EE3110 - Probability Foundations for Electrical Engineers
Tutorial - Week 2

Please submit solutions to the 2 starred questions in moodle for assignment submission by **Aug 30, 11:59 PM**.

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1. A batch of one hundred items is inspected by testing four randomly selected items. If one of the four is defective, the batch is rejected. What is the probability that the batch is accepted if it contains five defectives?

2.* *The prisoner's dilemma*: The release of two out of three prisoners has been announced, but their identity is kept secret. One of the prisoners considers asking a friendly guard to tell him who is the prisoner other than himself that will be released, but hesitates based on the following rationale: at the prisoner's present state of knowledge, the probability of being released is $\frac{2}{3}$, but after he knows the answer, the probability of being released will become $\frac{1}{2}$, since there will be two prisoners(including himself) whose fate is unknown and exactly one of the two will be released. What is wrong with this line of reasoning?

3. A coin is tossed twice. Shipra claims that the event of two heads is at least as likely if we know that the first toss is a head than if we know that at least one of the tosses is a head. Is she right? Does it make a difference if the coin is fair or unfair? How can we generalize Shipra's reasoning?

4. Let A and B be events with $\mathbb{P}(A) > 0$ and $\mathbb{P}(B) > 0$. We say that an event B suggests an event A if $\mathbb{P}(A|B) > \mathbb{P}(A)$, and does not suggest event A if $\mathbb{P}(A|B) < \mathbb{P}(A)$.
   1. Show that B suggests A if and only if A suggests B.
   2. Show that B suggests A if and only if B^c does not suggest A. Assume that $\mathbb{P}(B^c) > 0$.
   3. We know that a treasure is located in one of two places, with probabilities β and $1-\beta$, respectively, where $0 < \beta < 1$. We search the first place and if the treasure is there, we find it with probability $p > 0$. Show that the event of not finding the treasure in the first place suggests that the treasure is in the second place.

5. Alice searches for her term paper in her filing cabinet. which has several drawers. She knows that she left her term paper in drawer j with probability $p_j > 0$. The drawers are so messy that even if she correctly guesses that the term paper is in drawer i. the probability that she finds it is only d_i. Alice searches in a particular drawer. say drawer i. but the search is unsuccessful. Conditioned on this event, show that the probability that her paper is in drawer j, is given by
$$\frac{p_j}{1-p_i d_i}. \quad \text{if } j \neq i. \quad \frac{p_i\,(1-d_i)}{1-p_i d_i}, \quad \text{if } j = i$$

6. Consider a coin that comes up heads with probability p and tails with probability $1-p$. Let q_n be the probability that after n independent tosses, there have been an even number of heads. Derive a recursion that relates q_n to q_{n-1}, and solve this recursion to establish the formula
$$q_n = \left(1 + (1-2p)^n\right)/2$$

7. We roll two fair 6-sided dice. Each one of the 36 possible outcomes is assumed to be equally likely.
   (a) Find the probability that doubles are rolled.
   (b) Given that the roll results in a sum of 4 or less, find the conditional probability that doubles are rolled.
   (c) Find the probability that at least one die roll is a 6.
   (d) Given that the two dice land on different numbers, find the conditional probability that at least one die roll is a 6.

8. *False positives*: A rare disease affects one person in 10^5. A test for the disease shows positive with probability $\frac{99}{100}$ when applied to an ill person, and with probability $\frac{1}{100}$ when applied to a healthy person. What is the probability that you have the disease given that the test shows positive?

9. Suppose that a family has exactly n children ($n \geq 2$). Assume that the probability that any child will be a girl is $\frac{1}{2}$ and that all births are independent. Given that the family has at least one girl, determine the probability that the family has at least one boy.

10. Suppose that a fair coin is tossed until a head is obtained and that this entire experiment is then performed independently a second time. What is the probability that the second experiment requires more tosses than the first experiment?

11. For events A, B and C with $C \subseteq B$ and $P(A)P(B)P(C) > 0$, $P(A|B) = \alpha P(A|B \cap C) + (1-\alpha)P(A|B \cap C^c)$. Find α in terms of $P(A), P(B)$ and $P(C)$.

12. There are N urns of which the rth urn contains $r-1$ red balls and $N-r$ blue balls. You pick an urn at random and remove two balls at random without replacement. Find the probability that:
    (a) the second ball is blue.
    (b) the second ball is blue, given that the first is blue.

13. **Communication through a noisy channel:** A source transmits a string of symbols through a noisy channel. Each symbol is 0 or 1 with probability $1-p$ and p, respectively. The transmitted symbols 0 and 1 are correctly received over the channel with probability $1-\epsilon_0$ and $1-\epsilon_1$, respectively. Assume that the errors in different symbol transmissions are independent.
    1. What is the probability that the k^{th} symbol is received correctly?
    2. What is the probability that the string of symbols 1101 is received correctly?
    3. Suppose you transmit every symbol three times. Then, what is the probability that the k^{th} symbol is received correctly if the receiver uses the majority decision to decode the symbol?

14. If an aircraft is present in a certain area, a radar detects it and generates an alarm signal with probability 0.99.
    If an aircraft is not present, the radar generates a **false** alarm, with probability 0.10.

1. Assume that aircraft is present with probability 0.165. Find the probability of no aircraft presence and a false alarm. Also, the probability of aircraft presence and no detection was found.
2. Repeat the above question when the aircraft is present with probability 0.35.
3. Find the efficiency of Radar in both cases.
   (**Hint:** Efficiency is defined as the probability that an aircraft is present given radar generates an alarm)

15. Suppose the test for HIV is 99% accurate in both directions and 0.3% of the population is HIV positive. If someone tests positive, what is the probability they actually are HIV positive? Justify why you are getting a low probability even though the test appears to be very accurate.

16. Suppose you play the game by tossing a fair coin repeatedly and independently. If it comes up heads, you win a dollar, and if it comes up tails, you lose a dollar. Suppose you start with \$50. What's the probability you will get to \$200 without first getting ruined (running out of money)?

17.* A large class in probability theory is taking a multiple-choice test. For a particular question on the test, the fraction of examinees who know the answer is p; $1 - p$ is the fraction that will guess. The probability of answering a question correctly is unity for an examinee who knows the answer and $1/m$ for a guesser; m is the number of multiple choice alternatives. Compute the probability that an examinee knew the answer to a question, given that he or she has answered it correctly.