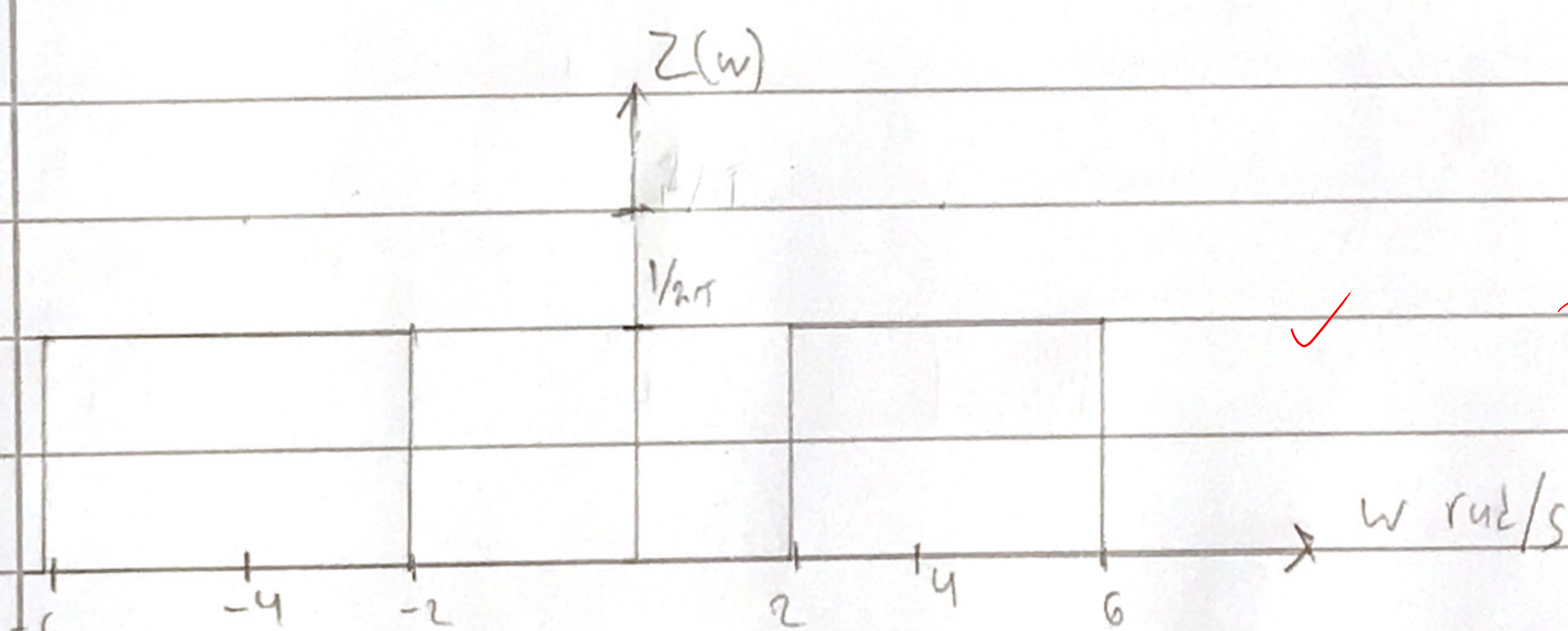
1/7

1a) The Fourier transform can be seen as the fourier series but when the period T_0 moves towards infinity, I.e a decoposition ✓ of hermonic components when their period is infinite

↳ which yields what frequencies? 1,5p

b) The fourier and laplace transforms are equivalent if
1: $X(t) = 0$ for all $t < 0$ ✓ what do we call this?
2: $s = \sigma + wj = wj \Rightarrow \sigma = 0$ ✓ 2p
3:

c)



✓ 2p

$X_1(t) = \frac{2}{\pi} \operatorname{sinc} \frac{2t}{\pi} \circ\!\!-\!\!\bullet X_1(w) = \operatorname{rect}\left(\frac{w}{2 \cdot 2}\right) = \operatorname{rect}\left(\frac{w}{4}\right)$

$z(t) = X_1(t) X(t) \circ\!\!-\!\!\bullet Z(w) = \frac{1}{2\pi} X_1(w) * X(w) = \frac{1}{2\pi} \operatorname{rect}\left(\frac{w}{4}\right) * \left(\delta(w+4) - \delta(w-4)\right)$

(used Transform table)

2/2

2 a) Yes, it's linear since: the super position principle holds:

