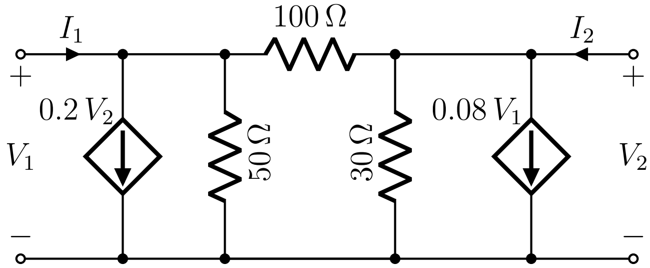
EE2015: Electric Circuits and Networks

Tutorial 4

(September 6, 2024)

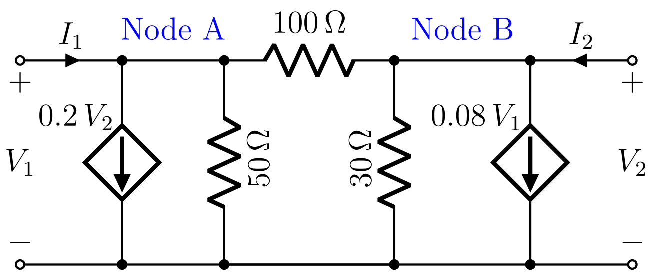
1.



Obtain the impedance and admittance parameters for the two-port network shown on the left.

Solution:

Consider nodes A and B as shown below. Let the negative terminals of both ports be grounded.



For this network it is easy to compute Y parameters, Set $V_2 = 0$ i.e., short circuit port 2 and connect a voltage source V_1 to port 1.

$$I_1 = V_1\left(\frac{1}{50} + \frac{1}{100}\right)$$
$$\implies y_{11} = 0.03$$
$$I_2 = 0.08V_1 - \frac{V_1}{100}$$
$$\implies y_{21} = 0.07$$

Set $V_1 = 0$ i.e., short circuit port 1 and connect a voltage source V_2 to port 2.

$$I_2 = V_2\left(\frac{1}{30} + \frac{1}{100}\right)$$
$$\implies y_{22} = 0.0433$$
$$I_1 = 0.2V_2 - \frac{V_1}{100}$$
$$\implies y_{21} = 0.19$$

This gives us the admittance ($\mathbf{Y}$) matrix (in S):

$$\boxed{\begin{bmatrix} 0.03 & 0.19 \\ 0.07 & 0.0433 \end{bmatrix}}$$

Alternate method using nodal analysis:

Alternately, you can use nodal analysis

Writing KCL at node A:

$I_1 = 0.2V_2 + \frac{V_1}{50} + \frac{V_1 - V_2}{100}$

Writing KCL at node B:

$I_2 = 0.08V_1 + \frac{V_2}{30} + \frac{V_2 - V_1}{100}$

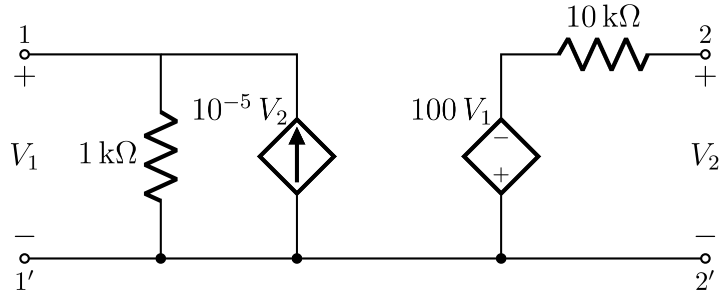
The admittance ($\mathbf{Y}$) matrix can be obtained from this (in S):

$$\boxed{\begin{bmatrix} 0.03 & 0.19 \\ 0.07 & 0.0433 \end{bmatrix}}$$

Inverting this gives us the impedance ($\mathbf{Z}$) matrix (in Ω):

$$\boxed{\begin{bmatrix} -3.61 & 15.833 \\ 5.833 & -2.5 \end{bmatrix}}$$

2. (a) Find the h-parameters of the two-port network shown below.
   (b) Find $\mathbf{Z}_{\text{out}}$ if an input $\mathbf{V}_s$ having source resistance of $R_s = 200\,\Omega$ is connected at $11'$.



Solution:

Set $V_2 = 0$ and connect a current source I_1 to port 1. Therefore,

- $V_1 = 1000 I_1 \implies h_1 1 = 1k\Omega$
- $I_2 = 10^5 I_1/10^4 \implies h_{21} = 10.$

Set $I_1 = 0$ and connect V_2 to port 2.

