FENICSTOOLS
SMART ADD-ONS FOR FENICS

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fenicstools is a collection of extensions to the FEniCS library with particular focus on efficient visualization and postprocessing of large scale computations.

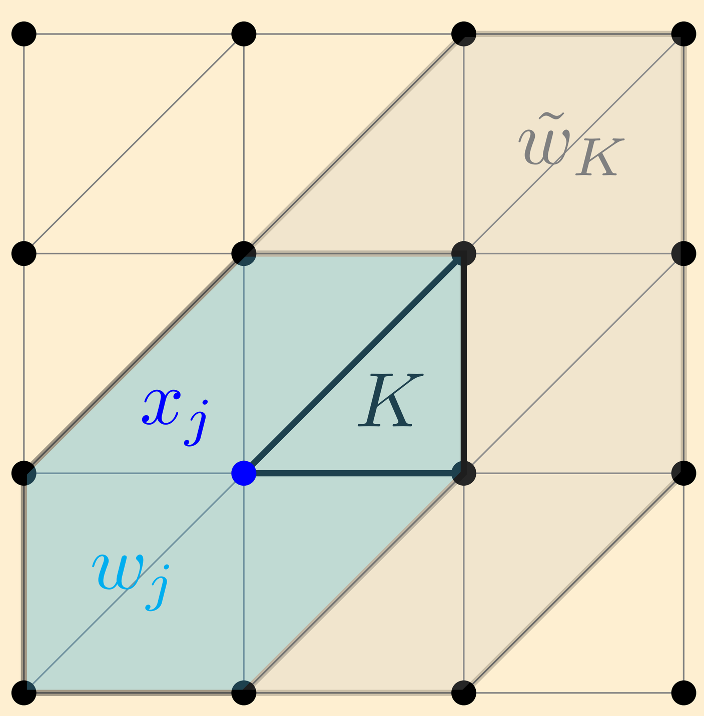
Clément interpolation

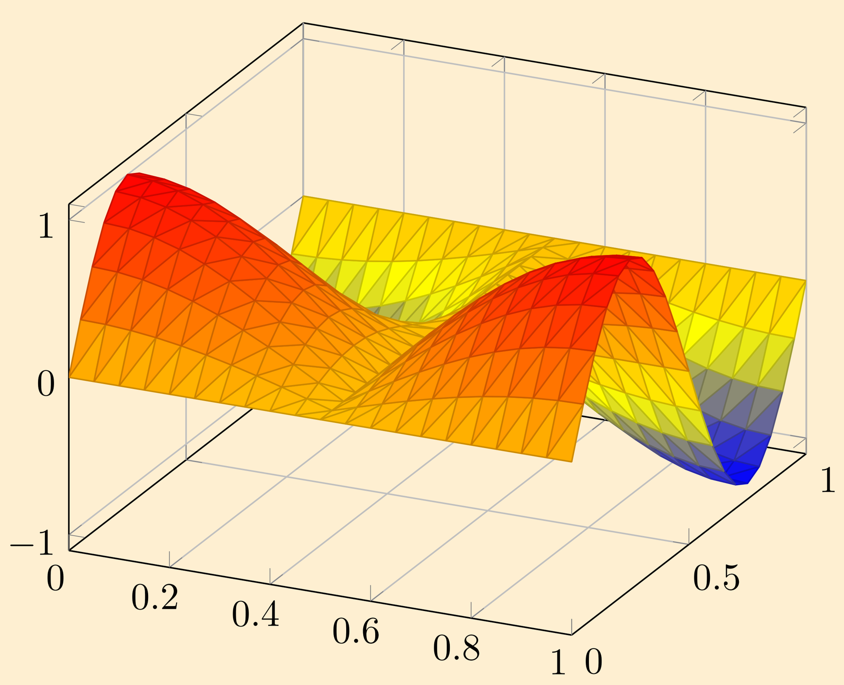
Evaluating quantities derived from primary unknowns, e.g. strain from displacement, is a frequent part of computational loops. Such quantities often lack the H^2 regularity required for nodal interpolation and therefore can be computed in FEniCS only by L^2 projection. An alternative method is the Clément interpolation - a numerical technique for constructing interpolants of H^1 functions based on local regularization.