$1 \cdot x_1(t) + 1 \cdot x_2(t) + 1 \cdot x_3(t) \mapsto 1 \cdot y_1(t) + 1 \cdot y_2(t) + 1 \cdot y_3(t)$ ✓

and its time invariant since ✓ 1p

$x_1(t-2) \mapsto y_1(t-2)$. (seen by first component in $x_2(t)$)

b) The impulse response is given when $\delta(t)$ is applied to the system. Since the system is a LTI system we can get it from:

$$\delta(t) = -x_2(t) \mapsto -y_2(t) = \mathrm{rect}\left(\frac{t-1}{2}\right) = h(t)$$ ✓ 2p

c) From the transform table we know that:

$x(t) = 2\cos\left(\frac{\pi}{2}t\right)$

$X(\omega) = 2\pi\left(\delta\left(\omega+\frac{\pi}{2}\right) + \delta\left(\omega - \frac{\pi}{2}\right)\right)$

$h(t) = \mathrm{rect}\left(\frac{t-1}{2}\right)$

$H(\omega) = e^{-j\omega} \cdot \frac{1}{2}\mathrm{sinc}\left(\frac{\omega \cdot 2}{2\pi}\right)$ ✓

$* = 0$ if $\omega \neq -\pi/2$

$** = 0$ if $\omega \neq \pi/2$

$***$ $\mathrm{sinc}(x) = \mathrm{sinc}(-x)$

Furtermore

$$Z(\omega) = X(\omega)H(\omega) = 2\pi\left(\overbrace{\delta\left(\omega+\frac{\pi}{2}\right)}^{*} + \overbrace{\delta\left(\omega-\frac{\pi}{2}\right)}^{**}\right) \cdot e^{-j\omega} \cdot \frac{1}{2}\mathrm{sinc}\left(\frac{\omega \cdot 2}{2\pi}\right)$$

$$= 2\pi e^{-j\omega}\left(\mathrm{sinc}\left(\frac{-1}{2}\right)\delta(\omega+\pi/2) + \mathrm{sinc}\left(\frac{1}{2}\right)\delta(\omega-\pi/2)\right)$$

$$\overset{***}{=} 2\pi\,\mathrm{sinc}\left(\frac{1}{2}\right) e^{-j\omega}\left(\delta(\omega+\pi/2) + \delta(\omega-\pi/2)\right)$$

(Transform table)

$$z(t) = 2\,\mathrm{sinc}\left(\frac{1}{2}\right)\cos\left(\frac{\pi}{2}(t-1)\right)$$ (v) 3,5p

3/2

3 a) $H(s) = \frac{1}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3} = \frac{As+3A+Bs+2B}{(s+2)(s+3)} =$

$= \left[\begin{matrix} A+B=0 \\ 3A+2B=1 \end{matrix} \Rightarrow \begin{matrix} A=1 \\ B=-1 \end{matrix}\right] \Rightarrow$

$H(s) = \frac{1}{s+2} - \frac{1}{s+3}$

(Transform tables)

$h(t) = e^{-2t}u(t) - e^{-3t}u(t) = u(t)\left(e^{-2t} - e^{-3t}\right)$ ✓ 2p

b) The systems frequency response is given by:

$H(\omega) = \frac{1}{(j\omega+2)(j\omega+3)} = \frac{1}{-\omega^2+3j\omega+2j\omega+6} = \frac{1}{6-\omega^2+5j\omega}$ ✓ 2p

c) We first write the transferfunction $H(s)$ in bode form:

$H(s)\Big|_{s=j\omega} = \frac{1}{(j\omega+2)(j\omega+3)} = \frac{1}{6} \cdot \frac{1}{\left(\frac{j\omega}{2}+1\right)\left(\frac{j\omega}{3}+1\right)}$

We then write the magnitude as dB:

$|H(j\omega)|_{dB} = 20\log_{10}\left|\frac{1}{6}\right| - 20\log_{10}\left|\frac{j\omega}{2}+1\right| - 20\log_{10}\left|\frac{j\omega}{3}+1\right|$

$\approx -16 - 20\log_{10}\left|\frac{j\omega}{2}+1\right| - 20\log_{10}\left|\frac{j\omega}{3}+1\right|$ ✓

The magnitude plot will for low ω be -16 dB. When $\omega = 2$ rad/s it droppes to -19 dB ($-16-3$ dB) and have a slope with -20 dB/dekade. When $\omega = 3$ the magnitude begins to decrease with -40 dB/dekade. ✓ See included plot on page 5

3c) For the phase plot we first write the phase function:

$$\angle H(j\omega) = \angle \frac{1}{6} \cdot \frac{1}{\left(\frac{j\omega}{2}+1\right)\left(\frac{j\omega}{3}+1\right)} = 0 - \operatorname{atan}\left(\frac{\omega}{2}\right) - \operatorname{atan}\left(\frac{\omega}{3}\right)$$

