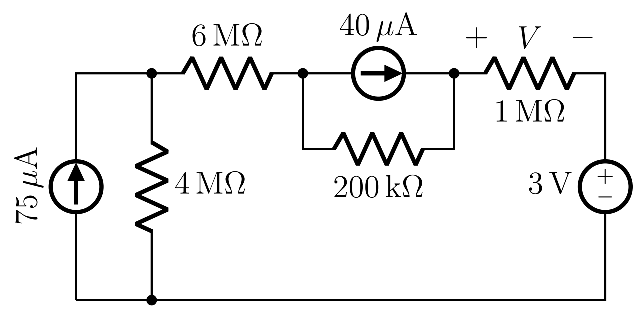
EE2015: Electric Circuits and Networks

Tutorial 3: Solutions

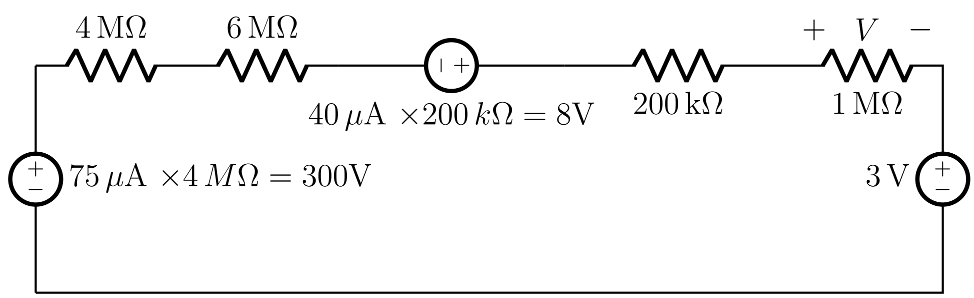
(August 24, 2024)

1. For the circuit shown on the right compute the voltage V across the $1\,\mathrm{M}\Omega$ resistor by using repeated source transformation.

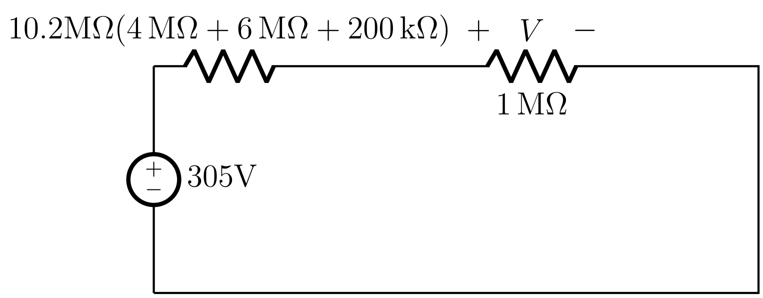


Solution:

- Step 1 :

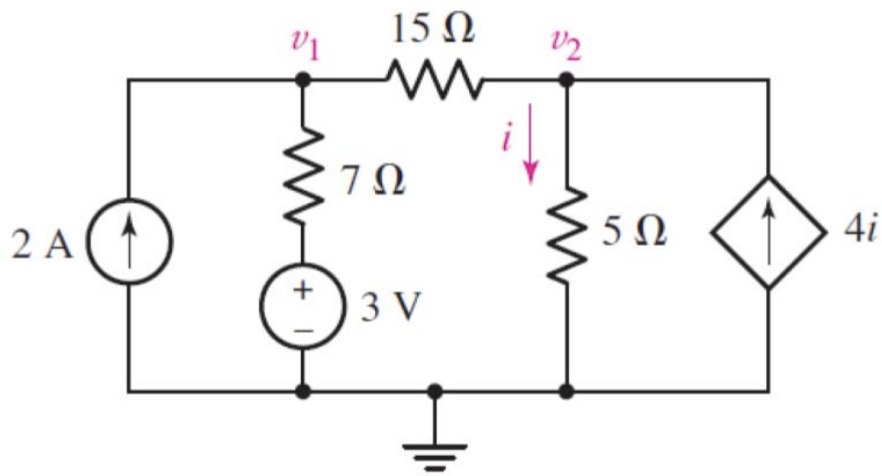


- Step 2 :



By voltage division among the resistors - $10.2\mathrm{M}\Omega$ and $1\mathrm{M}\Omega$, voltage difference across the $1\mathrm{M}\Omega$ resistor is $\frac{1}{10.2+1} \times 305 = 27.232\mathrm{V}$.

