

LINEAR-TIME SORTING

Hybrid Quicksort

What is the time complexity of quick sort where if size smaller than S insertion sort is applied to the subset?

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What is the time complexity of quick sort where if size smaller than S insertion sort is applied to the subset?

- Partitioning to S sized subsets:
- Sorting each subset:
- Total cost:

Counting Sort

A 5 3 0 1 3 2 5 3 1 0

1 2 3 4 5 6 7 8 9 10

B 0

B 0 1

B 0 1 3

B 0 1 3 5

B 0 1 2 3 5

B 0 1 2 3 3 5

0 1 2 3 4 5

C 2 2 1 3 0 2

C 2 4 5 8 8 10

C 1 4 5 8 8 10

C 1 3 5 8 8 10

C 1 3 5 7 8 10

C 1 3 5 7 8 9

C 1 3 4 7 8 9

C 1 3 4 6 8 9

Counting Sort

A

5	3	0	1	3	2	5	3	1	0
---	---	---	---	---	---	---	---	---	---

1	2	3	4	5	6	7	8	9	10
---	---	---	---	---	---	---	---	---	----

B

	0		1	2		3	3		5
--	---	--	---	---	--	---	---	--	---

B

	0	1	1	2		3	3		5
--	---	---	---	---	--	---	---	--	---

B

0	0	1	1	2		3	3		5
---	---	---	---	---	--	---	---	--	---

B

0	0	1	1	2	3	3	3		5
---	---	---	---	---	---	---	---	--	---

B

0	0	1	1	2	3	3	3	5	5
---	---	---	---	---	---	---	---	---	---

0	1	2	3	4	5
---	---	---	---	---	---

C

1	3	4	6	8	9
---	---	---	---	---	---

C

1	2	4	6	8	9
---	---	---	---	---	---

C

0	2	4	6	8	9
---	---	---	---	---	---

C

0	2	4	5	8	9
---	---	---	---	---	---

C

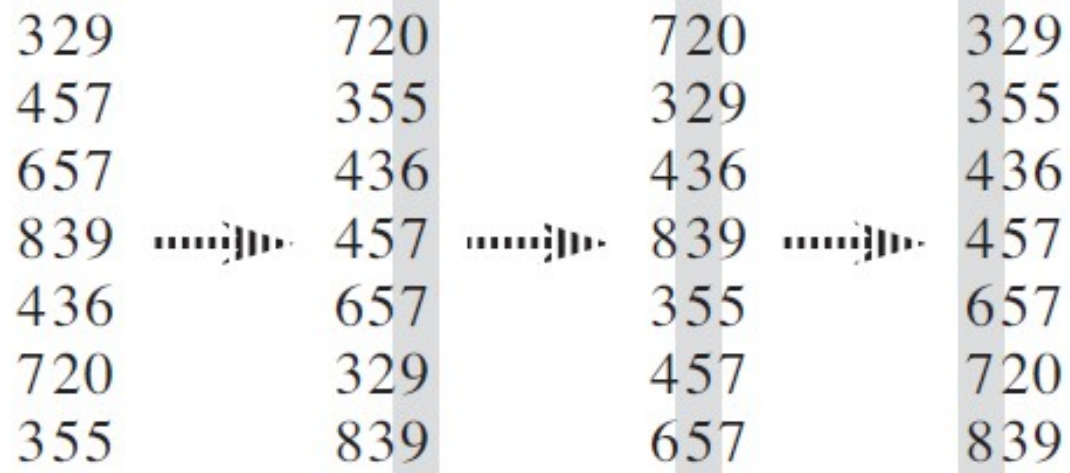
0	2	4	5	8	8
---	---	---	---	---	---

Counting Sort

COUNTING-SORT(A, B, k)

```
1  let  $C[0..k]$  be a new array
2  for  $i = 0$  to  $k$                                  $\Theta(k)$ 
3       $C[i] = 0$ 
4  for  $j = 1$  to  $A.length$ 
5       $C[A[j]] = C[A[j]] + 1$                          $\Theta(n)$ 
6  //  $C[i]$  now contains the number of elements equal to  $i$ .
7  for  $i = 1$  to  $k$ 
8       $C[i] = C[i] + C[i - 1]$                          $\Theta(k)$ 
9  //  $C[i]$  now contains the number of elements less than or equal to  $i$ .
10 for  $j = A.length$  downto 1
11      $B[C[A[j]]] = A[j]$                              $\Theta(n)$ 
12      $C[A[j]] = C[A[j]] - 1$ 
```

