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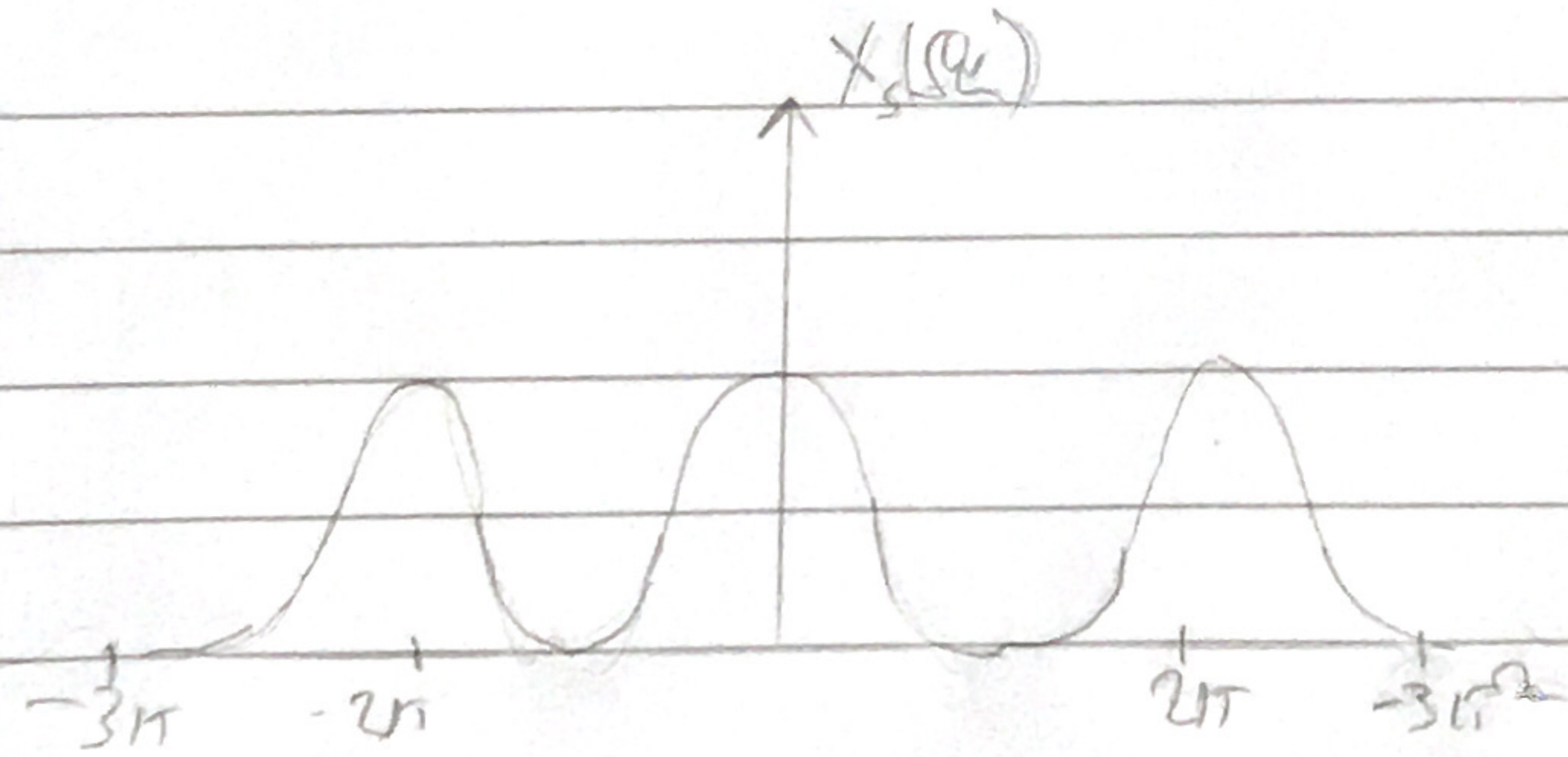
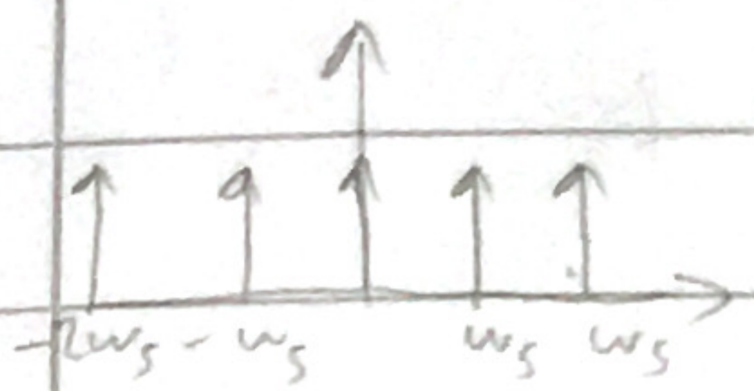
1 a) No, since the Nyquist-shannon theorem says that $w_s > 2w_b$ (where w_b is the signals bandwith) is needed to avoid aliasing. And $X(w)$ have a higer bandwith than 50 rad/s. Approximate $w_s = 120$ rad/s is needed to be sure that no aliasing ocurs.

b) Generally Filter 1 is prefered since filter 2 will "cut" away the signals frequencies above 50 rad/s which will lead to that information gets loosed. With filter 1 the signal is preserved, se b)*

c)



Sampling will in the freq domain yield a convolution between the original signal and the pulsetrain:



This will therefor give the original signal timeshifted to all pulses.