- $V_1 = 10^{-5} V_2 \cdot 1000 \implies h_{12} = 0.01.$
- $I_2 = (V_2 + 100 \times 10^{-2} V_2)/10^4 \implies h_{22} = 2 \times 10^{-4} S.$

To find Z_{out}, we can short V_s, connect a voltage source V_2 at port 2 and find I_2.

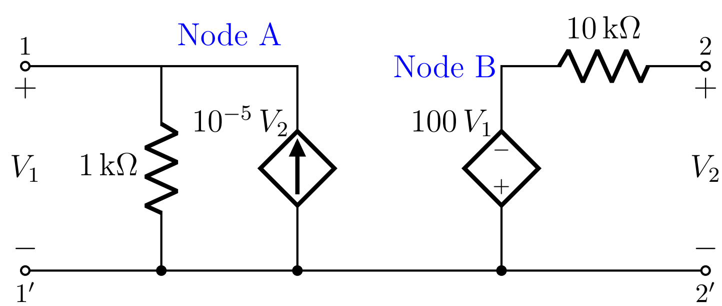
$$\begin{aligned} V_1 &= h_{11} I_1 + h_{12} V_2 \\ I_2 &= h_{21} I_1 + h_{22} V_2 \\ I_1 &= -R_s V_1 \end{aligned}$$

We can solve this to get:

$$Z_{out} = \frac{h_{11} + R_s}{(h_{22}(h_{11} + R_s) - h_{12} h_{21}} = 8576\Omega$$

Alternate Method:

For part (a), consider nodes A and B as below, also ground the 1'-2' terminals.



Writing KCL at node A:

$I_1 = \frac{V_1}{1000} - 10^{-5} V_2$

Writing KVL across the VDVS and $10k\Omega$ resistor:

$V_2 = 10000 I_2 - 100 V_1$

Solving these two equations gives us the matrix:

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1000\Omega & 0.01 \\ 10 & 0.0002\Omega^{-1} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

This gives us the h-parameters: $\boxed{\begin{bmatrix} 1000\Omega & 0.01 \\ 10 & 0.0002\Omega^{-1} \end{bmatrix}}$

For part (b), we can assume that the ratio $-\frac{V_1}{I_1}$ is equal to 200Ω, since that would be the case if a voltage source with input impedance 200Ω was connected.

Now, we have three equations:

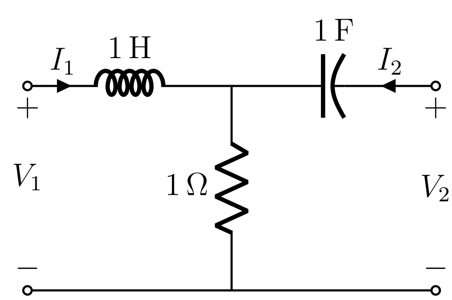
$I_1 = \frac{V_1}{1000} - 10^{-5}V_2$

$V_2 = 10000I_2 - 100V_1$

$-\frac{V_1}{I_1} = 200$

Solving this, we get $\frac{V_2}{I_2} = \boxed{8576\Omega}$.

3.



Consider the two-port network shown on the left. Find its g-parameters.

Solution:

Set $I_2 = 0$ and connect a voltage source V_1 to port 1. This will give $g_{11} = \frac{1}{s+1}$ and $g_{21} = \frac{1}{s+1}$.

Now set $V_1 = 0$ and connect a current source I_2 to port 2. This will give $g_{12} = \frac{-1}{s+1}$ and $g_{22} = \frac{s^2+s+1}{s(s+1)}$

This gives us the g-matrix:

$$\boxed{\begin{bmatrix} \frac{1}{1+s}\Omega^{-1} & \frac{-1}{1+s} \\ \frac{1}{1+s} & \frac{s^2+s+1}{s(1+s)}\Omega \end{bmatrix}}$$

Alternate Method:

Writing KVL across the $1H$ inductor and 1Ω capacitor loop:

$V_1 = sI_1 + 1(I_1 + I_2)$

Writing KVL across the $1F$ capacitor and 1Ω capacitor loop:

$V_2 = \frac{1}{s}I_2 + 1(I_1 + I_2)$

Solving this, we get the following equations:

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{1+s} & \frac{-1}{1+s} \\ \frac{1}{1+s} & \frac{s^2+s+1}{s(1+s)} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

This gives us the g-matrix:

$$\boxed{\begin{bmatrix} \frac{1}{1+s}\Omega^{-1} & \frac{-1}{1+s} \\ \frac{1}{1+s} & \frac{s^2+s+1}{s(1+s)}\Omega \end{bmatrix}}$$

