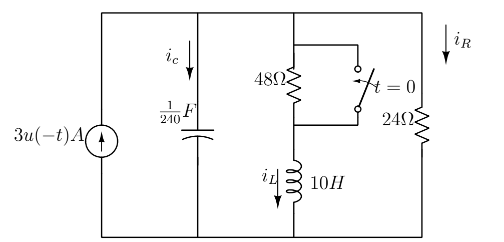
EE2015 Electric Circuits and Networks - Tutorial 6

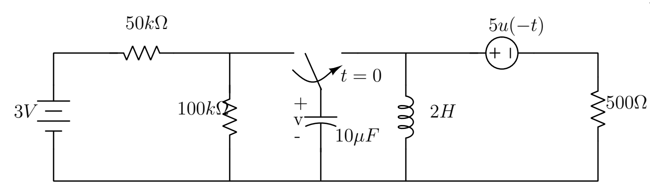
Sept. 20^{th}, 2024

1. All plots must be roughly to scale
2. Key x and y axis values must be marked
3. Time constant must be shown

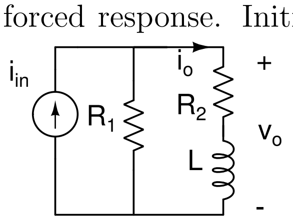
1. After being open for a long time, the switch in the network closes at $t = 0$. Find (a) $i_L(o^-)$ (b) $v_C(0^-)$ (c) $i_R(0^+)$ (d)$i_C(0^+)$

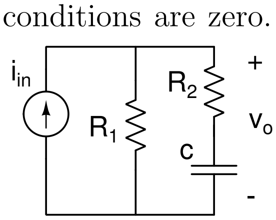


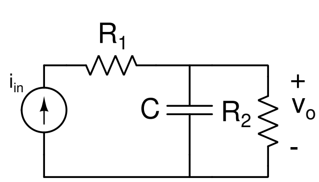
2. The switch is in the left position for a long time and is moved to the right at $t = 0$. Find $\frac{dv}{dt}$ at $t = 0^+$

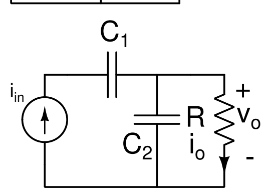


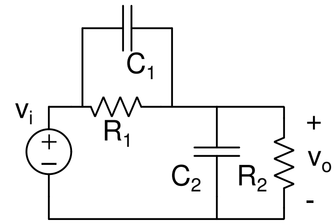
3. Evaluate and sketch the step response for the following all of the ten circuits using time domain techniques. Write the differential equation for each of the outputs shown in the figure and find the (a) Zero state and zero input response and (b) natural and forced response. Initial conditions are zero.

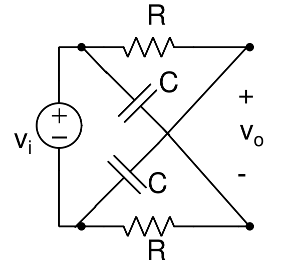


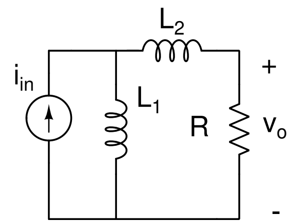


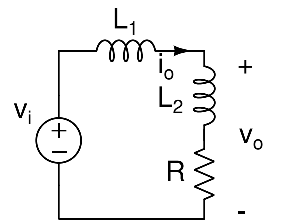


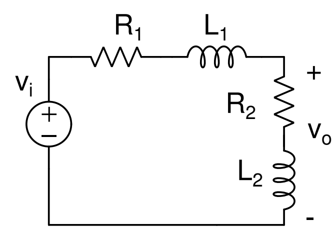


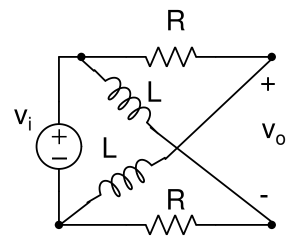












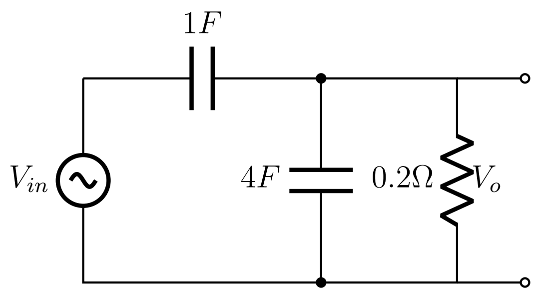
4. (a) For each of the ten circuits, find the impulse response by differentiating the step response