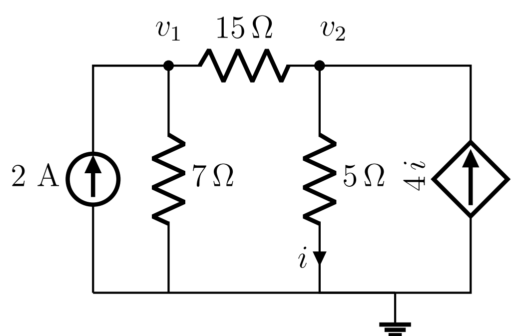
2. For the circuit below, use superposition to obtain the voltages v_1 and v_2.



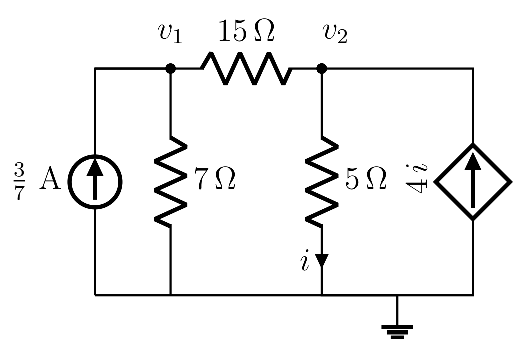
Solution:

By superposition theorem, consider only one independent source at a time. Short the terminals of other independent voltage sources and remove the terminals of other independent current sources.

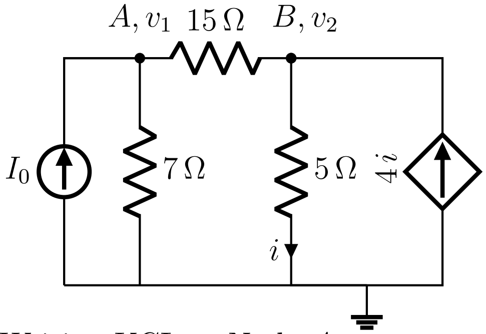
- Equivalent circuit with only $2A$ current source,



- Equivalent circuit with only $3V$ voltage source, using Source transformation



As both the above circuits are similar except the current of the independent current source, solve the circuit considering a current input I_0 first.



Writing KCL at Node A,

$$I_0 = \frac{v_1}{7} + \frac{v_1 - v_2}{15}$$

Writing KCL at Node B,

$$\frac{v_1 - v_2}{15} + 4i = i, \text{ where } i = \frac{v_2}{5}$$

Matrix from nodal analysis,

$$\begin{bmatrix} \frac{22}{105} & \frac{-1}{15} \\ \frac{1}{15} & \frac{8}{15} \end{bmatrix} \times \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 4.5901 & 0.0382 \\ -0.573 & 0.1202 \end{bmatrix} \times \begin{bmatrix} I_0 \\ 0 \end{bmatrix}$$

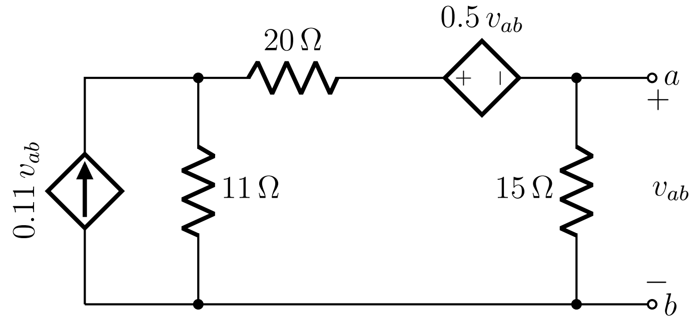
We get $v_1 = 4.5901 I_0$ and $v_2 = -0.5734 I_0$.