Radix Sort



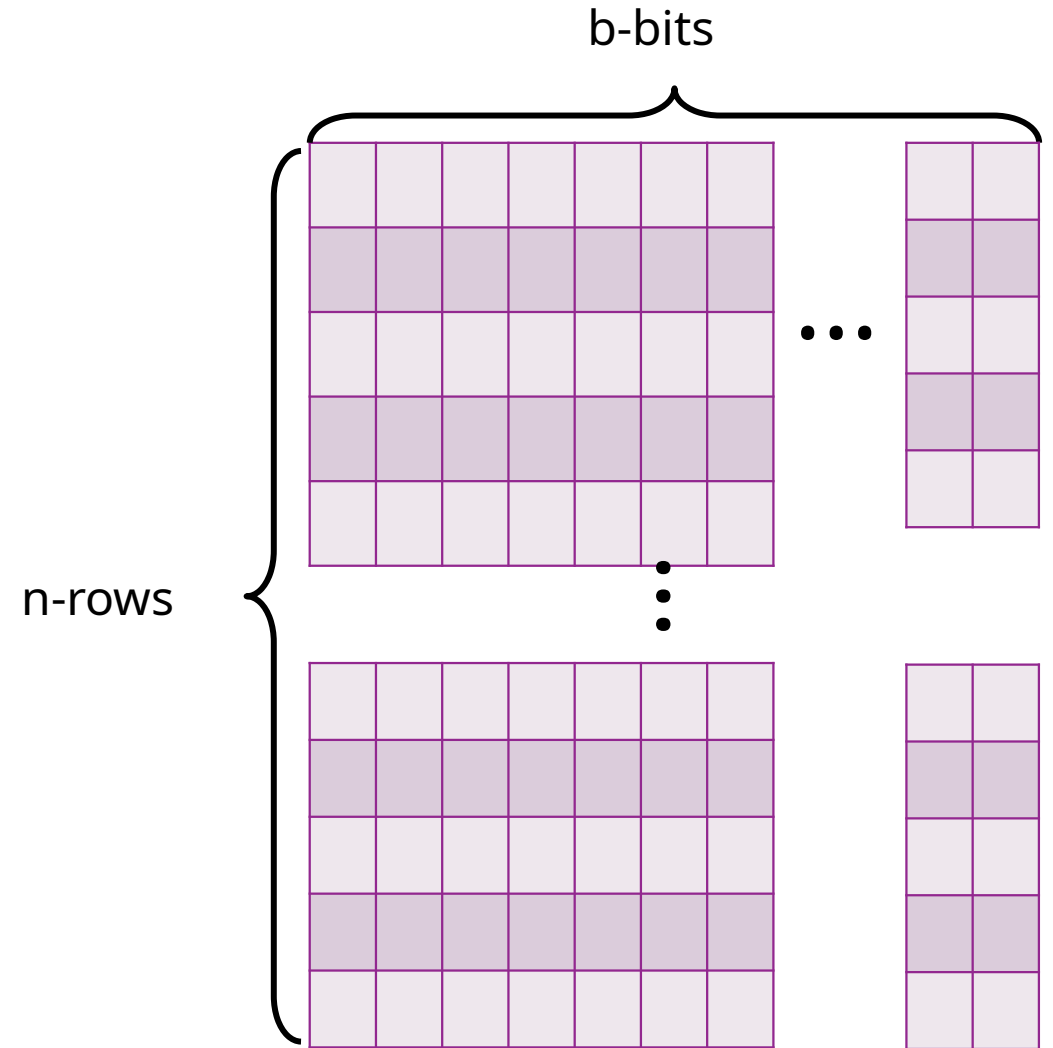
What would be if we start from MSD-to-LSD rather than LSD-to-MSD?

