MANUFACTORING DESING II COURSEWORK

GEARBOX FOR MACHINING CENTER

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1 Requirements

Design a 2-speed gearbox for a vertical machining center. The speeds needn't be changed at running motor.

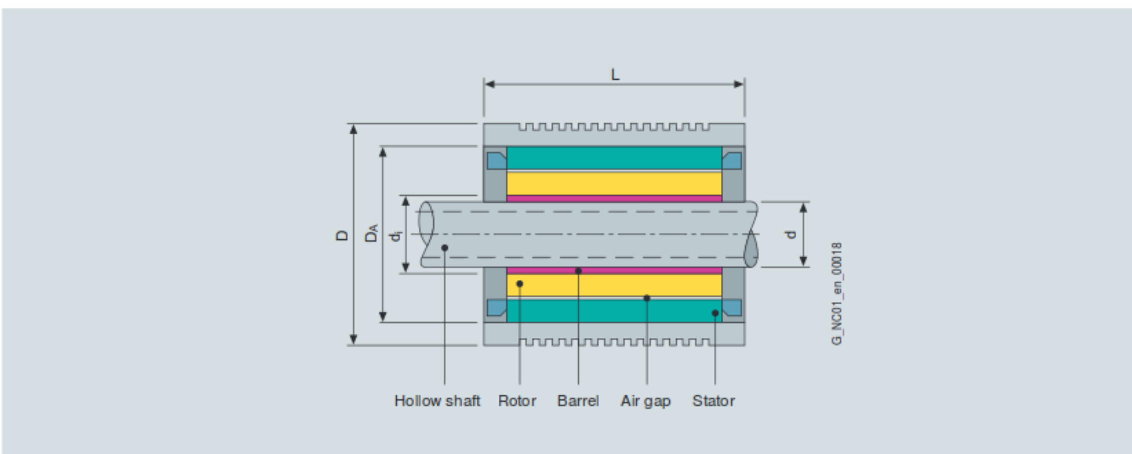
$$P_{output,max} = 11\text{kW}$$
$$n_{output,max} = 5000 \text{ rpm}$$

2 Motor selection

From [4] we select the Siemens 1PH209 spindle motor. It is the recommended built-in motor for standard machine tool spindle. Outside diameter is 205 mm, output shaft diameter 67 mm. Nominal speed 1500 rpm, maximum 10000 rpm. As the motor can be requested in different power ratings, we purchase one for 12kW as to compensate frcition losses.

Motor type	Rated power S1 kW	Rated torque range S1 Nm	Rated power S6-40% kW	Rated torque range S6-40% Nm	Rated speeds n_{rated} rpm	Maximum speeds n_{rated} rpm	L mm	D mm	D_A mm	d_i mm
1PH209	7.5 ... 13	48 ... 83	9 ... 15.4	57 ... 98	1,500	up to 10,000	250... 300	205	180	67
1PH211	15.1 ... 31	95 ... 197	19 ... 38.6	119 ... 245	1,500	up to 10,000	290... 390	250	220	82

Dimension drawings



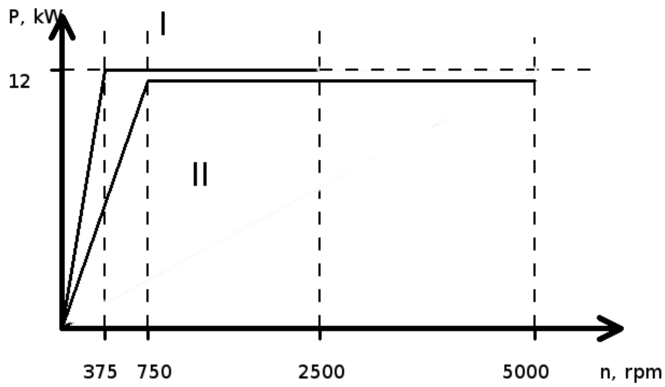
3 Kinematic diagram

The goal of the gearbox is to provide sufficient torque to the spindle at low motor speeds (below $n_{nominal}$). For selecting the low gear we utilize the rule of thumb to choose a transfer ratio between 1/3 and 1/4:

$$TR_1 = 1/4$$

For selecting the high gear, we try to match the maximum speed of the motor with the required maximum spindle speed by specification:

$$TR_2 = n_{spindle,max}/n_{motor,max} = 5000/10000 = 1/2$$



4 Components sizing

4.1 Gears

Requirements: Design input shaft gears G11 and G12 and output shaft gears G21 and G22 Both center distances (G11-G21 and G12-G22) must be equal. All gears must be capable of transmitting the rated power at all speeds.. Gear teeth must be shaped with a cutoff or round-off to assist gear engagement. Conventional spur gears with involute teeth profile have been selected for this application.

