IN FLUID DOMAIN AT TIME t, $\Omega_f(t)$, WANT TO SOLVE

$$-\mathrm{div}(\sigma_f(u_f, p_f)) = 0$$
$$-\mathrm{div}(u_f) = 0$$

- IMPORTANT BC HERE IS THAT ON THE MOVING WALL $\Gamma_w(t)$ u_f = WALL VELOCITY

IN SOLID DOMAIN WHICH WILL ALSO DEFORM WE SOLVE

$$-\mathrm{div}(\sigma_p(\xi_p, p_p)) = 0$$
$$u_p = -\alpha \nabla p_p \quad \text{in } \Omega_p(t)$$
$$\frac{\partial}{\partial t}\left(s_0 p_p + \alpha \nabla \cdot \xi_p\right) + \nabla \cdot u_p = 0$$

INTERFACE CONDITIONS ON INTERFACE $\Gamma(t)$ READ

$$\sigma_f \cdot n_f + \sigma_p \cdot n_p = 0$$
$$-(\sigma_f \cdot n_f) n_f = p_p$$
$$u_f \cdot n_f + \left(\frac{\partial \xi}{\partial t} + u_p\right) \cdot n_p = 0$$
$$u_f \cdot \tau_f + \frac{\partial \xi}{\partial t} \cdot \tau_p = 0$$

DOMAIN UPDATES

IN SOLID: WITH ξ_p^t THE COMPUTED DISPLACEMENT
$\Omega_p^{t+1} = \{x + \xi_p^t(x) \; ; \; x \in \Omega_p^t\}$

IN FLUID: FLUID DOMAIN DISPLACEMENT ξ_f^t

$$-\Delta \xi_f^t = 0 \quad \text{in } \Omega_{\xi_f}^t$$
$$\xi_f^t = \xi_p^t \quad \text{on } \Gamma^t$$
$$\xi_f^t = \text{WALL DISPLACEMENT} \quad \text{on } \Gamma_w^t$$

SOLUTION STEPS

FOR DOMAIN COUPLING NOTE THAT

$$u_f \cdot n_f + \left(\frac{\partial \xi}{\partial t} + u_p\right) \cdot n_p = 0$$

$$u_f \cdot \tau_f + \frac{\partial \xi}{\partial t} \cdot \tau_p = 0$$

CAN BE WRITTEN AS

$$u_f \cdot n_f - \left(\frac{\partial \xi}{\partial t} + u_p\right) \cdot n_f = 0 \quad \Leftarrow \quad u_f - \frac{\partial \xi}{\partial t} - (u_p \cdot n_f) \cdot n_f = 0$$

$$u_f \cdot \tau_f - \frac{\partial \xi}{\partial t} \cdot \tau_f = 0$$

1) GIVEN σ_p SOLVE ON Ω_f^t

$$-\nabla \cdot (\sigma_f) = 0$$

$$-\nabla \cdot u_f = 0$$

WITH BC ON Γ^t:

$$\sigma_f \cdot n_f + \sigma_p \cdot n_p = 0$$

2) NOW u_f, p_f IS KNOWN SOLVE

$$-\mathrm{div}\left(\sigma_p(\xi_p, p_p)\right) = 0$$

$$u_p = -\alpha \nabla p_p \qquad \text{in } \Omega_p^t$$

$$\frac{\partial}{\partial t}\left(S_0 p_p + \alpha \nabla \cdot \xi_p\right) + \nabla \cdot u_p = 0$$

WHERE ON Γ^t WE SET (WEAKLY)

$$p_p = -(\sigma_f \cdot n_f) \cdot n_f$$

AND FIX (STRONGLY) DISPLACEMENT USING

$$\frac{\partial \xi}{\partial t} = u_f - (u_p \cdot n_f) n_f$$

TAKEN FROM PREV STEP

3) DOMAIN UPDATES AS DESCRIBED BEFORE