EE2025: Engineering Electromagnetics

Tutorial 1: Transmission Lines

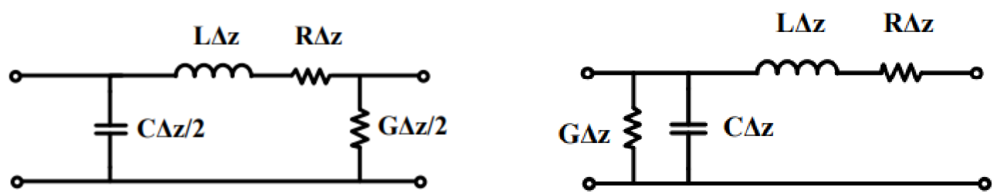
July-Nov 2024

- All transmission lines are lossless unless mentioned otherwise.
- If $x << y$, a good engineering approximation is $10x \leq y$.
- Electrical signals in coaxial cables travel at 2×10^8 m/s unless mentioned otherwise.

Motivating and Modeling Transmission Lines

1. Oniond is a high school student who can use only the lumped circuit model to solve circuits. He wants to install a CCTV camera right outside his circular house. The video feed is then transmitted through a coaxial cable at radio frequencies (assume the wavelength of transmission is 0.3 m). He processes this signal in the center of the house. How big can the house be if he doesn't have to learn any new concepts? Can Oniond live in such a house?

2. Observe the following models for an element (of length Δz) of a transmission line.



Model (a) Model (b)

Models for a Transmission Line

Derive the governing equation and the corresponding propogation constant for voltage and current variation across the transmission line in model (a) and model (b). How do the propogation constants differ between model (a) and (b)? Hence find their ratio assuming that all the primary constants remain the same.

Parameters of Transmission Lines

3. A 170 mm long lossless coaxial transmission line is characterized by the primary constants $L = 245$ nH/m and $C = 200$ pF/m ($R = 0\ \Omega/m$ and $G = 0\ \mho/m$). The transmission line is connected to a satellite dish which offers a purely resistive load $Z_L = 100\ \Omega$. The electrical signals are transmitted at 1 GHz.

   (a) Determine the characteristic impedance of the line. Is it purely resistive?

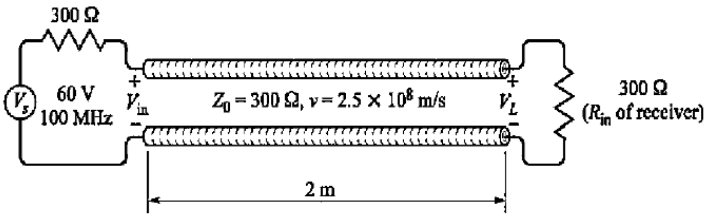
   (b) Determine the impedance looking into the input terminals of the transmission line. Think about why this impedance isn't purely resistive even though both Z_0 and Z_L are.

   (c) Determine the Reflection coefficient and the VSWR.

(d) I am an engineer who can modify the primary constants of my transmission line. What should I do to optimize performance? Calculate optimal L and C values.

4. A transmission line operating at 500 MHz has $Z_0 = 80\ \Omega$, $\alpha = 0.04$ Np/m, $\beta = 1.5$ rad/m. Find the line parameters R, L, G and C.

5. The transmission line shown in the below figure is a two-wire line lead-in from an antenna to a television receiver, where the input impedance offered to the voltage source is 300 Ω



Transmission Line

(a) Find the phase difference between the voltage at the source (V_{in}) and the voltage at the load (V_L).

(b) Find the reflection coefficient and voltage standing wave ratio on the line.

(c) Suppose a second receiver, also having an impedance of 300 Ω, is connected across the line in parallel with the first receiver. Find the new reflection coefficient and the voltage standing wave ratio on the line.

(d) Calculate the impedance seen by the AC voltage source before and after the introduction of the second reciever.