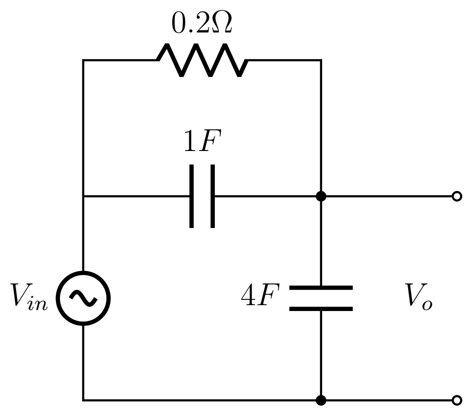
   (b) Verify your answer by finding the inverse Laplace transform of the transfer function.

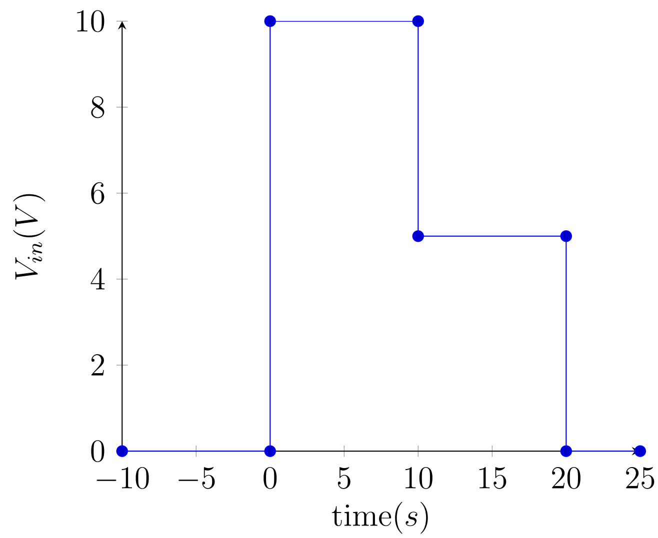
   (c) In each circuit, find the poles and zeros and plot it in the complex frequency plane.

5. In the above questions find the steady state response by open circuiting the capacitor/short circuiting inductors. How are the values obtained related to the $v_o(t)$ and $i_o(t)$ you calculated earlier.

6. A pulse with the following amplitude is applied to the two circuits given below at V_{in}. Find and plot V_o.

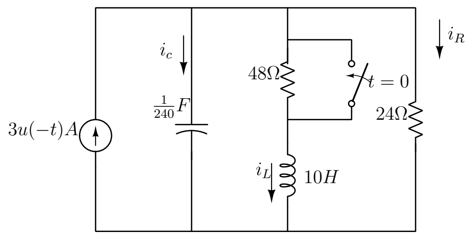

$1F$
V_{in}
$4F$
0.2Ω
V_o


0.2Ω
$1F$
V_{in}
$4F$
V_o

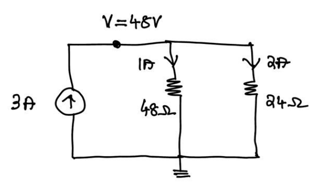

10
8
6
4
2
0
$V_{in}(V)$
−10
−5
0
5
10
15
20
25
time(s)

Solutions

1. After being open for a long time, the switch in the network closes at $t = 0$. Find (a) $i_L(o^-)$ (b) $v_C(0^-)$ (c) $i_R(0^+)$ (d)$i_C(0^+)$



