1a) Assume C to be discharged at $t \le 0$.

- $U_C = 1 - R I_C = 1 - RC \frac{dU_C}{dt}$

$\rightarrow$ $$-\frac{1}{RC}\, dt = \frac{1}{U_C - 1}\, dU_C$$

$$-\frac{1}{RC} \int dt = \int \frac{1}{U_C - 1}\, dU_C$$

$$-\frac{t}{RC} = \ln(U_C - 1) + \alpha_0$$

$$e^{-t/RC} = e^{\alpha_0}(U_C - 1)$$

$$U_C = 1 + \alpha_1 e^{-t/RC}$$

Initial cond. $U_C(0) = 0 \longrightarrow \alpha_1 = -1$

Answer: $U_C(t) = \begin{cases} 0, & t \le 0 \\ 1 - e^{-t/RC}, & t > 0 \end{cases}$ [Volt]

1b) Answer: $-10 \le U_{out}(t) \le 10$ [Volt]

1c)
- $1 - U_{IN}^{+} = RC \frac{dU_{IN}^{+}}{dt}$
- $U_{IN}^{-} = RC \frac{d}{dt}(U_{out} - U_{IN}^{-})$
- $U_{IN}^{+} = U_{IN}^{-}$ (Ideal OpAmp + neg. feedback)

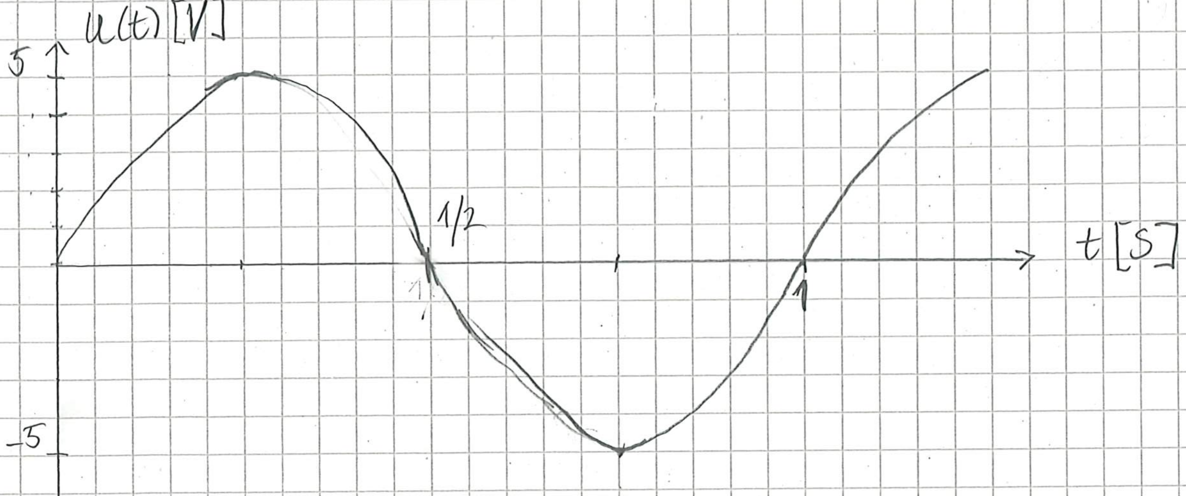
$\longrightarrow$ $$1 - RC\frac{d}{dt}(U_{out} - U_{IN}^{-}) = RC\frac{dU_{IN}^{+}}{dt}$$

