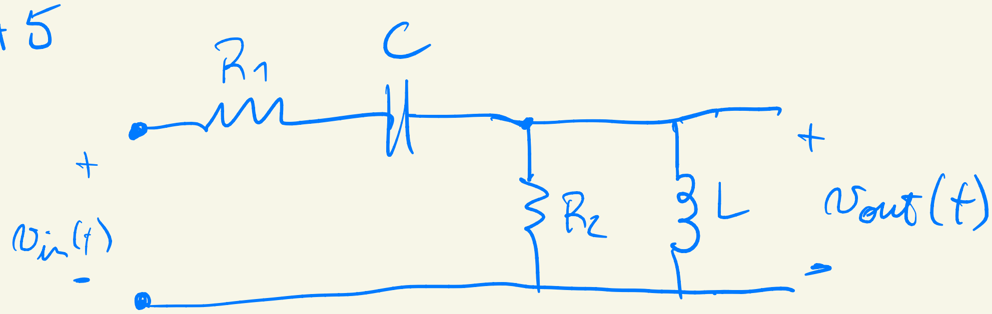
q_2q_1 \ q_0x	00	01	11	10
00	–		1	1
01	1			–
11	1			
10			–	1

$$q_0^+ = \overline{q_1} \cdot q_0 + q_1 \cdot \overline{q_0} \cdot \overline{x}$$





A5



$v_{in}(t) = 2\sin\left(10^5\pi t - \frac{\pi}{3}\right)$

- Find $v_{out}(t)$
- classify the type of filter

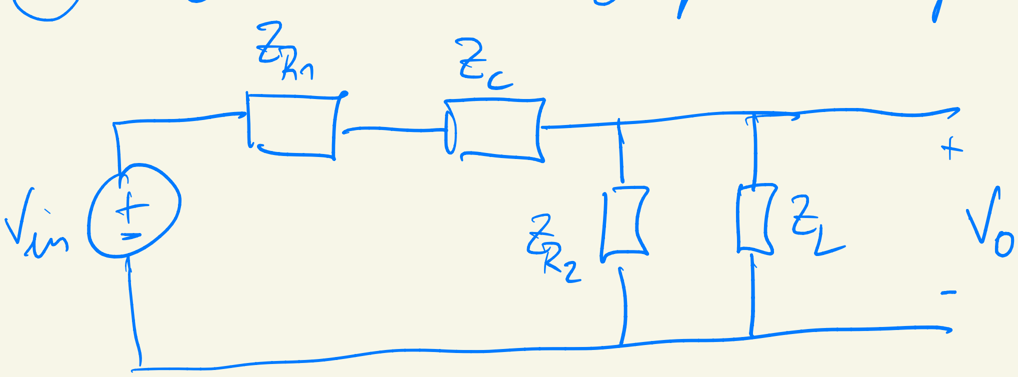
① Convert all voltages to phasors.

$$v_{in}(t) = \mathrm{Im}\left(V_{c_{in}}(t)\right) \;;\; V_{c_{in}}(t) = 2e^{j\omega t}e^{j\left(-\frac{\pi}{3}\right)}$$

$$\text{Phasor: } V_{in} = 2e^{-j\frac{\pi}{3}} \quad \left(\text{so } V_{c_{in}}(t) = V_{in}\, e^{j\omega t}\right)$$

$\omega = 10^5\pi$ rad/s.

