

Equations for system

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We follow [1] very very closely.

Domain

Our domain Ω is $d = 2$ or 3 -dimensional, and partitioned into Ω_f and Ω_p , with $\Gamma_{fp} = \Omega_f \cap \Omega_p$ being the $(d - 1)$ -dimensional interface. The boundary $\partial\Omega$ is partitioned into $\Gamma_f = \partial\Omega \cap \partial\Omega_f$ and $\Gamma_p = \partial\Omega \cap \partial\Omega_p$. We assume each region is connected, reasonably smooth and all that.

Unknowns

The unknowns of the system and the corresponding test functions are:

- $\mathbf{u}_f, \mathbf{v}_f$: free flow fluid velocity. Defined on Ω_f .
- $\mathbf{u}_p, \mathbf{v}_p$: porous flow fluid velocity. Defined on Ω_p .
- p_f, w_f : free flow fluid pressure. Defined on Ω_f .
- p_p, w_p : porous flow fluid pressure. Defined on Ω_p .
- $\boldsymbol{\eta}_p, \boldsymbol{\xi}_p$: displacement. Defined on Ω_p .
- $\lambda_\Gamma, \mu_\Gamma$: normal stress balance Lagrange multiplier. Defined on Γ_{fp} . In [1], denoted λ, μ_h .

Parameters

μ_f fluid viscosity (denoted μ in the [1])

λ_p, μ_p Lamé parameters. Denoted μ in [1].

α Biot-Willis constant

K Permeability tensor. Symmetric, bounded, positive definite. I take it to be scalar.

α_{BJS} Friction coefficient

s_0 Storage coefficient

$B = \frac{\mu_f \alpha_{BJS}}{\sqrt{K}}$ The coefficient in the BJS condition. Could be zero.

Notation

- $\mathbf{n}_f, \mathbf{n}_p$ are the outward unit normal vectors to $\partial\Omega_f, \partial\Omega_p$.
- $\boldsymbol{\tau}_{f,j}, j = 1, \dots, d-1$ is an orthogonal system of unit tangent vectors at Γ_{fp} .
- $\mathbf{D}(\mathbf{v}) := \frac{1}{2}(\nabla \mathbf{v} + \nabla \mathbf{v}^T)$
- $\boldsymbol{\sigma}_f(\mathbf{u}_f, p_f) := -p_f \mathbf{I} + 2\mu_f \mathbf{D}(\mathbf{u}_f)$
- $\boldsymbol{\sigma}_p(\boldsymbol{\eta}_p, p_p) := \lambda_p(\nabla \cdot \boldsymbol{\eta}_p) \mathbf{I} + 2\mu_p \mathbf{D}(\boldsymbol{\eta}_p) - \alpha p_p \mathbf{I}$

Next, here are a bunch of bilinear forms used in the problem:

- $a_f(\mathbf{u}_f, \mathbf{v}_f) = (2\mu \mathbf{D}(\mathbf{u}_f), \mathbf{D}(\mathbf{v}_f))_{\Omega_f}$
- $a_p^d(\mathbf{u}_p, \mathbf{v}_p) = (\mu K^{-1} \mathbf{u}_p, \mathbf{v}_p)_{\Omega_p}$
- $a_p^e(\boldsymbol{\eta}_p, \boldsymbol{\xi}_p) = (\mu_p \mathbf{D}(\boldsymbol{\eta}_p), \mathbf{D}(\boldsymbol{\xi}_p))_{\Omega_p} + (\lambda_p \nabla \cdot \boldsymbol{\eta}_p, \nabla \cdot \boldsymbol{\xi}_p)_{\Omega_p}$
- $b_f(\mathbf{v}_f, w_f) = -(\nabla \cdot \mathbf{v}_f, w_f)_{\Omega_f}$
- $b_p(\mathbf{v}_p, w_p) = -(\nabla \cdot \mathbf{v}_p, w_p)_{\Omega_p}$
- $a_{BJS}(\mathbf{u}_f, \boldsymbol{\eta}_p; \mathbf{v}_f, \boldsymbol{\xi}_p) = \frac{\mu \alpha_{BJS}}{\sqrt{K}} \sum_{j=1}^{d-1} ((\mathbf{u}_f - \boldsymbol{\eta}_p) \cdot \boldsymbol{\tau}_j, (\mathbf{v}_f - \boldsymbol{\xi}_p) \cdot \boldsymbol{\tau}_j)_{\Gamma_{fp}}$
- $b_{\Gamma_{fp}}(\mathbf{v}_f, \mathbf{v}_p, \boldsymbol{\xi}_p; \mu_\Gamma) = (\mathbf{v}_f \cdot \mathbf{n}_f + (\boldsymbol{\xi}_p + \mathbf{v}_p) \cdot \mathbf{n}_p, \mu_\Gamma)_{\Gamma_{fp}}$

Strong formulation

These are all ignoring body force and source terms. So it's okay to put stuff on the right hand sides if you want to.

