EE2025 Engineering Eelectromagnetics
Tutorial 3 (RKS 5.0 to 5.7)

Part A

1. The electric field of a plane wave in a non-magnetic, charge-free medium is given by

$$\vec{E} = (a\hat{x} - 2\hat{y} + 5\hat{z})e^{j(\omega t - 2x - 10y + 6z)} \text{ [V/m]}$$

(a) Write down the wave vector $\vec{k}$ of the plane wave.

(b) Calculate the unknown constant a.

(c) The corresponding magnetic field $\vec{H}$.

(d) From the $\vec{H}$ field, prove that the angle between $\vec{E}$ and $\vec{H}$ fields is equal to 90^o.

Consider $f = 1$ GHz and $\epsilon_r = 2.5$.

2. A light beam is incident from air to a medium with a dielectric constant of 4 and relative permeability of 100. If the angle of incidence is 60°, find the angle of reflection and angle of refraction.

3. A light beam is incident from air to a medium with a dielectric constant ϵ and relative permeability μ at an angle to the interface. Prove that the incident ray, reflected ray and transmitted ray lie on the same plane of incidence.

4. A uniform plane wave is incident from the air onto glass at an angle from the normal of 30°. Determine the fraction of the incident power reflected and transmitted for

(a) parallel polarization and

(b) perpendicular polarization.

Glass has a refractive index $n_2 = 1.45$.

5. The electric field of a uniform plane in the air is given by

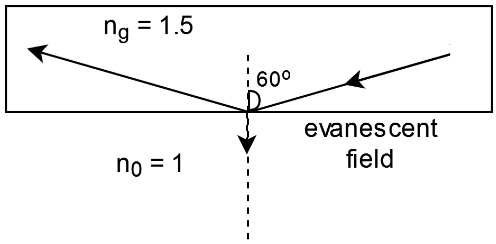
$$\vec{E}_i = 100\cos(2\pi f t - 3x - 4z)\hat{y} \text{ [V/m]},$$

is incident on a dielectric slab ($z \geq 0$) with $\mu_r = 1, \epsilon_r = 3$ and $\sigma = 0$, at an angle θ_i.

(i) Sketch the dielectric interface with corresponding orientations of electric field and magnetic fields for incident $(\vec{E}_i, \vec{H}_i)$, reflected $(\vec{E}_r, \vec{H}_r)$ and transmitted wave $(\vec{E}_t, \vec{H}_t)$ and find the polarization of the wave.

(ii) Calculate the frequency f, angle of incidence (θ_i), reflection (θ_r) and transmission (θ_t).

6. A beam of light from an argon laser ($\lambda = 500$ nm) traveling in a glass block ($n_g = 1.5$) is totally internally reflected at the flat air–glass interface. If the beam strikes the interface at 60° to the normal, how deep will the light penetrate the air before its amplitude drops to about 36.8% of its value at the interface?



7. A left-hand circularly polarized wave impinges at an interface between two different media with an angle of 45°. Determine the state of polarisation of the reflected and transmitted wave:

   1. If the wave travels from air to a perfect conductor.
   2. If the wave travels from air to a dielectric medium (non-magnetic) of $\epsilon_r = 2.5$.

8. A right circularly polarised plane wave in air is incident at Brewster's angle on a semi-infinite slab of plexiglass ($\epsilon_r = 3.45$). Determine the fraction of incident power that is reflected and transmitted at the interface.

Part B

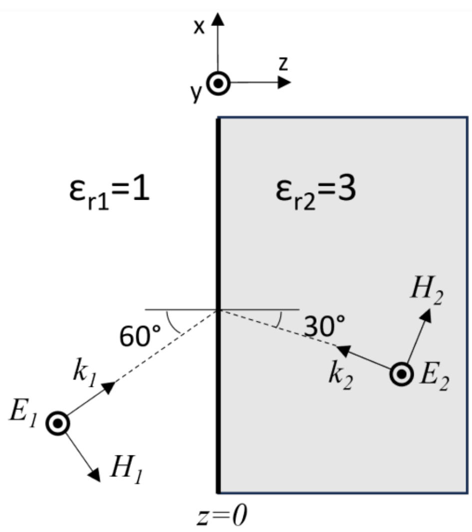
9. In question 5, calculate the reflected electric field ($\vec{E}_r$) and transmitted magnetic field ($\vec{H}_t$).