RADIX-SORT(A, d)

- 1 **for** $i = 1$ **to** d
- 2 use a stable sort to sort array A on digit i $\Theta(n+k)$

Radix Sort

- For a given n b -bit number, how we should divide numbers to r -bits?
- What would be complexity if we choose $r=b$ or $r=1$? What would be the optimal value for r ?



Radix Sort

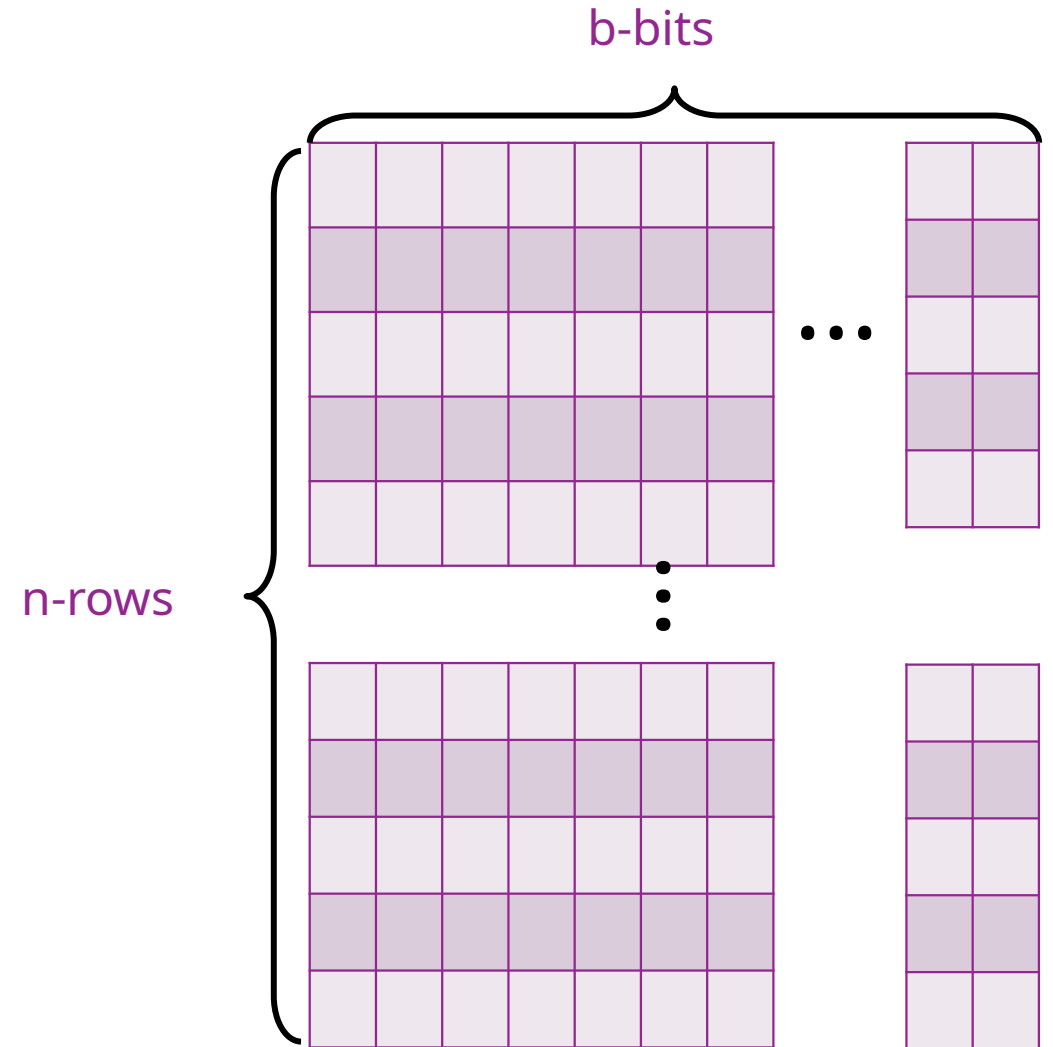
General formula for complexity becomes;

(

For $r=b$ ()

For $r=1$ (n)