Stokes (applies in Ω_f):

$$-\nabla \cdot \boldsymbol{\sigma}_f(\mathbf{u}_f, p_f) = 0 \quad (1a)$$

$$\nabla \cdot \mathbf{u}_f = 0 \quad (1b)$$

Darcy (applies in Ω_p):

$$\mathbf{u}_p = -\frac{K}{\mu} \nabla p_p \quad (2)$$

Biot (applies in Ω_p):

$$-\nabla \cdot \boldsymbol{\sigma}_p(\boldsymbol{\eta}_p, p_p) = 0 \quad (3a)$$

$$\partial_t (s_0 p_p + \alpha \nabla \cdot \boldsymbol{\eta}_p) + \nabla \cdot \mathbf{u}_p = 0 \quad (3b)$$

Interface conditions

Conservation of mass:

$$\mathbf{u}_f \cdot \mathbf{n}_f + (\partial_t \boldsymbol{\eta}_p + \mathbf{u}_p) \cdot \mathbf{n}_p = 0 \quad (4)$$

Balance of stress :

$$-(\boldsymbol{\sigma}_f \mathbf{n}_f) \cdot \mathbf{n}_f = p_p + C_p = \lambda \quad (5)$$

$$\boldsymbol{\sigma}_f \mathbf{n}_f + \boldsymbol{\sigma}_p \mathbf{n}_p = 0 \quad (6)$$

BJS condition:

$$-(\boldsymbol{\sigma}_f \mathbf{n}_f) \cdot \boldsymbol{\tau}_j = B (\mathbf{u}_f - \partial_t \boldsymbol{\eta}_p) \cdot \boldsymbol{\tau}_j \quad (7)$$

[1] does not have the constant C_p in (5), but their interface is 'actually flat' and not a vessel wall with muscles exerting force of their own, so that's reasonable.

Boundary conditions

In their numerical experiment (section 7.2), [1] use the domain shown in figure The Darcy boundary Γ_p is partitioned into the left part Γ_p^L and the remainder

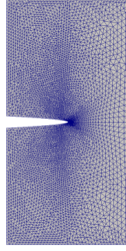


Figure 1: Darcy domain from [1]. Stokes domain is the removed 'finger'.

Γ_p^L in the obvious way. Physically, I think Γ_p^L is above ground and the other part is below ground or something.

As boundary conditions, they use:

- $\mathbf{u}_f = 10\mathbf{n}_f$ on Γ_f
- $\mathbf{u}_p \cdot \mathbf{n}_p = 0$ on Γ_p^{left}
- $p_p = 1000$ on $\Gamma_p^{\neg \text{left}}$
- $\boldsymbol{\eta}_p \cdot \mathbf{n}_p = 0$ on $\Gamma_p^{\neg \text{left}}$
- $(\boldsymbol{\sigma}_p \mathbf{n}_p) \cdot \boldsymbol{\tau}_p = 0$ on $\Gamma_p^{\neg \text{left}}$

I think I can use essentially the same boundary conditions, although maybe I should prescribe the pressure on Γ_f instead of the flow.

Variational formulation

Having used a backward Euler discretization of the time derivative, [1] obtain the following variational formulation