10. Consider two perpendicularly polarized plane waves, of the same frequency ω, incident from either side of the dielectric ($\sigma = 0$ and $\mu_r = 1$) interface as shown in the figure. The electric field of the two incident waves are out of phase as in the following expressions.

$$\vec{E}_1 = E_0 e^{-j\mathbf{k_1}\cdot\mathbf{r}}\hat{y}$$

$$\vec{E}_2 = -E_0 e^{-j\mathbf{k_2}\cdot\mathbf{r}}\hat{y}$$

Find the net electric field in the region $z > 0$ after interaction with the dielectric interface.



11. Given three materials fiberglass ($n = 1.6$), rain erosion paint ($n = 1.6$, thickness=$\lambda_3/16$) and primer ($n = 2.56$, thickness=$\lambda_2/2$). Where n is the relative refractive index of the material.

    Design a cascade of these three materials (propose an arrangement and the thickness of the fiberglass), such that overall transmission coefficient for a normally incident wave is unity. The wave propagates through the air ($n = 1$) into the cascade and then leaves back into the air. Suppose now you use this configuration for a radome which must transmit at least 95% of the incident signal power. Find the value of n for the atmosphere (n of atmosphere varies with height). Assume that the air between the aircraft antenna and the radome walls has $n = 1$ always, i.e air at sea level.

12. Light from a red LED (free space wavelength = 650 nm) is incident normally on an optical sensor embedded in a thick glass slab with relative permeability of 3/8 and dielectric constant of 8.

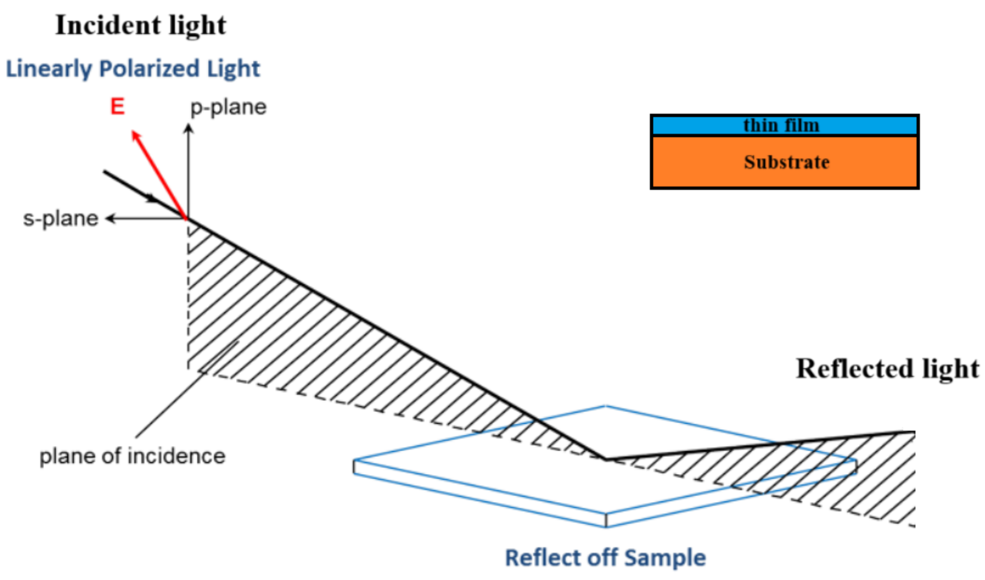
    a. What color of light in glass would the sensor detect?

    b. What percentage of incident power is reflected at the glass interface?

    c. To eliminate reflections, the glass is coated with a material of dielectric constant of 2, and thickness of 400 nm. Do you think this coating would act as an anti-reflection coating? If yes, justify your answer. If not, suggest an alternative coating (thickness and relative permittivity) to replace this coating.

    d. If the red LED is replaced by a green LED (525 nm) in the above question, quantify the reflection coefficient.

13. At the CNNP lab in IIT Madras, a spectroscopic ellipsometer is used to characterize thin films by measuring the polarization state of light after reflection from the sample. In this tool, a randomly polarized light from the light source is converted into linearly polarized light at 45°, with respect to the plane of incidence, before being directed onto the sample (see figure). The reflected light's polarisation state is measured using a rotating analyzer and a detector.



(a) Determine the typical polarization state of the reflected light if the angle of incidence ranges between 55° and 75°. Explain how the reflection affects the polarization state of the light.

(b) The primary data in an ellipsometer are the amplitude ratio (ψ) and phase difference (Δ) between parallel and perpendicular components of the reflected light. How can these raw data be used to determine the thickness and refractive index of the film?