# Exam in Automatic Control II
# Reglerteknik II 5hp (1RT495)

**Date:** August 18, 2023
**Time:** 14:00 – 19:00
**Venue:** Danmarksgatan 30

**Responsible teacher:** Sérgio Pequito

**Aiding material**: Calculator, mathematical handbooks, textbooks by Glad & Ljung (Reglerteori/Control theory & Reglerteknik). Additional notes in the textbooks are allowed.

**Preliminary grades**: $\geq$ 43p for grade 5, $\geq$ 33p for grade 4, and $\geq$ 23p for grade 3.

**Use separate sheets** for each problem, i.e., only one problem per sheet. Write your exam code on every sheet.

**Important:** Your solutions should be well motivated unless else is stated in the problem formulation. Vague or lacking motivation may lead to a reduced number of points.

Good luck!

**Please use English in your solutions!**

**Problem 1** Consider the following discrete-time state-space representation:

$$\begin{pmatrix} qx_1 \\ qx_2 \end{pmatrix} = \begin{pmatrix} 0.5 & 0 \\ 1 & 0.9 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} e + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u. \tag{1}$$

**(a)** Provide the block diagram representation associated with the state-space representation in (1). **(3p)**

**(b)** Suppose that there is no error term in (1), i.e., $e = 0$. What is the transfer function of the system associated with (1) considering that the output is $x_2$? **(2p)**

**(c)** Is the system associated with (1) asymptotically stable? Explain your answer. **(2p)**

**(d)** Is the error-free system, i.e., $e = 0$, associated with (1) controllable? Explain your answer. **(2p)**

**Please use English in your solutions!**

**Problem 2** Consider the following state space representation

$$\dot{x}(t) = \begin{bmatrix} -2 & 1 \\ -1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t), \tag{2}$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t). \tag{3}$$

**(a)** Is the state-space representation of the system (2)-(3) in a canonical form? Explain your answer. **(2p)**

**(b)** Is the system represented by the following state-space representation

$$\dot{x}(t) = -x(t) + u(t),$$
$$y(t) = x(t),$$

the same as the one underlying the state-space representation (2)-(3)? Explain your answer. **(3p)**

**(c)** Can the system in (2)-(3) be a minimum realization? Explain your answer. **(3p)**

**(d)** Compute the zero-order hold (Z.O.H.) realization of the system represented by (2)-(3) with a sampling step $h > 0$. **(4p)**

**(e)** Is a Z.O.H. realization of the system (2)-(3) a minimum realization? Explain your reasoning. **(2p)**

**Please use English in your solutions!**

**Problem 3** Consider the following stochastic process

$$x(k+1) = \begin{bmatrix} 0 & 0 \\ 0 & -0.9 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} v_1(k), \tag{4}$$

where $v_1(k)$ is independent and identically distributed (standard) white noise.

**(a)** Is $x(k)$ a stationary stochastic process as time goes to infinity? Explain your answer. **(4p)**

**(b)** What is the spectral density $\Phi_z(\omega)$ associated with the reference $z(k) = \begin{bmatrix} 1 & -1 \end{bmatrix} x(k)$? **(4p)**

**(c)** Suppose that we perform measurements of the system dynamics (4) as follows

$$y(k) = \begin{bmatrix} 1 & -1 \end{bmatrix} x(k) + v_2(k),$$

such that the disturbance and sensor noise are uncorrelated, and are independent and identically distributed over time. Describe the Kalman filter (or estimator) of the state of the system described in (4), i.e., $\hat{x}(k+1|k)$, and compute the optimal steady-state gain for such estimator. **(6p)**

**Please use English in your solutions!**

**Problem 4** Consider the following state-space representation

$$\begin{cases} x(k+1) = x(k) + u(k) + v_1(k), \\ z(k) = x(k), \\ y(k) = x(k) + v_2(k), \end{cases} \tag{5}$$

where the disturbance and noise are independent and identically distributed over time but not mutually independent. Furthermore, we seek to determine a control law that minimizes the control objective associated with a linear quadratic Gaussian optimal control problem, i.e.,

$$V = E[z^2 + \rho u^2], \quad \rho > 0. \tag{6}$$

**(a)** Determine the control gain $L$. **(3p)**

**(b)** Indicate how to compute the $m$ time-steps ahead estimates $\hat{x}(k+m|k)$, with $m > 0$, using the Kalman filter. **(3p)**

**(c)** Suppose that one seeks to deploy a model predictive control (MPC) strategy for the state-space representation in (5) with objective (6). Additionally, accuracy is of utmost importance. What would be the values assigned to $\rho$, the control horizon, and the estimation horizon? **(3p)**

**(d)** Consider the MPC strategy used in (c), and suppose that despite correct choices (by assumption) of $\rho$, the control and estimation horizon, as well as proper implementations of the MPC, the system does not satisfy the specification of having its state at the pre-specified reference. What could be the causes for such unfortunate outcome? **(4p)**