TOSHKENT SHAHRIDAGI INHA UNIVERSITETI
INHA UNIVERSITY IN TASHKENT

STUDENT NAME:	
STUDENT ID NUMBER:	DEPARTMENT:

Solution for the recently held

FINAL EXAMINATION (FALL 2017)

COURSE NAME:	**PHYSICS 1**
COURSE NUMBER:	MSC1021
EXAMINATION DATE:	TIME:
EXAMINATION DURATION:	**90 Minutes**
ADDITIONAL MATERIALS ALLOWED TO USE:	**Own brain, own memory and logics.** **CALCULATORS ARE ALLOWED.**
SPECIAL INSTRUCTIONS:	**No Cheating and Good Luck!**

Please do not open the examination paper until directed to do so.

READ INSTRUCTIONS FIRST:

Desks should be free from all unnecessary items (books, notes, technology, food, water, clothes);

Use of any electronic device (Phone, iPod, iPad, laptop) is not allowed during the examination;

Cheating, talking to fellow students, singing, turning back are not allowed;

Write your Name (*capital letters*), ID number and Department name in each page of your examination paper;

Final answers must be written by only blue or black, non-erasable pen. Do not use highlighters or correction pen;

All answers should be written in the space provided for each question, unless specified the other way;

If additional space is required, you should notify Proctors;

If you have a problem please raise your hand and wait quietly for a Proctor;

You are not allowed to leave the examination room until you submit the examination papers.

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Important: Please, write your solutions inside rectangular boxes, drawn below each question. Please, write your final answers after the word "Answer", given in the last line of boxes. You can use the backside of pages for your intermediate calculations *etc*, but you have to write your final answers after the word "Answer". There are 9 (nine) problems in the paper.

(Total: 40 Points, Obtained:__________)

[1] A block whose mass ***m*** is **800 g** is attached to a spring having spring constant k=80 N/m. The block is pulled a distance x=**10 *cm*** from its equilibrium position at x=0 *cm* on a frictionless surface and released from rest at t=0 *s*. **Find the angular frequency (w), the frequency (f), the period (T), the magnitudes of the maximum speed (v_m) (in *m/s*) and maximum acceleration (a_m) of the oscillating block (in m/s^2). Finally determine the displacement function $x(t)$ for the spring-block system.**

(Max: 3 Points, Obtained:______)

$x = x_m \cdot \cos(\omega t + \varphi) \quad x(t=0) = x_m = 0.1 \Rightarrow \varphi = 0.$

$\boxed{x = 0.1 \cdot \cos(\omega t)} \quad T = 2\pi\sqrt{\frac{m}{k}} = 6.28\sqrt{\frac{0.8\,kg}{80\,N/m}} = 6.28 \cdot \frac{1}{10} = 0.628\,s.$

$T = 0.628\,s. \quad f = \frac{1}{T} = \frac{1}{0.628\,s} = 1.59\,Hz$

$\omega = 2\pi \cdot f = 6.28 \cdot 1.59\,\frac{rad}{s} \simeq 10\,\frac{rad}{s} \Rightarrow \boxed{x(t) = 0.1 \cdot \cos(10 \cdot t)}$

$V_{max} = \omega \cdot X_m = 10 \cdot 0.1\,m/s = 1\,m/s$

$a_{max} = \omega^2 \cdot X_m = 100 \cdot 0.1\,m/s^2 = 10\,m/s^2$

Displacement function for the spring-block system.

Answer: $\omega = 10\,\frac{rad}{s}$, $f \simeq 1.6\,Hz$, $T \simeq 0.6\,s$, $V_{max} = 1\,m/s$, $a_{max} = 10\,m/s^2$, $x(t) = 0.1 \cdot \cos(10 \cdot t)$

[2] Two gliders move toward each other on a linear air track, which we assume is frictionless. Glider A has a mass of **0.5 kg**, and glider B has a mass of **0.3 kg**; both gliders move with an initial speed of **2 *m/s***. After they collide, glider B moves away with a final velocity whose x component is **+2 *m/s***. **What is the final velocity of the glider A?**

(Max: 3 Points, Obtained:______)