4. Find the z and g parameters of a network if the T parameters are

$$T = \begin{bmatrix} 10 & 1.5\Omega \\ 2S & 4 \end{bmatrix}$$

Solution: From the T matrix A = 10, B = 1.5, C = 2 and D = 4. $|T| = 37$

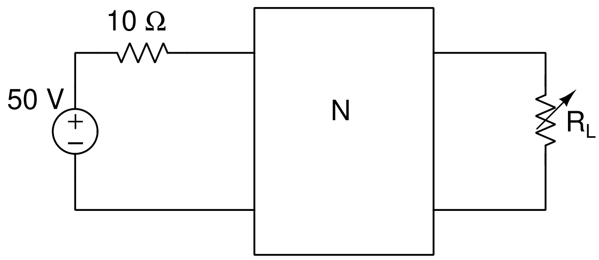
$$Z = \begin{bmatrix} \frac{A}{C} & \frac{|T|}{C} \\ \\ \frac{1}{C} & \frac{D}{C} \end{bmatrix} = \begin{bmatrix} 5 & 18.5 \\ 0.5 & 2 \end{bmatrix}$$

$$G = \begin{bmatrix} \frac{C}{A} & \frac{-|T|}{A} \\ \\ \frac{1}{A} & \frac{B}{A} \end{bmatrix} = \begin{bmatrix} 0.2 & -3.7 \\ 0.1 & 0.15 \end{bmatrix}$$

5. The T parameters of the network N in the figure below are

$$T = \begin{bmatrix} 10 & 1.5\Omega \\ 2S & 4 \end{bmatrix}$$

The output port is connected to a variable load resistor R_L. Find R_L for maximum power transfer. What is the maximum power transferred?



Solution: This can be solved by finding the Thevenin equivalent circuit looking from R_L. From the T matrix A = 20, B = 1.5, C = 2 and D = 4 and ,

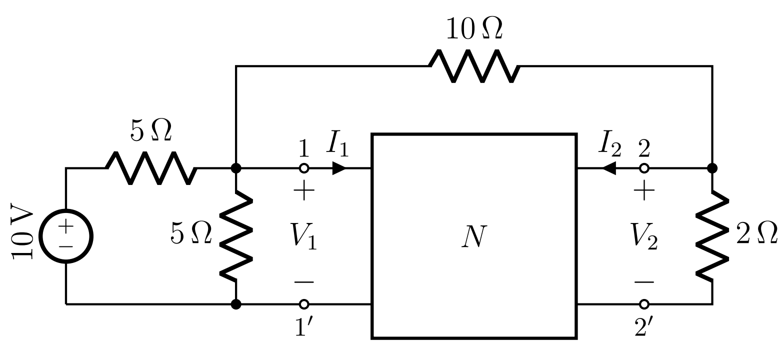
$$\begin{cases} V_1 = AV_2 - BI_2 = 20V_2 - 1.5I_2 \\ I_1 = CV_2 - DI_2 = 2V_2 - 4I_2 \end{cases} \tag{1}$$

From circuit,

$$50 - 10I_1 = V_1 \tag{2}$$

Finding $V_2 = V_{OC}$, $I_2 = 0$, on solving 1 and 2 gives $V_{OC} = 5/3V$.
Finding I_{SC}, $V_2 = 0$, on solving 1 and 2 gives $I_{SC} = 50/38.5A$.
$\mathrm{R}_{th} = 1.28\Omega$, and $R_L = 1.28\Omega$. and $P_{max} = \frac{V^2}{4R} = 0.54\mathrm{W}$

6. A resistive symmetric two-port network N is shown on the right. It was observed that $y_{11} = 0.2\,\mathrm{S}$ and $y_{12} = -0.05\,\mathrm{S}$. Find the port voltages.



Solution:

Since the network is symmetric, $y_{21} = y_{12}$ and $y_{22} = y_{11}$. The equations at node 1 and 2 are

$$2 = V_1\left(\frac{1}{2.5}+\frac{1}{10}+y_{11}\right) - V_2\left(\frac{1}{10}+y_{12}\right)$$
$$0 = V_1\left(-\frac{1}{10}+y_{21}\right) + V_2\left(\frac{1}{10}+\frac{1}{2}+y_{22}\right)$$

