



TOSHKENT SHAHRIDAGI INHA UNIVERSITETI
INHA UNIVERSITY IN TASHKENT

STUDENT NAME:

STUDENT ID NUMBER:

DEPARTMENT:

Solution of the recently held

FINAL EXAMINATION FALL 2018

COURSE NAME:

COURSE NUMBER:

EXAMINATION DATE:

TIME:

EXAMINATION DURATION:

ADDITIONAL MATERIALS
ALLOWED TO USE:

Own brain, own memory and logics.
CALCULATORS ARE ALLOWED

SPECIAL INSTRUCTIONS:

No Cheating and Good Luck!

Please do not open the examination paper until directed to do so.

READ INSTRUCTIONS FIRST:

Desks should be free from all unnecessary items (books, notes, technology, food, water, clothes);

Use of any electronic device (Phone, iPod, iPad, laptop) is not allowed during the examination;

Cheating, talking to fellow students, singing, turning back are not allowed;

Write your Name (capital letters), ID number and Department name in each page of your examination paper;

Final answers must be written by only blue or black, non-erasable pen. Do not use highlighters or correction pen;

All answers should be written in the space provided for each question, unless specified the other way;

If additional space is required, you should notify Proctors;

If you have a problem please raise your hand and wait quietly for a Proctor;

You are not allowed to leave the examination room until you submit the examination papers.

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Important: Please, write your solutions inside rectangular boxes, drawn below each question. You have to write your final answers after the word "Answer", given in the last line of boxes. You can use the backside of pages for your intermediate calculations etc. There are 8 problems in the paper.

(Total: 40 Points, Obtained: _____)

[1] Two gliders move toward each other on a linear air track, which we assume is frictionless. Glider **A** has a mass of **0.5 kg**, and glider **B** has a mass of **0.3 kg**; both gliders move with an initial speed of **2 m/s**. After they collide, glider **B** moves away with a final velocity whose x component is **+2 m/s**. What is the final velocity of glider **A**?

(Max: 4 Points, Obtained: _____)

(a) Before collision

(b) Collision

(c) After collision

Let us write a total momentum conservation law for X axis:

$P_{ix} = P_{fx}$, where *i* and *f* stand for initial and final total momenta, respectively.

$$P_{ix} = m_A \cdot V_{Ai} + m_B \cdot V_{Bi}$$

$$P_{fx} = m_A \cdot V_{Af} + m_B \cdot V_{Bf}$$

$$V_{Af} = \frac{m_A \cdot V_{Ai} + m_B \cdot V_{Bi} - m_B \cdot V_{Bf}}{m_A}$$

$$V_{Af} = \frac{0.5 \text{ kg} (2 \frac{\text{m}}{\text{s}}) + 0.3 \text{ kg} (-2 \frac{\text{m}}{\text{s}}) - 0.3 \text{ kg} (2 \frac{\text{m}}{\text{s}})}{0.5 \text{ kg}}$$

$$V_{Af} = -0.4 \text{ m/s}$$

Answer: $V_{A,f,x} = -0.4 \text{ m/s}$

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[2] Two blocks with masses $m_1=2.0 \text{ kg}$ and $m_2=9.0 \text{ kg}$ are attached over a pulley with mass $M=3.0 \text{ kg}$, hanging straight down as in Atwood's machine (see Figure below). The pulley is a solid cylinder with radius $R=0.05 \text{ m}$, and there is some friction in the axle. The system is released from rest, and the string moves without slipping over the pulley. **If the larger mass is traveling at a speed of 2.5 m/s when it has dropped 1.0 m , how much mechanical energy was lost due to friction in the pulley's axle?** The moment of inertia of a solid cylinder of radius R and mass M with respect to rotation axis going through its center is $I = 0.5(MR^2)$. Use $g=9.8 \text{ m/s}^2$.