6. A transmission line of characteristic impedance 600 Ω is terminated by a reactance of $j150$ Ω. Find the input impedance of a section 25 cm long at a frequency of 300 MHz. Use the velocity of the wave as 3×10^8 m/s. Is this impedance capacitive or inductive? Can I change the characteristic impedance of my trasmission line to change that?

Voltages and Currents in Transmission Lines

7. A 300 Ω transmission line is 0.8 m long and terminated with a short circuit. The line operates in the air with a wavelength of 0.3 m and is lossless.

(a) If the input voltage amplitude is 10 V, what is the maximum value of voltage amplitude on the line?

(b) What is the current amplitude in the short circuit?

8. A 50 Ω transmission line is terminated to a load with an unknown impedance. The voltage standing wave ratio on the line is $\rho = 2.4$ and a voltage maximum occurs at a distance $\lambda/8$ from the load.

(a) Determine the load impedance.

(b) How far is the first minimum from the load (write the answer as a multiple of λ).

9. On a lossless 50 Ω transmission line, measurements at the load gave VSWR = 5. The results of three voltage measurements made on the line are as follows:

(a) The first maximum from the load is found at a distance of 0.25 m from the load.

(b) The next maximum is found at 0.75 m from the load.

(c) $V_L = 100\ V$.

Calculate Z_L, V^+, V^-,I^+,I^-.

10. The Γ_L on a 500 m long transmission line has a phase angle of 210^0. If the operating wavelength is 150m, what would be the number of voltage maxima formed on the line?

11. You have two pieces of coaxial cable both of length $\lambda/8$ (call them cables A and B) where λ is the operating wavelength. One of them is short-circuited and the other is left open. Both of them are connected in parallel at the end of yet another coaxial line (call it cable C). What will be the VSWR seen on the cable C? Show the configuration using a diagram.

Lossy Transmission Lines

12. A 50 Ω low-loss transmission line has a loss of 1.5 dB/m. The velocity of the voltage wave is 2×10^8 m/sec. A parallel resonant circuit at 2 GHz is to be designed with the section of the line.

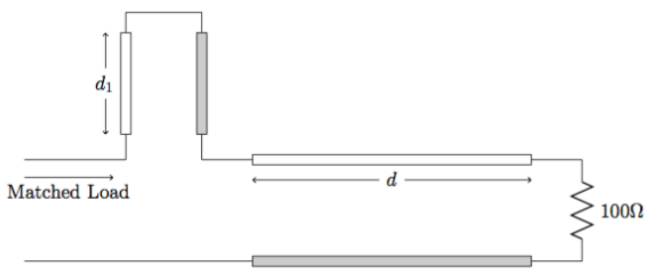
    (a) Find the input impedance of the line of the above design.

    (b) Find the quality factor and 3 dB bandwidth.

13. An RG-59U coaxial cable has a (power) loss of 10 dB per 100 m of length. A 10 V - 3 A (both peak-to-peak values) signal is generated using a function generator and connected to one end of 50 meters of the above cable. On the other side, the cable is impedance matched to a set top box unit. Find the power delivered to the load. If the cable was not impedance matched can you solve the problem?

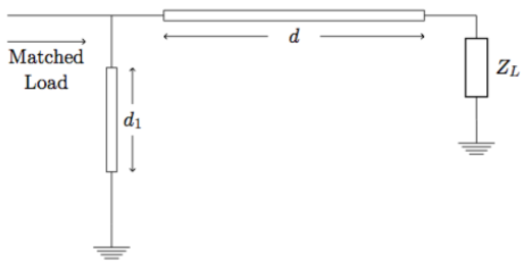
Impedance Matching

14. Two lossless two-wire lines of length d and d_1 are added together as shown in Fig. **??** for impedance matching at $\lambda = 100$ cm. If the characteristic impedance of both the lines are $Z_0 = 200\Omega$, find the possible values of d and d_1 to provide a matched load. (Note that the un-shaded and shaded conductor are both parts of the same transmission line, for example they can be the inner and outer conductor of a coaxial cable.)