$t < 0$

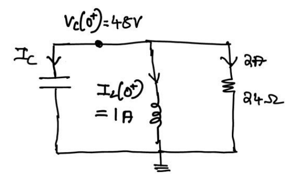


$V_C(0^-) = 48V$

$I_L(0^-) = 1A$

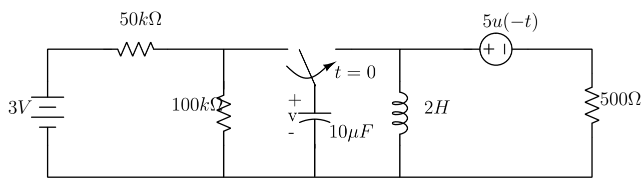
$I_R(0^-) = 2A$

$t = 0^+$

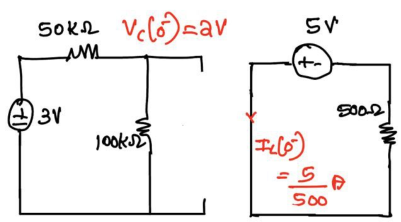


$I_c(0^+) = -1 - 2 = -3A$

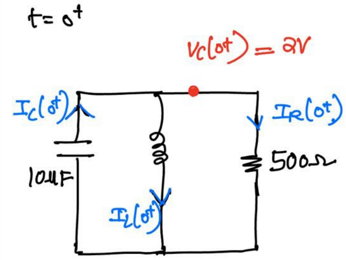
2. The switch is in the left position for a long time and is moved to the right at $t = 0$. Find $\frac{dv}{dt}$ at $t = 0^+$



$t < 0$



$t = 0^+$



$$\left.\frac{dv}{dt}\right|_{t=0^+} = \frac{I_c(0^+)}{c}$$

$$I_c(0^+) = \frac{I_L(0^+) + I_R(0^+)}{C}$$

$$= \frac{5/500 + 2/500}{10 \times 10^{-6}}$$

$$= \underline{\underline{1400\ V/s}}$$

3.

4.

5. For questions 3, 4, and 5, only the differential equation is provided. The other subparts can be solved from this.

   Circuit 1 :

$$i_{in} = i_0(1 + \frac{R_2}{R_1}) + \frac{L}{R_1}\frac{\mathrm{d}i_0}{\mathrm{d}t}$$
$$i_{in} = v_0\frac{R_1 + R_2}{R_1 R_2} + \frac{L}{R_1 R_2}\frac{\mathrm{d}v_0}{\mathrm{d}t}$$

   Circuit 2 :

$$i_{in} + R_2 C\frac{\mathrm{d}i_{in}}{\mathrm{d}t} = v_0 R_1 + \frac{R_1 + R_2}{R_1}C\frac{\mathrm{d}v_0}{\mathrm{d}t}$$

   Circuit 3 :

$$i_{in} = \frac{v_0}{R_2} + C\frac{\mathrm{d}v_0}{\mathrm{d}t}$$

   Circuit 4 :

$$i_{in} = \frac{v_0}{R} + C_2\frac{\mathrm{d}v_0}{\mathrm{d}t}$$

   Circuit 5:

$$(C_1 + C_2)\frac{\mathrm{d}V_o}{\mathrm{d}t} + V_o\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{V_i}{R_1} + C_1\frac{\mathrm{d}V_i}{\mathrm{d}t}$$

   Circuit 6:

$$C\frac{\mathrm{d}V_o}{\mathrm{d}t} + \frac{V_o}{R} = \frac{V_i}{R} - C\frac{\mathrm{d}V_i}{\mathrm{d}t}$$

   Circuit 7: Get using mesh analysis

$$(L_1 + L_2)\frac{\mathrm{d}i_{L_2}}{\mathrm{d}t} + Ri_{L_2} = L_1\frac{\mathrm{d}i_{in}}{\mathrm{d}t}$$

$$\frac{(L_1 + L_2)}{R}\frac{\mathrm{d}V_0}{\mathrm{d}t} + V_o = L_1\frac{\mathrm{d}i_{in}}{\mathrm{d}t}$$

   Circuit 8: Let the loop current be i_o.

$$(L_1 + L_2)\frac{\mathrm{d}i_o}{\mathrm{d}t} + i_o R = V_i$$

   Once we've obtained the loop current i_o,

$$V_o = L_2\frac{\mathrm{d}i_o}{\mathrm{d}t} + i_o R$$

   Circuit 9:

Let the loop current be i_o.

$$-V_i + i_o(R_1 + R_2) + (L_1 + L_2)\frac{\mathrm{d}i_o}{\mathrm{d}t}V_o = i_0 R_2 + L_2\frac{\mathrm{d}i_o}{\mathrm{d}t}$$

Once we've obtained the loop current i_o,

$$V_o = L_2\frac{\mathrm{d}i_o}{\mathrm{d}t} + i_o R_2$$

Circuit 10:

First note that the currents in both of the arms will be equal since the impedances are equal. Now, let this current be $i_o(t)$.

$$V_{in} = Ri + L\frac{\mathrm{d}i}{\mathrm{d}t}$$

Once we've obtained the loop current i_o,

$$V_o = L\frac{\mathrm{d}i}{\mathrm{d}t} - iR$$

6. **Part (a):**

Let $C_1 = 1F$, $C_2 = 4F$

Using KCL, observe down the following differential equation:

$C_1\frac{d(V_o - V_{in})}{dt} + C_2\frac{dV_o}{dt} + \frac{V_0}{R} = 0.$

$C_1\frac{d(V_{in})}{dt} = (C_1 + C_2)\frac{d(V_o)}{dt} + \frac{V_0}{R}$

$\frac{d(V_o)}{dt} + \frac{V_0}{R(C_1+C_2)} = \frac{C_1}{(C_1+C_2)}\frac{d(V_{in})}{dt}$

This gives us the time constant, $\tau = R(C_1 + C_2) = 1s$.

Solving a specific case:

Now, assume that our input is $Au(t)$.

This gives us $\frac{d(V_o)}{dt} + \frac{V_o}{R(C_1+C_2)} = \frac{C_1}{(C_1+C_2)}A\delta(t)$

For this, the corresponding response is $V_o(t) = \frac{C_1}{(C_1+C_2)}Ae^{-\frac{t}{\tau}}u(t)$.

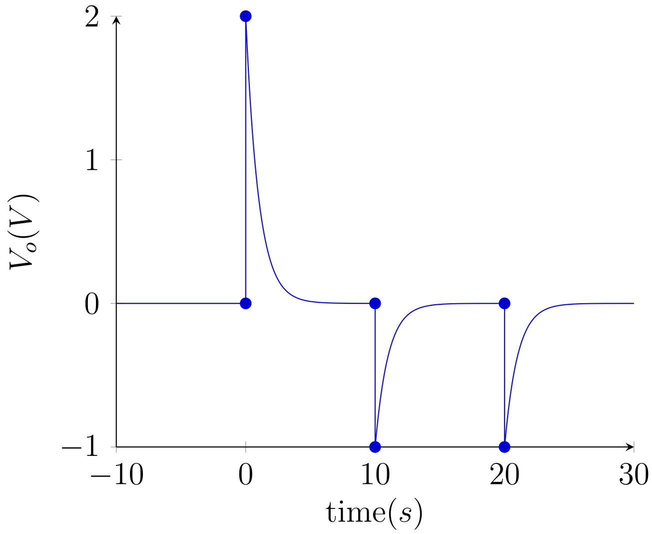
Now, the input $V_{in}(t) = 10u(t) - 5u(t-10) - 5u(t-20)$.

Using linearity and superposition,

$V_o(t) = \frac{C_1}{(C_1+C_2)}(10e^{-\frac{t}{\tau}}u(t) - 5e^{-\frac{t-10}{\tau}}u(t-10) - 5e^{-\frac{t-20}{\tau}}u(t-20))$.

Plugging in the values of C_1 and C_2, this is equal to:

$V_o(t) = 2e^{-\frac{t}{1}}u(t) - 1e^{-\frac{t-10}{1}}u(t-10) - 1e^{-\frac{t-20}{1}}u(t-20)$



Part (b):

Note that the time constant is still the same (this can be verified by writing the differential equations).

Note that in the first case, the voltage drop across the C_1 capacitor is just equal to

$V_o(t) = (10 - 10\frac{C_1}{C_1+C_2}e^{-\frac{t}{1}})u(t) + (-5 + 5\frac{C_1}{C_1+C_2}e^{-\frac{t-10}{1}})u(t-10) + (-5 + 5\frac{C_1}{C_1+C_2}e^{-\frac{t-20}{1}})u(t-20)$

The second part of the question is essentially the same circuit, with $C_1 = 4F$ and $C_2 = 1F$.

Plugging in the values, we get

$V_o(t) = (10 - 8e^{-\frac{t}{1}})u(t) + (-5 + 4e^{-\frac{t-10}{1}})u(t-10) + (-5 + 4e^{-\frac{t-20}{1}})u(t-20)$