Fixed Income Assignment

Jeroen Kerkhof

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1 Practicalities

Assignment for the Fixed Income class. Due on Sunday 22 March 2026, 23.59h. Assignments can be made in groups of maximum 3 people. Discussions between groups regarding general content and programming techniques is allowed (actually encouraged). However, copy-pasting of material (code or report) is NOT allowed and will be reported. You are expected to hand-in a report with discussion of the questions and results, tables and figures. No code should be present in the report. This should be provided separately in a **working** Python (.py) file. I need to be able to run that file if the data set is in the same folder as the .py file. DO NOT LINK TO folders like `'C:\Dropbox\blablabla\fixed_income\assignment\'`

For the report, you are expected to clearly translate your technical findings into plain English. Just reporting tables with estimatess and graphs is NOT enough. If you are unsure, what is expected, you might want to watch the plain_english.mp4 (again) and put yourself in the role of the analyst (though you might want to leave out the part about me being a golden retriever).

2 Exercises

exercise 1. This question is about Black-Scholes-Merton Hedging.

```
v = bs_value(S, T, K, r, q, sigma,
                OptionTypes.EUROPEAN_CALL.value)
delta = bs_delta(S, T, K, r, q, sigma,
                     OptionTypes.EUROPEAN_CALL.value)
```

(a) Install FinancePy and for speed issues, in the following, call directly into the model library rather than go via the EquityOptions class. See the example code above. This will be about 60 times faster than going via pure Python code.

(b) Write a Python function called **DeltaHedge** that simulates the delta hedging of a European put option from trade date until expiry using a self-financing portfolio. It should use the function in (a) for calculating the option price and delta. The function inputs must include the option strike K, spot price of the stock S, risk-free rate r, the stock price drift μ (we do not necessarily assume that the stock grows at r), volatility σ and years to expiry T. The other input must be the hedging frequency per year N.

The dynamics of the stock price should be assumed to be lognormal with a drift μ (hedging does not have to set equal $\mu = r$ as the true stock price evolution is not risk-neutral) and a volatility σ_{Real}. The output of your function should be a tuple that has 4 elements:

- The terminal stock price $S(T)$ in the simulation path;
- The option payoff;
- The realized variance of returns during the hedging period;
- The replicating error which is the difference between the total value of the hedging portfolio and the option payoff.

Make sure your code is clear and well-commented with good variable names.

(c) Write another function that calls the previous DeltaHedge function and which can then be used to calculate the hedging error over $10,000$ different paths. You must provide a clear and easy-to-understand listing of your code in the answer.

(d) Consider a put option with $S(0) = 100$, $K = 100$, $r = 4\%$, $T = 1.0$ and $\sigma = 20\%$. Assume here that $\mu = 5\%$. For this option, make a scatterplot of the hedging error (y-axis) versus the terminal stock price (x-axis) for $N = 12$ (monthly), $N = 52$ (weekly) and $N = 252$ (daily). Use different symbols or colours to distinguish the points.

(e) For each value of N also calculate the mean and variance of this option hedging error over 10,000 different paths. You can use this to generate the answers to the remaining parts of this question. Present this in a simple table format.

(f) Create a scatterplot of the realized volatilities vs the replication error. Explain the pattern.

(g) For the same put option, calculate the mean absolute error value and the variance of the hedging error for $\mu = 2.5\%, 5.0\%, 7.5\%, 10\%$ by sampling $10,000$ hedging paths using $N = 52$. Show the results in a table. What does this tell you? Does the value of the drift change the hedging by a little or a lot?

exercise 2. This question is about time-varying volatility.

(a) We start with the code of Question 1. However, we make one adjustment. Instead of having a constant volatility of 20%, we now have a volatility period of 4% for the first half of T and 28% for the second half. First, value a put option with $S(0) = 100$, $K = 100$, $r = 4\%$, $T = 1.0$ (same as in Q1). Compare this to the value of a put option with $S(0) = 100$, $K = 100$, $r = 4\%$, $T = 1.0$ and $\sigma = 20\%$. [Hint: There should be no difference]

(b) Now create an updated delta hedging function DeltaHedgeTV allowing for time-varying volatility. [Hint: What happens to the average volatility when the time to maturity decreases?]

(c) Perform the delta hedge exercise from 1(d) again. How does the performance of the hedge compare with the constant volatility case?

(d) Now consider the case where the first half has a volatility of 28% and the second half has a volatility of 4%.

(e) Explain (if any) the differences between 1) constant volatility 2) low volatility first half 3) low volatility second half.

exercise 3. This question is about determining the implied density of the terminal stock price from the volatility skew.

(a) Suppose that we have managed to fit the 1-year volatility smile of the equity option market using a function

$$\sigma(x) = ax^2 + bx + c$$

where x is the "moneyness" ($x = K/S(0)$) and the initial stock price $S(0) = 100$ and $a = 0.025$, $b = -0.23$ and $c = 0.55$. Build a Python function that extracts the market-implied distribution of $S(T)$ at a 1-year horizon from $\sigma(x)$ using the Breeden-Litzenberger formula we derived in class. You can (should) use FinancePy to calculate the option prices. Assume that $r = 4\%$ and $q = 0$.

(b) A digital call option pays \$1 if $S(T) > K$ and zero otherwise. Using the probability density function implied by this volatility smile, calculate a table of prices of the 1-year European digital call option with strikes at $K = 60, 80, 100, 120, 140$. Assume that $r = 4\%$ and $q = 0.0\%$.