$$\begin{aligned}
& a_f(\mathbf{u}_f, \mathbf{v}_f) + b_f(\mathbf{v}_f, p_f) + a_p^e(\boldsymbol{\eta}_p, \boldsymbol{\xi}_p) \\
& \quad + \alpha b_p(\boldsymbol{\xi}_p, p_p) + a_p^d(\mathbf{u}_p, \mathbf{v}_p) + b_p(\mathbf{v}_p, p_p) \\
& \quad + b_{\Gamma_{fp}}(\mathbf{v}_f, \mathbf{v}_p, \boldsymbol{\xi}_p; \lambda) + a_{BJS}\left(\mathbf{u}_f, \frac{\boldsymbol{\eta}_p}{\Delta t}; \mathbf{v}_f, \boldsymbol{\xi}_p\right) \\
& = a_{BJS}\left(\mathbf{u}_f, \frac{\boldsymbol{\eta}_p^{n-1}}{\Delta t}; \mathbf{v}_f, \boldsymbol{\xi}_p\right) + (\mathbf{v}_p, C_p \mathbf{n}_p)_{\Gamma_{fp}} + (\boldsymbol{\sigma}_f \mathbf{n}_f, \mathbf{v}_f)_{\Gamma_f} \\
& \quad + (\boldsymbol{\sigma}_p \mathbf{n}_p, \boldsymbol{\xi}_p)_{\Gamma_p} + (p_p \mathbf{n}_p, \mathbf{v}_p)_{\Gamma_p}
\end{aligned} \tag{8a}$$

$$\begin{aligned}
& \left(s_0 \frac{p_p}{\Delta t}, w_p\right)_{\Omega_p} - \alpha b_p\left(\frac{\boldsymbol{\eta}_p}{\Delta t}, w_p\right) - b_p(\mathbf{u}_p, w_p) - b_f(\mathbf{u}_f, w_f) \\
& = \left(s_0 \frac{p_p^{n-1}}{\Delta t}, w_p\right)_{\Omega_p} - \alpha b_p\left(\frac{\boldsymbol{\eta}_p^{n-1}}{\Delta t}, w_p\right)_{\Omega_p}
\end{aligned} \tag{8b}$$

$$b_{\Gamma_{fp}}(\mathbf{u}_f, \mathbf{u}_p, \boldsymbol{\eta}_p; \mu_\Gamma) = b_{\Gamma_{fp}}(\mathbf{u}_f, \mathbf{u}_p, \boldsymbol{\eta}_p^{n-1}; \mu_\Gamma) \tag{8c}$$

Here the unknowns with no superscript mean the unknowns at time n (e.g. $\mathbf{u}_f = \mathbf{u}_f^n$).

Derivation of weak form

Briefly, (8a) is (1a) multiplied by \mathbf{v}_f integrated over Ω_f ; (3a) multiplied by $\boldsymbol{\xi}_p$ integrated over Ω_p ; and (2) multiplied \mathbf{v}_p integrated over Ω_p . The interface conditions are also all used.

To be more detailed, multiply (1a) by \mathbf{v}_f and integrate over Ω_f . By standard vector calculus,

$$\int_{\Omega_f} -\mathbf{v}_f \cdot \nabla \cdot \boldsymbol{\sigma}_f = \int_{\Omega_f} \boldsymbol{\sigma}_f : \nabla \mathbf{v}_f - \int_{\partial\Omega_f} \boldsymbol{\sigma}_f \mathbf{n}_f \cdot \mathbf{v}_f.$$

Now, as the $\partial\Omega_f$ consists of Γ_f and Γ_{fp} , the boundary term splits into an integral over Γ_f which we can move to the RHS by applying boundary conditions¹, and an interface term $-I_{\mathbf{v}_f} = - \int_{\Gamma_{fp}} \boldsymbol{\sigma}_f \mathbf{n}_f \cdot \mathbf{v}_f$.

¹Specifically, Dirichlet conditions on \mathbf{u}_f or Neumann conditions on $\boldsymbol{\sigma}_f$.

Before proceeding, we expand²

$$\begin{aligned}
\int_{\Omega_f} \boldsymbol{\sigma}_f : \nabla \mathbf{v}_f &= \int_{\Omega_f} (-p_f \mathbf{I} + 2\mu_f \mathbf{D}(\mathbf{u}_f)) : \nabla \mathbf{v}_f \\
&= \int_{\Omega_f} -p_f \nabla \cdot \mathbf{v}_f + \int_{\Omega_f} 2\mu_f \mathbf{D}(\mathbf{u}_f) : \nabla \mathbf{v}_f \\
&= b_f(\mathbf{v}_f, p_f) + a_f(\mathbf{u}_f, \mathbf{v}_f)
\end{aligned}$$

So the contribution here is

$$a_f(\mathbf{u}_f, \mathbf{v}_f) + b_f(\mathbf{v}_f, p_f) - I_{\mathbf{v}_f} - (\boldsymbol{\sigma}_f \mathbf{n}_f, \mathbf{v}_f)_{\Gamma_f}.$$