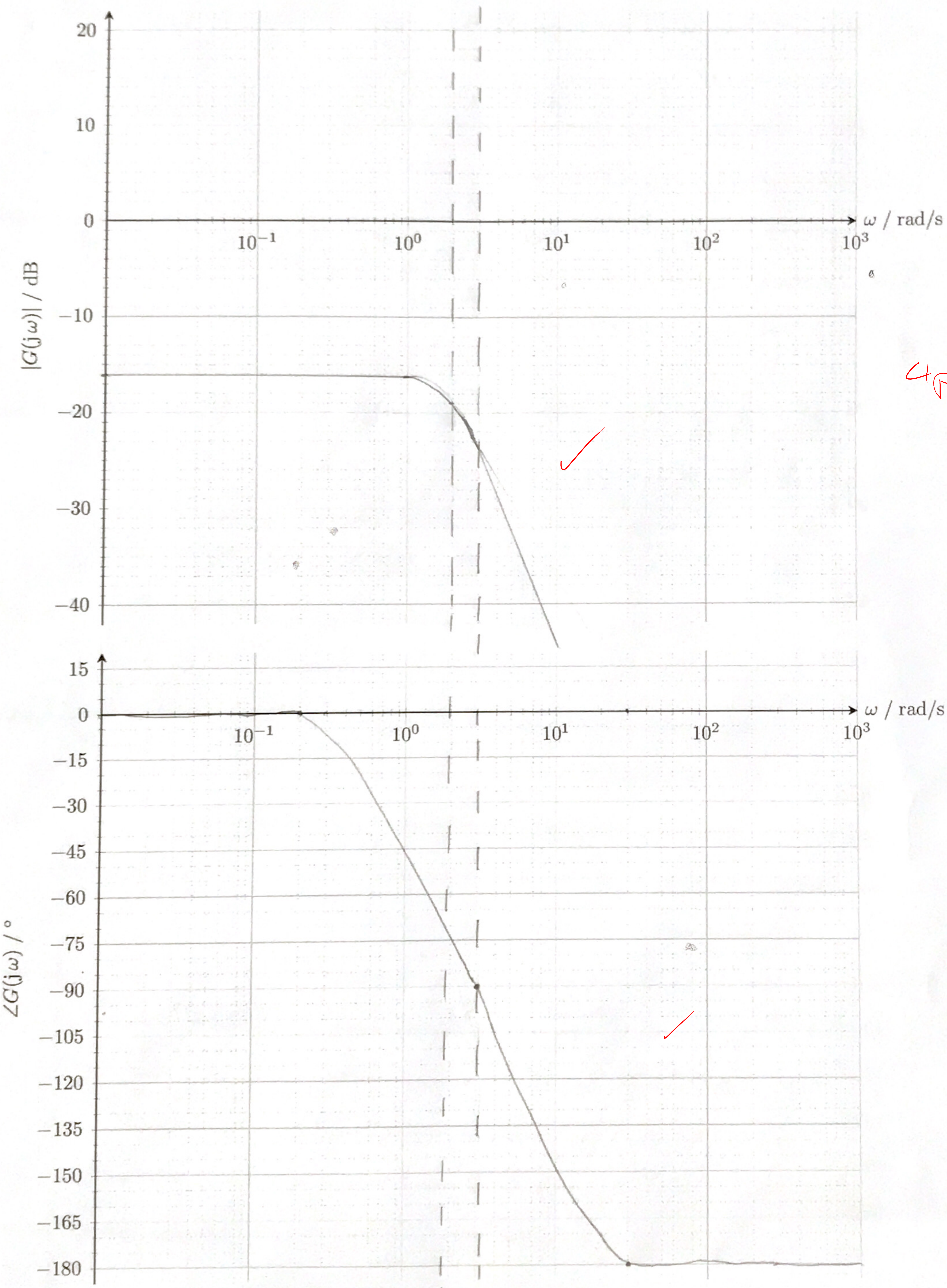
$$-\operatorname{atan}\left(\frac{\omega}{2}\right) = \begin{cases} 0 & \text{for } \omega \ll 2 \text{ rad/s} \\ -45^\circ & \text{for } \omega = 2 \text{ rad/s} \\ -90^\circ & \text{for } \omega \gg 2 \text{ rad/s} \end{cases}$$ ✓

$$-\operatorname{atan}\left(\frac{\omega}{3}\right) = \begin{cases} 0 & \text{for } \omega \ll 3 \text{ rad/s} \\ -45^\circ & \text{for } \omega = 3 \text{ rad/s} \\ -90^\circ & \text{for } \omega \gg 3 \text{ rad/s} \end{cases}$$ ✓

Which gives the plot shown on page 5.