*b) But if w_s is fixed chosen to be $w_s = 100$ rad/s, aliasing will occur and distort the signal above 50 rad/s. Then it might be good to use filter 2 since this will remove the distortion from the aliasing.

2a) Yes, the system is stable since all poles are within the unit circle, $|P_1| = |P_2| < 1$

b) Given by the pole-zero map is that:

$z_1 = -0{,}75 + 0{,}75j$, $z_2 = -0{,}75 - 0{,}75j$

$P_1 = 0.5 + 0{,}5j$, $P_2 = 0.5 - 0{,}5j$

$(z + 0{,}75 - 0{,}75j)(z + 0{,}75 + 0{,}75j) = z^2 + 1{,}5z + 1{,}125$

$(z - 0{,}5 - 0{,}5j)(z - 0{,}5 + 0{,}5j) = z^2 - z + 0{,}5$

$$\Rightarrow H(z) = a \cdot \frac{z^2 + 1{,}5z + 1{,}125}{z^2 - z + 0.5}$$

$$H(1) = 1 \Rightarrow a \cdot \frac{1 + 1{,}5 + 1{,}125}{1 - 1 + 0.5} = 1 \Rightarrow a = \frac{1}{7{,}25}$$

$$\Rightarrow \boxed{H(z) = \frac{1}{7.25} \cdot \frac{z^2 + 1.5z + 1.125}{z^2 - z + 0.5} = \frac{1}{7.25} \cdot \frac{1 + 1.5z^{-1} + 1.125z^{-2}}{1 - z^{-1} + 0.5z^{-2}}}$$

c) $$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{7{,}25} \cdot \frac{1 + 1.5z^{-1} + 1{,}125z^{-2}}{1 - z^{-1} + 0{,}5z^{-2}} \Rightarrow$$

$$Y(z)(7{,}25 - 7{,}25z^{-1} + 3{,}625z^{-2}) = X(z)(1 + 1{,}5z^{-1} + 1{,}125z^{-2}) \Rightarrow$$

$$7.25Y(z) - 7{,}25z^{-1}Y(z) + 3{,}625z^{-2}Y(z) = X(z) + 1{,}5z^{-1}X(z) + 1{,}125z^{-2}X(z) \Rightarrow$$

$$\mathcal{Z}^{-1}\{\} \Rightarrow$$

$$\boxed{7{,}25y[k] - 7{,}25y[k-1] + 3{,}625y[k-1] = x[k] + 1{,}5x[k-1] + 1{,}125x[k-2]}$$

is the signals difference eqation.

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2d) We know that $y[k] = x[k] * h[k] \Rightarrow Y(z) = X(z)H(z)$
Since the system is causal, we can use the transform tables and apply z-transform to $x[k]$:

$$x[k] = \cos(\tfrac{\pi}{2}k) + \cos(\tfrac{3\pi}{4}k)$$

$$\updownarrow$$

$$X(z) = \frac{1 - z^{-1}\cos(\frac{\pi}{2})}{1 - 2z^{-1}\cos(\frac{\pi}{2}) + z^{-2}} + \frac{1 - z^{-1}\cos(\frac{3\pi}{4})}{1 - 2z^{-1}\cos(\frac{3\pi}{4}) + z^{-2}}$$

$$= \frac{1}{1 + z^{-2}} + \frac{1 + \frac{1}{\sqrt{2}}z^{-1}}{1 + \frac{2}{\sqrt{2}}z^{-1} + z^{-2}}$$

$\Rightarrow$

$$Y(z) = H(z)X(z) = \frac{1 + 1{,}5z^{-1} + 1{,}125z^{-2}}{7{,}25 - 7{,}25z^{-1} + 3{,}625z^{-2}} \cdot \left(\frac{1}{1+z^{-2}} + \frac{1 + \frac{1}{\sqrt{2}}z^{-1}}{1 + \frac{2}{\sqrt{2}}z^{-1} + z^{-2}}\right)$$

= ... times up!

When $Y(z)$ is derived, inverse z-transform would be used to obtain $y[k]$.

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3 a) We know that we can extract all nessecary information from the signal by using a DFT with $L=k$, further more, to obtain a spectral resolution of $\Delta f = 0.25 Hz$ k need to be chosen as:

$$K \geq \frac{f_s}{\Delta F} = \frac{100 Hz}{0.25 Hz} = 400$$

therefore, 400 samples of the signal is needed to obtain the spectral resolution of $\Delta f = 0.25$.
And since the signal is time limited we know that:

$$X(\Omega_l) = X[l] \quad \text{and} \quad X(\omega_l) \approx T_s X[l]$$

which can be used to approximate $X(F)$

b) By using a rectangular window in a) we are receiving a narrow mainlobe but with strong sidelobes, i.e more leakage. By instead using a Barlet/triangle window the sidelobes would be smaller, but then the mainlobe would be wider, more smearing.
By using a window with less leakage the distortion of the sidelobes would be decreased.

3c) We know that $Y(\omega) = H(\omega)X(\omega)$.
Since the output $y(t)$ is also time limited we also know that $Y(\Omega_l) = X[l]$ and $Y(\omega_l) \approx T_s Y[l]$.
By then using the DFT from both $X(t)$ and $y(t)$ we can derive an approximation of $H(\omega)$ by using that $Y(\omega_l) \approx T_s Y[l]$ and $X(\omega_l) \approx T_s X[l]$