② Convert to complex impedances and phasors

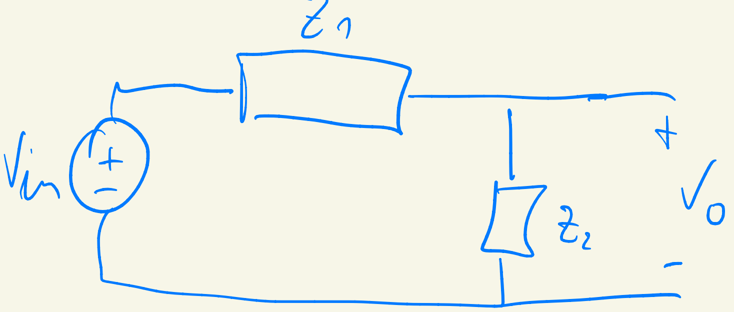


$Z_{R_1} = R_1$, $Z_{R_2} = R_2$

$Z_C = \dfrac{1}{j\omega C}$

$Z_L = j\omega L$

③ (Simplify and) Analyze the circuit



$$Z_1 = Z_{R_1} + Z_C = R + \frac{1}{j\omega C}$$

$$Z_2 = Z_{R_2} \,//\, Z_L = \frac{Z_{R_2} Z_L}{Z_{R_2} + Z_L}$$

$$V_o = \frac{Z_2}{Z_1 + Z_2} V_{in}$$ (It is a voltage divider)

OK to stop ③ here & go to ④

$$\boxed{H(j\omega) = \frac{V_o}{V_{in}} = \frac{Z_2}{Z_1 + Z_2} = \frac{1}{\frac{Z_1}{Z_2} + 1}} = \frac{1}{\frac{(Z_{R_2}+Z_L)(Z_{R_1}+Z_C)}{Z_{R_2} Z_L} + 1} =$$

$$= \frac{Z_{R_2} Z_L}{(Z_{R_1}+Z_C)(Z_{R_2}+Z_L) + Z_{R_2} Z_L} = \frac{j\omega R_2 L}{Z_{R_1} Z_{R_2} + Z_{R_1} Z_L + Z_C Z_{R_2} + Z_C Z_L + Z_{R_2} Z_L} =$$

$$= \frac{j\omega R_2 L}{R_1 R_2 + j\omega R_1 L + \frac{R_2}{j\omega C} + \frac{L}{C} + j\omega R_2 L} = \quad (R_1 = R_2 = R)$$

$$= \frac{-\omega^2 RLC}{j\omega C R^2 - \omega^2 RLC + R + j\omega L - \omega^2 RLC} =$$

$$= \frac{-\omega^2 RLC}{R - 2\omega^2 RLC + j\omega(R^2 C + L)} = \frac{-\cancel{10^{10}} \pi^2 . 50 . \cancel{10^{-10}}}{50 - 2\, \cancel{10^{10}} . \pi^2 . 50 . \cancel{10^{-10}} + j 10^5 \pi (50^2 . 10^{-7} + 10^{-3})}$$

$$= \frac{-\pi^2 . 50}{(1 - 2\pi^2) . 50 + j \pi . 10^5 (1.25 \times 10^{-3})} = \frac{-50\pi^2}{(1 - 2\pi^2) . 50 + j \pi . 125}$$

$$H(j10^5\pi) = \frac{-2\pi^2}{(1-2\pi^2).2 + j5\pi} = |H(j10^5\pi)| . e^{j\angle H(j10^5\pi)}$$

④ Compute the phasor V_0

$$V_0 = H(j10^5\pi).V_{in} = |H(j10^5\pi)|.|V_{in}|.e^{j(\angle V_{in} + \angle H(j10^5\pi))}$$

⑤ Convert back from phasor to sinusoidal voltages

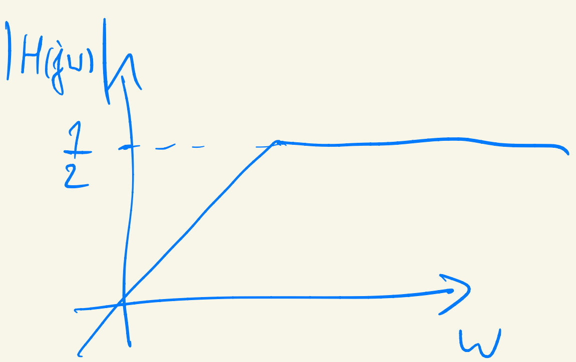
$$\Rightarrow v_{out}(t) = I_m\left(V_0 e^{j\omega t}\right) = 2.|H(j10^5\pi)|.\sin\left(10^5\pi t - \frac{\pi}{3} + \angle H(j10^5\pi)\right)$$