Next, multiply (3a) by $\boldsymbol{\xi}_p$ and integrate over Ω_p . By exactly the same argument,

$$\int_{\Omega_p} -\boldsymbol{\xi}_p \cdot \nabla \cdot \boldsymbol{\sigma}_p = \int_{\Omega_p} \boldsymbol{\sigma}_p : \nabla \boldsymbol{\xi}_p - \int_{\partial\Omega_p} \boldsymbol{\sigma}_p \mathbf{n}_p \cdot \boldsymbol{\xi}_p.$$

Again, the boundary term splits into an integral over Γ_p where we need boundary conditions³ and an interface term $-I_{\boldsymbol{\xi}_p} = - \int_{\Gamma_{fp}} \boldsymbol{\sigma}_p \mathbf{n}_p \cdot \boldsymbol{\xi}_p$. Expanding,

$$\begin{aligned}
\int_{\Omega_p} \boldsymbol{\sigma}_p : \nabla \boldsymbol{\xi}_p &= \int_{\Omega_p} (\lambda_p (\nabla \cdot \boldsymbol{\eta}_p) (\nabla \cdot \boldsymbol{\xi}_p) + 2\mu_p \mathbf{D}(\boldsymbol{\eta}_p) : \nabla \boldsymbol{\xi}_p) - \alpha p_p \nabla \cdot \boldsymbol{\xi}_p \\
&= a_p^e(\boldsymbol{\eta}_p, \boldsymbol{\xi}_p) + \alpha b_p(\boldsymbol{\xi}_p, p_p)
\end{aligned}$$

So the contribution is

$$a_p^e(\boldsymbol{\eta}_p, \boldsymbol{\xi}_p) + \alpha b_p(\boldsymbol{\xi}_p, p_p) - I_{\boldsymbol{\xi}_p} - (\boldsymbol{\sigma}_p \mathbf{n}_p, \boldsymbol{\xi}_p)_{\Gamma_p}.$$

Next, multiply (2) by $\frac{\mu}{K} \mathbf{v}_p$ and integrate over Γ_p . Integration by parts yields

$$\int_{\Omega_p} \frac{\mu}{K} \mathbf{v}_p \cdot \mathbf{u}_f - \int_{\Omega_p} p_p \nabla \cdot \mathbf{v}_p = \int_{\partial\Omega_p} p_p \mathbf{v}_p \cdot \mathbf{n}_p$$

The boundary term splits into an integral over Γ_p where we need boundary conditions⁴ and an interface term $I_{\mathbf{v}_p} = \int_{\Gamma_{fp}} p_p \mathbf{v}_p \cdot \mathbf{n}_p$, so the contribution is

$$a_p^d(\mathbf{u}_p, \mathbf{v}_p) + b_p(\mathbf{v}_p, p_p) + I_{\mathbf{v}_p} - (p_p \mathbf{n}_p, \mathbf{v}_p)_{\Gamma_p}.$$

²The equality $\mathbf{D}(\mathbf{u}_f) : \nabla \mathbf{v}_f = \mathbf{D}(\mathbf{u}_f) : \mathbf{D}(\mathbf{v}_f)$ looks false because $\nabla \mathbf{v}_f \neq \mathbf{D}(\mathbf{v}_f)$, but $\mathbf{D}(\mathbf{u}_f)$ is symmetric, so $\mathbf{D}(\mathbf{u}_f) : \nabla \mathbf{v}_f = \mathbf{D}(\mathbf{u}_f) : \nabla \mathbf{v}_f^T = \mathbf{D}(\mathbf{u}_f) : \mathbf{D}(\mathbf{v}_f)$.

³Specifically, Dirichlet conditions on $\boldsymbol{\eta}_p$ or Neumann conditions on $\boldsymbol{\sigma}_p$.

⁴Specifically, Dirichlet conditions on \mathbf{u}_p or Neumann conditions on p_p .