Transmission Line with matched loading

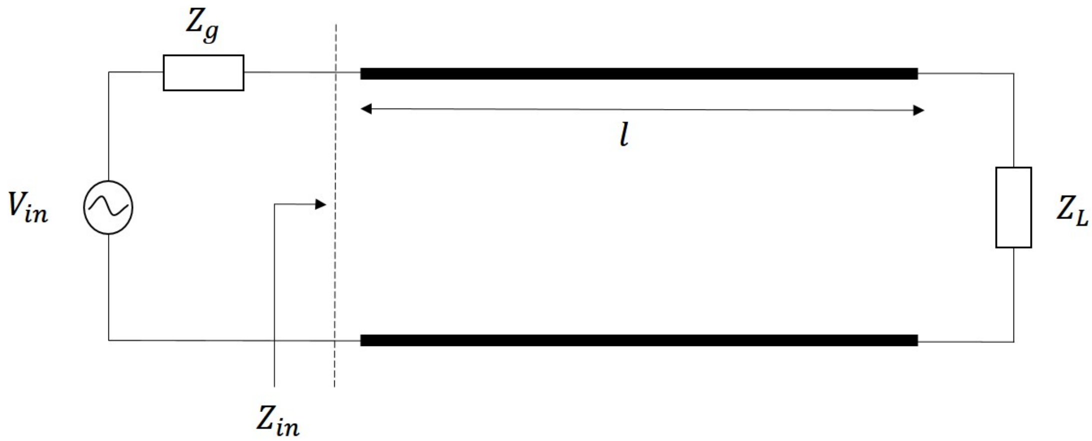
15. Given the system in (refer to the below figure) is operating with $\lambda = 100cm$ and $Z_0 = 300\Omega$. If $d_1 = 10cm$, $d = 25cm$, and the system is matched to 300 Ω, find Z_L?



Load attached to two Transmission Lines

Power Transfer

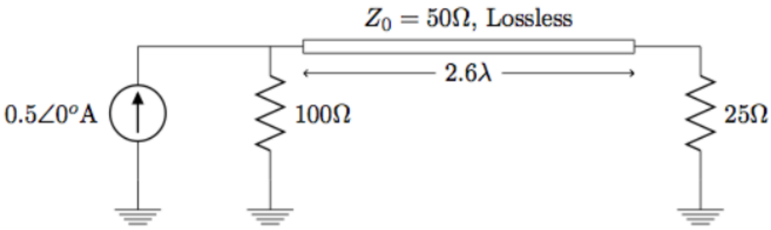
16. According to the maximum power transfer theorem, maximum time averaged power is transferred from a source with internal impedance Z_g to a load, Z_L when $Z_g = Z_L^*$. A 50 MHz generator with an internal impedance (Z_g) of 50 Ω is connected to an impedance 50−j25 Ω. How would you ensure maximum power transfer in this case using a lossless transmission line of characteristic impedance 100 Ω, and what should be the minimum length of the transmission line element ? Assume $v = 2 \times 10^8$m/s as wave velocity.



Impedance matching using a transmission line of length l

17. In the above question, is there some other method that can be employed for ensuring maximum power transfer?

18. Calculate the average power dissipated by each resistor in the circuit shown in the below figure.



Transmission Line connected to a constant current source

19. For a 50-Ω lossless transmission line terminated in a load impedance Z_L= (100+ j50) Ω, determine the fraction of the average incident power reflected by the load. What is the magnitude of the average reflected power if $|V_0^+| = 1$ V (peak)?

Quarter Wavelength Transformer

20. The 5G testbed of IIT Madras is developing hardware for the next generation of Indian cellular systems that operates at a center frequency of 6 GHz. The antennas they develop have an input impedance of 120 ohms, and the source has an internal impedance of 50 ohms. To test out the system, the source and antennas are kept in the lab, and a quarter wave transformer is used to match the source and load. What should be the optimal impedance of the quarter wave line and what is its minimum length? Assume velocity $v = 2.5 \times 10^8$ m/s wherever necessary.