- Voltages with only 2A source, $v_{1,2A} = 4.5901 \times 2 = 9.18V$ and $v_{2,2A} = -0.5734 \times 2 = -1.147V$.
- Voltages with only 3V source, $v_{1,3V} = 4.5901 \times \frac{3}{7} = 1.967V$ and $v_{2,3V} = -0.5734 \times \frac{3}{7} = -0.246V$.

By superposition theorem,

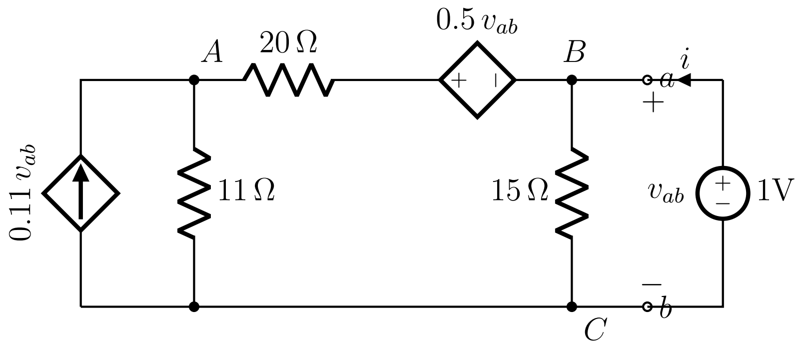
$$v_1 = v_{1,2A} + v_{1,3V} = 11.147\text{V}$$
$$v_2 = v_{2,2A} + v_{2,3V} = -1.393\text{V}$$

3. For the circuit shown on the right, find the Thevenin and Norton equivalents as seen across terminals ab.



Solution: As there are no independent voltage or current sources in the circuit, there will be no voltage source in the Thevenin equivalent circuit and no current source in Norton equivalent circuit. So, the problem of finding the equivalent circuit gets reduced to finding the Equivalent resistance across the terminals a and b.

As the voltage v_{ab} controls the VCVS and VCCS, apply $v_{ab} = 1V$, and find the current i, flowing out of the 1V source.



Consider the node C to be the reference node, so $V_C = 0$V and $V_B = 1$V.

The current through the VCCS $= 0.11 \times v_{ab} = 0.11A$.

The voltage difference across the VCVS $= 0.5 \times v_{ab} = 0.5$ V.

Let the voltage at node A wrt node C be V_A.

Writing KCL at Node A,

$$0.11 + \frac{(V_B + 0.5) - V_A}{20} = \frac{V_A}{11}$$

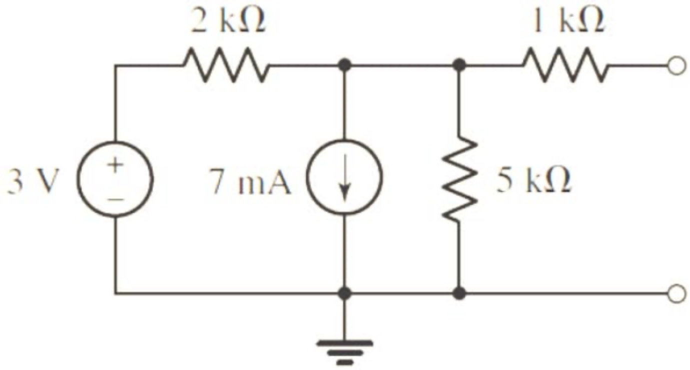
Solving the above equation, we get $V_A = \frac{40.7}{31}$.

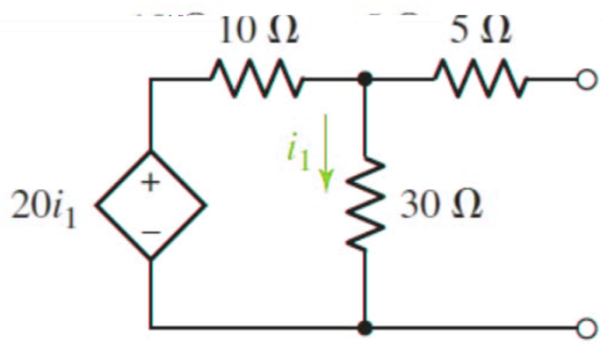
Writing KCL at C,

$$i + 0.11 = \frac{V_A}{11} + \frac{V_B}{15} \implies i = 0.076A.$$

Equivalent Resistance seen through the terminals a and b is $\frac{1}{0.076} = 13.154\Omega$. Hence, Both the Thevenin and Norton Equivalent circuits seen across a and b consists of only the resistance 13.154Ω.