(Max: 8 Points, Obtained: _____)

We will use conservation of energy

$E_{\text{lost}} = |W_{\text{friction}}|$, where W_{friction} = work done by friction force

$m_1 \cdot g \cdot y_{1i} + m_2 \cdot g \cdot y_{2i} + U_{\text{disk}} = \frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} + \frac{I \omega^2}{2} + m_1 g y_{1f} + m_2 g y_{2f} + |W_{\text{friction}}|$

$|W_{\text{friction}}| = E_{\text{Lost}} = - \left(\frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} + \frac{I \omega^2}{2} + m_1 g y_{1f} + m_2 g y_{2f} \right)$

$E_{\text{Lost}} = - \left(\frac{2 \cdot (2.5)^2}{2} + \frac{9 \cdot (2.5)^2}{2} + \frac{0.00375 \cdot (50)^2}{2} + 2 \cdot 9.8 \cdot 1 + 9.98 \cdot 1 \right) =$

$E_{\text{Lost}} = - (6.25 + 28.125 + 4.6875 + 19.6 + 9.98) = - 58.6625 \approx - 30 \text{ Joules}$

$V = \omega R \Rightarrow \omega = \frac{V}{R} = \frac{2.5 \text{ m/s}}{0.05 \text{ m}} = 50 \text{ rad/s}$

$I = \frac{1}{2} M R^2 = 0.00375 \text{ kg} \cdot \text{m}^2$

The easy way is to use conservation of energy:

$E_i = E_f + |W_{\text{friction}}|$

Answer: $E_{\text{Lost}} \approx 30$ Joules ($\approx 30 \text{ Joules}$)

[3] An acrobatic physics professor stands at the center of a turntable, holding his arms extended horizontally, with a 5 kg dumbbell in each hand (mass of a one dumbbell is 5 kg). Initially he is rotating about a vertical axis, making one revolution in 2 seconds. His moment of inertia (without the dumbbells) is 3 kg

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m^2 when his arms are outstretched, and his moment of inertia (without the dumbbells) is 2.2 kg m^2 when his arms are pulled in close to his chest. Each dumbbell is 1 m from the axis of rotation initially (when arms are outstretched) and is 0.2 m from the axis of rotation at the end (when arms are pulled in close to the chest). Neglect the friction in turntable and find professor's final angular velocity (in revolutions/sec and in rad/sec) when he pulls dumbbells close to his chest. Calculate the initial (rotational) kinetic energy and final (rotational) kinetic energy (in Joules).

(Max: 4 Points, Obtained:)

Because no friction in the turntable and no external torques with respect to the vertical axis \Rightarrow the angular momentum about this axis is conserved:

$I_i \cdot \omega_i = I_f \cdot \omega_f$. In each case $\begin{cases} i = \text{initial} \\ f = \text{final} \end{cases}$

$I = I_{\text{prof}} + I_{\text{dumb}}$, thus,

$I_i = 3 \text{ kg} \cdot \text{m}^2 + 2 \cdot (5.0 \text{ kg}) \cdot (1.0 \text{ m})^2 = 13 \text{ kg} \cdot \text{m}^2$

$I_f = 2.2 \text{ kg} \cdot \text{m}^2 + 2 \cdot (5.0 \text{ kg}) \cdot (0.2 \text{ m})^2 = 2.6 \text{ kg} \cdot \text{m}^2$

$\omega_i = 2\pi \frac{1 \text{ rev.}}{2.0 \text{ s}} = \pi \text{ rad/s}$

$(13 \text{ kg} \cdot \text{m}^2) \cdot (\pi \frac{\text{rad}}{\text{s}}) = (2.6 \text{ kg} \cdot \text{m}^2) \cdot \omega_f$

$\omega_f = 5.0 \pi \frac{\text{rad}}{\text{s}} = 2.5 \text{ rev/s}$

$K_i = \frac{1}{2} I_i \omega_i^2 = \frac{1}{2} (13 \text{ kg} \cdot \text{m}^2) \cdot (\pi \frac{\text{rad}}{\text{s}})^2$

$K_i \approx 64 \text{ Joules}$

$K_f = \frac{1}{2} I_f \omega_f^2 = \frac{1}{2} (2.6 \text{ kg} \cdot \text{m}^2) \cdot (5.0 \pi \frac{\text{rad}}{\text{s}})^2$

$K_f \approx 320 \text{ Joules}$

Answer: $K_{\text{rot}i} = 64$ Joules $K_{\text{rot}f} = 320$ Joules

[4] a) Find the position of the center of mass ($x_{\text{com}}=?$) of the three objects located in a coordinate system as shown in Figure (a) below. b) Find the position of the

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center of mass ($x_{\text{com}}=?$; $y_{\text{com}}=?$) of the three objects located in a coordinate system as shown in Figure (b) below. Treat the objects as point particles.

