



TOSHKENT SHAHRIDAGI INHA UNIVERSITETI
INHA UNIVERSITY IN TASHKENT

STUDENT NAME:

STUDENT ID NUMBER:

DEPARTMENT:

Solution of the recently held

FINAL EXAMINATION (FALL 2016)

COURSE NAME:

COURSE NUMBER:

EXAMINATION DATE:

TIME:

EXAMINATION DURATION:

ADDITIONAL MATERIALS
ALLOWED TO USE:

SPECIAL INSTRUCTIONS:

Please do not open the examination paper until directed to do so.

READ INSTRUCTIONS FIRST:

Desks should be free from all unnecessary items (books, notes, technology, food, water, clothes);

Use of any electronic device (Phone, iPod, iPad, laptop) is not allowed during the examination;

Cheating, talking to fellow students, singing, turning back are not allowed;

Write your Name (*capital letters*), ID number and Department name in each page of your examination paper;

Final answers must be written by only blue or black, non-erasable pen. Do not use highlighters or correction pen;

All answers should be written in the space provided for each question, unless specified the other way;

If additional space is required, you should notify Proctors;

If you have a problem please raise your hand and wait quietly for a Proctor;

You are not allowed to leave the examination room until you submit the examination papers.

STUDENT NAME: STUDENT ID NUMBER: DEPARTMENT:

Important: Please, write your solutions inside rectangular boxes, drawn below each question. Please, write your final answers after the word "Answer", given in the last line of boxes. You can use the backside of pages for your intermediate calculations etc, but you have to write your final answers after the word "Answer". There are 7 (seven) problems (questions) in the paper.

(Total: 40 Points, Obtained: _____)

[1] Two gliders move toward each other on a linear air track, which we assume is frictionless. Glider A has a mass of 0.5 kg, and glider B has a mass of 0.3 kg; both gliders move with an initial speed of 2 m/s. After they collide, glider B moves away with a final velocity whose x component is +2 m/s. **What is the final velocity of glider A?**

(Max: 5 Points, Obtained: _____)

(a) Before collision

(b) Collision

(c) After collision

Let us write a total momentum conservation law for X axis:

$P_{ix} = P_{fx}$, where i and f stand for initial and final total momenta, respectively.

$$P_{ix} = m_A \cdot V_{Ai} + m_B \cdot V_{Bi}$$

$$P_{fx} = m_A \cdot V_{Af} + m_B \cdot V_{Bf}$$

$$V_{Af} = \frac{m_A V_{Ai} + m_B V_{Bi} - m_B V_{Bf}}{m_A}$$

$$V_{Af} = \frac{0.5 \text{ kg} (2.0 \text{ m/s}) + 0.3 \text{ kg} (-2.0 \text{ m/s}) - 0.3 \text{ kg} (2.0 \text{ m/s})}{0.5 \text{ kg}} =$$

$$V_{Af} = -0.4 \text{ m/s}$$

Answer: $V_{Af} = -0.4 \text{ m/s}$

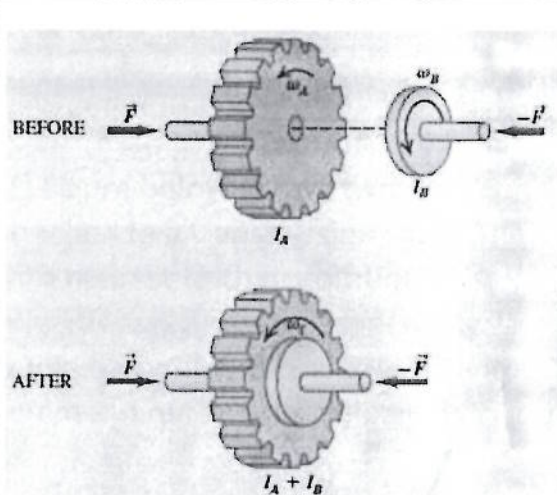
STUDENT NAME:

STUDENT ID NUMBER:

DEPARTMENT:

[2] Figure below shows two disks, one an engine flywheel, the other a clutch plate attached to a transmission shaft. Their moment of inertia are I_A and I_B , respectively, with respect to their rotation axis. Initially they are rotating with constant angular velocities ω_A and ω_B , respectively. We then push the disks together with the forces acting along the x axis only. Eventually the disks stick to each other and reach a common final angular velocity ω_f . Derive an expression for ω_f .