4. Determine the Thevenin and Norton equivalents of the circuits shown below.

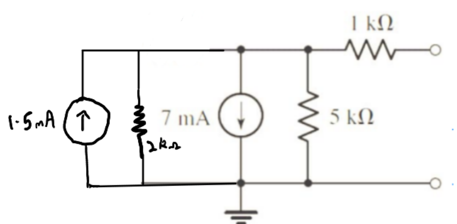




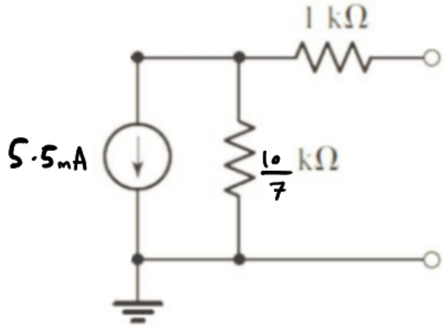
Solution:

Circuit 1

The $3V$ and $2k\Omega$ can be converted into a Norton equivalent, with $1.5mA$ and $2k\Omega$ values.



The parallel combination of these two current sources yields us a current source of $-5.5mA$ and a parallel resistor $\frac{10}{7}k\Omega$.

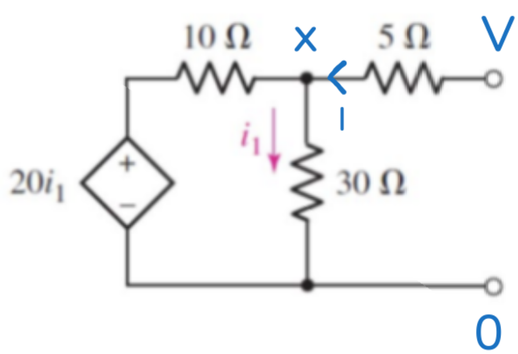


Now, the Thevenin equivalent of the above current source and resistor combination would be a Voltage source of $-\frac{55}{7}V$ and resistor $\frac{10}{7}k\Omega$, which in series with $1k\Omega$ resistor gives us a $\frac{17}{7}k\Omega$ resistor.

Thus, the overall Thevenin equivalent is voltage source $\boxed{\frac{-55}{7}V}$ and resistance $\boxed{\frac{17}{7}k\Omega}$.

The Norton equivalent is current source $\boxed{\frac{-55}{17}mA}$ and resistance $\boxed{\frac{17}{7}k\Omega}$.

Circuit 2



Consider a V volt source connected across the circuit, and let the current entering the circuit be I.

Across the 5Ω resistor,

$$I = \frac{V - x}{5}$$

From this, $x = V - 5I$.

Across 30Ω resistor,

$$i_1 = \frac{x}{30} = \frac{V - 5I}{30}$$

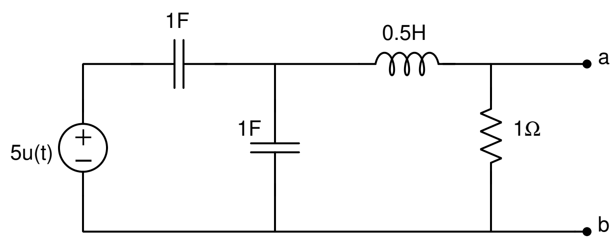
Writing KCL at node x:

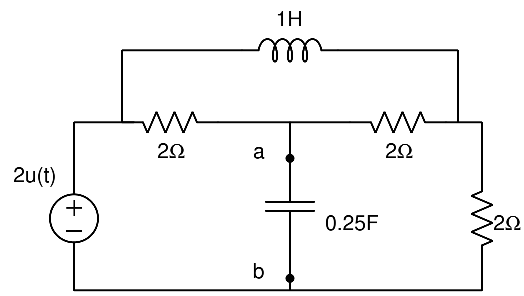
$$I = \frac{V - 5I}{30} + \frac{x - 20i_1}{10} = \frac{V - 5I}{30} + \frac{x}{30} = \frac{V - 5I}{15}$$

Solving this, we get $V = 20I$.