Next, once we add all these equations together, we will have to handle the sum of the interface terms

$$-I_{\mathbf{v}_f} - I_{\boldsymbol{\xi}_p} + I_{\mathbf{v}_p} = - \int_{\Gamma_{fp}} \boldsymbol{\sigma}_f \mathbf{n}_f \cdot \mathbf{v}_f - \int_{\Gamma_{fp}} \boldsymbol{\sigma}_p \mathbf{n}_p \cdot \boldsymbol{\xi}_p + \int_{\Gamma_{fp}} p_p \mathbf{v}_p \cdot \mathbf{n}_p$$

We start with $\int_{\Gamma_{fp}} p_p \mathbf{v}_p \cdot \mathbf{n}_p$. By (5), $p_p = \lambda_\Gamma - C_p$ on Γ_{fp} , so

$$I_{\mathbf{v}_p} = \int_{\Gamma_{fp}} p_p \mathbf{v}_p \cdot \mathbf{n}_p = \int_{\Gamma_{fp}} \lambda_\Gamma \mathbf{v}_p \cdot \mathbf{n}_p - \int_{\Gamma_{fp}} C_p \mathbf{v}_p \cdot \mathbf{n}_p$$

The second term can then go live on the right hand side. Next, we treat the other two interface terms. By (6), $\boldsymbol{\sigma}_f \mathbf{n}_f = -\boldsymbol{\sigma}_p \mathbf{n}_p$, so we have that

$$I_{\mathbf{v}_f} + I_{\boldsymbol{\xi}_p} = \int_{\Gamma_{fp}} (\mathbf{v}_f - \boldsymbol{\xi}_p) \cdot \boldsymbol{\sigma}_f \mathbf{n}_f$$

Next, note that the BJS condition (7) gives us information on the tangential component of $\boldsymbol{\sigma}_f \mathbf{n}_f$, while (5) gives us information on the normal component. As $\mathbf{n}_f, \boldsymbol{\tau}_1, \boldsymbol{\tau}_2$ form an orthonormal system, we have that⁵

$$\begin{aligned} \mathbf{v}_f - \boldsymbol{\xi}_p &= \mathbf{n}_f ((\mathbf{v}_f - \boldsymbol{\xi}_p) \cdot \mathbf{n}_f) + \sum_j \boldsymbol{\tau}_j ((\mathbf{v}_f - \boldsymbol{\xi}_p) \cdot \boldsymbol{\tau}_j) \\ \implies (\boldsymbol{\sigma}_f \mathbf{n}_f) \cdot (\mathbf{v}_f - \boldsymbol{\xi}_p) &= ((\boldsymbol{\sigma}_f \mathbf{n}_f) \cdot \mathbf{n}_f) ((\mathbf{v}_f - \boldsymbol{\xi}_p) \cdot \mathbf{n}_f) \\ &\quad + \sum_j ((\boldsymbol{\sigma}_f \mathbf{n}_f) \cdot \boldsymbol{\tau}_j) ((\mathbf{v}_f - \boldsymbol{\xi}_p) \cdot \boldsymbol{\tau}_j) \end{aligned}$$

By (5), $(\boldsymbol{\sigma}_f \mathbf{n}_f) \cdot \mathbf{n}_f = -\lambda$, and by (7), $(\boldsymbol{\sigma}_f \mathbf{n}_f) \cdot \boldsymbol{\tau}_j = -B(\mathbf{u}_f - \partial_t \boldsymbol{\eta}_p) \cdot \boldsymbol{\tau}_j$, so

$$\begin{aligned} (\boldsymbol{\sigma}_f \mathbf{n}_f) \cdot (\mathbf{v}_f - \boldsymbol{\xi}_p) &= -\lambda ((\mathbf{v}_f - \boldsymbol{\xi}_p) \cdot \mathbf{n}_f) \\ &\quad - \sum_j (B(\mathbf{u}_f - \partial_t \boldsymbol{\eta}_p) \cdot \boldsymbol{\tau}_j) ((\mathbf{v}_f - \boldsymbol{\xi}_p) \cdot \boldsymbol{\tau}_j) \end{aligned}$$

We can then use that $\mathbf{n}_f = -\mathbf{n}_p$ to write $\lambda(\mathbf{v}_f - \boldsymbol{\xi}_p) \cdot \mathbf{n}_f = \lambda(\mathbf{v}_f \cdot \mathbf{n}_f + \boldsymbol{\xi}_p \cdot \mathbf{n}_p)$. Putting it all together,