▷ **Figure**: fenicstools implements the lowest order Clément interpolation operator resulting in a CG_1 approximation of interpolated function f. The degree of freedom at x_j is computed as v minimizing $\|f - v\|^2_{0,w_j}$ over constant fields on patch w_j. The interpolation error is controlled on the union $\tilde{w}_K$. Therefore no power h is lost in the error estimates:

$$\|u - I_h u\|_{m,K} \leq C h_K^{1-m} \|u\|_{1,\tilde{w}_K}$$

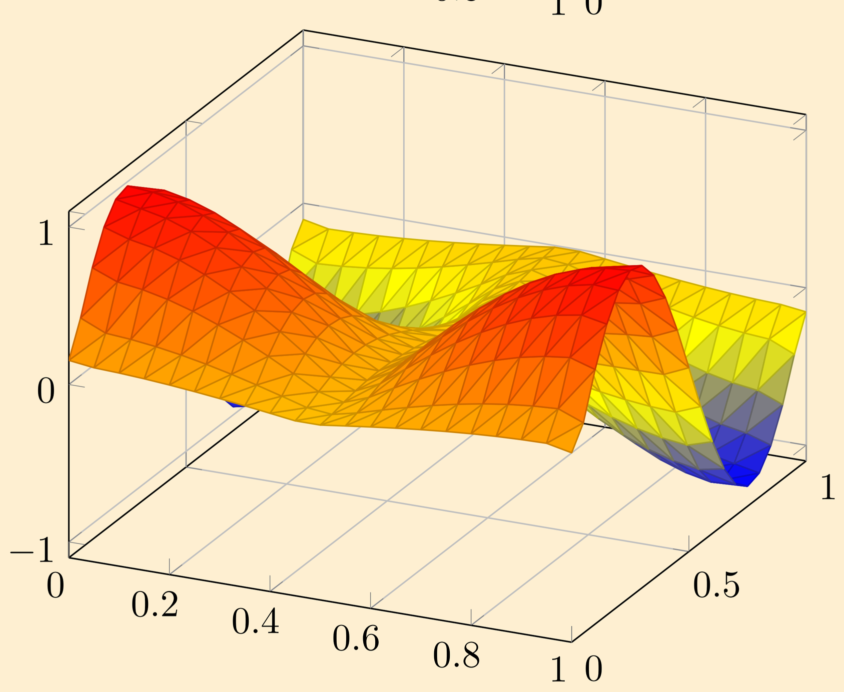
for $u \in H^1$, $m = 0, 1$ and K an element of triangulation.

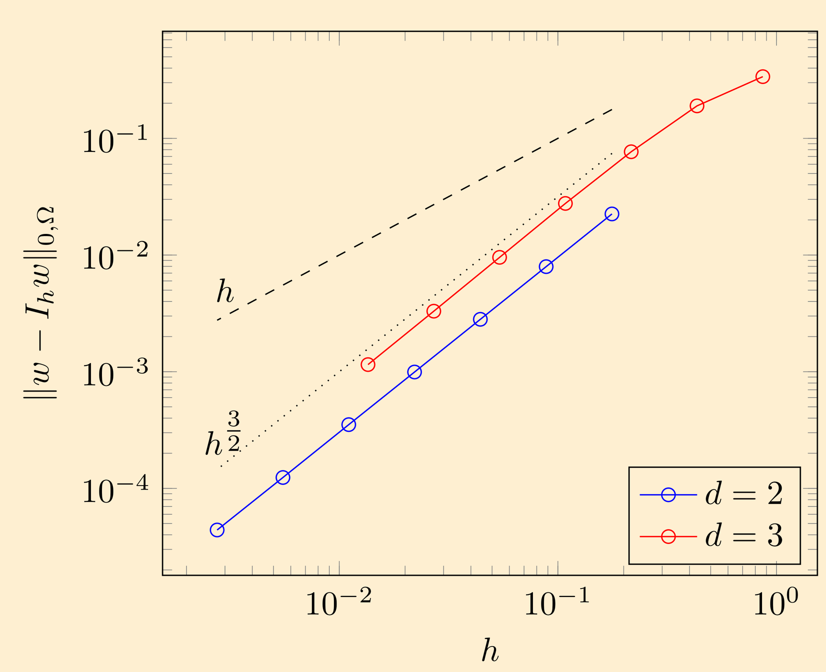




◁ **Figure**: The local regularization procedure results in smearing of gradients. However, the largest errors are localized near the boundaries where the interpolant(bottom figure) fails to preserve the boundary values.

▽ **Figure**:fenicstools supports Clément interpolation of all* valid UFL expressions. How about evaluating $\boldsymbol{w} = \nabla\cdot(\boldsymbol{u} \otimes \nabla v)$ for $\boldsymbol{u} \in [\text{CG}_1]^2$, $v \in \text{CG}_1$ or $\boldsymbol{w} = \sin(\det \nabla \boldsymbol{u})$ where $\boldsymbol{u}$ is a vector field in $\mathbb{R}^3$?