4.1.1 Standard Basic Rack Tooth Profile

Due to the numerous advantages of using standard tooth geometry, we decide to adhare to the Standard Basic Rack Tooth Profile [5].
Pressure angle: $\alpha = 20°$
Addendum circle: $h_a = 1.00m$
Bottom clearance: $c = 0.25m$
Daedendum circle: $h_f = 1.25m$
Fillet radius: $\rho = 0.38m$
Active tooth depth: $h_w = 2.00m$
Whole depth: $h = 2.25m$
Tooth thickness: $s = (\pi m)/2$

From [6] we select standard module and number of teeth for G11. The module is limitted to be select among the standard modules[6]: $1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10$ etc. The number of teeth on the pinion is limitted by $z_{min} = 17$ and $z_{max} = 33$ [6].

$$m_{11} = 6\text{mm}$$

$$z_{11} = 17$$

Now calculate G21, meshing with G11. For a transfer ratio of $TR_1 = 0.25$, we would need G21 to have number of teeth:

$$z_{21} = z_{11}/TR_1 = 17/0.25 = 68$$

$$m_{21} = 6\text{mm}$$

This results in a center distance for G11-G21 of:

$$a_1 = \frac{d_1 + d_2}{2} = \frac{m_{11}z_{11} + m_{21}z_{21}}{2} = 255\text{mm}$$

Now we need to design gear G12 and G22 so that they have the same center distance and equal to each other's modules. For brevity, allowable values are listed only once.

$$\begin{cases} a_2 = \frac{mz_{12}+mz_{22}}{2} \quad = \quad a_1 \quad = \quad 255 \\ z_{12} = z_{22}TR_2 \\ z_{12} \geq 17 \\ z_{12} \leq 33 \\ z_{12} \in \mathbb{N} \\ z_{22} \in \mathbb{N} \\ m_{12} = m_{22} \in \{1; 1.25; 1.5; 2; 2.5; 3; 4; 5; 6; 8; 10\} \end{cases}$$

$$\begin{cases} m(z_{12} + z_{22}) = 510 \\ z_{22}0.5 = z_{11} \end{cases}$$

$3mz_{12} = 510, z_{12} \in \mathbb{N}$

$mz_{12} = 170$

The following table lists all possible solutions:

m, mm	z_{12}
1	170.0000
1.25	136.0000
1.5	113.3333
2	85.0000
2.5	68.0000
3	56.6667
4	42.5000
5	34.0000
6	28.3333
8	21.2500
10	17.0000

We have only one feasable solution:

$$m_{12} = m_{22} = 10\text{mm}$$

$$z_{12} = 17$$

$$z_{22} = 34$$

The table below represents numerically the resulting geometry from module and teeth number choices.

Gear	α	h_a, [mm]	c, [mm]	h_f, [mm]	ρ, [mm]	h_w, [mm]	h, [mm]	s, [mm]
G11	20°	6.0000	1.5000	7.5000	2.2800	12.0000	13.5000	9.4247
G12	20°	6.0000	1.5000	7.5000	2.2800	12.0000	13.5000	9.4247
G21	20°	10.0000	2.5000	12.5000	3.8000	20.0000	22.5000	15.7079
G22	20°	10.0000	2.5000	12.5000	3.8000	20.0000	22.5000	15.7079

4.1.2 Contact ratio

From [5] we know about the number of simultaneously engaged teeth that "for smooth and quiet operation, the contact ratio should not be less than 1.2":

$$\epsilon_\alpha \approx 1.88 - 3.2(1/z_I + 1/z_{II}) \geq 1.2$$

$$\epsilon_{\alpha,1} \approx 1.88 - 3.2(1/z_{11} + 1/z_{21}) \approx 1.6447 \geq 1.2$$

$$\epsilon_{\alpha,2} \approx 1.88 - 3.2(1/z_{12} + 1/z_{22}) \approx 1.5976 \geq 1.2$$

4.1.3 Bending stress

From both [5] and [6] we have the formula for bendig stress in gear teeth. Each individual gear is calculated separately.