Therefore, the Thevinin equivalent and the Norton equivalent is a resistor of 20Ω value.

5. For the following circuits, find the Thevenin equivalent circuit as seen by (a) the 1Ω resistor and (b) 0.25F capacitor.





Solution:

Note: When we're considering the Thevenin equivalent as seen by a circuit element, we don't include that circuit element while calculating the Thevenin equivalent.

Part 1:

To find the Thevenin voltage:

We have a $5u(t)$ source across two capacitors of $1F$ each, and the inductor is not connected to anything (i.e.) it's a floating terminal. Thus, $V_{th} = 2.5u(t)V$.

To find R_{th}, we can short the voltage source. Now, two $1F$ capacitors are in parallel, and this combination is in series with a $0.5H$ inductor. Together, their equivalent combination is a $2F$ capacitor and $0.5H$ inductor in series. $R_{th}(s) = \frac{1}{2}\frac{s^2+1}{s}\Omega$.

Part 2:

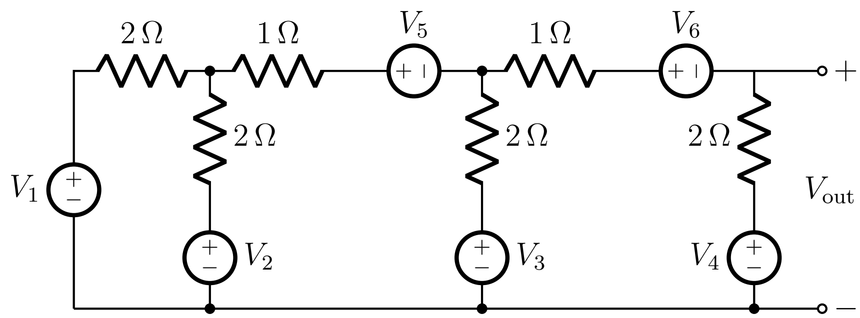
We can take the circuit to the Laplace domain. We can remove the capacitor connected between a and b and then find the Open circuit voltage.

To find Z_{th}, we short the voltage source. Now, the equivalent resistance from points a and b is: we have $2||(2 + (2||s))\Omega$, which reduces to $\boxed{4\frac{s+1}{3s+4}\Omega}$.

To find V_{th}, consider the Voltage source, let the end b be grounded, and let us find the potential at point a. Now, the current through the 2Ω resistor is simply $\frac{V_{source}}{\frac{4s}{s+4}+2} As$.

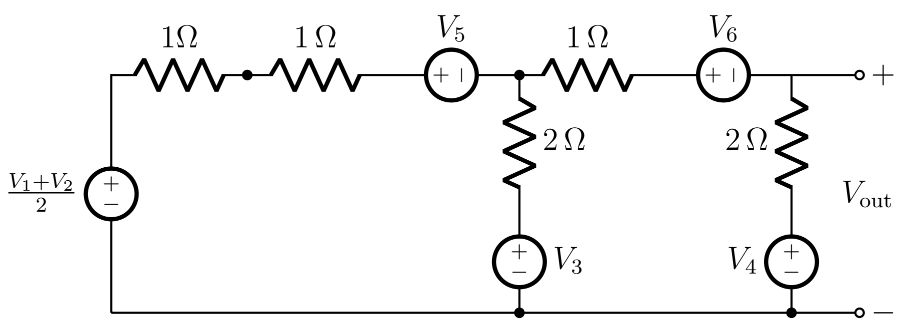
Thus, the potential difference at point a is $\left(I2 + \frac{1}{2} I \frac{4s}{4+s}\right) Vs = \boxed{\frac{4(s+2)}{s(3s+4)} Vs}$.