- Filter type:

$$|H(j0)| = \lim_{w \to 0} |H(jw)| = \frac{0}{R} = 0$$

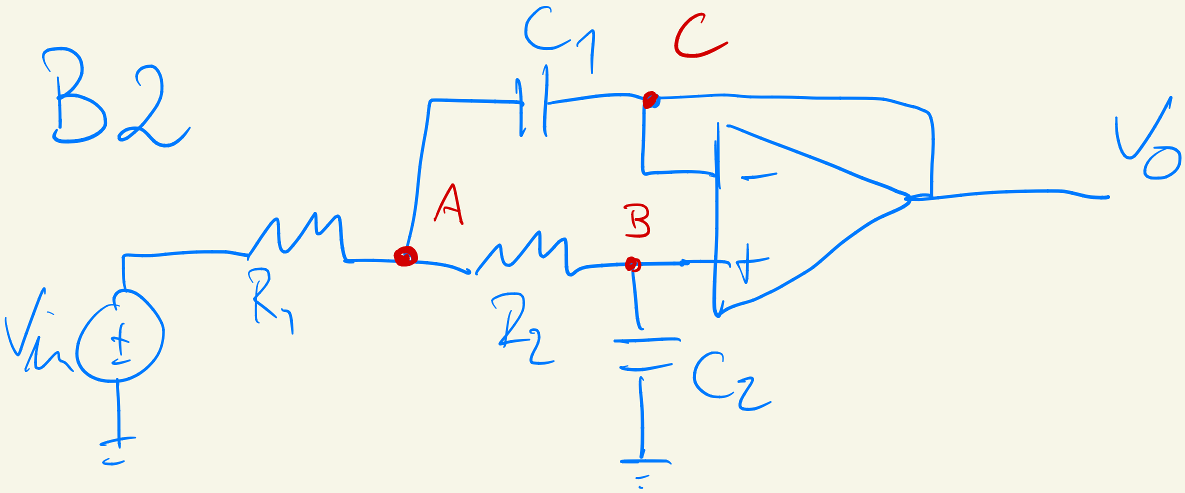
$$|H(j\infty)| = \lim_{w \to \infty} |H(jw)| = \lim_{w \to \infty} \left| \frac{-w^2 RLC}{R - 2w^2 RLC + jw(R^2C + L)} \right| =$$

$$= \lim_{w \to \infty} \left| \frac{-RLC}{\frac{R}{w^2} - 2RLC + j\frac{1}{w}(R^2C+L)} \right| = \frac{1}{2}$$



It is a high-pass filter

B2



$V_A = ?$ $\quad V_B = V_C = V_o$

↳ OPAmp in neg. feedback

KCL:

$$\begin{cases} \dfrac{V_{in} - V_A}{Z_{R_1}} + \dfrac{V_B - V_A}{Z_{R_2}} + \dfrac{V_o - V_A}{Z_{C_1}} = 0 \quad (i) \\[2ex] \dfrac{V_A - V_B}{Z_{R_2}} + \dfrac{0 - V_B}{Z_{C_2}} = 0 \\[2ex] V_o = V_B \end{cases}$$

$(\Leftrightarrow)$

$$\begin{cases} \dfrac{V_{in}}{Z_{R_1}} + \left(\dfrac{1}{Z_{R_2}} + \dfrac{1}{Z_{C_1}}\right) V_o - \left(\dfrac{1}{Z_{R_1}} + \dfrac{1}{Z_{R_2}} + \dfrac{1}{Z_{C_1}}\right) V_A = 0 \\[2ex] \dfrac{1}{Z_{R_2}} V_A = \left(\dfrac{1}{Z_{R_2}} + \dfrac{1}{Z_{C_2}}\right) V_o \Leftrightarrow V_A = \left(1 + \dfrac{Z_{R_2}}{Z_{C_2}}\right) V_o \end{cases} \quad (\Rightarrow)$$

$$\frac{V_{in}}{Z_{R_1}} = -\frac{Z_{R_2}+Z_{C_1}}{Z_{R_2}\,Z_{C_1}}V_0 + \left(\frac{1}{Z_{R_1}}+\frac{1}{Z_{R_2}}+\frac{1}{Z_{C_1}}\right)\left(1+\frac{Z_{R_2}}{Z_{C_2}}\right)V_0$$

