Retake Exam (Part 1) in
Program Design and Data Structures (1DL201)

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2016-08-15 / 8:00–13:00

Instructions

Read and follow these instructions carefully to increase your chance of getting good marks.

- This is a closed book exam. You may use a standard English dictionary. Otherwise, **no notes, calculators, mobile phones, or other electronic devices are allowed.** Cheating will not be tolerated.

- This is a multiple-choice exam. Each question has exactly **one** correct answer.

- You may keep these question sheets. **Only hand in the answer sheet.** Also read the instructions on the answer sheet before you start.

- Tjark Weber will come to the exam hall around 10:00 to answer questions.

Good luck!

Master Theorem

Given a recurrence of the form

$$T(n) = aT(n/b) + f(n)$$

Case 1: If $f(n) = O(n^c)$ where $c < \log_b a$
    then $T(n) = \Theta(n^{\log_b a})$.

Case 2: If $f(n) = \Theta(n^c \log^k n)$ where $c = \log_b a$ and $k \geq 0$
    then $T(n) = \Theta(n^c \log^{k+1} n)$.

Case 3: If $f(n) = \Omega(n^c)$ where $c > \log_b a$ and the regularity condition holds
    then $T(n) = \Theta(f(n))$.
    The regularity condition is that $a \cdot f(n/b) \leq k \cdot f(n)$ for some constant $k < 1$ and all sufficiently large n.

Common Material

Some of the exam questions refer to the following function:

```
{- evens xs
   PRE: ?PRE?
   POST: ?POST?
 -}
evens :: ?TYPE?
-- VARIANT: ?VARIANT?
evens (x:_:xs) = x : evens xs
evens xs       = xs
```

Questions

Please choose a single answer for each question. Read the questions carefully, and watch out for negations (*not*, *except*, etc.).

Question 1: What is the value of `evens [1,2,3,4]` ?

[A] `[1,3]`   [B] `[]`   [C] `Int`   [D] `4`   [E] `[2,4]`

Question 2: What is the type (`?TYPE?`) of `evens` ?

[A] `[a] -> [Int]`   [C] `[a] -> [a]`   [E] `a -> a`
[B] `[Int]`   [D] `Int`

**Question 3:** What is the most appropriate precondition (`?PRE?`) for `evens` ?

[A] `xs` is a list   [C] `False`   [E] `xs` is non-empty
[B] `True`   [D] `xs` is 0

Question 4: What is the most appropriate postcondition (`?POST?`) for `evens` ?

[A] `True`

[B] the head of `xs`, followed by `evens` applied to the tail of the tail of `xs`

[C] the elements at even indices in `xs`

[D] the even integers in `xs`

[E] `xs`

Question 5: Which of the following is a variant (`?VARIANT?`) for the function `evens` ?

[A] `xs`   [C] `length xs ` $\geq$ ` 2`   [E] `length xs`
[B] `x + length xs`   [D] `0`
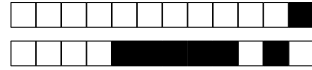
Question 6: The *semantics* of a programming language ...

A defines the meaning of programs.

B is usually defined very precisely, e.g., by formal grammars.

C defines what combinations of symbols constitute valid programs.

D describes the error messages obtained when compiling incorrect programs.

E defines the data types and operations that are available in the language.

Question 7: What is the value of the following expression?

```
let
  x = 0
  f y = let x = y+1 in x+y
  y = f x
in
  x+y
```

A 1   B 2   C 3   D 4   E 5

Question 8: What is the value of the following expression?

```
case (1, False) of
  (x, _) | x>1    -> "A"
  (_, y) | y      -> "B"
  (1, x) | not x -> "C"
  _               -> "D"
```

A "A"   C "C"   E The expression is not type-correct.

B "B"   D "D"

Question 9: Consider the following function:

```
f x y = if x<y then x*x else x+y
```

Which of the following expressions is **not** type-correct?

A f   C f 1 2   E All of these.

B f 'A' 'B'   D f 1.2 3.4

Question 10: Which of the following statements about recursive algorithms is **not** generally true?

A There is a base case that can be solved directly.

B The (worst-case) time complexity is exponential.

C A recursive function terminates if it has a variant.

D To find a suitable recursion scheme, it may be necessary to solve a more general problem.

E The solution to a problem is found by solving smaller versions of the same problem.

Question 11: Which of the following lists does **not** have length 3?

A `[1..3]` C `[[1,2,3]]` E `1:2:3:[]`

B `[[], [1], [2,3]]` D `[[], [], []]`

Question 12: Recall the quicksort algorithm. Suppose the algorithm is applied to the input list `[8,2,4,4,11,12,1,10]` and the value 8 is chosen as the pivot. What are the arguments to the immediate recursive calls of the quicksort algorithm?

A `[2,4,4,11]` and `[12,1,10]` D `8` and `[2,4,4,11,12,1,10]`

B `[1,2,4,4]` and `[10,11,12]`

C `[2,4,4,1]` and `[11,12,10]` E `[8,2]`, `[4,4]`, `[11,12]` and `[1,10]`

Question 13: What is the type of `map map`?

A The expression is not type-correct. D `[a] -> [b]`

B `[a -> b] -> [[a] -> [b]]`

C `(a -> b) -> [a] -> [b]` E `[a -> b]`

Question 14: Which is the **most precise** bound for the function $10n^3 + 5n + 3n^4 + n^3 \log n$?