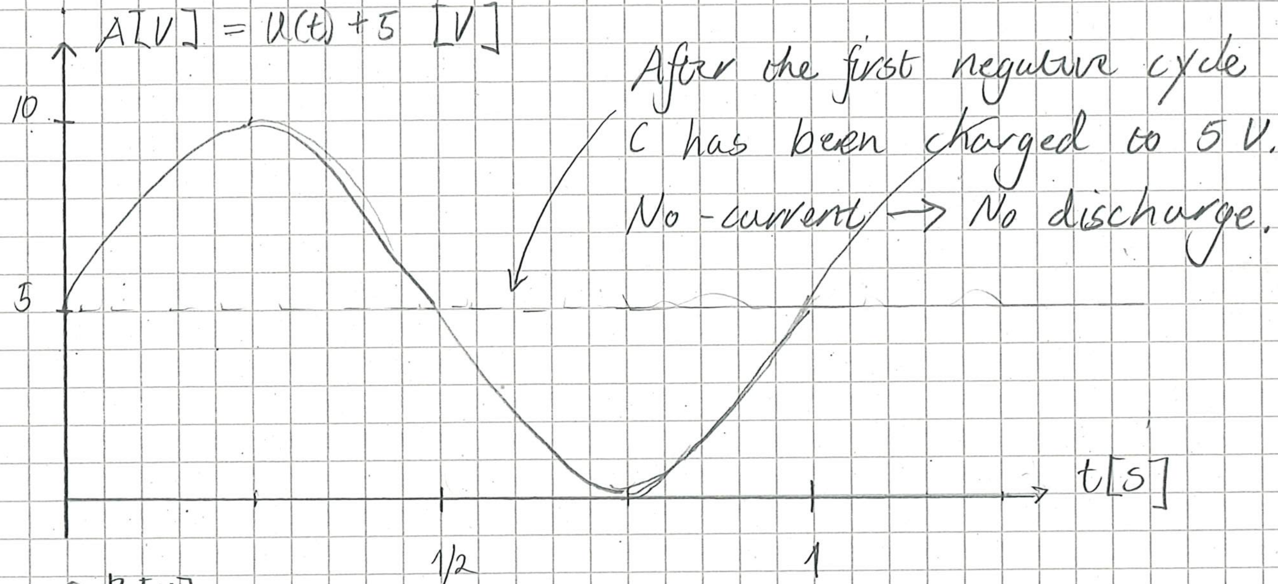
$\longrightarrow$ $$U_{out} = \frac{1}{RC}\int 1\, dt$$

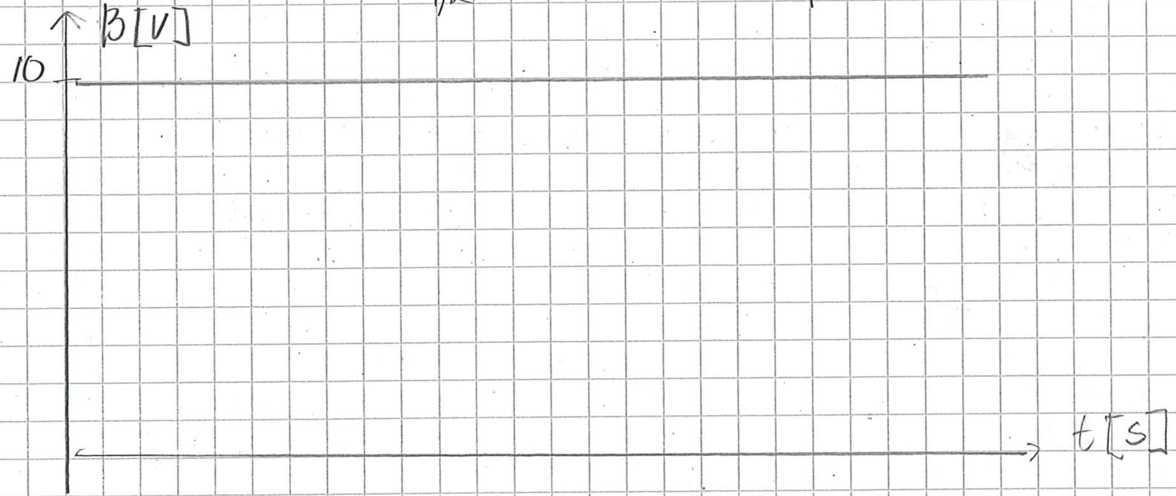
Answer: $u_{out}(t) = \begin{cases} 0, & t < 0 \\ t/RC, & 0 \leq t < 10RC \\ 10, & t \geq 10RC \end{cases}$ [Volt]

1d) Answer; Integration.

2a)