(Max: 4 Points, Obtained: _____)



In this problem, the total angular momentum of the system is conserved.

Conservation of angular momentum gives

$$I_A \cdot \omega_A + I_B \cdot \omega_B = (I_A + I_B) \cdot \omega_f$$

$$\omega_f = \frac{I_A \cdot \omega_A + I_B \cdot \omega_B}{I_A + I_B}$$

Answer: $\omega_f = \frac{I_A \cdot \omega_A + I_B \cdot \omega_B}{I_A + I_B}$

STUDENT NAME: STUDENT ID NUMBER: DEPARTMENT:

[3] An acrobatic physics professor stands at the center of a turntable, holding his arms extended horizontally, with a 5 kg dumbbell in each hand (mass of a one dumbbell is 5 kg.). Initially he is rotating about a vertical axis, making one revolution in 2 seconds. His moment of inertia (without the dumbbells) is 3 kg m^2 when his arms are outstretched, and his moment of inertia (without the dumbbells) is 2.2 kg m^2 when his arms are pulled in close to his chest. Each dumbbell is 1 m from the axis of rotation initially (when arms are outstretched) and is 0.2 m from the axis of rotation at the end (when arms are pulled in close to the chest).

Neglect the friction in turntable and find professor's final angular velocity (ω_f) (in both revolutions/sec and in rad/sec) when he pulls dumbbells close to his chest. Calculate the initial (rotational) kinetic energy and final (rotational) kinetic energy (in Joules).

(Max: 6 Points, Obtained:)

Note that Professor has only two dumbbells at both initial and final positions ("Before" and "After").

There is no friction in the turntable and there are no external torques with respect to the vertical rotation axis, and so the angular momentum about this axis is conserved:

$I_i \omega_i = I_f \omega_f$. In each case,

$i = \text{initial}$
 $f = \text{final}$

$I = I_{\text{prof}} + I_{\text{dumb}}$, thus,

$I_i = 3 \text{ kg} \cdot \text{m}^2 + 2 (5.0 \text{ kg}) \cdot (1.0 \text{ m})^2 = 13 \text{ kg} \cdot \text{m}^2$

$I_f = 2.2 \text{ kg} \cdot \text{m}^2 + 2 (5.0 \text{ kg}) \cdot (0.20 \text{ m})^2 = 2.6 \text{ kg} \cdot \text{m}^2$

$\omega_i = 2\pi \frac{1 \text{ rev.}}{2.0 \text{ s}} = \pi \text{ rad/s}$

$(13 \text{ kg} \cdot \text{m}^2) \cdot (\pi \frac{\text{rad}}{\text{s}}) = (2.6 \text{ kg} \cdot \text{m}^2) \cdot \omega_f$

$\omega_f = 5.0 \pi \frac{\text{rad}}{\text{s}} = 2.5 \text{ rev/s}$

Answer: $K_i = \frac{1}{2} I_i \omega_i^2 = \frac{1}{2} (13 \text{ kg} \cdot \text{m}^2) \cdot (\pi \frac{\text{rad}}{\text{s}})^2$

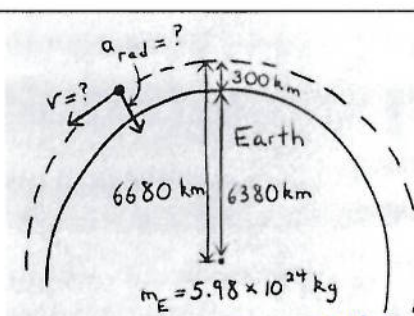
$K_i \approx 64 \text{ Joules}$

$K_f = \frac{1}{2} I_f \omega_f^2 = \frac{1}{2} (2.6 \text{ kg} \cdot \text{m}^2) \cdot (5.0 \pi \frac{\text{rad}}{\text{s}})^2$

$K_f \approx 320 \text{ Joules}$

STUDENT NAME: STUDENT ID NUMBER: DEPARTMENT:

[4] Suppose we want to place a weather satellite into a circular orbit 300 km above the earth's surface (see Figure below). What speed (v), period (T), and radial acceleration (a_r) must it have? The Earth's radius is $6380 \text{ km} = 6.38 \times 10^6 \text{ m}$, and its mass is $5.98 \times 10^{24} \text{ kg}$. You may use $G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$.

