$$M\ddot{\bar{x}} + K\bar{x} = 0$$

$\bar{x} = (L^T)^{-1}\bar{q}$

$\bar{x}$: state vector

$\bar{q}$: mass/inertia normalized coordinates

$M = LL^T$

L: cholesky decomposition

n eigenvalues

$\lambda_i \quad i = 1, \dots, n$

$\sqrt{\lambda_i}$ eigen frequencies (natural frequencies)

$\bar{v}_i$: eigenvectors

↳ orthonormal set

$c_i \sin(\omega t + \phi) = A \sin \omega t + B \cos \omega t$

$\ddot{\bar{q}} + \tilde{K}\bar{q} = 0$

$\tilde{K} = L^{-1} K (L^T)^{-1}$

↳ guaranteed to be symmetric

$\tilde{K}$: mass normalized stiffness matrix

$\tilde{K}\bar{v} = \lambda \bar{v} \leftarrow$ eigenvalue prob

$\det(\tilde{K} - I\lambda) = 0 \rightarrow$ characteristic eq

$$\bar{q}(t) = \sum_{i=1}^{n} c_i \bar{v}_i \sin(\omega_i + \phi_i)$$

$P = [\bar{v}_1, \dots, \bar{v}_n]$

$\bar{q} = P\bar{r}$

↳ modal coordinate

$\ddot{\bar{r}} + \Lambda \bar{r} = 0$

modal equations

Λ: spectral matrix

↑ diagonal!!

$$\Lambda = \begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{bmatrix} = \begin{bmatrix} \omega_1^2 & & 0 \\ & \ddots & \\ 0 & & \omega_n^2 \end{bmatrix}$$

$$\begin{bmatrix} \ddot{r}_1 \\ \vdots \\ \ddot{r}_n \end{bmatrix} + \begin{bmatrix} \omega_1^2 r_1 \\ \vdots \\ \omega_n^2 r_n \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\bar{r}(t) = \begin{bmatrix} r_1(t) \\ \vdots \\ r_n(t) \end{bmatrix} = \begin{bmatrix} A_1 \sin(\omega_1 t + \phi_1) \\ \vdots \\ A_n \sin(\omega_n t + \phi_1) \end{bmatrix}$$

$$\bar{x} = (L^T)^{-1}\bar{q} = (L^T)^{-1} P \bar{r}$$

$\tilde{K}$ is symmetric:

- all eigenvals are real
- all eigenvecs are real
- eigenvals are positive if and only if $\tilde{K}$ is positive definite

- eigenvectors can be chosen to be orthogonal, for repeated eigenvalues

$$S = (L^T)^{-1} P = [\bar{u}_1, \ldots, \bar{u}_n]$$

evecs in mass normalized

mode shapes

$$\bar{x}(t) = \sum_{i=1}^{n} \underbrace{c_i \bar{u}_i \sin(\omega_i t + \phi_i)}_{\text{determined initial conditions}}$$

$$\phi_i = \arctan\left(\frac{\omega_i \bar{v}_i^T \bar{q}_0}{\bar{v}_i^T \dot{\bar{q}}_0}\right)$$

mass normalized initial conditions

$$\bar{x}_0 = (L^T)^{-1} \bar{q}_0$$

$$c_i = \frac{\bar{v}_i^T \bar{q}_0}{\sin \phi_i}$$

modal participation factor