(c) Price the same set of digital call options using the Black-Scholes pricing formula for digital call options in the lecture notes or use FinancePy.

(d) Explain why the results of (b) and (c) do not agree with each other and explain which prices you think are more correct.

(e) Calculate the value of a put option with strike \$100 which only pays out if the stock falls below \$75 expiry (this is a European and not a path dependent option). Explain carefully and clearly how this was done.

Maturity	Quote
1M	1.1%
2M	1.15%
3M	1.23%
6M	1.32%
12M	1.40%
2Y	1.51%
3Y	1.53%
4Y	1.55%
5Y	1.59%
6Y	1.61%
7Y	1.65%
8Y	1.69%
9Y	1.71%
10Y	1.73%

Table 1: Quotes for swap rates and IBOR rates on 2-Feb-2026 (settlement on 4-Feb-2026).

Effective date	Maturity	Freq	Rate	Pay/Rec
3-Apr-2022	10Y	SA	1.40%	Rec
3-Oct-2023	4Y	A	1.52%	Pay
3-Oct-2021	12Y	A	2.00%	Rec
3-Apr-2024	5Y	SA	2.10%	Pay

Table 2: Historical IR swaps.

exercise 4. This question is about Swap and Swaption Pricing.

(a) You rotate on the swap desk for a day. Using FinancePy create an IBOR curve using swap rates with maturities of 2-10 years and the IBOR rates given in Table 1. Use a semi-annual fixed frequency and a daycount type of THIRTY_E_360_ISDA. Demonstrate that all swaps price to zero.

(b) You are asked to manage a (small) book of swaps given in Table 2. First, mark-to-market these swaps (i.e. determine the value of each swap and the whole portfolio). [Hint: Use the settlement date in swap.value]

(c) Now that you know the value, you are interested in the risk of the book. Calculate the change in value of the portfolio for with respect to a 1bp change in the 2Y-10Y swap rates. Explain how you would hedge this portfolio. Do not do any further calculations.

(d) Now you are moving to the swaption desk for a day. Calculate the forward rate for a 4×6 year swap with semi-annual fixed leg payments. Show the value for $Z(t, T_1), Z(t, T_2)$ and the swap PV01.

(e) Now using Black's model, value a 4×6 Payer and a 4×6 Receiver swaption with a strike 1.75%. Assume a swaption volatility of 25%. Explain the prices.

(f) Now that you know the value of the swaptions you are again focussed on the risk. Calculate the change in value for the payer and receiver swaption with respect to a 1bp change in the 2Y-10Y swap rates. Explain how these swaptions would be hedged? Show the results in a table. Do not do any further calculations.

Maturity	Quote (bp)
1Y	70
2Y	74
3Y	80
4Y	85
5Y	88

Table 3: CDS quotes

exercise 5. This question is about CDS Valuation and Risk.

(a) Today you move to the CDS desk. The first thing they ask you to do is build the curve based on the quotes for today for Company DoOrDie. Use FinancePy's CDS functions (CDS and CDSCurve) to build a cds curve for this company. Use a recovery rate of 40%.

(b) Determine the market spread of a 3.5-year CDS that matures on the 20 March 2029. Explain how the value is determined.

(c) Calculate the value of an existing long protection CDS contract traded with a contractual spread of 120bp with a maturity of 21 March 2029 and a notional of $20m.

(d) Recalculate the value of the contract for $R = 0\%, 10\%, 20\%, 30\%$. Are the changes significant or not? Can you explain why?

(e) Calculate the change in the CDS value to a 1bp increase in each of the 1Y, 2Y, 3Y, 4Y and 5Y CDS market rates. Show the results in a table. Explain what you find and how a dealer would hedge this 3.5 year CDS trade. Do not do any further calculations.

maturity	quote
1Y	6.05%
2Y	4.30%
3Y	3.50%
4Y	3.15%
5Y	2.95%
6Y	2.90%
7Y	2.70%
8Y	2.65%
9Y	2.60%
10Y	2.55%
11Y	2.50%
12Y	2.48%
15Y	2.46%
20Y	2.44%
25Y	2.42%
30Y	2.40%

Table 4: quotes for annual zero-coupon inflation swaps. Base month equals November-2025.

Nob-25	120.70
Dec-25	120.24

Table 5: Inflation History

exercise 6.

This question is about Inflation Markets

(a) calibrate the inflation curve $I(0,T)$ for all necessary T using the zero-coupon inflation swap quotes in Table 4. The latest inflation numbers are given in Table 5. Nominal interest rates are flat at 3%.

(b) Determine the forward CPI numbers for Nov-26, Nov-27, ..., Nov-37, Nov-40, Nov-55. Interpolate the curve using log-linear interpolation.

(c) Compute the inflation PV01 for all quoted inflation swaps.

(d) Now consider the liabilities of a pension fund given in Table 6. Determine the nominal and real value of these liabilities.

(e) Explain how you could hedge the interest-rate and inflation risk of these liabilities.

maturity	liability
1Y	1m
2Y	2m
3Y	4m
4Y	5m
5Y	8m
6Y	10m
7Y	12m
8Y	15m
9Y	18m
10Y	25m
11Y	23m
12Y	20m
15Y	15m
20Y	9m
25Y	6m
30Y	2m

Table 6: Real liability profile of a small pension fund (in mln).