(Max: 4 Points, Obtained:)

a

b

1) Figure (a)

$$x_{\text{com}} = \frac{\sum m_i x_i}{\sum m_i} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$x_{\text{com}} = \frac{(5 \text{ kg}) \cdot (-0.5 \text{ m}) + (2 \text{ kg}) \cdot (0 \text{ m}) + (4 \text{ kg}) \cdot (1.0 \text{ m})}{11 \text{ kg}} = \frac{1.5 \text{ kg} \cdot \text{m}}{11 \text{ kg}} = 0.136 \text{ m}$$

2) Figure (b)

$$y_{\text{com}} = \frac{\sum m_i y_i}{\sum m_i} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

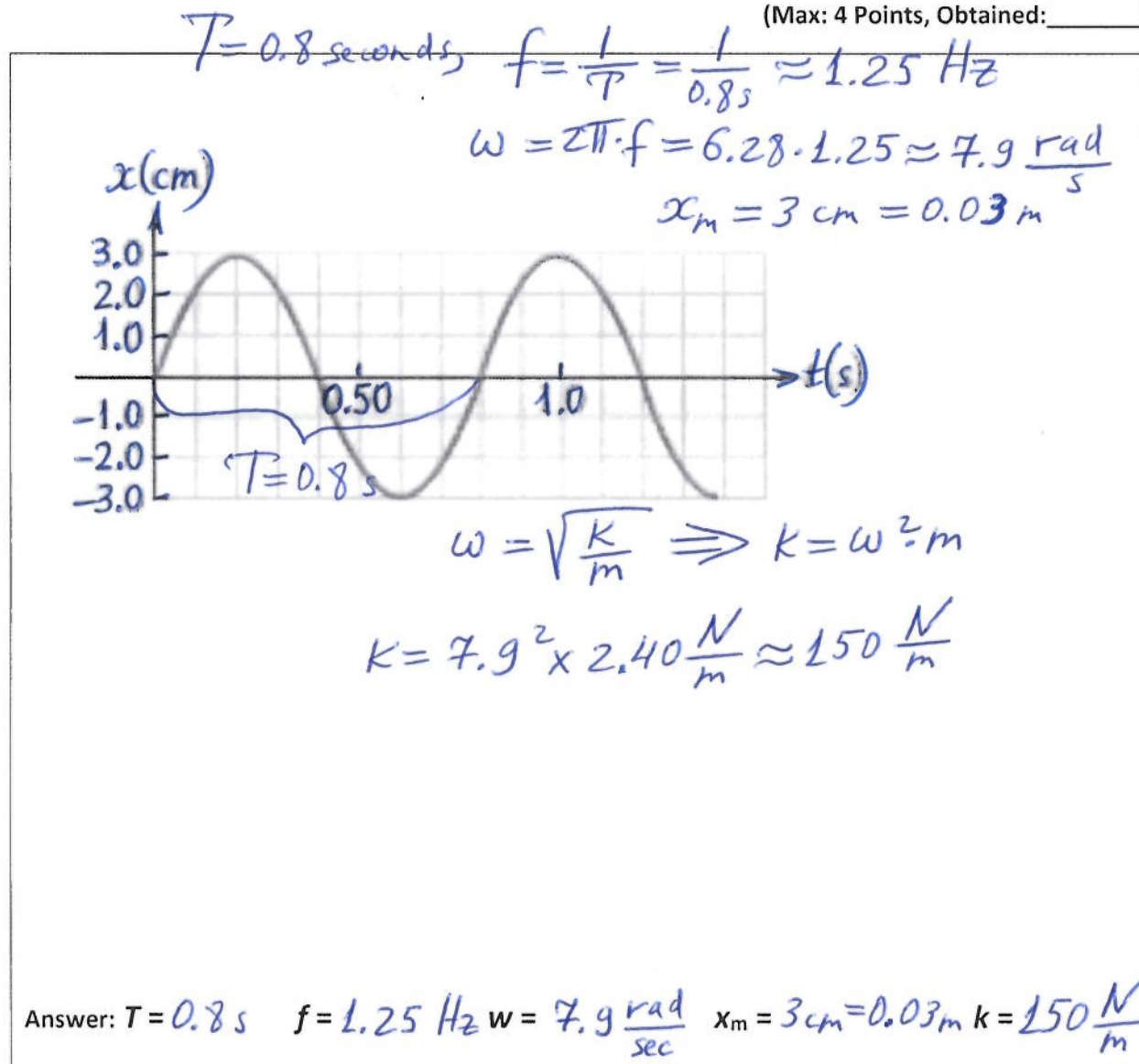
$$y_{\text{com}} = \frac{(5 \text{ kg}) \cdot (1 \text{ m}) + (2 \text{ kg}) \cdot (0 \text{ m}) + (4 \text{ kg}) \cdot (-0.5 \text{ m})}{5 \text{ kg} + 2 \text{ kg} + 4 \text{ kg}} = 0.273 \text{ m}$$