(Max: 6 Points, Obtained:)


Handwritten solution for Question 4:

$$\frac{G M_E m_s}{r^2} = \frac{m_s v^2}{r}$$

$$r = (6.38 \times 10^6 + 0.3 \times 10^6) \text{ m} = 6.68 \times 10^6 \text{ m}$$

$$v = \sqrt{\frac{G \cdot m_E}{r}} = \sqrt{\frac{6.67 \times 10^{-11} \cdot 5.98 \times 10^{24}}{6.68 \times 10^6}} \text{ m/s}$$

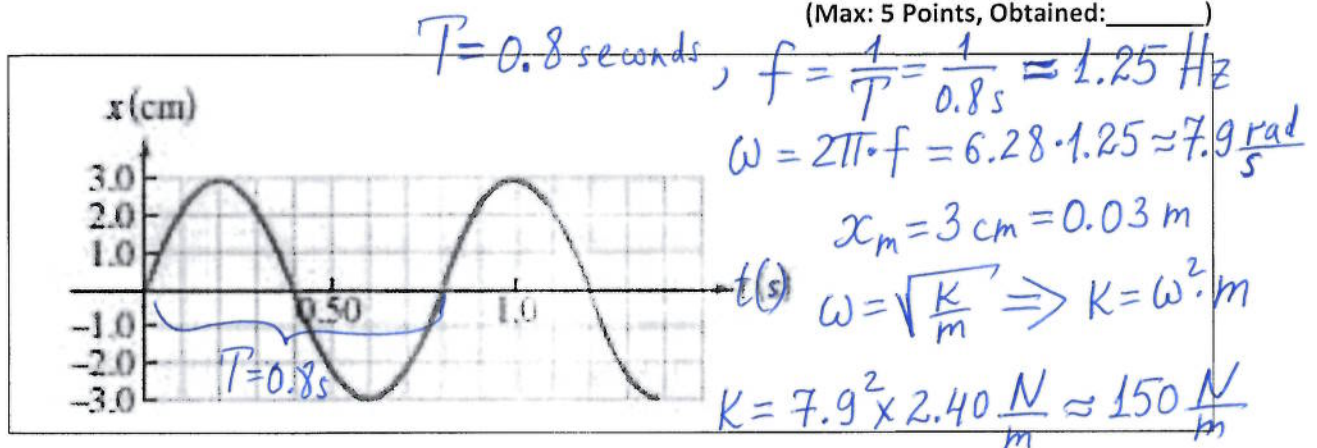
$$v \approx 7700 \text{ m/s}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi \times 6.68 \times 10^6 \text{ m}}{7700 \text{ m/s}} \approx 5400 \text{ seconds} \approx 91 \text{ minutes}$$

The radial acceleration $a_r = \frac{v^2}{r} = \frac{(7700 \text{ m/s})^2}{6.68 \times 10^6 \text{ m}} \approx 8.9 \text{ m/s}^2 \approx 9 \text{ m/s}^2$

Answer: $v \approx 7700 \text{ m/s}$; $T \approx 5400 \text{ seconds} \approx 91 \text{ minutes}$; $a_r \approx 9 \text{ m/s}^2$

[5] A 2.40 kg ball is attached to an unknown spring and is set to oscillate on a frictionless surface. Figure below shows a graph of the ball's position $x(t)$ as a function of time (in seconds). For this motion (SHM), find (a) its period (T), (b) its frequency (f), (c) its angular frequency (ω), (d) its amplitude (x_m), and (e) the spring constant (k).

(Max: 5 Points, Obtained:)

STUDENT NAME:

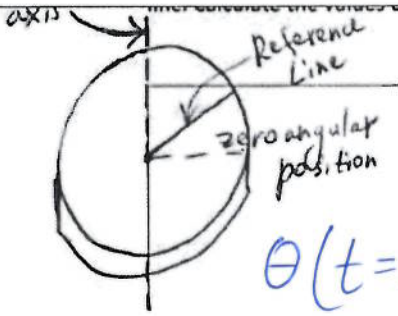
STUDENT ID NUMBER:

DEPARTMENT:

Answer: $T = 0.8$ seconds; $f = 1.25$ Hz; $\omega \approx 7.9 \frac{\text{rad}}{\text{sec}}$; $x_m = 3 \text{ cm} = 0.03 \text{ m}$;
 $K = 150 \frac{\text{N}}{\text{m}}$

[6] The disk is rotating about its central axis like a merry-go-round. The angular position $\theta(t)$ of a reference line on the disk is given by $\theta(t) = -1 - 2t + 5t^2$, with t in seconds and θ in radians. Determine the angular velocity function $w(t)$ and angular acceleration function $\alpha(t)$ for the reference line. Calculate the values of $\theta(t)$ (in rad), $w(t)$ (in rad/s), and $\alpha(t)$ (in rad/s²) at $t = 5$ s.