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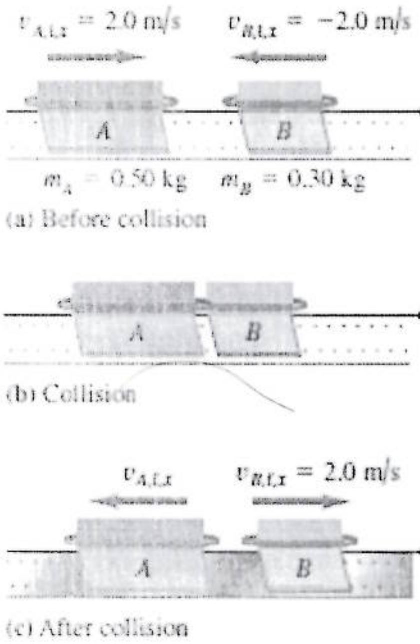

Let us write a total momentum conservation law for X axis:

$P_{ix} = P_{fx}$, where i and f stand for initial and final total momenta, respectively.

$$\left.\begin{array}{l} P_{ix} = m_A V_{Ai} + m_B \cdot V_{Bi} \\ P_{fx} = m_A \cdot V_{Af} + m_B \cdot V_{Bf} \end{array}\right\} \Longrightarrow$$

$$V_{Af} = \frac{m_A \cdot V_{Ai} + m_B \cdot V_{Bi} - m_B \cdot V_{Bf}}{m_A}$$

$$V_{Af} = \frac{0.5\,kg\,(2\,\frac{m}{s}) + 0.3\,kg\,(-2\,\frac{m}{s}) - 0.3\,kg\,(2\,\frac{m}{s})}{0.5\,kg}$$

$$V_{Af} = -0.4\ m/s$$

Answer: $V_{Af} = -0.4\ m/s$

[3] A rocket is launched from Earth (mass $\mathbf{M_E}$, radius $\mathbf{R_E}$) with velocity $\mathbf{v_0}$, and reaches the radial distance $\mathbf{r = 6\ R_E}$ with velocity $\mathbf{v_0/10}$. **a)** Express $\mathbf{v_0}$ in terms of $\mathbf{M_E}$ and $\mathbf{R_E}$. **b)** What would be the maximum height ($\mathbf{h_{max}}$) that the rocket in the above problem could reach if launched vertically?

(Max: 4 Points, Obtained:_______)

a) According to the Law of conservation of mechanical energy: $\frac{\cancel{m}V_0^2}{2} - \frac{GM_E\cancel{m}}{R_E} = \frac{\cancel{m}\left(\frac{V_0}{10}\right)^2}{2} - \frac{GM_E\cancel{m}}{6R_E}$

$$\frac{V_0^2}{2} - \frac{V_0^2}{200} = \frac{6GM_E}{6R_E} - \frac{GM_E}{6R_E}$$

$$\frac{99}{200} V_0^2 = \frac{5GM_E}{6 \cdot R_E} \Longrightarrow V_0 = \sqrt{\frac{5GM_E}{6R_E} \cdot \frac{200}{99}} \simeq 1.3 \cdot \sqrt{\frac{GM_E}{R_E}}$$

$$V_0 \simeq 1.3 \cdot \sqrt{\frac{GM_E}{R_E}}$$

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b) At the maximum height h_{max} the velocity will be zero. According to the Law of conservation of Energy:

$$\frac{mV_0^2}{2} - \frac{GM_E \cdot m}{R_E} = 0 - \frac{GM_E \cdot m}{h_{max}}$$

$$\frac{GM_E}{h_{max}} = \frac{GM_E}{R_E} - \frac{V_0^2}{2} \Rightarrow \frac{\cancel{GM_E}}{h_{max}} = \frac{\cancel{GM_E}}{R_E} - \frac{1}{2}(1.3)^2\frac{\cancel{GM_E}}{R_E}$$

$$\frac{1}{h_{max}} = \frac{1}{R_E}(1-0.845) \Rightarrow \boxed{h_{max} = 6.45 R_E}$$

Answer: $h_{max} = 6.45\, R_E$

[4] Spring with a spring constant $k = 1\ N/m$ and an attached mass m oscillate on a smooth (frictionless) horizontal table. When the mass is at position $x_1 = 0.1\ m$ its velocity $v_1 = -2\ m/s$, and at the position $x_2 = -0.2\ m$ it has velocity $v_2 = 1\ m/s$. Find the mass m (in kg) and the Amplitude (in m) of the motion.

(Max: 5 Points, Obtained:_______)

Energy conservation requires that $E = \frac{mV^2}{2} + \frac{KX^2}{2}$ remains constant.

