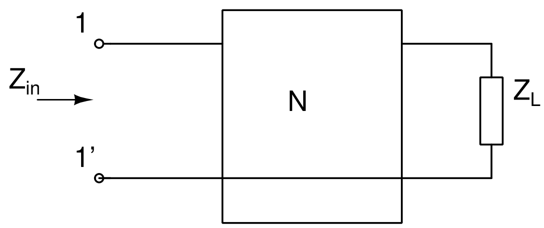
EE2015: Electric Circuits and Networks

Tutorial 5

(September 13, 2024)

1. Let the open circuit impedance matrix of network N be

$$\begin{bmatrix} 0 & -r \\ r & 0 \end{bmatrix}.$$



Find $Z_{in}(s)$ in terms of r and $Z_L(s)$. If $Z_L(s)$ is $1/sC$, show that the network behaves like an inductor at the input terminals.

Solution: From the z matrix $z_{11} = 0$, $z_{12} =$ -r, $z_{21} =$ r and $z_{22} = 0$ and assume V_1 and I_1 at input port and V_2 and I_2 at output port.

which implies,

$$V_1 = Z_{11}I_1 + Z_{12}I_2 = -rI_2 \tag{1}$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2 = rI_1 \tag{2}$$

and from circuit,

$$V_2 = -I_2 Z_L, \tag{3}$$

by solving 1, 2, 3 gives

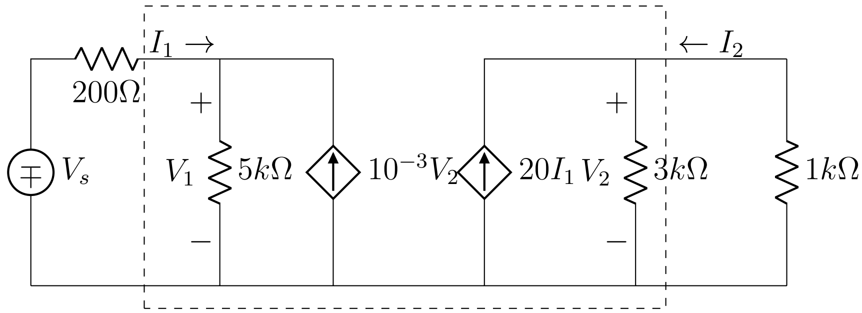
$$Z_{\text{in}} = \frac{V_1}{I_1} = \frac{r^2}{Z_L},$$

and if $Z_L = \frac{1}{sC}$, then

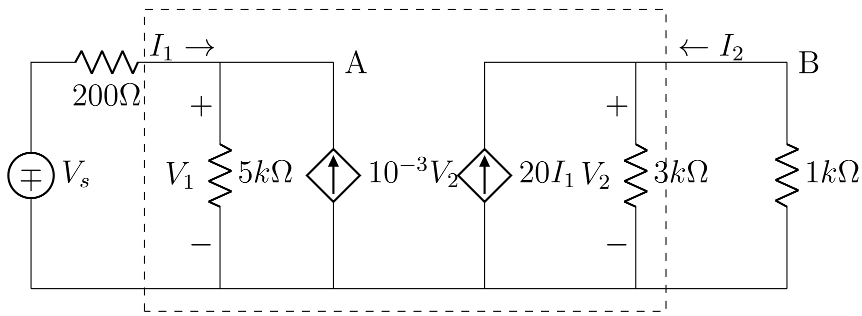
$$Z_{\text{in}} = r^2 Cs = L's,$$

where $L' = r^2 C$.

2. Find the Z parameters for the 2 port network marked with dotted lines and then find the Z_{out} (impedance seen by the $1k\Omega$ resistor) for the terminated two port network.



