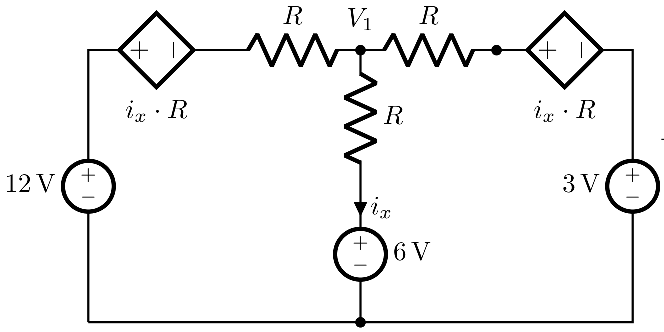
Tutorial 2

August 16, 2024

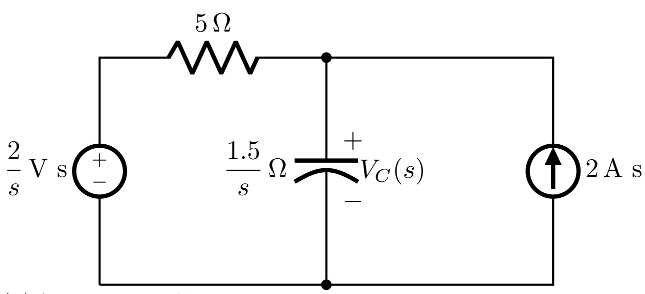
1.



$$\frac{12-10ki_x-V_1}{\cancel{10k}}+\frac{3+10ki_x-V_1}{\cancel{10k}}=\frac{V_1-6}{\cancel{10k}}$$
$$15-2V_1=V_1-6$$
$$\Rightarrow V_1=7V$$
$$i_x=\frac{V_1-6}{10}mA$$
$$i_x=\frac{1}{10}mA$$

Power absorbed by the 6V source $= 6 \times i_x = 6 \times 0.1 = 0.6mW$
Power delivered $= -0.6mW$

2.



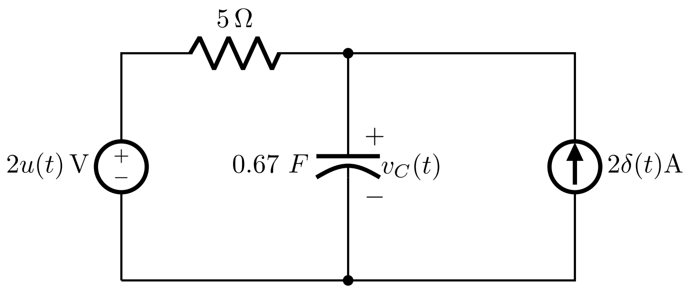
(a)Applying KCL at the capacitor node,

$$\frac{V_C(s)-\frac{2}{s}}{5}+\frac{V_C(s)}{\frac{1.5}{s}}-2=0$$
$$\frac{V_C(s)s-2}{5s}+\frac{V_C(s)s}{1.5}=2$$
$$\frac{1.5V_C(s)s-3+5V_C(s)s^2}{7.5s}=2$$
$$1.5V_C(s)s-3+5V_C(s)s^2=15s$$
$$V_C(s)\times \cancel{5}s(s+0.3)=\cancel{5}(3s+0.6)$$
$$V_C(s)=\frac{3s+0.6}{s(s+0.3)}$$

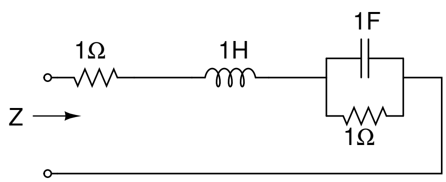
split into partial fractions

$$V_C(s)=\frac{2}{s}+\frac{1}{s+0.3}$$

(b) $v_C(t)=\mathcal{L}^{-1}(V_C(s))=2u(t)+e^{-0.3t}u(t)$
(c) time-domain representation of the circuit,

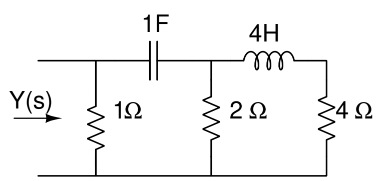


3. To find Z(s):



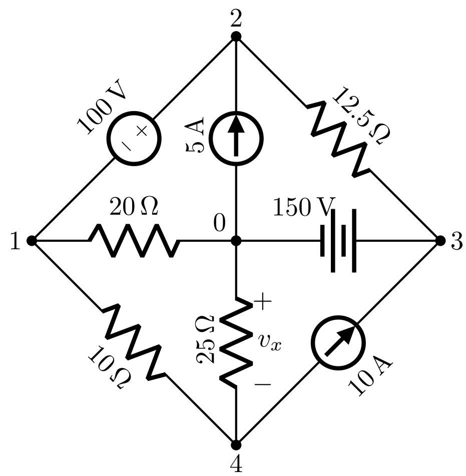
$$\begin{aligned} Z(s) &= 1+s+\left(\tfrac{1}{s}||1\right) \\ &= 1+s+\tfrac{\frac{1}{s}\times 1}{1+\frac{1}{s}} \\ &= 1+s+\tfrac{1}{s+1} \\ &= \tfrac{s^2+2s+2}{s+1} \end{aligned}$$