Thus $mV_1^2 + KX_1^2 = mV_2^2 + KX_2^2$

$$\Rightarrow m = K(X_2^2 - X_1^2)/(V_1^2 - V_2^2)$$

$$m = 1 \cdot ((-0.2)^2 - (0.1)^2)/((-2)^2 - 1^2)\ kg$$

$$\Rightarrow m = 0.09\ kg$$

The amplitude A can be found by setting $V_2 = 0$, $x_2 = A$, so that

$$KA^2 = mV_1^2 + KX_1^2$$

$$\Rightarrow A = \sqrt{\frac{mV_1^2 + KX_1^2}{K}} = \sqrt{\frac{0.09 \cdot (-2)^2 + 0.09 \cdot (0.1)^2}{1}}$$

$$\Rightarrow A = 0.6\ m$$

Answer: $m = 0.09\ kg$; $A = 0.6\ m$

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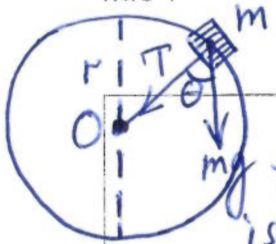
[5] A mass $m = 0.5$ **kg** is attached to a string and whirled (rotated) in a vertical circle at a constant speed. The radius of the circle is $\mathbf{r} = 3\ m$. Calculate the minimum speed $\mathbf{v_{min}}$ required to keep the string taut (tight). You can use $\mathbf{g=9.8}$ m/s^2.

(Max: 3 Points, Obtained:_______)



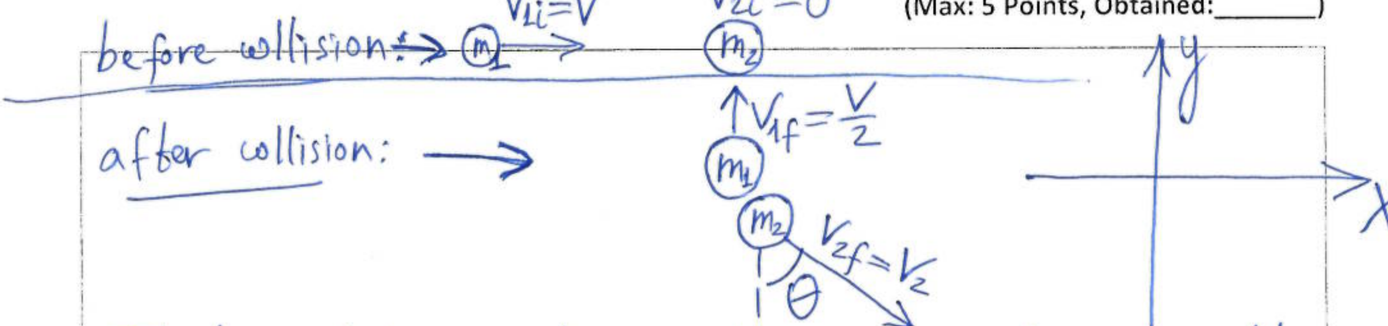
~~When the string makes an angle θ to the vertical,~~ the centripetal force is $T+mg\cdot\cos\theta$, this must be constant in uniform circular motion, so the minimum tension T is reached when $\cos\theta = 1 \Rightarrow \theta = 0$, i.e., at the highest point. Here $T+mg=\frac{mv^2}{r}$. To keep the string taut requires $T>0$, i.e., $\frac{mv^2}{r} > mg$, i.e., $v^2 > rg$ or

$v > \sqrt{r\cdot g}$ or $v > \sqrt{3m\cdot 9.8\frac{m}{s^2}} \simeq 5.4\ m/s \Rightarrow V_{min} \simeq 5.4\ m/s$

Answer: $V_{min} \simeq 5.4\ m/s$

[6] A ball with mass m_1, having a speed **v**, collides with a second ball, which is at rest. After the completely elastic collision, the ball with mass m_1 moves with a speed $\mathbf{v}/2$ along the direction, perpendicular to its initial direction (i.e. the ball gets scattered by 90^0). Find the mass of the second ball in terms of m_1.

(Max: 5 Points, Obtained:_______)



The Law of conservation of Linear Momentum along the X and y axes: (1) $m_1\cdot V = m_2\cdot V_2\cdot\sin\theta$ (X axis)

(2) $0 = m_1\frac{V}{2} - m_2\cdot V_2\cdot\cos\theta$ (y axis)

(3) $\frac{m_1\cdot V^2}{2} = \frac{m_1\cdot\left(\frac{V}{2}\right)^2}{2} + \frac{m_2\cdot V_2^2}{2}$ (conservation of kinetic energy for completely elastic collision)

Answer:

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~~From (1) $\Rightarrow V_2 = \frac{m_1 \cdot V}{m_2 \cdot \sin\theta}$ (4) and From (2) $\Rightarrow V_2 = \frac{m_1 \cdot V}{2 m_2 \cdot \cos\theta}$ (5).~~