$R_1 = R_2 = Z_{R_1} = Z_{R_2} = Z_R$

$Z_{C_1} = \frac{1}{j\omega C_1} = \frac{1}{2j\omega C_2} = \frac{1}{2}Z_{C_2}$

$Z_{C_1} = Z_C$, $Z_{C_2} = 2Z_C$

$$\frac{V_{in}}{Z_R} = -\frac{Z_R+Z_C}{Z_R\cdot Z_C}V_0 + \left(\frac{2}{Z_R}+\frac{1}{Z_C}\right)\left(1+\frac{Z_R}{2Z_C}\right)V_0$$

$$V_{in} = -\frac{Z_R+Z_C}{Z_C}V_0 + \left(2+\frac{Z_R}{Z_C}\right)\left(1+\frac{Z_R}{2Z_C}\right)V_0$$

$$V_{in} = \left(-\frac{Z_R+Z_C}{Z_C} + \left(2+\frac{Z_R}{Z_C}+\frac{Z_R}{Z_C}+\frac{Z_R^2}{2Z_C^2}\right)\right)V_0$$

$$= \left(1+\frac{Z_R}{Z_C}+\frac{Z_R^2}{2Z_C^2}\right)V_0$$

$$= \left(\frac{2Z_C^2+2Z_RZ_C+Z_R^2}{2Z_C^2}\right)V_0$$

$$\Leftrightarrow V_0 = \frac{2Z_C^2}{2Z_C^2+2Z_RZ_C+Z_R^2}V_{in}$$

$$H(j\omega) = \frac{V_0}{V_{in}} = \frac{2\left(\frac{1}{j\omega C}\right)^2}{2\left(\frac{1}{j\omega C}\right)^2+\frac{2R}{j\omega C}+R^2} = \frac{1}{1+j\omega RC+\frac{1}{2}(j\omega RC)^2} =$$

$$= \frac{1}{1-\frac{1}{2}\omega^2R^2C^2+j\omega RC} \; ; \; |H(j\omega C)| = \frac{1}{\sqrt{\left(1-\frac{1}{2}\omega^2R^2C^2\right)^2+\omega^2R^2C^2}} =$$

$$\lim_{\omega\to 0}|H(j\omega)| = 1$$

$$\lim_{\omega\to\infty}|H(j\omega)| = 0$$

$$= \frac{1}{\sqrt{1^2-\cancel{\omega^2R^2C^2}+\frac{1}{4}\omega^4R^4C^4+\cancel{\omega^2R^2C^2}}}$$

$$= \frac{1}{\sqrt{1^2+\frac{1}{4}\omega^4R^4C^4}}$$

$RC = 10^4 \times 20 \times 10^{-9} = 2 \times 10^{-4}$

$$H(j\,1.4 \times 10^4) = \frac{1}{\sqrt{1^2 - \frac{1}{4}\,1.4^4 \times 10^{16} \times (R.C)^4}} =$$

$$= \frac{1}{\sqrt{1^2 + \frac{1.4^4}{4} \times \cancel{10^{16}} \times 16 \times \cancel{10^{-16}}}} = \frac{1}{\sqrt{1^2 + 4 \times 1.4^4}} \simeq \frac{1}{\sqrt{1^2 + 15.4}} =$$

$$= \frac{1}{\sqrt{16.4}} \simeq \frac{1}{4.05} \simeq 0.247$$