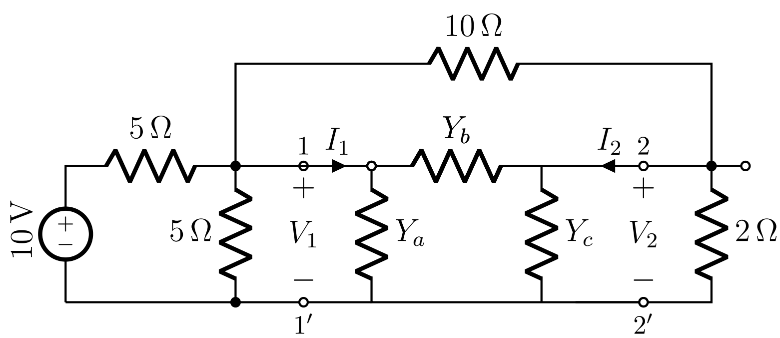
Substituting h-parameters, we get

$$2 = V_1\left(\frac{1}{2.5}+\frac{1}{10}+0.2\right) - V_2\left(\frac{1}{10}-0.05\right)$$
$$0 = V_1\left(-\frac{1}{10}-0.05\right) + V_2\left(\frac{1}{10}+\frac{1}{2}+0.2\right)$$

On solving $V_1 = 2.97V$ and $V_2 = 0.558V$

Alternate Method:

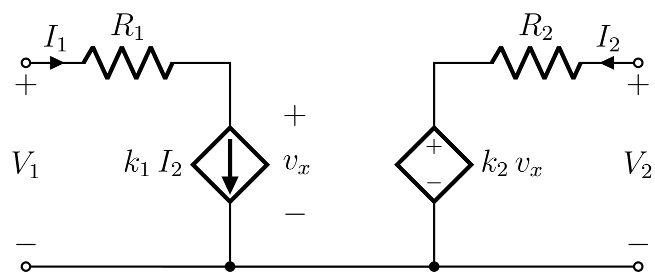
Since the two-port network is symmetric and Y parameters are given it can be replaced by equivalent π network. $Y_{11} = Y_{22}$



$Y_a = Y_{11} + Y_{12} = 0.15; R_a = 20/3\Omega$
$Y_b = -Y_{12} = 0.05; R_b = 20\Omega$
$Y_c = Y_{11} + Y_{12} = 0.15; R_c = 20/3\Omega$

On solving $V_1 = 2.97V$ and $V_2 = 0.558V$

7.



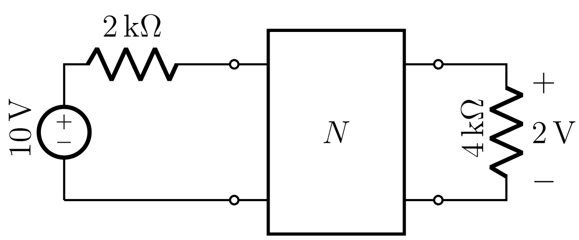
Consider the two-port network shown on the left. Find the condition that k_1, k_2, R_1, and R_2 should satisfy for the network to be reciprocal.

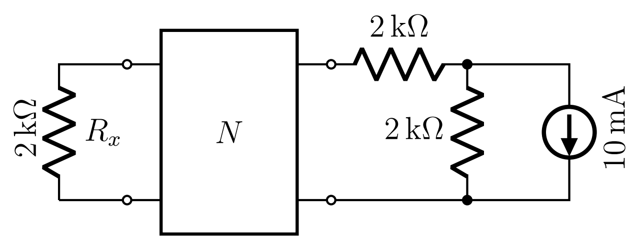
Solution:

Finding the g parameters : $g_{11} = 0$, $g_{21} = k_2$, $g_{12} = k_1$ and $g_{22} = R_2 - k_1 k_2 R_1$. For reciprocity, $g_{12} = -g_{21} \implies k_1 = -k_2$

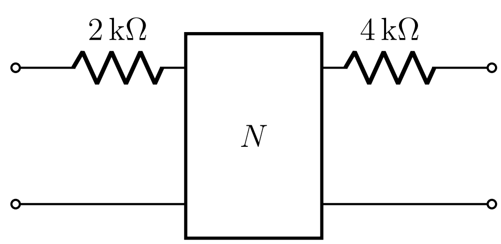
All parameters need not exist for all circuits. If $R_1 = R_2 = 0$, then y parameters will not exist, but g parameters will.

8. Consider the resistive two-port network N shown below on the left. When an independent source of 10 V was connected as shown, the measured voltage at port 2 was 2 V. The same network N is now connected in the configuration shown on the right. Find the power dissipated in R_x in this configuration.





Solution: A purely resistive network obeys reciprocity. Consider the modified two port network N_1.



In case 1, N1 is excited by voltage source 10V from the left. So, $V_1 = 10$V. The current at the output $I_2 = \frac{-2}{4\times10^3} = -0.5$mA, following the general two port sign convention.

In case 2, after source transformation at the right side, we get a 4kΩ resistor and a 20V source.
Now, $V_2 = -20$V. By reciprocity theorem, $I_1 = \frac{-20}{10} \times -0.5$ mA $= 1$ mA. So the current through 2 kΩ is 1 mA and the power dissipated is 2 mW.