⁵This is just use of the fact when a vector \mathbf{v} is written in a basis \mathbf{e}_i , the coefficient of \mathbf{e}_i

$$\begin{aligned}
-(I_{\mathbf{v}_f} + I_{\boldsymbol{\xi}_p}) + I_{\mathbf{v}_p} &= \int_{\Gamma_{fp}} \lambda (\mathbf{v}_f \cdot \mathbf{n}_f + (\boldsymbol{\xi}_p + \mathbf{v}_p) \cdot \mathbf{n}_p) \\
&\quad + \int_{\Gamma_{fp}} (B (\mathbf{u}_f - \partial_t \boldsymbol{\eta}_p) \cdot \boldsymbol{\tau}_j) ((\mathbf{v}_f - \boldsymbol{\xi}_p) \cdot \boldsymbol{\tau}_j) \\
&\quad - \int_{\Gamma_{fp}} C_p \mathbf{v}_p \cdot \mathbf{n}_p \\
&= b_{\Gamma_{fp}} (\mathbf{v}_f, \mathbf{v}_p, \boldsymbol{\xi}_p; \lambda) + a_{BJS} (\mathbf{u}_f, \partial_t \boldsymbol{\eta}_p; \mathbf{v}_f, \boldsymbol{\xi}_p) - (\mathbf{v}_p, C_p \mathbf{n}_p)_{\Gamma_{fp}}
\end{aligned}$$

We have now derived all the necessary identities, and summing them all yields the following:

$$\begin{aligned}
&a_f(\mathbf{u}_f, \mathbf{v}_f) + b_f(\mathbf{v}_f, p_f) + a_p^e(\boldsymbol{\eta}_p, \boldsymbol{\xi}_p) \\
&\quad + \alpha b_p(\boldsymbol{\xi}_p, p_p) + a_p^d(\mathbf{u}_p, \mathbf{v}_p) + b_p(\mathbf{v}_p, p_p) \\
&\quad + b_{\Gamma_{fp}} (\mathbf{v}_f, \mathbf{v}_p, \boldsymbol{\xi}_p; \lambda) + a_{BJS} (\mathbf{u}_f, \partial_t \boldsymbol{\eta}_p; \mathbf{v}_f, \boldsymbol{\xi}_p) \\
&= (\mathbf{v}_p, C_p \mathbf{n}_p)_{\Gamma_{fp}} + (\boldsymbol{\sigma}_f \mathbf{n}_f, \mathbf{v}_f)_{\Gamma_f} + (\boldsymbol{\sigma}_p \mathbf{n}_p, \boldsymbol{\xi}_p)_{\Gamma_p} + (p_p \mathbf{n}_p, \mathbf{v}_p)_{\Gamma_p}
\end{aligned}$$

If $\partial_t \boldsymbol{\eta}_p$ is now discretized by a backward Euler difference, this is exactly (8a).

The next two are not as bad. (8b) is just (1b) multiplied by w_f integrated over Ω_f plus (3b) multiplied by w_p integrated over Darcy.

Doing the above (no integration of parts needed) yields

$$s_0 (\partial_t p_p, w_p)_{\Omega_p} - \alpha b_p(\partial_t \boldsymbol{\eta}_p, w_p) - b_p(\mathbf{u}_p, w_p) - b_f(\mathbf{u}_f, w_f)$$

Once the ∂_t 's are discretized by a backward Euler difference, this is exactly (8b).

Thoughts

- Kent offered very gentle scepticism about using a 3-field formulation, and suggested not having \mathbf{u}_p as an unknown, using $\mathbf{u}_p = \nabla p_p$ to remove it. I thought [1] had some opinion on this, but on closer reading I can't find it, so maybe that's from another article. I should read up on this.
- I derived the weak formulation by noting that [1] had $\lambda_\Gamma = -(\boldsymbol{\sigma} \mathbf{n}_f) \cdot \mathbf{n}_f = p_p$, while I want $\lambda_\Gamma = -(\boldsymbol{\sigma} \mathbf{n}_f) \cdot \mathbf{n}_f = p_p + C_p$, so I figured I could get my weak formulation by taking [1]'s and replacing λ_Γ by $\lambda_\Gamma + C_p$. I should rederive it myself to see whether this is actually true.
- Why is there a 2 in front of μ in equation (1a)? If μ is just divided by 2 that's fine, but then I need to divide my choice by μ accordingly.

References

- [1] AMBARTSUMYAN, ILONA, ET AL. , "*A Lagrange multiplier method for a Stokes-Biot fluid-poroelastic structure interaction model.*" , arXiv preprint arXiv:1710.06750 (2017).