Answer: a) $x_{\text{com}} = 0.136 \text{ m}$ b) $x_{\text{com}} = 0.136 \text{ m}$; $y_{\text{com}} = 0.273 \text{ m}$

[5] A 2.40 kg ball is attached to an unknown spring and is allowed to oscillate. Figure below shows a graph of the ball's position $x(t)$ as a function of time (in

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seconds). For this motion (SHM), find (a) its period (T), (b) its frequency (f), (c) its angular frequency (ω), (d) its amplitude (x_m) (in meters), and (e) the spring constant (k).

(Max: 4 Points, Obtained:)

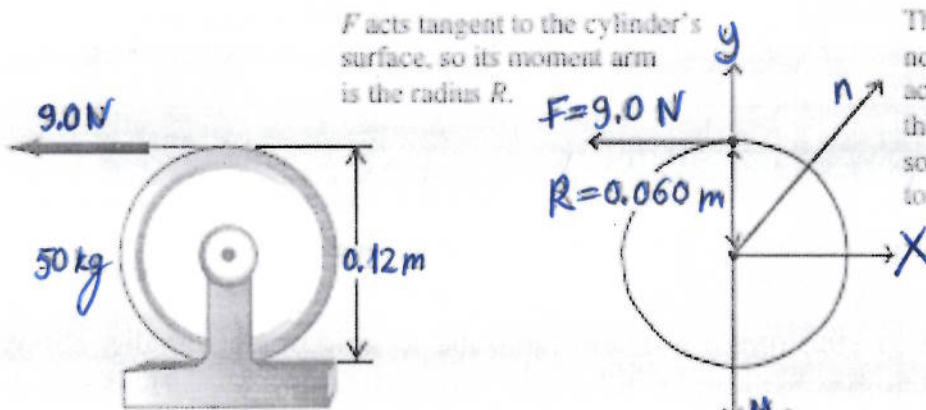
[6] An astronaut standing on the surface of Ceres, the largest asteroid, drops a rock from a height of **10.0 m** (assume it travels in a vacuum). It takes **8.06 seconds** for the rock to hit the ground. (a) Calculate the acceleration of gravity ($g_c = ?$) (in m/s^2) on the surface of Ceres. (b) Find the mass of Ceres ($M_c = ?$) (in kg) if the radius of Ceres is $R_c = 5.10 \times 10^5 \text{ m}$. (c) Calculate the gravitational acceleration (in m/s^2) at the height of **50 km** above the surface of Ceres ($g_c' = ?$).

(Max: 6 Points, Obtained:)

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- a) Apply the kinematic displacement equation to the falling rock to solve for the gravitational acceleration on Ceres, g_c :
- $$\Delta x = \frac{1}{2}at^2 + v_0t \Rightarrow -10\text{m} = -\frac{1}{2}g_c \cdot (8.06\text{s})^2 \Rightarrow g_c = 0.308\text{m/s}^2$$
- b) Find the mass of Ceres, M_c , equating the weight of the rock on Ceres to the gravitational force acting on the rock:
- $$mg_c = G \frac{M_c m}{R_c^2} \Rightarrow M_c = \frac{g_c R_c^2}{G} = 1.20 \times 10^{21}\text{kg}$$
- c) Calculate the acceleration of gravity at a height of 50 km above the surface of Ceres, equating the weight at 50 km to the gravitational force:
- $$m \cdot g_c' = G \frac{M_c m}{r^2} \Rightarrow g_c' = G \frac{M_c}{r^2} = (6.67 \times 10^{-11}\text{kg}^{-1}\text{m}^3\text{s}^{-2}) \frac{1.20 \times 10^{21}\text{kg}}{(5.60 \times 10^5\text{m})^2}$$
- $$g_c' = 0.255\text{m/s}^2$$
- Answer: $g_c = 0.308\text{m/s}^2$ $M_c = 1.20 \times 10^{21}\text{kg}$ $g_c' = 0.255\text{m/s}^2$