Solution:



From the circuit, applying KCL at points A and B gives the following equations:

$$-I_1 - 10^{-3}V_2 + \frac{V_1}{5k\Omega} = 0$$
$$-20I_1 - I_2 + \frac{V_2}{3k\Omega} = 0$$

Rearranging these equations yields:

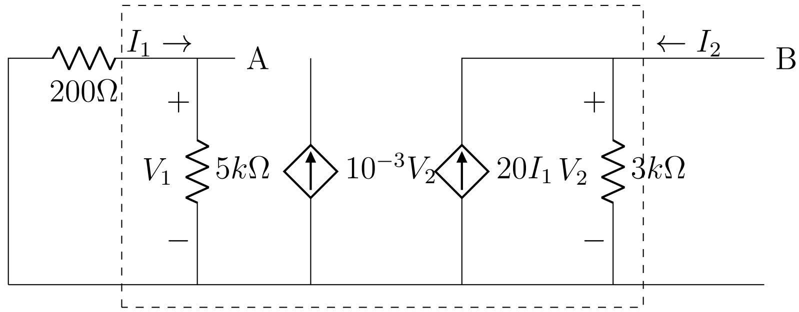
$$V_1 = 305k\Omega I_1 + 15k\Omega I_2$$
$$V_2 = 60k\Omega I_1 + 3k\Omega I_2$$

Thus, the impedance matrix Z is:

$$Z = \begin{bmatrix} 305k & 15k \\ 60k & 3k \end{bmatrix}$$

For Z_{out}, when $V_s = 0$, we have:

$$Z_{out} = \frac{V_2}{I_2}$$



From circuit,

$$V_1 = -200I_1$$

$$V_1 = 305k\Omega I_1 + 15k\Omega I_2$$

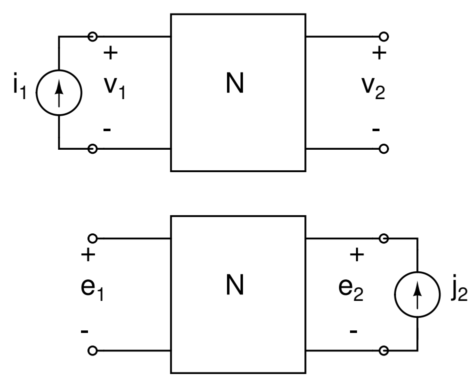
$$V_2 = 60k\Omega I_1 + 3k\Omega I_2$$

On solving all three $Z_{out} = 51.11\Omega$

3. The two port network N is an RLC network. If $i_1(t) = \delta(t)$, $v_2(t) = (3e^{-t} + 5e^{-t})u(t)$.

   a. If $j_2(t) = u(t)$, find $e_1(t)$

   b. If $j_2(t) = \cos 500tu(t)$, find the response $e_1(t)$.

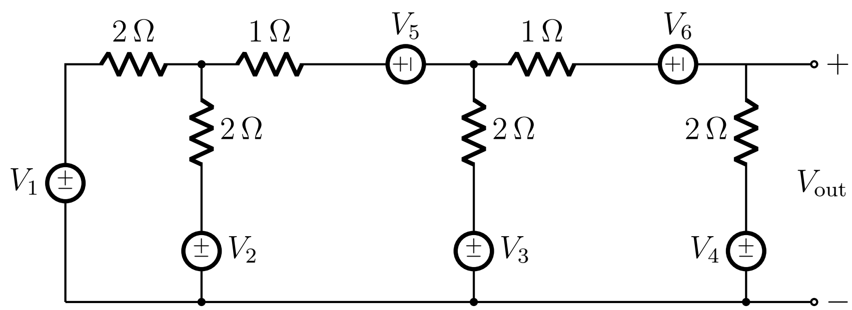


Solution: This can be solved by reciprocity,

For a), according to reciprocity If $i_1(t) = \delta(t)$, $v_2(t) = (8e^{-t})u(t)$, then if $j_2(t) = \delta(t)$, $e_1(t) = (8e^{-t})u(t)$ it implies $j_2(s) = 1$, $e_1(s) = \frac{8}{s+1}$ and according to linearity, $j_2(s) = \frac{1}{s}$, $e_1(s) = \frac{8}{s(s+1)}$. and $e_1(t) = (8 - 8e^{-t})u(t)$

For b), according to reciprocity If $i_1(t) = \delta(t)$, $v_2(t) = (8e^{-t})u(t)$, then if $j_2(t) = \delta(t)$, $e_1(t) = (8e^{-t})u(t)$ it implies $j_2(s) = 1$, $e_1(s) = \frac{8}{s+1}$ and according to linearity, $j_2(s) = \frac{s}{s^2+500^2}$, $e_1(s) = \frac{8s}{(s^2+500^2)(s+1)}$.