For $r \approx \log(n)$ ()

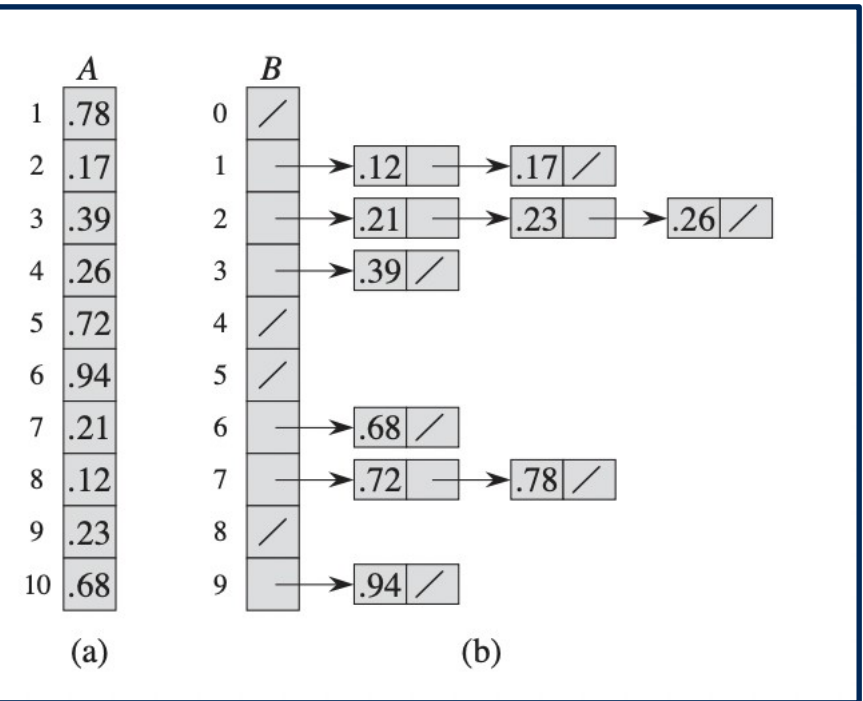


Bucket Sort

- Assumptions: Uniform distribution over $[0, 1)$

BUCKET-SORT(A)

```
1  let  $B[0 \dots n - 1]$  be a new array
2   $n = A.length$ 
3  for  $i = 0$  to  $n - 1$ 
4      make  $B[i]$  an empty list
5  for  $i = 1$  to  $n$ 
6      insert  $A[i]$  into list  $B[\lfloor nA[i] \rfloor]$ 
7  for  $i = 0$  to  $n - 1$ 
8      sort list  $B[i]$  with insertion sort
9  concatenate the lists  $B[0], B[1], \dots, B[n - 1]$  together in order
```



Bucket Sort

- Illustration of the BUCKET-SORT operation on the array
 - {0.79; 0.13; 0.16; 0.64; 0.39; 0.20; 0.89; 0.53; 0.71; 0.42}

IndexBucket

0	0		
1	0.1	0.13	0.16
2	0.2	0.20	
3	0.3	0.39	
4	0.4	0.42	
5	0.5	0.53	
6	0.6	0.64	
7	0.7	0.71	0.79
8	0.8	0.89	

- Step 1: Form sublists
- Step 2: Sort and combine sublists with insertion sort

Index of 0.13:
 $I = \text{Floor}(0.13 * 10)$
 $I = \text{Floor}(1.3)$
 $I = 1$

Bucket Sort

Explain why the worst-case running time for bucket sort is $\Theta(n^2)$. What simple change to the algorithm preserves its linear average-case running time and makes its worst-case running time $\Theta(n)$?

Bucket Sort

Explain why the worst-case running time for bucket sort is $\Theta(n^2)$. What simple change to the algorithm preserves its linear average-case running time and makes its worst-case running time $\Theta(n \log n)$?

- Worst Case: Bucket which contains all n values of the array.
- Since insertion sort has worst case running time $\Theta(n^2)$, so does Bucket sort.
- We can avoid this by using merge sort to sort each bucket instead, which has worst case running time $\Theta(n \log n)$.

Can we do better?