Exam in Automatic Control II
Reglerteknik II 5hp (1RT495)

Date: October 25, 2023
Time: 14:00 – 19:00
Venue: Bergsbrunnagatan 15: Sal 2

Responsible teacher: Sérgio Pequito

Aiding material: Calculator, mathematical handbooks, textbooks by Glad & Ljung (Reglerteori/Control theory & Reglerteknik). Laplace table and the automatic control glossary between Swedish and English. Additional handwritten notes in the textbooks are allowed.

Preliminary grades: $\geq$ 43p for grade 5, $\geq$ 33p for grade 4, and $\geq$ 23p for grade 3.

Use separate sheets for each problem, i.e., only one problem per sheet. Write your exam code on every sheet.

Important: Your solutions should be well motivated unless else is stated in the problem formulation. Vague or lacking motivation may lead to a reduced number of points.

Good luck!

Please use English in your solutions!

Problem 1 Consider the following continuous-time state-space representation of a single-input single-output system:

$$\begin{pmatrix} px_1 \\ px_2 \end{pmatrix} = \begin{pmatrix} -0.1 & 0.8 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u, \tag{1}$$

$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + u. \tag{2}$$

(a) Is the state-space representation in (1)-(2) in a canonical form? Explain your answer. **(3p)**

(b) Provide the block diagram representation associated with the state-space representation in (1)-(2). **(3p)**

(c) Is the system associated with (1)-(2) asymptotically stable? Explain your answer. **(3p)**

Please use English in your solutions!

Problem 2 Consider the following continuous-time state-space representation of a single-input single-output system:

$$\begin{pmatrix} px_1 \\ px_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u, \tag{3}$$

$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + u. \tag{4}$$

(a) Obtain the zero-order holder approximation of (3)-(4) with a discretization step $h > 0$. **(3p)**

(b) Determine the state feedback control $u = -Kx$ that makes the poles of the closed-loop to be at $-0.5i$ and $+0.5i$. **(3p)**

Please use English in your solutions!

Problem 3 Consider the following discrete-time state-space representation of a single-input single-output system of a continuous-time system:

$$\begin{pmatrix} qx_1 \\ qx_2 \end{pmatrix} = \begin{pmatrix} h & 0 \\ 1 & e^{-h} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u, \tag{5}$$

$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + u. \tag{6}$$

(a) State the values of $h > 0$ that make the system (5)-(6) controllable. Explain your answer. **(3p)**

(b) State the values of $h > 0$ that make the system (5)-(6) observable. Explain your answer. **(3p)**

(c) State the values of $h > 0$ that make the system (5)-(6) a minimal realization. Explain your answer. **(3p)**

Please use English in your solutions!

Problem 4

(a) What is a stochastic process? **(2p)**

(b) Consider the following stochastic process

$$x_1(k+1) = ax_1(k) + v(k),$$

where $|a| < 1$ and $v(k)$ is standard white Gaussian noise. In the steady-state setting, is it a weakly stationary stochastic process? **(4p)**

(c) Now, suppose a more realistic setting where

$$x_1(k+1) = ax_1(k) + w(k),$$

where $|a| < 1$ where $w(k)$ is a stochastic process that can be modeled as a response to standard white Gaussian noise and $v(k)$, i.e., $w(k) = G(q)v(k)$, with spectral density

$$\Phi_w(\omega) = \frac{0.19}{1.81 - 1.8\cos(\omega)}.$$

Determine the transfer operator $G(q)$. **(4p)**
Hint: You may want to recall Euler's formula: $e^{i\theta} = \cos(\theta) + i\sin(\theta)$.

(d) Consider a similar realistic setting

$$x_1(k+1) = ax_1(k) + w(k), \tag{7}$$

where $|a| < 1$ where $w(k)$ is a stochastic process that can be modeled as a response to standard white Gaussian noise and $v(k)$, i.e., $w(k) = G(q)v(k)$, where

$$G(q) = \frac{0.6}{q + 0.8}.$$

Additionally, assume that reference and output are given respectively as follows:

$$z(k) = x_1(k), \quad \text{and} \quad y(k) = z(k) + v(k). \tag{8}$$

Rewrite (7)-(8) in the standard form

$$\begin{cases} qx = Fx + Gu + Nv_1, \\ z = Mx, \\ y = Hx + v_2, \end{cases}$$

where both v_1 and v_2 are white Gaussian noise. Do not forget to indicate the intensities for R_1, R_2, and R_{12}. **(4p)**

Please use English in your solutions!

Problem 5 Suppose that

$$\begin{cases} \dot{x} = -x + u + 0.2v_1, \\ z = \dfrac{1}{2}x, \\ y = 2x + v_2, \end{cases}$$

where both v_1 and v_2 are independent and identically distributed white Gaussian noise with intensities $R_1 = 0.1$ and $R_2 = 5$. Provide an explicit solution to an estimator that minimizes the steady-state variance of the estimation error. **(4p)**

Please use English in your solutions!

Problem 6 Consider a controllable large dimensional multivariate continuous-time single-input system with very fast dynamics where the natural frequency is $\omega_n >> 0$. The dynamics have bounded uncertainty (i.e., are affected by disturbance with bounded variance). The sensors can be assumed to be noiseless and capable of measuring all the states individually. Additionally, there are no constraints on states or actuation capabilities, and the actuation cost is negligible. Nonetheless, the solution must be implemented using a digital controller.

(a) What would be the approach you would consider? Why? **(2p)**

(b) How would you implement it? **(4p)**

(c) Why should it work (or not)? **(2p)**