Equations for system

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We follow [1] very very closely.

Domain

Our domain Ω is $d = 2$ or 3-dimensional, and partitioned into $\Omega_f$and Ω_p, with $\Gamma_{fp} = \Omega_f \cap \Omega_p$ being the $(d-1)$-dimensional interface. We assume each region is connected, reasonably smooth and all that.

Unknowns

The unknowns of the system and the corresponding test functions are:

- $\mathbf{u}_f$, $\mathbf{v}_f$: free flow fluid velocity. Defined on Ω_f.
- $\mathbf{u}_p$, $\mathbf{v}_p$: porous flow fluid velocity. Defined on Ω_p.
- p_f, q_f: free flow fluid pressure. Defined on Ω_f.
- p_p, q_p: porous flow fluid pressure. Defined on Ω_p.
- $\boldsymbol{\eta}_p$, $\boldsymbol{\xi}_p$: displacement. Defined on Ω_p.
- λ_Γ, μ_Γ: normal stress balance Lagrange multiplier. Defined on Γ_{fp}. In [1], denoted λ, μ_h.

Parameters

μ_f fluid viscosity (denoted μ in the [1])

λ_p, μ_p Lamé parameters. Denoted μ in [1].

α Biot-Willis constant

K Permeability tensor. Symmetric, bounded, positive definite. Probably scalar.

Notation

- $\mathbf{D}(\mathbf{v}) := \frac{1}{2}(\nabla\mathbf{v} + \nabla\mathbf{v}^T)$
- $\sigma_f(\mathbf{u}_f, p_f) := -p_f\mathbf{I} + 2\mu_f\mathbf{D}(\mathbf{u}_f)$
- $\sigma_p(\boldsymbol{\eta}_p, p_p) := \lambda_p(\nabla\cdot\boldsymbol{\eta}_p) + 2\mu_p\mathbf{D}(\boldsymbol{\eta}_p) - \alpha p_p\mathbf{I}$
- $a_f(\mathbf{u}_f, \mathbf{v}_f) = (2\mu\mathbf{D}(\mathbf{u}_f), \mathbf{D}(\mathbf{v}_f))_{\Omega_f}$
- $a_p^d(\mathbf{u}_p, \mathbf{v}_p) = (\mu K^{-1}\mathbf{u}_p, \mathbf{v}_p)_{\Omega_p}$
- $a_p^e(\boldsymbol{\eta}_p, \boldsymbol{\xi}_p) = (\mu_p\mathbf{D}(\boldsymbol{\eta}_p), \mathbf{D}(\boldsymbol{\xi}_p))_{\Omega_p} + (\lambda_p\nabla\cdot\boldsymbol{\eta}_p, \nabla\cdot\boldsymbol{\xi}_p)_{\Omega_p}$

Strong formulation

Boundary conditions

Interface conditions

Variational formulation

Thoughts

- Kent offered very gentle scepticism about using a 3-field formulation, and suggested not having $\mathbf{u}_p$as an unknown, using $\mathbf{u}_p = \nabla p_p$ to remove it. I thought [1] had some opinion on this, but on closer reading I can't find it, so maybe that's from another article. I should read up on this.

References

[1] Ambartsumyan, Ilona, et al. , *"A Lagrange multiplier method for a Stokes-Biot fluid-poroelastic structure interaction model."* , arXiv preprint arXiv:1710.06750 (2017).