6. For the circuit given below, determine V_{out} in terms of V_k, $k = 1, 2, \ldots, 6$.

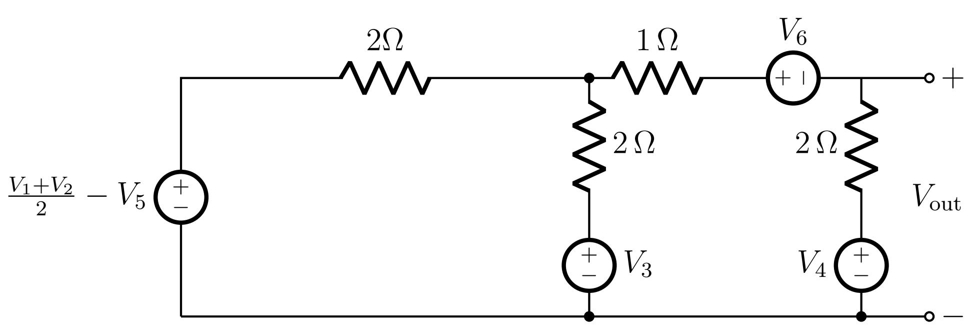


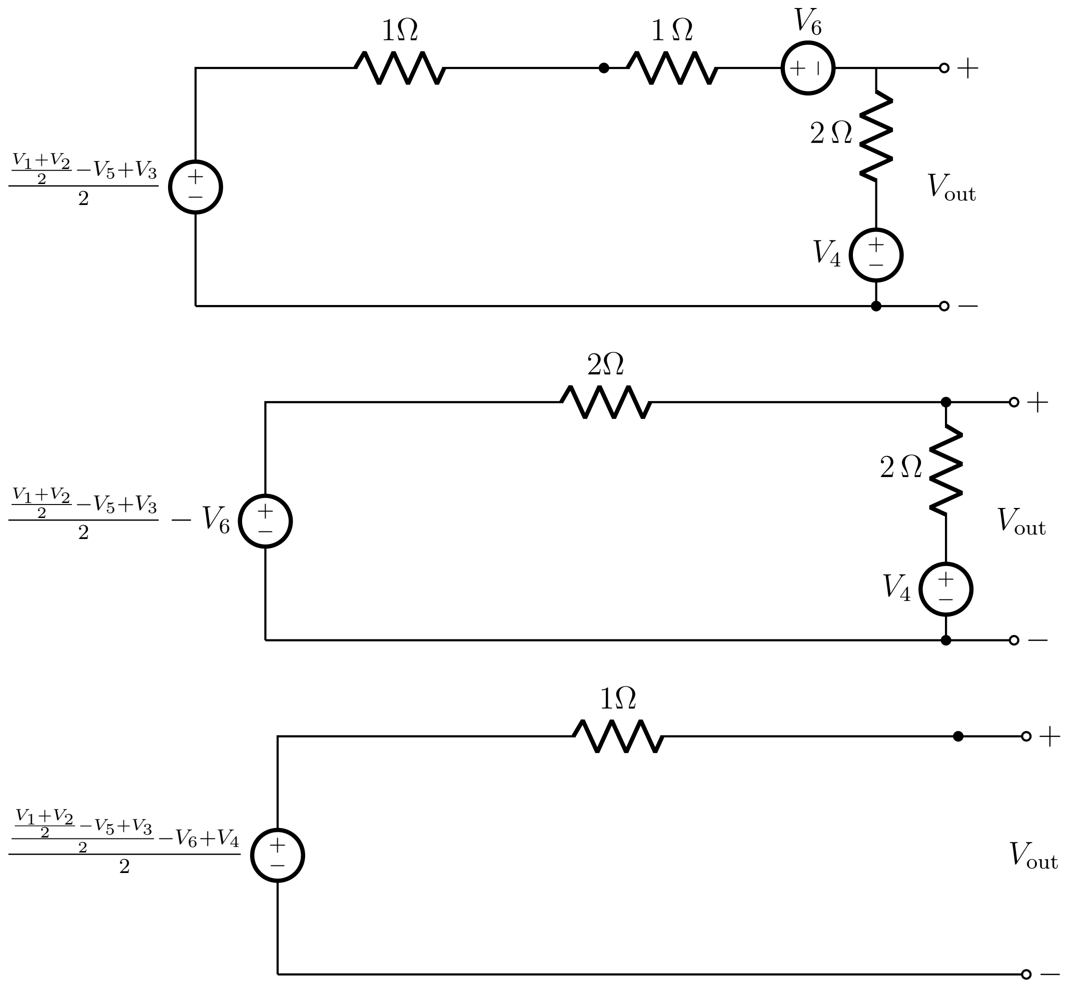
Solution:

This network can be simplified by repeated source transformation.



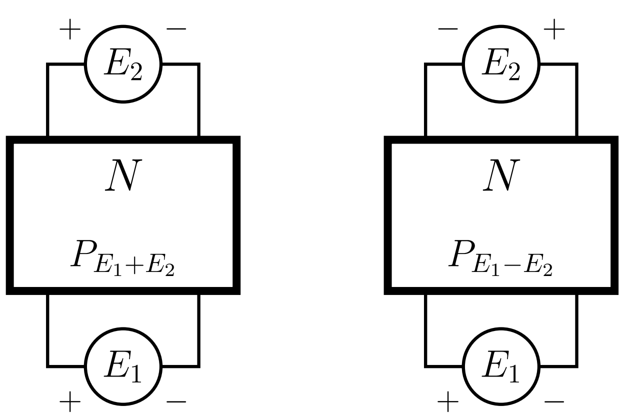
Now it's just a series combination of the two voltage sources.





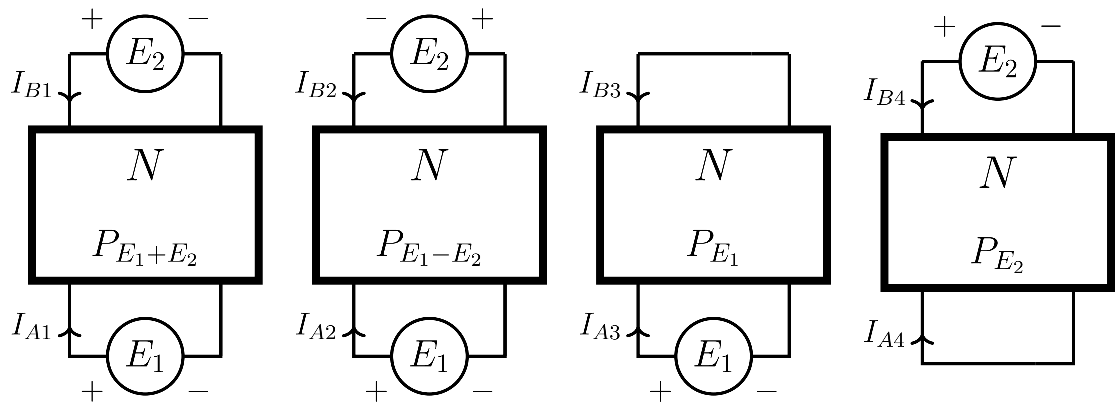
That gives us the Thevinin equivalent: a Voltage source of $\frac{V_1}{8}+\frac{V_2}{8}+\frac{V_3}{4}+\frac{V_4}{2}-\frac{V_5}{4}-\frac{V_6}{2}$ and 1Ω resistance.

7.



A network N consists entirely of linear resistors and excited by two voltage sources E_1 and E_2, as shown in the figure. When E_2 is connected as shown on the left, the power dissipated is $P_{E_1+E_2}$, whereas it is $P_{E_1-E_2}$ when E_2 is connected as shown on the right (i.e., its polarity is reversed). Let P_{E_1} and P_{E_2} be the powers dissipated when E_1 and E_2 are acting alone (i.e., the other source is set to zero). Show that $P_{E_1}+P_{E_2}=\frac{1}{2}\left(P_{E_1+E_2}+P_{E_1-E_2}\right)$.