$$\sigma_F = Y_{FS} Y_\epsilon Y_\beta \frac{F_t}{bm} K_A K_V K_{F\alpha} K_{F\beta} \leq \sigma_{FP} \tag{1}$$

where:
Tip factor: Y_{FS} - taken from graph 8.13 in [6] with shift coefficient $x = 0$ [7].
Contact ratio factor:

$$Y_\epsilon = \frac{1}{\epsilon_\alpha}$$

Helical angle factor: $Y_\beta = 1$ for spur gears.
Tangential force:

$$T = \frac{P}{\omega} = \frac{P}{\frac{2\pi}{60}n} = \frac{30P}{\pi n}$$

$$F_t = \frac{2}{1000}\frac{T}{d} = \frac{60}{1000}\frac{P}{\pi n d} = \frac{60}{1000}\frac{P}{\pi n m z}$$

Gear thickness: $b = 10$mm - freely selected.
Application factor: $K_A = 1.25$ from table 8.8 in [6].
Dynamic factor: $K_V = 1.3$ - an intermediate-high value [6].
Transverse load factor: $K_{F\alpha} = 1.3$ - an intermediate-high value [6].
Load distribution factor $K_{F\beta}$ - taken from figure 8.14 [6].
Allowable tooth-root stress:

$$\sigma_{FP} = \frac{\sigma_{F,lim} Y_N}{S_F}$$

Tooth-root stress endurance limit for the chosen material: $\sigma_{F,lim}$ - from table 8.10 [6].
Stress cycle factor: $Y_N = 1.7$ - by advice in [6].
Safety factor : $S_F = 1.7$ - by advice in [6].

We select steel 18ХГТ with cementation and tempering treatment. In the below table, the coefficients for each gear are estimated, before being substituted into (1).

$$\frac{F_t}{bm} = \frac{\frac{60}{1000}\frac{P}{\pi n m z}}{1000 * 1000 b m} = 60000\frac{Pb}{\pi n z}$$

$$n_{2,min} = 375 \text{ rpm}$$

$$n_{1,min} = n_{2,min}/TR_1 = 1500 \text{ rpm}$$

	$G11$	$G12$	$G21$	$G22$
Y_{FS}	4.25	4.25	3.70	3.85
Y_ϵ	0.6080	0.6259	0.6080	0.6259
Y_β	1	1	1	1
$\frac{F_t}{bm}$	82388.5180	20597.1295	329554.0721	164777.0361
K_A	1.25	1.25	1.25	1.25
K_V	1.3	1.3	1.3	1.3
$K_{F\alpha}$	1.3	1.3	1.3	1.3
$K_{F\beta}$	1.35	1.35	1.35	1.35
$\sigma_{F,lim}$	$700e6$	$700e6$	$700e6$	$700e6$
Y_N	2.5	2.5	2.5	2.5
S_F	1.7	1.7	1.7	1.7

We can generalise that

$$Y_{FS}Y_\epsilon \frac{F_t}{bm} * 1.25 * 1.3 * 1.3 * 1.35 \leq \frac{700e6 * 1.7}{1.7}$$

$$Y_{FS}Y_\epsilon \frac{F_t}{bm} 2.8519 \leq 700e6$$

$$607142 < 700e6$$

$$156254 < 700e6$$

$$2114297 < 700e6$$

$$1132391 < 700e6$$

All gears are durable within a significant factor of safety.

4.1.4 Gear-Tooth Surface Durability

The contact surfaces of gear teeth are subjected to Hertz contact stresses [6]. ISO 6336 provides the following equation [6]:

$$\sigma_H = Z_E Z_H Z_\epsilon Z_\beta \sqrt{\frac{F_t}{b_1 d}\frac{u+1}{u} K_A K_V K_{H\alpha} K_{H\beta}} \leq \sigma_{HP} \tag{2}$$

,where:
$K_A, K_V, K_{H\alpha} = K_{F\alpha}, K_{H\beta} = K_{F\beta}$ have already been defined.
Elasticity factor (for identical Young modulus and Pisson ratio):

$$Z_E = \sqrt{\frac{1}{2\pi \frac{1-v^2}{E}}}$$

Geometry factor:

$$Z_H = \frac{1}{cos\alpha}\sqrt{\frac{2cos\beta}{\tan\alpha}} = \frac{1}{cos20^\circ}\sqrt{\frac{2}{\tan 20^\circ}} = 2.4946$$

Contact ratio factor: $Z_\epsilon = \sqrt{\frac{4-\epsilon_\alpha}{3}}$ [7]
Helical angle factor: $Z_\beta = 1$.
Allowable stress:

$$\sigma_{HP} = \frac{\sigma_{H,lim} Z_N}{S_H} = \frac{1.3\sigma_{H,lim}}{1.7} = 0.76\sigma_{H,lim}$$

