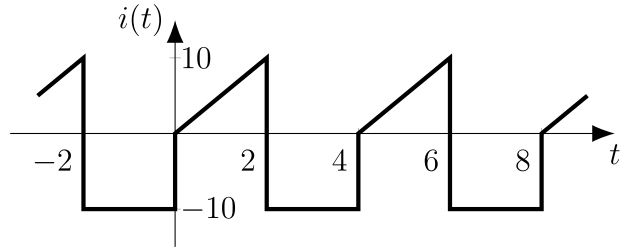
EE2015: Electric Circuits and Networks

Tutorial 9

(October 18, 2024)

1. Consider the periodic signal shown on the right, which represents the current passing through a $10\,\Omega$ resistor.



   (a) Find the RMS value of $i(t)$.

   (b) Find the average power absorbed by the resistor.

   (c) Find the value of the energy dissipated in the resistor over one cycle.

Solution:

(a) RMS value of i(t) = 8.165 V.

(b) Average power dissipated by a 10Ω resistor = $i^2_{rms} \times 10 = 666.67W$.

(c) Energy dissipated by resistor in one cycle = Average power dissipated by the resistor × Time period = $2.67kJ$.

2. (a) A given circuit consists of a series combination of a sinusoidal voltage source $3\,\cos(100t - 3°)$ V, a $500\,\Omega$ resistor, a 30 mH inductor, and an unknown impedance. If it is observed that voltage source is delivering maximum power to the unknown impedance, what is its value?

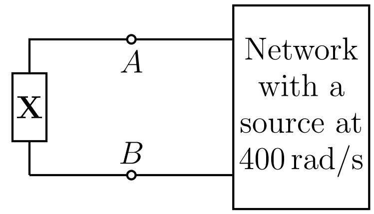
   (b) If in the previous part, the 30 mH inductor is replaced by a $10\,\mu$F capacitor, what is the value of the inductive component of the unknown impedance **Z** if it is known that **Z** is absorbing maximum power?

Solution:

(a) For maximum power transfer, $Z_L = Z^*_{th}$. $Z_{th} = 500 + j3\ \Omega$, so the value of the unknown impedance should be $500 - j3\ \Omega$.

(b) Now, the $Z_{th} = 500 - j1000\ \Omega$, so the value of the unknown impedance should be $500 + j1000\ \Omega$ for maximum power transfer. The inductive component of Z $= \frac{Imag(Z)}{\omega} = 10H$

3. Consider the network shown on the right. When $\mathbf{X}$ is a $2.5\,\mu$F capacitor, the voltage V_{AB} equals E_1, where $E_1 = 100\,\text{V}$. When $\mathbf{X}$ is a $2\,\text{H}$ inductor, V_{AB} equals $E_2 = 40\,\text{V}$, and leads E_1 by $90°$. Suppose now $\mathbf{X}$ is replaced by a short-circuit. Find the current $\mathbf{I}_{SC}$ flowing through AB under these conditions, and its phase angle with respect to E_1.



Solution:

$Z_{cap} = -j1000\Omega$

$Z_{ind} = j800\Omega$

Consider the Thevenin equivalent of the Network. Let the Thevenin voltage be V and its Thevenin impedance be $Z\Omega$.

Without loss of generality, since the phase of I_{sc} is asked w.r.t. E_1 and all the angles are relative to E_1, we can assume that phase of $E_1 = 0$.

$100 = V\frac{-j1000}{-j1000+Z}$

$40\angle 90° = V\frac{j800}{j800+Z}$

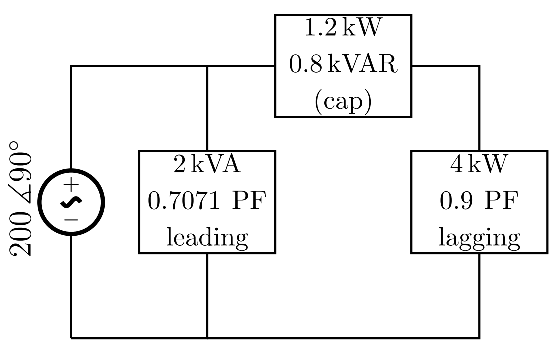
Dividing the two equations,

$2.5\angle(-90) = -1.25\frac{j800+Z}{-j1000+Z}$

Solving this, $Z = 720 + 640j\Omega$.

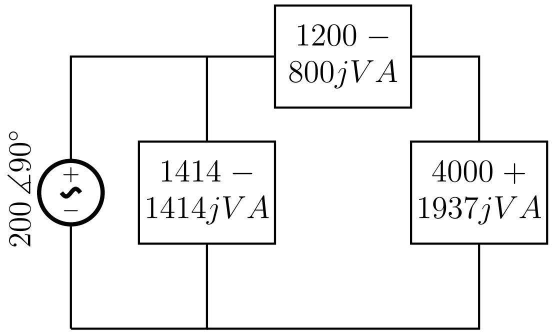
Plugging this into the above equation, we get $V = 36 + 72jV$.

$I_{sc} = \frac{V}{Z} = 0.0775 + 0.031j = \boxed{0.0835\angle 21.801A}$.

4. 

For the network shown on the left, calculate the following: (a) power factor, (b) average power, (c) reactive power, (d) apparent power, and (e) complex power delivered by the source.

Solution:



The complex powers have been displayed in the circuit.

$P_{total} = 6614 - 277jVA$

1. $PF = 0.999$
2. $P_{av} = 6614W$
3. $277VAR$
4. $6619VA$
5. $P_{total} = 6614 - 277jVA$

5. A sub-station operating at 50 Hz provides power to a large data center through a 20 km long high voltage line that can be modeled by a resistance of 5 Ω and an inductive reactance of 20 Ω. The voltage magnitude at the receiving end has to be maintained at 63.5 kV rms. If the consumer draws 20 MW of power at 0.707 lag power factor, find (i) line current (ii) active and reactive power absorbed by the line impedance (iii) the voltage and power factor at the sub-station end (iv) the active and reactive power at the sub-station end .

   If a capacitor is connected directly across the load at customer side such that the receiving end current is at unity power factor with respect to receiving end voltage, recalculate the quantities (i)-(iv) for this case and the value of the capacitor.

   Find the area of this capacitor, if we are planning to build a simple parallel plate capacitor with air as dielectric that can support 63.5 kV rms. Please note that breakdown field of air at atmospheric pressure is 30 kV/cm.

Solution:

i) $I_{line} = 445.42$A
ii) $P_{line} = 0.99$ MW + 3.968j MVA
iii) $V_s = 71.61$ kV, power factor = 0.658 lag
iv) $P_{total} = 20.99$ MW + 23.968j MVA

i) $I_{line} = 314.96$A
ii) $P_{line} = 0.496$ MW + 1.98j MVA
iii)$V_s = 65.38$ kV, power factor = 0.9953 lag
iv) $P_{total} = 20.496$ MW + 1.98j MVA

C = $\frac{1}{-201.6j*2\pi*50}$ and $d = 3\,cm$ (integer value chosen above 2.1 cm), $A = 5.32*10^4 m^2$