(Max: 4 Points, Obtained: _____)



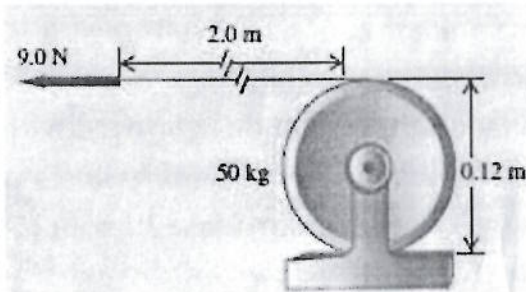
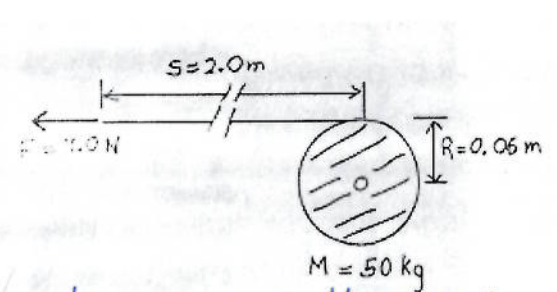
$\omega(t) = \theta'(t) = -2 + 10t$
 $\alpha(t) = \omega'(t) = 10$
 $\theta(t = 5 \text{ s}) = -1 - 2 \cdot 5 + 5 \cdot 5^2 \text{ rad} = 114 \text{ radians}$
 $\omega(t = 5 \text{ s}) = -2 + 10 \cdot 5 \frac{\text{rad}}{\text{s}} = 48 \frac{\text{rad}}{\text{second}}$
 $\alpha(t = 5 \text{ s}) = 10 \frac{\text{rad}}{\text{s}^2}$

Answer: $\omega(t) = -2 + 10 \cdot t$; $\alpha(t) = 10$; $\theta(t = 5 \text{ s}) = 114 \text{ radians}$;
 $\omega(t = 5 \text{ s}) = 48 \frac{\text{rad}}{\text{sec.}}$; $\alpha(t = 5 \text{ s}) = 10 \frac{\text{rad}}{\text{s}^2}$

STUDENT NAME: STUDENT ID NUMBER: DEPARTMENT:

[7] A light, flexible, nonstretching cable (rope) is wrapped several times around a winch drum – a solid cylinder with mass $M = 50 \text{ kg}$ and diameter 0.12 m ($R = 0.06 \text{ m}$), that rotates about a stationary horizontal axis that turns without a friction. The free end of the cable is pulled with a constant force of magnitude $F = 9.0 \text{ N}$ over a distance of $s = 2 \text{ meters}$ (see Figure below). It unwinds without slipping. If the cylinder is initially at rest, find its final angular velocity (ω_f) and the final speed (v_f) of the cable.

You may use $I = (1/2)MR^2$ for a moment of inertia of the cylinder with respect to rotation axis.

(Max: 10 Points, Obtained:)



According to work-kinetic energy theorem:

$$F \cdot s = \frac{I \omega_f^2}{2} - \left(\frac{I \omega_i^2}{2} \right) = \frac{I \omega_f^2}{2} \quad \text{because it is initially at rest}$$

$$\omega_f = \sqrt{\frac{2 \cdot F \cdot s}{I}} = \sqrt{\frac{2 \cdot F \cdot s}{\frac{1}{2} M R^2}} = \sqrt{\frac{4 \cdot F \cdot s}{M \cdot R^2}}$$

$$\omega_f = \sqrt{\frac{4 \cdot 9 \cdot 2}{50 \cdot (0.06)^2}} \frac{\text{rad}}{\text{second}} = 20 \frac{\text{rad}}{\text{sec.}}$$

$$V_f = \omega \cdot R = (20 \frac{\text{rad}}{\text{s}}) \cdot (0.06 \text{ m}) = 1.2 \text{ m/s}$$

Answer: $\omega_f = 20 \frac{\text{rad}}{\text{sec}} ; V_f = 1.2 \text{ m/s}$