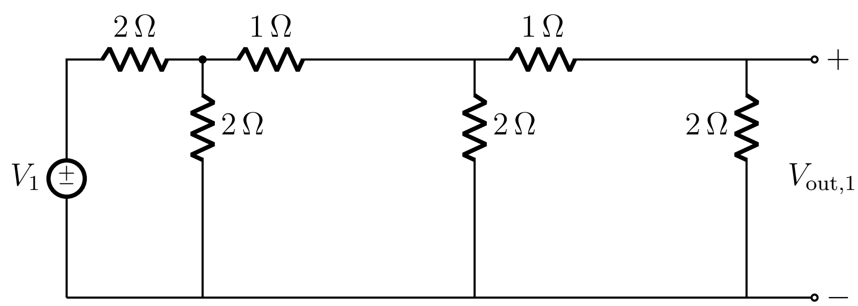
4. For the circuit given below, use reciprocity and superposition to determine V_{out} in terms of V_k, $k = 1, 2, \ldots, 6$.



Solution:

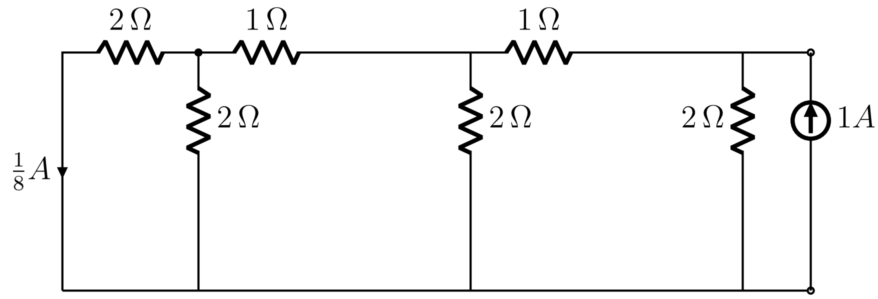
First, let us use superposition. The response we get at V_{out} will be the superposition of the responses due to each of the V_i's.

Consider $V_{\text{out},1}$, the response at V_{out} due to only V_1, in the below circuit:



Now, let us invoke reciprocity across the V_1 port (the 'input') and the $V_{\text{out},1}$ port (the 'output'). We are allowed to do this since it is a purely resistive network.

Since we're using reciprocity, connect a $1A$ source across $V_{\text{out, 1}}$, and observe what response we get at the other port (where V_1 was initially).



This current ends up being $\frac{1}{8}A$.

Now, for a $1A$ excitation we provided at the output port, we received a $\frac{1}{8}$ response at the input port. Therefore, for a V_1 excitation we provide at the input port, we will receive a $\frac{V_1}{8}$ response at the output port. Therefore, $V_{\text{out},1} = \frac{V_1}{8}$.

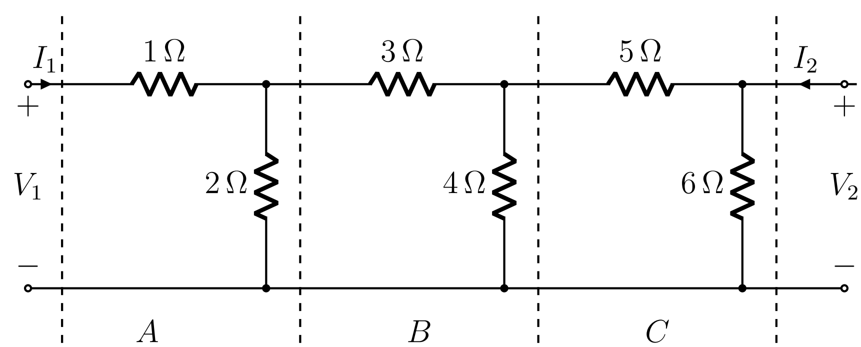
We can proceed the same way for all of the other voltage sources. Even though we solve the circuit only once (in the above figure), we get all the branch currents and hence we can apply reciprocity in one shot.