Now the actual calculation [8] [9]:

$$Z_E = \sqrt{\frac{1}{2\pi \frac{1-0.3^2}{200e9}}} = 468807.2309$$

$$Z_{\epsilon,1} = 0.8861$$

$$Z_{\epsilon,2} = 0.8949$$

$$\sigma_{HP} = 0.76 * 1300 * 10^6 = 988 * 10^6 \text{Pa}$$

Now substitute back into (2):

$$468807 * 2.4946 Z_\epsilon \sqrt{\frac{F_t}{0.01d} \frac{u+1}{u} 1.25 * 1.3 * 1.3 * 1.35} \leq 988 * 10^6$$

$$19749688 * Z_\epsilon \sqrt{\frac{F_t}{d} \frac{u+1}{u}} \leq 988 * 10^6$$

$$19749688 * Z_\epsilon \sqrt{\frac{\frac{60}{1000} \frac{P}{\pi n m z}}{d} \frac{u+1}{u}} \leq 988 * 10^6$$

$$4837665 * Z_\epsilon \sqrt{\frac{Pd}{\pi n m z} \frac{u+1}{u}} \leq 988 * 10^6$$

$$2729400 * Z_\epsilon \sqrt{\frac{P}{n} \frac{u+1}{u}} \leq 988 * 10^6$$

$$286261982 * Z_\epsilon \sqrt{\frac{1}{n} \frac{u+1}{u}} \leq 988 * 10^6 \tag{3}$$

This yields our wokind fromula (3). Now substitute for each gear individually:

$$286261982 * 0.8861 \sqrt{\frac{1}{1500}} = 6549388 \leq 988 * 10^6$$

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$$286261982 * 0.8949 \sqrt{\frac{1}{375}} = 13228863 \leq 988 * 10^6$$

$$286261982 * 0.8949 \sqrt{\frac{1}{375}} = 13228863 \leq 988 * 10^6$$

All inequalities are true within a significatn margin of error. Therefore, the gear train is expected to last for tens of years of operation [6].

4.2 Shafts

Requirements: desing Input shaft and Output shaft with appropriate key / spline / press joints, capable of transmitting 11kW plus efficiency losses. As the requirements match, we will callculate only a single shaft design.

4.2.1 Static loading

Maximum load magnitude:

$$T = P/\omega = \frac{30 * 11000}{375\pi} = 280\text{Nm}$$

$$F_r = F_t \tan\alpha = \frac{2 * 1000 * T}{d} \tan\alpha = \frac{2 * 1000 * 280}{68} \tan 20^\circ = 2997.4018 \approx 3\text{kN}$$

Select shaft material and its yield strength σ_y.
Same as gears - 700MPa

5. Select a safety factor S.
Same as gears S = 1.7

Axial forces are negligeable and shall be neglected. Now we determine the reaction forces in the supports.

$$\begin{cases} \sum F_y = 0 \\ \sum M_z = 0 \end{cases}$$

$$\begin{cases} + \qquad F_C = F_B + F_D \\ + F_C * CD = F_B * BD \end{cases}$$

$$\begin{cases} + \qquad 3000 = F_B + F_D \\ + 3000 * 22.5 = F_B * 100 \end{cases}$$