4. To find Y(s):



$$\begin{aligned} Z(s) &= 1||\left(\frac{1}{s}+2||(4s+4)\right) \\ &= 1||\left(\frac{1}{s}+\frac{2\times 4(s+1)}{(2+4s+4)}\right) \\ &= 1||\left(\frac{1}{s}+\frac{8(s+1)}{(4s+6)}\right) \\ &= 1||\left(\frac{4s+6+8s^2+8s}{(4s^2+6s)}\right) \\ &= \frac{\frac{(8s^2+12s+6)}{(4s^2+6s)}\times 1}{1+\frac{(8s^2+12s+6)}{(4s^2+6s)}} \\ &= \frac{(8s^2+12s+6)}{(4s^2+6s+8s^2+12s+6)} \\ &= \frac{(8s^2+12s+6)}{(12s^2+18s+6)} \\ \Longrightarrow\ Y(s) &= \frac{1}{Z(s)} \\ &= \frac{(12s^2+18s+6)}{(8s^2+12s+6)} \\ &= \frac{(6s^2+9s+3)}{(4s^2+6s+3)} \end{aligned}$$

5. Nodal analysis: Assume that the current through the 100V source from node 1 to node 2 is i_1. The voltage at node 3 is known; need not write an equation at this node. The nodal equations at nodes 1, 2 and 4 can be written as follows.



$$-i_1 + \frac{V_1}{20} + \frac{V_1 - V_4}{10} = 0 \quad (1)$$
$$i_1 - 5 + \frac{V_2 - 150}{12.5} = 0 \quad (2)$$

Adding equations (1) and (2), we get

$$\frac{V_1}{20} - 5 + \frac{V_1 - V_4}{10} + \frac{V_2 - 150}{12.5} = 0 \quad (3)$$
$$\frac{V_4 - V_1}{10} + \frac{V_4}{25} + 10 = 0 \quad (4)$$
$$V_2 - V_1 = 100V \quad (5)$$

Arranging equations (3), (4) and (5) in matrix form,

$$\begin{pmatrix} -1 & 1 & 0 \\ 10 & 0 & -14 \\ 15 & 8 & -10 \end{pmatrix} \cdot \begin{pmatrix} V_1 \\ V_2 \\ V_4 \end{pmatrix} = \begin{pmatrix} 100 \\ 1000 \\ 1700 \end{pmatrix}$$

$V_1 = 11.71\,\mathrm{V}$, $V_2 = 111.71\,\mathrm{V}$,$V_3 = 150\,\mathrm{V}$, $V_4 = -63.06\,\mathrm{V}$
$v_x = -V_4 = 63.06\,\mathrm{V}$

6. Using the circuit model, we have

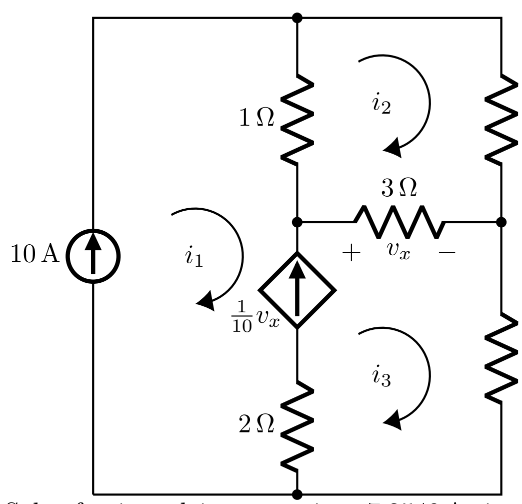
$$v_{ab} = \frac{1M\Omega}{1M\Omega + 500\Omega} \times 200mV = 199.9mV$$

Input voltage to the speaker $= 120 v_{ab} = 23.988V$
By applying voltage division rule across speaker resistor,

$$16 = \frac{10}{10 + R} \times 23.988$$
$$\Rightarrow R = 4.9925\Omega \approx 5\Omega$$

Power delivered to the speaker,

$$P_D = \frac{V^2}{R} = \frac{16^2}{10} = 25.6W$$

7. 