Based on the above discussion, we get:

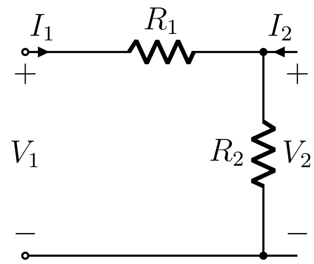
$V_{\text{out, 2}} = \frac{V_2}{8}$, $V_{\text{out, 3}} = \frac{V_3}{4}$, $V_{\text{out, 4}} = \frac{V_4}{2}$, $V_{\text{out, 5}} = -\frac{V_5}{4}$, $V_{\text{out, 6}} = -\frac{V_6}{2}$.

Using superposition, we get $V_{\text{out}} = \sum_{i=1}^{6} V_{\text{out, i}} = \boxed{\frac{V_1}{8} + \frac{V_2}{8} + \frac{V_3}{4} + \frac{V_4}{2} - \frac{V_5}{4} - \frac{V_6}{2}}$.

5. The two-port network shown below can be viewed as a cascaded combination of three two-port networks A, B, and C.
   (a) Find the transmission parameters of each network.
   (b) Find the transmission parameters of the overall network.



Solution:



This question can be solved by a knowledge of cascading of two-port networks. Firstly, consider the general two-port as shown below, which for appropriate values of R_1 and R_2 can be used to represent the two-port networks A, B, and C.

To find the T matrix, first set $I_2 = 0$.

We get $V_1 = \frac{R_1+R_2}{R_2}V_2$ and $I_1 = \frac{V_2}{R_2}$

Now set $V_2 = 0$.

We get $V_1 = -R_1 \cdot I_2$ and $I_1 = -I_2$

Based on this, we get the T matrix to be:

$$\begin{bmatrix} 1+\frac{R_1}{R_2} & R_1 \\ \frac{1}{R_2} & 1 \end{bmatrix}$$

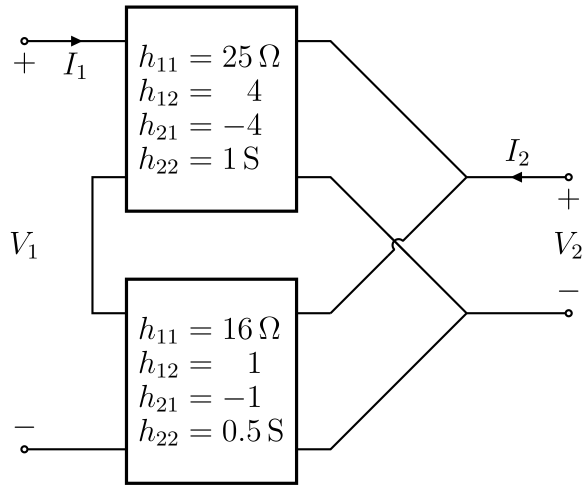
Using this, we can compute the T matrices of each of the parts A, B, and C:

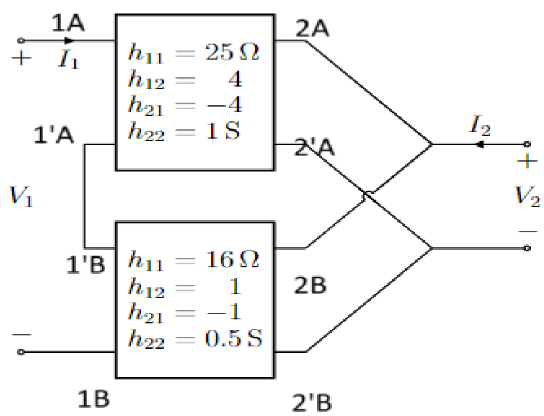
$$T_A = \begin{bmatrix} 1.5 & 1 \\ 0.5 & 1 \end{bmatrix}, T_B = \begin{bmatrix} 1.75 & 3 \\ 0.25 & 1 \end{bmatrix}, T_C = \begin{bmatrix} 1.84 & 5 \\ 0.16 & 1 \end{bmatrix}$$

Using cascading of transfer function, the net transfer function for the cascade is simply the product of the T-matrices of the individual parts,

$$T_{eff} = T_A \cdot T_B \cdot T_C = \begin{bmatrix} 1.5 & 1 \\ 0.5 & 1 \end{bmatrix} \begin{bmatrix} 1.75 & 3 \\ 0.25 & 1 \end{bmatrix} \begin{bmatrix} 1.84 & 5 \\ 0.16 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 6.17 & 19.875 \\ 2.47 & 8.125 \end{bmatrix}}$$

6. A series-parallel connection of two two-port networks is shown on the right. Terminals 1 and 1' and 2 and 2' are connected. Determine the z-parameters of the overall network.