Now (4) = (5) $\Rightarrow \frac{\cancel{m_1} \cdot \cancel{V}}{\cancel{m_2} \cdot \sin\theta} = \frac{\cancel{m_1} \cdot \cancel{V}}{2 \cancel{m_2} \cdot \cos\theta} \Rightarrow \sin\theta = 2\cos\theta$

$\Rightarrow \tan\theta = 2 \Rightarrow \theta = \arctan(2) = 63.435^\circ$

From (4) $\Rightarrow V_2 = \frac{m_1 \cdot V}{m_2 \cdot \sin\theta} = \frac{m_1 \cdot V}{m_2 \cdot \sin(63.435^\circ)} = \frac{m_1 \cdot V}{m_2 \cdot 0.8944} \Rightarrow$

$\boxed{V_2 \approx 1.118 \frac{m_1}{m_2} \cdot V}$ (6) Substituting (6) into (3) we obtain:

$$\frac{m_1 \cancel{V^2}}{2} = \frac{m_1 \cdot \cancel{V^2}}{8} + \frac{\cancel{m_2}}{2} \cdot (1.118)^2 \cdot \frac{m_1^2}{m_2^{\cancel{2}}} \cancel{V^2}$$

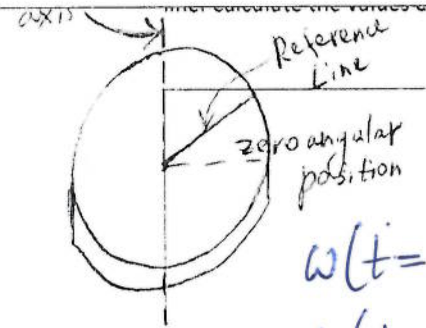
$$\frac{3 \cancel{m_1}}{8} = 0.624962 \frac{m_1^{\cancel{2}}}{m_2} \Rightarrow m_2 = \frac{8 \cdot 0.624962}{3} \cdot m_1 = \frac{4.999}{3} m_1$$

$$\Rightarrow \boxed{m_2 \simeq \frac{5}{3} m_1}$$

Answer: $m_2 \simeq \frac{5}{3} m_1$

[7] The disk is rotating about its central axis like a merry-go-round. The angular position $\theta(t)$ of a reference line on the disk is given by $\theta(t) = -1 - 2t + 5t^2$, with t in seconds and θ in radians. **Determine the angular velocity function $w(t)$ and angular acceleration function $\alpha(t)$ for the reference line. Calculate the values of $\theta(t)$ (in rad), $w(t)$ (in rad/s), and $\alpha(t)$ (in rad/s^2) at t=5 s.**

(Max: 3 Points, Obtained:_______)



$\omega(t) = \theta'(t) = -2 + 10 \cdot t$

$\alpha(t) = \omega'(t) = 10$

$\theta(t = 5s) = -1 - 2 \cdot 5 + 5 \cdot 5^2$ rad $= 114$ radians

$\omega(t = 5s) = -2 + 10 \cdot 5 \frac{rad}{s} = 48 \frac{rad}{second}$

$\alpha(t = 5s) = 10 \frac{rad}{s^2}$

Answer: $\omega(t) = -2 + 10 \cdot t$; $\alpha(t) = 10$;

$\theta(t = 5s) = 114$ radians; $\omega(t = 5s) = 48 \frac{rad}{seconds}$;

$\alpha(t = 5s) = 10 \frac{rad}{s^2}$

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Answer: $\omega(t) = -2 + 10t$; $\alpha(t) = 10$; $\theta(t=5s) = 114$ radians;
$\omega(t=5s) = 48 \frac{rad}{seconds}$; $\alpha(t=5s) = 10 \frac{rad}{s^2}$

[8] String fixed at both ends is **10 *m*** long (***L*=10 *m***) and has a mass of **0.150 kg**. It is subjected to a tension force ***T* = 100 *N*** and set oscillating. **a)** What is the speed (**v**) of the waves on the string? **b)** What is the longest possible wavelength (λ) for a standing wave? **c)** Find the frequency (*f*) corresponding to this longest possible wavelength (λ) for a standing wave.

(Max: 4 Points, Obtained:________)

a) the wave speed $V = \sqrt{\frac{T}{\mu}}$, T = tension force, $\mu = \frac{m}{L}$ = linear mass density.