Solution: Assume the following current are flowing through the branches of each the figures.



$$\begin{cases} I_{A1} = I_{A3} + I_{A4} & I_{B1} = I_{B3} + I_{B4} \\ I_{A2} = I_{A3} - I_{A4} & I_{B2} = I_{B3} - I_{B4} \end{cases} \quad \text{(due to superposition theorem)} \tag{1}$$

According to Tellegen's Theorem,

$$-I_{A1}E_1 - I_{B1}E_2 + P_{E_1+E_2} = 0 \tag{2}$$

$$-I_{A2}E_1 + I_{B2}E_2 + P_{E_1-E_2} = 0 \tag{3}$$

$$-I_{A3}E_1 + P_{E_1} = 0 \tag{4}$$

$$-I_{B4}E_2 + P_{E_2} = 0 \tag{5}$$

(3) + (4) =

$$-I_{A1}E_1 - I_{B1}E_2 + P_{E_1+E_2} - I_{A2}E_1 + I_{B2}E_2 + P_{E_1-E_2} = 0$$

on substituting (1) gives,

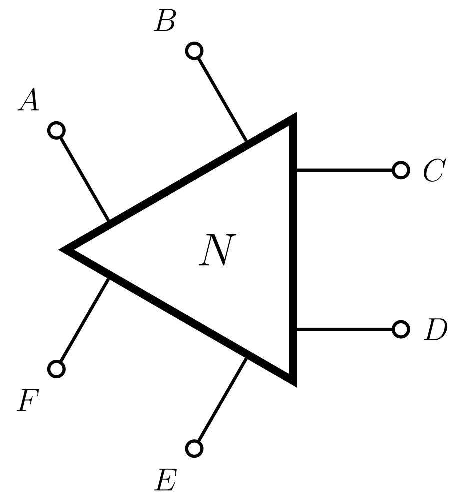
$$-(I_{A3}+I_{A4})E_1 - (I_{B3}+I_{B4})E_2 + P_{E_1+E_2} - (I_{A3}-I_{A4})E_1 + (I_{B3}-I_{B4})E_2 + P_{E_1-E_2} = 0$$

$$-2I_{A3}E_1 - 2I_{B4}E_2 + P_{E_1+E_2} + P_{E_1-E_2} = 0$$

$$-2(P_{E_1} + P_{E_2}) + P_{E_1+E_2} + P_{E_1-E_2} = 0$$

$$P_{E_1} + P_{E_2} = \frac{P_{E_1+E_2} + P_{E_1-E_2}}{2}$$

8. The network N shown on the right is composed entirely of resistors.
   (a) An ammeter is connected across FE and voltage sources of 6 V and 2 V across AB and CD. When A and C are positive, $I_{FE} = 14$ A; when B and C are positive, $I_{EF} = 22$ A.
   (b) The ammeter is now replaced by a voltmeter and voltage sources of 4 V and 5 V are connected across AB and CD, with A and C being positive. The voltmeter reads 8 V, F being at a higher potential



With conditions as in the first part of (a), a variable resistance connected across EF is adjusted until it consumes maximum power. What is this power?

Solution:

Step 1: As the network N has only resistors with no independent sources, one can write any branch current as a superposition of sources connected across AB and CD

$$k_1 V_{AB} + k_2 V_{CD} = I_{EF}$$

$$6k_1 + 2k_2 = 14$$
$$-6k_1 + 2k_2 = -22$$

on solving $k_1 = 3$ and $k_2 =$ -2

$$3V_{AB} - 2V_{CD} = I_{EF}$$

For $V_{AB} = 4V$ and $V_{CD} = 5V$, $I_{EF} = 2A$

Step 2: Finding maximum Power

Maximum power is dissipated when external resistance is same as Thevenin resistance. So $I_{SC} = 2$A and $V_{OC} = 8$V and $R_{TH} = 8/2 = 4\Omega$, From first part of a) it is given that I_{SC} through FE is 14 A and we have found that $R_{TH} = 4\Omega$. We can get maximum power dissipation if a 4Ω resistor is connected across EF. Then Pmax = $(14/2)^2 * 4 = 196W$