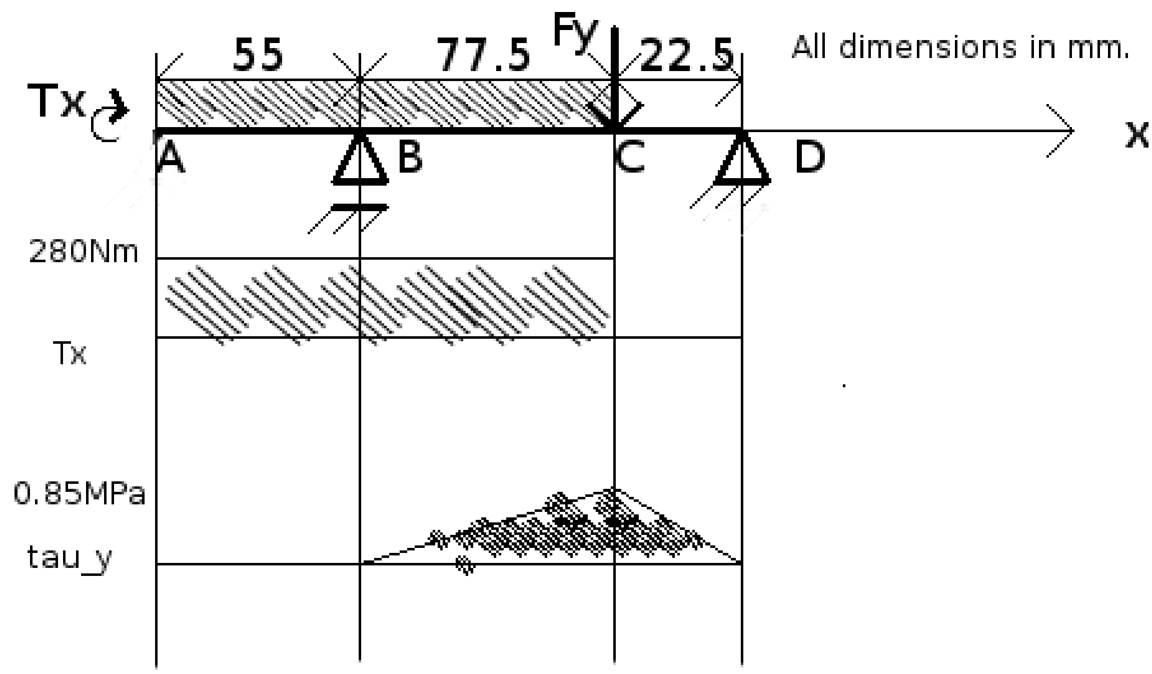
$$F_B = 675\text{N}$$

$$F_D = 2325\text{N}$$

We have pure shear stress:

$$\tau_{shear} = \frac{F}{A} = \frac{F_C}{\frac{1}{4*10^6}\pi d^2} = \frac{3000 * 4 * 10^6}{\pi 67^2} = 850931\text{Pa} \approx 0.85\text{MPa}$$

$$\tau_{torsion} = \frac{16 * 30^9 * T}{\pi d^3} = 4741508\text{Pa} \approx 4.74\text{MPa}$$



The critical section is at the gear G21 [10]:

$$\sigma_{eq} = \sqrt{0.85^2 + 4.73^2} = 4.08\text{MPa} \leq 0.58 * 700\text{MPa}$$

The inequality is true within significant factor of safety ≥ 1.7.

4.3 Couplings

Requirements: Design a way to connect the Input shaft to the driving motor and the output shaft to the spindle.

We select a rigid coupling, because a flexible coupling could reduce the accuracy of the spindle rotation. Furthermore, we narrow down to flange couplings for their superior vibration resistence and rigidity. We select flange width $D =$ 40mm. The stresses in the bolt bodies are [6]:

$$\tau_{av} = \frac{8T_{max}}{D\pi d^2} \leq \tau_{all}$$

$$\tau_{av} = \frac{8 * 280 * 10^9}{40 * \pi * 67^2} \leq 700 * 10^6\text{Pa}$$

$$3971013\text{Pa} \approx 4\text{MPa} \leq 700\text{MPa}$$

4.4 Bearings

Requirements: Select bearings of appropriate dimensions, precision, axial and radial stifness and sufficient vibration resistence.

Because sliding bearing cannot provide adequate wear resistence and heat rejection, we need rolling element bearing. On the other hand, due to the complexity of designing a full-film lubrication system, we will rely on mixed film lubrication - the same used for the gear train. We can expect a coefficient of friction between 0.04 and 0.1. Because of the negligable axial load and the nessecity for high rigidity of the Output shaft, we select radial roller bearing type. Bearing loads were already determined in the Shaft section.

5 References

1. Machine Desing II, Module 2 - GEARS, Lecture 17 – DESIGN OF GEARBOX, Prof. K.Gopinath & Prof. M.M.Mayuram
2. Design Basics of Industrial Gear Boxes, Andrzej Maciejczyk, Zbigniew Zdziennicki, Technical University of Lodz
3. Example of gearbox calculation, www.mitcalc.com
4. https://www.industry.usa.siemens.com/drives/us/en/electric-motor/mc-motors/direct-drive-motors/Documents/mtr-direct-drive-brochure.pdf (page 16)
5. Principles of mechanical engineering, Liubomir Dimitrov
6. Ръководство за конструктивни упражнения по машинни елементи, Доц Николай Николов и други
7. http://www.gearcalc.com/downloads/manual/manualse48.html
8. https://en.wikipedia.org/wiki/Young's_modulus
9. https://en.wikipedia.org/wiki/Poisson's_ratio
10. https://en.wikipedia.org/wiki/Shear_strength
11. http://www.skf.com/binary/57-129877/Super-precision-bearings—13383_1-EN.pdf

Съдържание