Unlike L^2 projection, which requires solution of large linear system, Clément interpolant is constructed from local linear systems of size 1 assembled over patches surrounding mesh vertices. In fenicstools the mapping is realized more efficiently using a precomputed averaging operator A such that $Ab(u) = I_h u$. Due to this choice the setup cost of the interpolant is higher (4x) than that of L^2 projector. However, the subsequent (repetitive) evaluation comes at a cost of a matrix-vector product. ▷ **Table**: MPI.MAX-ed timings (in seconds) of action of Clément interpolator and L^2 projector.

CPUs	Degrees of freedom		
	2101250	8396802	14612418
1	(1.0, 5.2)	(4.1, 21.0)	(7.5, 37.2)
2	(0.7, 3.5)	(2.7, 14.0)	(4.0, 26.4)
4	(0.7, 2.2)	(1.5, 10.8)	(2.0, 19.1)
8	(0.3, 1.6)	(0.7, 8.5)	(1.8, 13.9)
16	(0.2, 1.1)	(0.7, 7.5)	(1.7, 13.4)

Development cycle

fenicstools is developed using Travis Continuous Integration and Anaconda Cloud. With Travis CI all tests are automatically executed in an Ubuntu environment on travis-ci.org for each new commit or pull request to origin/master. A tailored Anaconda build of a recent FEniCS version is then used in a MiniConda environment and tests start within minutes.

```
conda config --add channels mikaem/label/travis
conda config --add channels mikaem
conda install fenics=1.7.0 pyvtk h5py=2.6.0
```

△ **Code**: Setting up FEniCS with conda.

ANACONDA

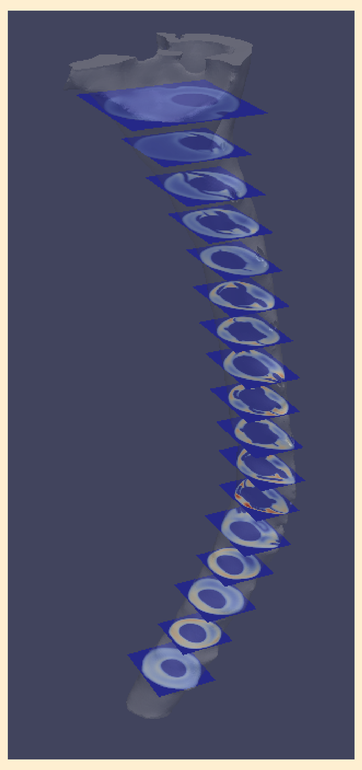
Probes & Structured grids

A turbulent flow simulation often requires setting a probe at a certain location inside the flow, where regular samplings are made over time. This can be done efficiently with *Probes* and *StatisticsProbes* classes.

```
from dolfin import *
from numpy import array
from fenicstools import Probes
mesh = UnitSquareMesh(10, 10)
V = FunctionSpace(mesh, "CG", 1)
x = array([[0.1, 0.1], [0.5, 0.5]])
probes = Probes(x.flatten(), V)
u = project(Expression("x[0]*x[1]"), V)
probes(u)    # evaluate probes once
print probes.array() # [ 0.01   0.25]
```

△ **Code**: Probe two locations defined by the x array.

▷ **Figure**: The *StructuredGrid* class allows you to set probes in a $2d$ slice through a $3d$ (or $2d$) geometry - or it can be used to set probes in a $3d$ box for some interesting part of the simulated geometry. The slice may be stored as VTK file and viewed, e.g., in Paraview.



Lagrangian tracking

Transport problems $C_t + \boldsymbol{u}\cdot\nabla C = 0$ with dominant convection can often become more numerically tractable by employing the Lagrangian description of flow. The field C then remains constant in time along the characteristics

$$\boldsymbol{x}_t = \boldsymbol{u}(\boldsymbol{x}, t)$$

and the problem reduces to computing the curves tangent to the velocity field $\boldsymbol{u}$. In fenicstools the Lagrangian tracking is implemented via *LagrangianParticles* class.

```
lp = fenicstools.LagrangianParticles(V)
x0, C0 = rand((10, 2)), rand(10)
lp.add_particles(x0, {C: C0})
lp.step(u, dt=dt)  # Time integration
```

△ **Code**: Single step of the transport problem.

▽▷ **Figure:** *LagrangianParticles* were successfully used to study drug delivery and mixing in a cerebrospinal fluid flow.