A $O(n^3)$ C $\Theta(n)$ E $\Theta(n^4)$

B $O(n^4)$ D $\Omega(n^4)$

Question 15: What is the closed form of the following recurrence?

$$T(0) = 10$$
$$T(n) = T(n-1) + 5$$

A $T(n) = 5n + 10$ C $T(n) = 10n^2 + 5n$ E $T(n) = 10n + 5$

B $T(n) = 5n^2 + 10n$ D $T(n) = \frac{10n(n+1)}{2} + 5$

Question 16: Recall that $O(g(n))$, $\Theta(g(n))$ and $\Omega(g(n))$ actually represent *sets* of functions related in the appropriate way to $g(n)$. What is the relationship between $O(g(n))$, $\Theta(g(n))$ and $\Omega(g(n))$?

A $\Omega(g(n)) \subseteq \Theta(g(n))$ and $\Theta(g(n)) \subseteq O(g(n))$

B $\Omega(g(n)) \cap O(g(n)) = \Theta(g(n))$

C $O(g(n)) \subseteq \Theta(g(n))$ and $\Theta(g(n)) \subseteq \Omega(g(n))$

D $O(g(n)) = \Omega(g(n)) \cup \Theta(g(n))$

E None of the above.

Question 17: Use the Master Theorem to find a closed form for the following recurrence:

$$T(n) = 16T(n/4) + n!$$

The closed form is:

[A] $\Theta(n^2 \log n)$.

[B] $\Theta(n!)$.

[C] $\Theta(n^2)$.

[D] $\Theta(n! \log n)$.

[E] The Master Theorem does not apply.

Question 18: Consider the buggy function:

```
{- drop n xs
   PRE: n >= 0
   POST: the suffix of xs after dropping n elements,
         or the empty list itself if n > length xs:
 -}
drop :: Int -> [a] -> [a]
drop 0 xs = xs
drop n (_:xs) = drop (n-1) xs
```

Which of the following test cases fails due to the bug?

[A] TestCase (assertEqual "A" [] (drop 4 [1,2,3,4]))

[B] TestCase (assertEqual "B" ([] :: [Int]) (drop (-1) [1,2,3,4]))

[C] TestCase (assertEqual "C" [3] (drop 2 [1,2,3]))

[D] TestCase (assertEqual "D" [] (drop 4 [1,2,3]))

[E] TestCase (assertEqual "E" ([] :: [Int]) (drop 0 []))

Question 19: Consider the following recurrence:

$$\begin{aligned}
T(0) &= \Theta(1) \\
T(1) &= \Theta(1) \\
T(n) &= T(n-1) + T(n-2) + \Theta(1)
\end{aligned}$$

Which of the following Haskell functions' runtime function is given by this recurrence?

A
```
lamp [] = 0
lamp [a] = a
lamp (a : b : as) = a + lamp (b:as) + lamp as
```

B
```
crush [] = 0
crush [a] = a
crush l = crush left + crush right + 1
  where
    (left, right) = split l
    split l = let n = length l `div` 2 in (take n l, drop n l)
```

C
```
fuzz [] = 0
fuzz [a] = a
fuzz (a : _ : as) = a + fuzz as
```

D
```
bib [] = 1
bib (a : as) = a - bub as
bub [] = 0
bub (a : as) = a + bib as
```

E
```
bar [] = []
bar (a : as) = [a] : bar as
```

Question 20: The purpose of *stepwise refinement* (also known as *cheating*) is:

A To ensure that the implementation details of functions are hidden.

B To make sure that all functions are thoroughly tested.

C To break a program down into smaller functions.

D To give functions the correct type signature.

E To solve a problem by first solving a simplified version of the problem.

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*Do **not** write above this line!*

Answer Sheet — Exam 1DL201 of 2016-08-15

Instructions: Using a **dark** color, fill in **at most one** answer box (A to E) per question. Fill the answer box **entirely** (■)—we will use an optical character recognition (OCR) system that may not recognize ticks, crosses, circles, etc.

If you think that a question is ambiguous or has no correct answer, mark the question number with a ⋆ and explain **on the backside of this sheet** what the problem is and what assumptions you have made to answer the question.

Transfer your answers from the question sheets to this answer sheet **just before handing in**. If you want to change an answer, then please request a new answer sheet. You may keep the question sheets; at the end of the exam, **only hand in this answer sheet**.

Also fill in your **exam code** in clear handwriting at the bottom of this page.

Grading:

Correct answers	≤ 9	10 − 13	14 − 16	17 − 20
Grade	U	3	4	5

Question 1: ■ B C D E   **Question 11:** A B ■ D E

Question 2: A B ■ D E   **Question 12:** A B ■ D E

Question 3: A ■ C D E   **Question 13:** A ■ C D E

Question 4: A B ■ D E   **Question 14:** A B C D ■

Question 5: A B C D ■   **Question 15:** ■ B C D E

Question 6: ■ B C D E   **Question 16:** A ■ C D E

Question 7: ■ B C D E   **Question 17:** A ■ C D E

Question 8: A B ■ D E   **Question 18:** A B C ■ E

Question 9: A ■ C D E   **Question 19:** ■ B C D E

Question 10: A ■ C D E   **Question 20:** A B ■ D E

Again: Please fill your chosen boxes **entirely** and in **dark** color!

Your exam code: | | | | | | |