$$\text{Clearly, } i_1 = 10\text{A} \quad (1)$$
$$\text{and } i_3 - i_1 = \frac{v_x}{10}$$
$$\text{where } v_x = 3(i_3 - i_2)$$
$$\implies 0.7i_3 + 0.3i_2 = 10 \quad (2)$$

Using KVL for mesh (2)

$$2i_2 + 3(i_2 - i_3) + 1(i_2 - i_1) = 0 \quad (3)$$

From (2) and (3),

$$6i_2 - 3i_3 = 10$$
$$0.3i_2 + 0.7i_3 = 10$$

Solve for i_2 and i_3 to get, $i_2 = 7.2549\,\mathrm{A}$, $i_3 = 11.1765\,\mathrm{A}$

8. Since the current through the 2Ω resistor is $\frac{2}{s}$, we need to write the nodal equations only at nodes 2 and 3. V_1 can be obtained as $V_2 + 100\frac{2}{s}$. Similarly $V_4 = V_3 - 2\frac{5}{s}$.

$$\frac{-2}{s} + V_2\left(1+\frac{s}{5}\right) - V_3 \cdot \frac{s}{5} = 0 \quad (1)$$
$$\frac{s}{5}(V_3 - V_2) + \frac{V_3}{2s} + \frac{s}{5} = 0 \quad (2)$$

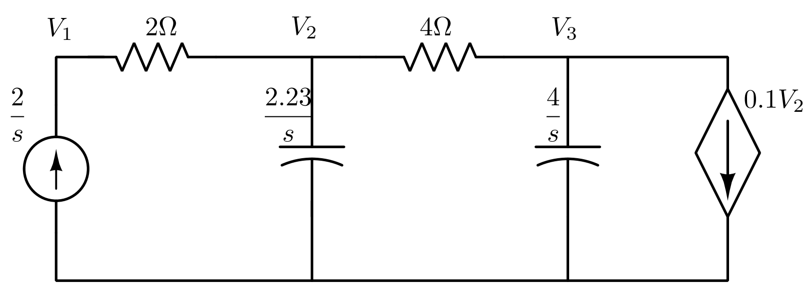
Using equations (1) and (2), we have

$$\begin{pmatrix} 1+\frac{s}{5} & -\frac{s}{5} \\ -\frac{s}{5} & \left(\frac{1}{2s}+\frac{s}{5}\right) \end{pmatrix} \begin{pmatrix} V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} \frac{2}{s} \\ -\frac{5}{s} \end{pmatrix}$$

$$V_2(s) = -\frac{6\,s^2 - 10}{s\,(2s^2+s+5)} \longleftrightarrow v_2(t) = 2\,u(t) - 5\,e^{-t/4}\left[\cos\left(\frac{\sqrt{39}}{4}\,t\right) - \frac{\sqrt{39}}{195}\sin\left(\frac{\sqrt{39}}{4}\,t\right)\right] u(t)\,\mathrm{V}$$

$$V_3(s) = -\frac{6\,s + 50}{2s^2+s+5} \longleftrightarrow v_3(t) = -3\,e^{-t/4}\left[\cos\left(\frac{\sqrt{39}}{4}\,t\right) + \frac{97\,\sqrt{39}}{117}\sin\left(\frac{\sqrt{39}}{4}\,t\right)\right] u(t)\,\mathrm{V}$$

9. s-domain representation of circuit,



Once again write equations only for V_2 and V_3.

$$V_2 \cdot 0.45s + \frac{V_2 - V_3}{4} = \frac{2}{s} \quad (1)$$
$$\frac{V_3 - V_2}{4} + V_3 \cdot \frac{s}{4} + 0.1V_2 = 0 \quad (2)$$

$$\begin{pmatrix} (0.45s+0.25) & -0.25 \\ -0.15 & 0.25(1+s) \end{pmatrix} \cdot \begin{pmatrix} V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} \frac{2}{s} \\ 0 \end{pmatrix}$$
$$\implies \begin{pmatrix} V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} \frac{0.5(s+1)}{s(0.1125s^2+0.175s+0.025)} \\ \frac{-0.3}{s(0.1125s^2+0.175s+0.025)} \end{pmatrix}$$

Expanding using partial fractions, we get

$$V_2(s) = \frac{22.53}{s} + \frac{136.32}{(s+0.159)} - \frac{158.85}{(s+0.1396)} \longleftrightarrow v_2(t) = 22.53u(t) + 136.32e^{-0.159t}u(t) - 158.85e^{-0.1396t}u(t)$$
$$V_3(s) = \frac{-13.51}{s} - \frac{97.26}{(s+0.159)} + \frac{110.77}{(s+0.1396)} \longleftrightarrow v_3(t) = -13.52u(t) - 97.26e^{-0.159t}u(t) + 110.78e^{-0.1396t}u(t)$$