

# STABILIZATION BY NEURAL NETS

- WE INVESTIGATE WHETHER/HOW UNSTABLE DISCRETIZ. OF THE PDE EFFECTS HYBRID PDE-NN MODEL

WE HAVE 3 TYPES OF PDE/ODE CONSTRAINTS IN MIND

1)  $\min_u \int_L (u - u_d)^2$  with  $-\Delta u = f$  in  $\Omega$   
 $u = 0$  on  $\partial\Omega$

Here we want to use DG, elementwise and  
constrain the learning problem for NN if when

A) THE PDE IS ENFORCED "CORRECTLY" i.e.  
 $\sum_K \int_{\Omega_K} + \text{OTHER TERMS OF}$   
 $\mathcal{C}^\alpha \mathcal{D}^\beta \text{ CYLINDRICAL INTERIOR PENALTY}$

B) WE ONLY TRY TO GET PDES RIGHT  
 $\sum_K \int_{\Omega_K} \nabla u \cdot \nabla v + \text{NOTHING}!!$

---

• NOTE:  $P_2$  WITH BIHARMONIC EQ IS A  
SYNTHETIC PROBLEM

$$2) \min \int (u - u_d)^2 + \left( \frac{1}{(\rho - \rho_d)^2} \right) \text{ with}$$

$$\begin{aligned} \Delta u + \nabla p &= f \quad \text{or} \quad \Delta u + \nabla p = 0 \\ \nabla \cdot u &= 0 \end{aligned}$$

Here we want to control 2 different element types

A) STABLE  $P_2 - P_0$ ,  $P_L - P_0$

B) UNSTABLE  $P_0 - P_0$ ,  $P_0 - P_1$

- THE POD WITH CONTROL ON MASS-CSRV EQ SHOULD INSTEAD LEARN PRESSURE STABILIZATION. ONE OPTION IS  $f = \epsilon(h) \Delta p$

- MOMENTUM CONTROL MIGHT BE MORE DIFFICULT TO STABILIZE

- MADE USE IDENTIFY MOMENTUM CONTROL

- THE POD OF NNL FOR MASS-CRW STRG

$\text{inner}(NNL(\text{input}), \nabla q) * dx$  } effect on which stabilization is used?

$\text{inner}(NNL(\text{input}), q) * dx$  }

$$3) \min_u \int (u - u_d)^2 \quad \text{with} \quad \frac{du}{dt} = Au \quad \text{in } \Omega$$

$u(0) = u_0$

Here we want to consider two schemes

A) STABLE  $\rightarrow$  BACKWARD Euler

B) UNSTABLE  $\rightarrow$  FORWARD Euler

$\hookrightarrow$  change dt to make things worse

- WILL **(B)** WORK ON LONG TIME HORIZON  
(EXTRACTION BEYOND  $S_L$ )

### GENERAL COMMENTS

- |  |   |
|--|---|
| • KEEP THE ARCHITECTURE SIMPLE<br>BUT MAKE LARGE CONSIDERATIONS<br>A LA KENTHORN   | <ul style="list-style-type: none"> <li>• WE WOULD LIKE TO LEARN SUCH THAT THE NET WORKS OVER MANY RESOLUTIONS OF <math>S_L</math></li> </ul>                                |
|  | <ul style="list-style-type: none"> <li>• IDEALLY TRAIN ON COARSE AND APPLY ON FINE</li> <li>• CAN RE DIRECT THE NETWORK, i.e. ISOLATE STANDARDIZATION TENDENCIES</li> </ul> |
| <ul style="list-style-type: none"> <li>• FOR PROBLEMS WHERE WE KNOW INSTITUTIONS WE SHOULD ADD NOISE LIKE THAT TO OBSERVATIONS</li> <li>• MAKE THE NN CAN BE LINEAR</li> </ul> |   |