$V = \sqrt{\frac{T \cdot L}{m}} = \sqrt{\frac{100N \cdot 10m}{0.150\,kg}} = \sqrt{6666.67}\ m/s \simeq 82\ m/s$

b) The longest possible wavelength λ for a standing wave is related to the length of the string by

$L = \frac{\lambda}{2}$, so $\lambda = 2L = 2 \cdot 10m = 20m$

c) The corresponding frequency is

$f = \frac{V}{\lambda} = \frac{(82\ m/s)}{(20\ m)} \simeq 4.1\ Hz$

Answer: a) $V = 82\ m/s$; b) $\lambda = 20m$; c) $f = 4.1\ Hz$.

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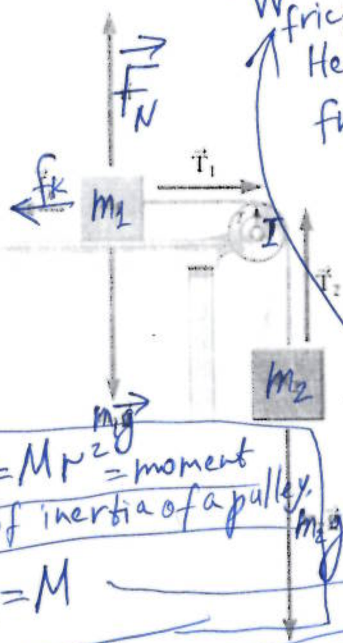
[9] Two blocks with masses m_1=5 kg and m_2=7 kg are attached by a string, as shown in the figure below, over a pulley with mass M=2.00 kg. The pulley, which turns on a frictionless axle, is a hollow cylinder with radius r=0.05 m over which the string moves without slipping. The horizontal surface has coefficient of kinetic friction μ_k=0.350. Find the speed of the system when the block of mass m_2 has dropped 2.0 m. The moment of inertia of a hollow cylinder is $I = M^*r^2$. (g=9.8 m/s^2. Hint: consider the work-energy theorem). $(h = 2\ m)$

(Max: 10 Points, Obtained:______)

~~Apply the work-energy theorem for the system:~~ *

$$W_{frict.} = \Delta K_1 + \Delta K_2 + \Delta K_{rot.} + \Delta U$$

Here W_{frict} = is the work of non-conservative frictional force (on mass m_1), which decreases the total mechanical energy of the system. ΔK_1 and ΔK_2 = changes in kinetic energies of m_1 and m_2. ΔK_{rot} = change in rotational kinetic energy of a pulley. ΔU = change in potential energy of a mass m_2.



$$-\mu_k (m_1 \cdot g) \cdot h = \left(\frac{m_1 V^2}{2} - 0\right) + \left(\frac{m_2 \cdot V^2}{2} - 0\right) + \left(\frac{1}{2} I \omega^2 - 0\right) + (0 - m_2 \cdot g h)$$

$I = M r^2$ = moment of inertia of a pulley.

$\frac{I}{r^2} = M$

$\boxed{V = r \cdot \omega}$

$$-\mu_K (m_1 g) h = \frac{1}{2} m_1 V^2 + \frac{1}{2} m_2 V^2 + \frac{1}{2} \left(\frac{I}{r^2}\right) r^2 \omega^2 - m_2 \cdot g h$$

$$-\mu_K (m_1 \cdot g) \cdot h = \frac{1}{2} m_1 \cdot V^2 + \frac{1}{2} m_2 V^2 + \frac{1}{2} M V^2 - m_2 \cdot g h$$

$$m_2 \cdot g \cdot h - \mu_K \cdot (m_1 \cdot g) h = \frac{1}{2}(m_1 + m_2 + M) V^2$$

$$V = \sqrt{\frac{2 \cdot g \cdot h \cdot (m_2 - \mu_K \cdot m_1)}{m_1 + m_2 + M}}$$

$$V = \sqrt{\frac{2 \cdot 9.8 \cdot 2 \cdot (7 - 0.35 \cdot 5)}{5 + 7 + 2}}\ m/s \simeq 3.83\ m/s$$

Answer:

$V \simeq 3.83\ m/s$

* Please, note if all ~~the forces were~~ conservative (if friction force was absent) then we would have $0 = \Delta K_1 + \Delta K_2 + \Delta K_{rot} + \Delta U$

GOOD LUCK!

or $0 = (K_{1f} - K_{2i}) + (K_{2f} - K_{2i}) + (K_{rotf} - K_{roti}) + (U_f - U_i)$ or

$K_{1f} + K_{2f} + K_{rotf} + U_f = K_{1i} + K_{2i} + K_{roti} + U_i$, which is the Law of conservation of mechanical energy.