Exercise sessions 7 and 8 - AC circuits and filters

1TE717 Digital Technologies and Electronics

Question 1

This question is a short recap and practice of complex numbers.

(a) Rewrite the following complex numbers in polar coordindates.

(1) $z = 10 + j5$.

(2) $z = j3$.

(3) $z = -j5$.

(4) $z = -5 + j2.5$.

(b) Compute the following operations with complex numbers.

(1) $z_1 = 10 + j5$, $z_2 = -4 + j8$. $z = z_1 + z_2$.

(2) $z_1 = 10 + j5$, $z_2 = -4 + j8$. $z = z_1 z_2$.

(3) $z_1 = 10 + j5$, $z_2 = z_2 = -4 + j8$, $z_3 = 2 - j4$. $z = z_1 z_2 z_3$.

(4) $z_1 = 10 + j5$, $z_2 = z_2 = -4 + j8$, $z_3 = 2 - j4$. $z = z_1/(z_2 + z_3)$.

(5) $z = (j4)^2 + j2 - 3$.

(6) $z = \frac{1}{j4} + j5 + 1$.

(c) Compute the magnitude and phase of the following complex numbers.

(1) $z = j3$.

(2) $z = 2 - j6$.

(3) $z = \frac{1+j}{3+j6}$.

(4) $z = \frac{(1-j2)(3-j2)}{(1+j4)(3+j2)}$.

(5) $z = \frac{2+j3+(j4)^2}{1-j3-(j4)^2}$.

Question 2

Determine the time-signal $i(t)$ corresponding to the following complex currents.
Note: Indicate which transformation you have considered to go from complex current to current signals.

(a) $I = 5mA$.

(b) $I = 8e^{j\frac{\pi}{3}}mA$.

(c) $I = j2mA$.

(d) $I = (1 + j)mA$.

Question 3

Determine the complex current corresponding to the current signals:

(a) $i(t) = 4\cos(wt)\ mA$.

(b) $i(t) = 2\cos(wt + \frac{\pi}{6})\ mA$.

(c) $i(t) = 5\cos(wt - \frac{\pi}{3})\ mA$.

(d) $i(t) = 4\sin(wt)\ mA$.

Question 4

A AC voltage course with voltage $v(t) = V_0\cos(wt)$ is connected to a capacitor with capacitance C. Determine $i(t)$ using the jw-method and complex voltages.

Question 5

Consider a high-pass filter composed by a capacitor C connected in series with a resistor R. The input voltage $v_{in}(t) = V_0\cos(wt)$ is applied to this series of components, while the output voltage $v_{out}(t)$ is measured as the voltage drop across the resistor. Suppose that R and C are known.

(a) Determine the expression for the complex voltage V_{out}.

(b) What is the phase shift (or phase difference) between $v_{out}(t)$ and $v_{in}(t)$?

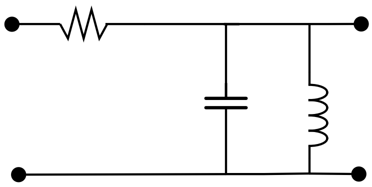
(c) Determine the voltage signal $v_{out}(t)$.

(d) Suppose that $R = 1\,k\Omega$ and $C = 1\,\mu F$. Determine the frequency f in Hz (or w in rad/s) for which the peak-amplitude of the output voltage signal is $\frac{1}{\sqrt{2}}$ of peak-amplitude of the input voltage. Determine the phase difference between the output and input voltages at this frequency.

Question 6

For each of the RLC circuits below, derive the transfer function $H(jw)$ from the input to the output Determine what is the gain $|H(jw)|$ for frequencies close to $w = 0$ and $w = \infty$.

a)



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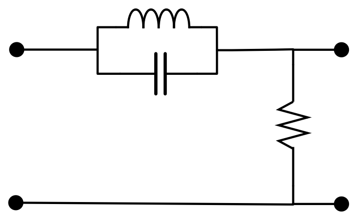

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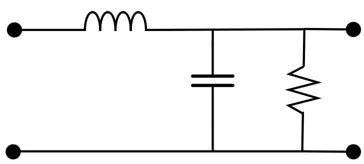

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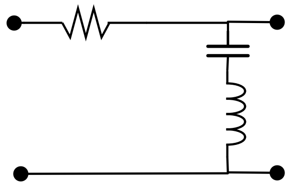

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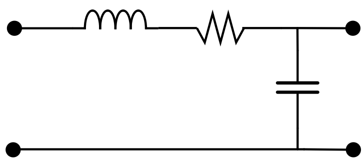

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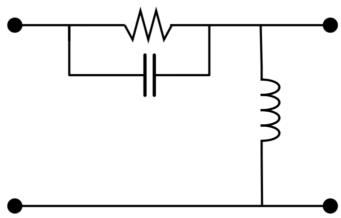

Question 7

For each of the RLC circuits in Question 6, use the respective component values indicated below and compute the amplitude gain $|H(jw)|$ and phase shift $\arg H(jw)$ of the output when a sinusoidal signal with a specific frequency is applied at the input.
Note: match the circuit in Question 6.a) with the parameters in Question 7.a), and so on for the other circuits.

(a) $R = 1\,\mathrm{k\Omega},\ C = 1\,\mu\mathrm{F},\ L = 1.0134\,\mathrm{mH},\ f = 5000\,\mathrm{Hz}.$

(b) $R = 1\,\mathrm{k\Omega},\ C = 1\,\mu\mathrm{F},\ L = 1.0134\,\mathrm{mH},\ f = 2000\,\mathrm{Hz}.$

(c) $R = 100\,\Omega,\ C = 1\,\mu\mathrm{F},\ L = 10\,\mathrm{mH},\ f = 4000\,\mathrm{Hz}.$

(d) $R = 20\,\Omega,\ C = 100\,\mathrm{nF},\ L = 100\,\mathrm{mH},\ f = 1600\,\mathrm{Hz}.$

(e) $R = 1000\,\Omega,\ C = 100\,\mathrm{nF},\ L = 100\,\mathrm{mH},\ f = 1000\,\mathrm{Hz}.$ (test also with $f = 1590$ and $f = 1580$)

(f) $R = 1000\,\Omega,\ C = 1\,\mu\mathrm{F},\ L = 1\,\mathrm{mH},\ f = 2000\,\mathrm{Hz}.$

Question 8

- Derive the transfer functions H(jw), i.e. Vo / Vi, of the following circuits.
  Determine the gain |H(jw)| for frequencies close to w = 0 and w = ∞ .