Solution:

Let the two-port network on the top be A, and the network below be B. It can be observed that,

$I_{1A} = -I_{1B} = I_1$
$I_{2A} + I_{2B} = I_2$
$V_{1A} - V_{1B} = V_1$
$V_{2A} = V_{2B} = V_2$

Using the h-parameters,
$V_{1A} = h_{11,A}I_1 + h_{12,A}V_2$
$I_{2A} = h_{21,A}I_1 + h_{22,A}V_2$
$V_{1B} = -h_{11,B}I_1 + h_{12,B}V_2$
$I_{2B} = -h_{21,B}I_1 + h_{22,B}V_2$

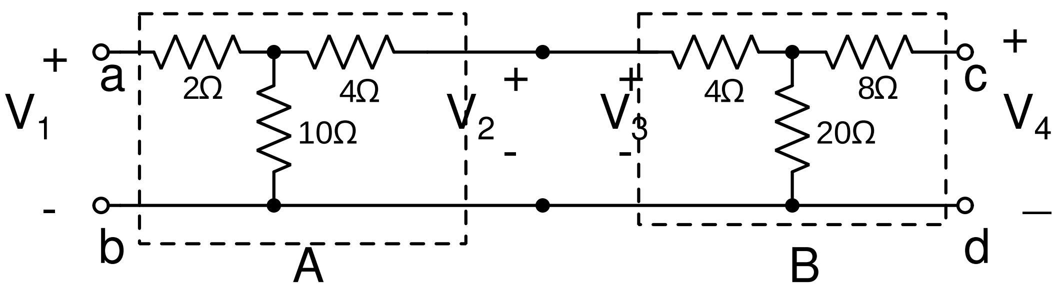
Hence, we get h-parameters as,

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11,A} + h_{11,B} & h_{12,A} - h_{12,B} \\ h_{21,A} - h_{21,B} & h_{22,A} + h_{22,B} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

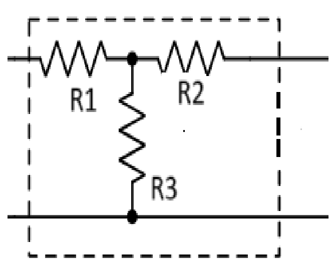
$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 41 & 3 \\ -3 & 1.5 \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Rearrange the equations in terms of I_1 and I_2 to get the z-parameters as
$z = \begin{bmatrix} 47 & 2 \\ 2 & 0.667 \end{bmatrix}$

7. In the circuit below, find the **z** parameters individually for the networks A and B. Now, if a voltage source V_s is connected between terminals a and b and a load resistor $R_L = 10\Omega$ is connected between terminals c and d, find the voltage gain $\frac{V_4}{V_1}$ using the **z** parameters of the networks A and B.



Solution:



z-parameters of the above T-network is $\begin{bmatrix} R_1 + R_3 & R_3 \\ R_3 & R_2 + R_3 \end{bmatrix}$.

We get z-parameters of network A as $\begin{bmatrix} 12 & 10 \\ 10 & 14 \end{bmatrix}$. Similarly, for B we get z-parameters as $\begin{bmatrix} 24 & 20 \\ 20 & 24 \end{bmatrix}$.

Also, we get $I_2 = -I_3$ and $V_2 = V_3$. When a load resistor of 10Ω is connected across c and d, we get $V_4 = -10I_4$.
Using the z-parameters of B, we get $V_3 = V_2 = 2.56V_4$ and $I_3 = -I_2 = 0.19V_4$.
Now using z-parameters of A, we get $I_1 = 0.522V_4$ and $V_1 = 4.364V_4$.
The voltage gain $\frac{V_4}{V_1} = 0.229$.