5/7 3c)

Bode Diagram



4P

6/7

4a) We first apply Laplace transform which gives:

$$s^2 Y(s) + 6s Y(s) + 13 Y(s) = s X(s) + 2 X(s)$$

$$\Leftrightarrow$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s+2}{s^2+6s+13}$$

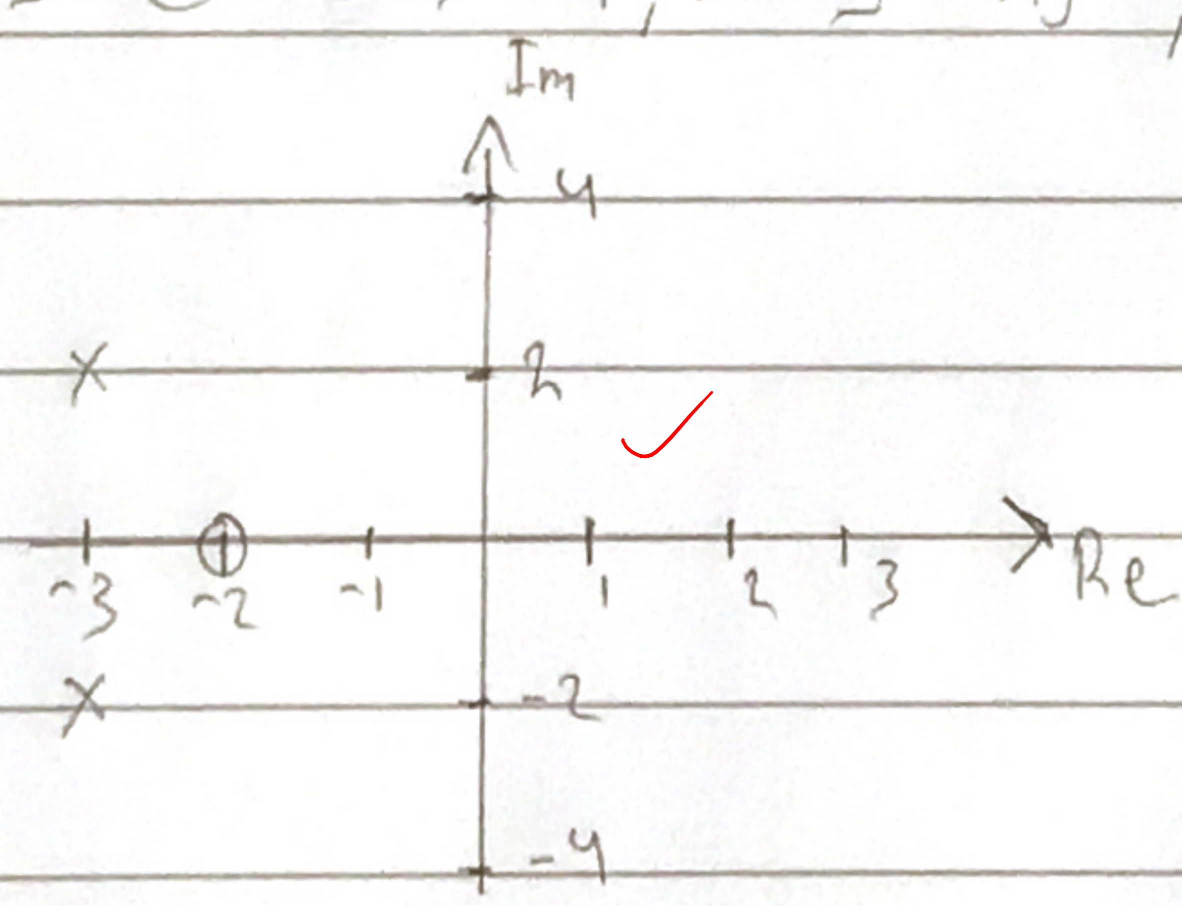
2p

b) The zeros are given by the s where the numerator is 0, i.e:

$s + 2 = 0 \Leftrightarrow s = -2 \Rightarrow z_1 = -2$

The poles are given by the s wher the denumerator is 0, ie:

$s^2 + 6s + 13 = 0 \Rightarrow p_1 = -3 + 2j, \; p_2 = -3 - 2j$



The system is stable since the real part of all poles is less than 0!

$Re\{p_i\} = -3 < 0, \; i = 1, 2$

2p

7/7

4c) We know that $x(t) = u(t) \circ\!\!-\!\!\bullet X(s) = 1/s$.

Then $Z(s)$ is given by

$$Z(s) = H(s)X(s) = \frac{s+2}{s^2+6s+13} \cdot \frac{1}{s} = \frac{s+2}{s(s^2+6s+13)}$$ 0,5p

To determine the output signal we meed to find the transform pair

$Z(s) \circ\!\!-\!\!\bullet z(t)$

which was hard...

Another way is to find $h(t)$, and

then use the def. for utput of system

$y(t) = h(t) * x(t)$

But time's up...