[7] A cable is wrapped several times around a uniform solid cylinder with diameter **0.12 m** and mass **$M=50\text{ kg}$** that can rotate freely about its axis (see Figure below). The cable is pulled by a force with magnitude **$F=9.0\text{ N}$** . Assuming that the cable unwinds without stretching or slipping, find the magnitude of acceleration of the cable ($a=?$). The moment of inertia of a solid cylinder of radius **R** and mass **M** with respect to rotation axis going through its center is **$I = 0.5(MR^2)$** .

(Max: 6 Points, Obtained:)


F acts tangent to the cylinder's surface, so its moment arm is the radius R .

The weight and normal force both act on a line through the axis of rotation, so they exert no torque.

The torque of the force F relative to axis of rotation is $\tau = F \cdot R = (9\text{ N}) \cdot (0.060\text{ m}) = 0.54\text{ N}\cdot\text{m}$ $\tau = 0.54\text{ N}\cdot\text{m}$

The moment of inertia of a solid cylinder with respect to axis of rotation is $I = \frac{1}{2}MR^2 = \frac{1}{2}(50\text{ kg}) \cdot (0.060\text{ m})^2 = 0.090\text{ kg}\cdot\text{m}^2$

$$I = 0.090\text{ kg}\cdot\text{m}^2$$

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Using $\tau = I \cdot \alpha$, we can find the angular acceleration α :

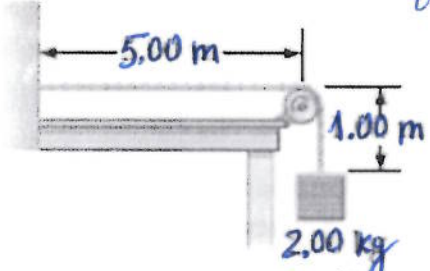
$$\alpha = \frac{\tau}{I} = \frac{0.54 \text{ N}\cdot\text{m}}{0.090 \text{ kg}\cdot\text{m}^2} = 6.0 \frac{\text{rad}}{\text{s}^2}$$

The magnitude of acceleration a of the cable is given by

$$a = R \cdot \alpha = (0.060 \text{ m}) \cdot (6.0 \text{ rad/s}^2) = 0.36 \text{ m/s}^2 \quad a = 0.36 \text{ m/s}^2$$

Answer: $a = 0.36 \text{ m/s}^2$

[8] A uniform string has a mass $M=0.03 \text{ kg}$ and a length $L=6.0 \text{ m}$. Tension is maintained in the string by suspending a block of mass $m=2.0 \text{ kg}$ from one end (see Figure below). a) Find the speed v of a transverse wave pulse on this string. (b) Find the time t it takes the pulse to travel from the wall to the pulley. Neglect the mass of the hanging part of the string. Use $g=9.8 \text{ m/s}^2$.

(Max: 4 Points, Obtained:)


Apply Newton's 2nd Law to find tension force F_T in the string!

tension force $F_T = mg$

Speed of the wave on a stretched string is found using $v = \sqrt{\frac{F_T}{\mu}}$

where $\mu = \frac{M}{L}$ = linear mass density of the string.

Hence $v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{mg}{M/L}} = \sqrt{\frac{2 \text{ kg} \cdot 9.8 \text{ m/s}^2}{0.03 \text{ kg}/(6 \text{ m})}} =$

$$v = \sqrt{\frac{19.6 \text{ N}}{0.005 \text{ kg/m}}} = 62.6 \text{ m/s} \quad v = 62.6 \text{ m/s}$$

The time it takes the pulse to travel from the wall to the pulley: $t = \frac{d}{v} = \frac{5 \text{ m}}{62.6 \text{ m/s}} = 0.0799 \text{ s}$

Answer: $v = 62.6 \text{ m/s} \approx 63 \text{ m/s} \quad t = 0.0799 \text{ s} \approx 0.08 \text{ s}$

GOOD LUCK!