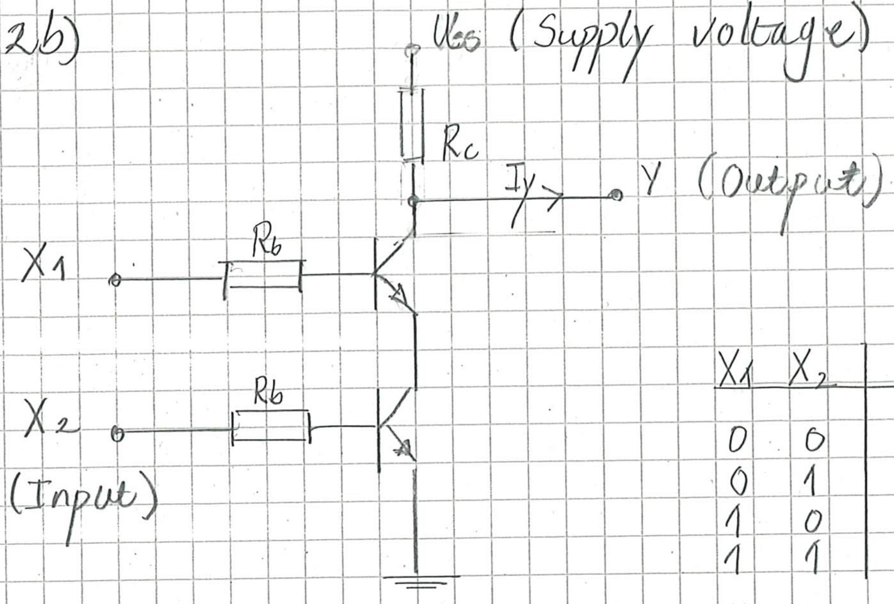






2b) The circuit rectifies the AC-input and double its peak voltage.

2b)



X_1	X_2	Y
0	0	1
0	1	1
1	0	1
1	1	0

$U_{high} > 4\,V$ $\quad$ $I_{Cmax} = 10\,mA$

$U_{low} < 1\,V$

- Assume X_1 or X_2 is low $\longrightarrow I_C \approx 0$

$\longrightarrow Y = U_{SS} - R_C I_y > U_{high}$ & $I_y \leq 1\,mA$

$\longrightarrow R_C < (5-4)/10^{-3} = 1\,k\Omega$

$\longrightarrow$ Select $\underline{R_C = 820\,\Omega}$. (E12 series of values)

- Minimum I_C when X_1 & X_2 are high.

$U_y = U_{SS} - R_C(I_y + I_C) \leq U_{SS} - R_C I_C < U_{low}$

$\longrightarrow I_C > (U_{SS} - U_{low})/R_C \approx 5\,mA$

- Maximum I_C when X_1 & X_2 are high

$I_C \leq U_{SS}/R_C \approx 6\,mA$

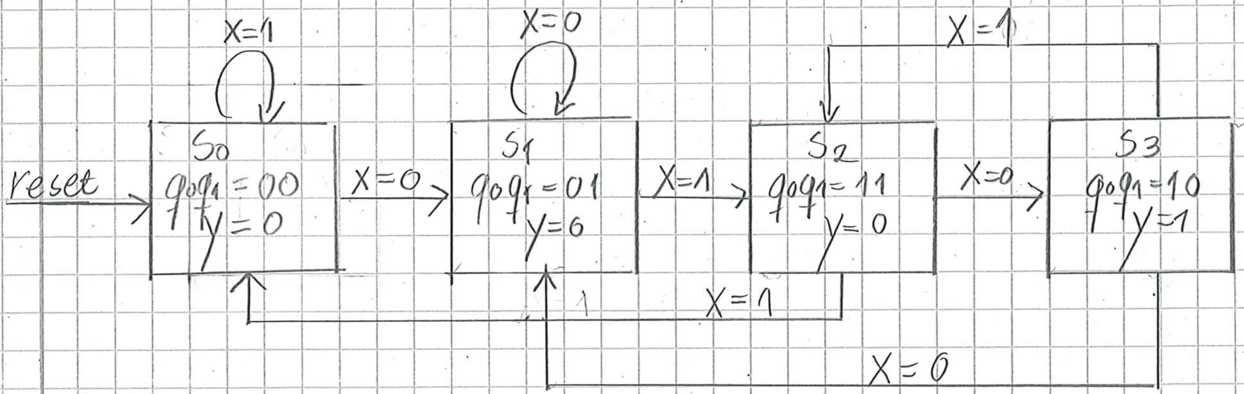
- Minimum bias current $I_b > I_C/h_{FE} = 6\cdot 10^{-3}/100 = 6\cdot 10^{-5}$.
- Maximum $I_b \leq 1\,mA$

Upper bound: $I_b < (U_{SS} - V_{BE})/R_b < 1\,mA \rightarrow R_b > 4{,}3\,k\Omega$

Lower bound: $I_b > (U_{low} - V_{BE} - V_{CE})/R_B > 60\,\mu A \rightarrow R_b < 53\,k\Omega$

$\longrightarrow$ Select $\underline{R_b = 10\,k\Omega}$

3 a) $q_0 q_1$ - state coding, y - output, x - input



Truth tables

x	q_0	q_1	q_0^+	q_1^+
0	0	0	0	1
0	0	1	0	1
0	1	1	1	0
0	1	0	0	1
1	0	0	0	0
1	0	1	1	1
1	1	1	0	0
1	1	0	1	1

q_0	q_1	y
0	0	0
0	1	0
1	0	1
0	0	0

K-map q_0^+

$x \backslash q_0q_1$	00	01	11	10
0	0	0	1	0
1	0	1	0	1

K-map q_1^+

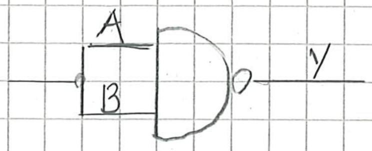
$x \backslash q_0q_1$	00	01	11	10
0	1	1	0	1
1	0	1	0	1

$$q_0^+ = x\bar{q}_0 q_1 + \bar{x} q_0 q_1 + x q_0 \bar{q}_1$$

$$q_1^+ = \bar{x}\bar{q}_0 + \bar{q}_0 q_1 + q_0 \bar{q}_1$$

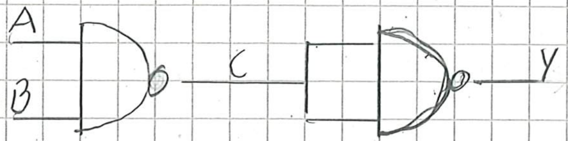
$$y = q_0 \bar{q}_1$$

3b) NOT-gate from NAND-gate



$Y = \overline{AB} = /A=B/ = \overline{A}$

AND-gate from NAND-gates

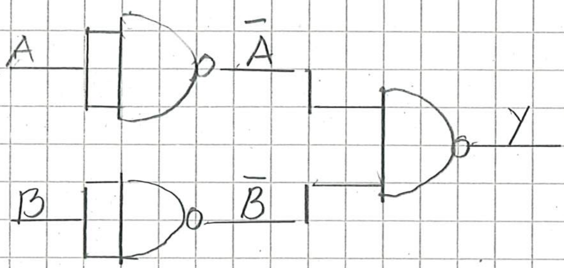


$Y = \overline{\overline{AB}} = AB$

OR-gate from NAND-gates

$Y = \overline{\overline{A}\,\overline{B}}$ = /De'Morgons/

$= \overline{\overline{A+B}} = A+B$



4a) Assume ideal Op Amp.

- Negative feedback $\rightarrow u_{IN}^- = u_{IN}^+ = 0$

- $\frac{u_{out}}{Z_{C1}//Z_{R2}} = -\frac{u_{IN}}{Z_{R1}} \longrightarrow H(j\omega) = -\frac{Z_{C1}//Z_{R2}}{Z_{R1}}$

- $Z_{C1} = 1/j\omega C_1$, $Z_{R1} = R_1$, $Z_{R2} = R_2$

$$\Rightarrow H(j\omega) = -\frac{\frac{Z_{C1} \cdot Z_{R1}}{Z_{C1} + Z_{R1}}}{Z_{R2}} = -\frac{1}{R_1} \cdot \frac{R_2/j\omega C_1}{\frac{1}{j\omega C_1} + R_2} =$$

$$= -\frac{R_2}{R_1} \cdot \frac{1}{1 + j\omega R_2 C_1}$$

4b)

- $\lim_{\omega \to 0} |H(j\omega)| = R_2/R_1$
- $\lim_{\omega \to \infty} |H(j\omega)| = 0$
- Filter has one pole (one cut'of freq)

$\Longrightarrow$ LP - filter

4c) With component values $H(j\omega)$ becomes

$$H(j\omega) = \frac{-10}{1 + j0{,}1\omega}$$

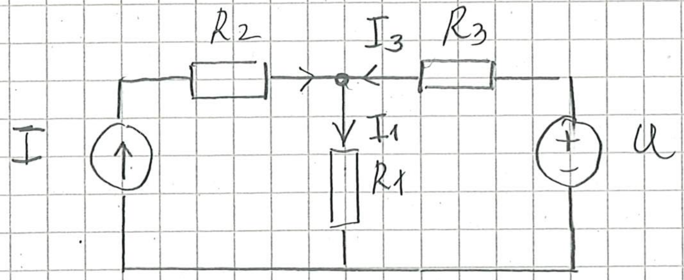
- $|H(j\omega)|_{\omega=10} = 10/\sqrt{2}$

- $\angle H(j\omega)|_{\omega=10} = \pi - \angle(1+j) = \pi - \pi/4 = 3\pi/4$

- SIN-IN $\rightarrow$ SIN OUT

$$\Longrightarrow u_{out}(t) = \frac{10}{\sqrt{2}} \cos(10t + \frac{3\pi}{4}) \text{ [Volt]}$$

5a)



$$\left|\begin{array}{l} I_1 = I + I_3 \\ U_{R1} = R_1 I_1 = U - R_3 I_3 \end{array}\right.$$

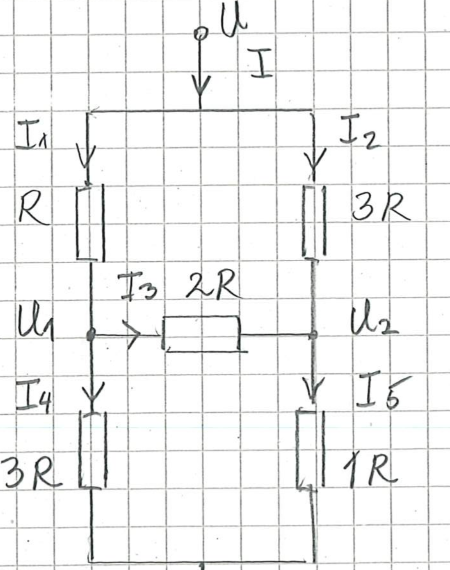
$$\longrightarrow \quad R_1 I_1 = U - R_3 (I_1 - I)$$

$$(R_1 + R_3) I_1 = U + R_3 I$$

$$I_1 = (U + R_3 I)/(R_1 + R_3)$$

$$\longrightarrow U_{R1} = R_1 I_1 = \underline{\frac{R_1}{R_1 + R_3} U + \frac{R_1 R_3}{R_1 + R_3} \cdot I} \quad [\text{Volt}]$$

5b)



$$I = I_1 + I_2 = I_4 + I_5$$

$$I_1 = I_3 + I_4$$

$$I_5 = I_3 + I_2$$

$$R_{eqv} = U / I \; . \longrightarrow \text{Find } I$$

$$\left|\begin{array}{l} U = R I_1 + 3R I_4 \\ U = 3R I_2 + R I_5 \\ U = R I_1 + 2R I_3 + R I_5 \end{array}\right.$$

$$U = RI_1 + 3R(I_1 - I_3) = 4RI_1 - 3RI_3 \quad (a)$$
$$U = 3RI_2 + R(I_2 + I_3) = 4RI_2 + RI_3 \quad (b)$$
$$U = RI_1 + 2RI_3 + R(I_2 + I_3) = RI_1 + RI_2 + 3RI_3 \quad (*)$$

$$(*) \longrightarrow U = R\underbrace{(I_1 + I_2)}_{=I} + 3RI_3 \longrightarrow I_3 = \frac{U - RI}{3R}$$

Insert (*) into (a) yields:

$$\longrightarrow U = 4RI_1 - 3RI_3 = 4RI_1 - 3R\left(\frac{U - RI}{3R}\right)$$

$$\longrightarrow 2U = 4RI_1 + RI \longrightarrow I_1 = \frac{2U - RI}{4R} = \frac{U}{2R} - \frac{I}{4}$$

Insert (*) into (b) yields:

$$\longrightarrow U = 4RI_2 + R\left(\frac{U - RI}{3R}\right) = 4RI_2 + \frac{U}{3} - \frac{RI}{3}$$

$$\longrightarrow I_2 = \left(\frac{RI}{3} + \frac{2U}{3}\right)/4R = \frac{I}{12} + \frac{U}{6R}$$

This yields the eqv. resistance

$$I = I_1 + I_2 = \frac{U}{2R} - \frac{I}{4} + \frac{I}{12} + \frac{U}{6R}$$

$$\frac{7}{6}I = \frac{2U}{3R}$$

$$R_{eqv} = \frac{U}{I} = \frac{3 \cdot 7R}{6 \cdot 2} = \frac{21}{12}R = \underline{\underline{\frac{7R}{4}}}$$