EC2015 Electric Circits and Networks – Tutorial 1 Solutions

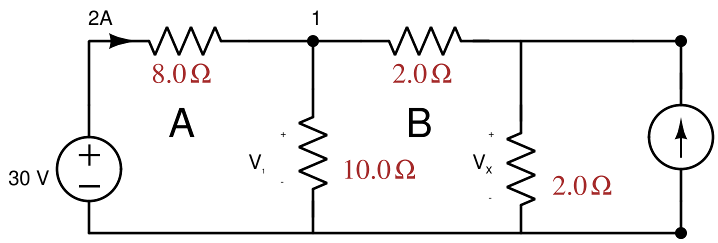
August 9th, 2024

1.

$$V_{8\Omega} = 2 \times 8 = 16V$$

Applying KVL in loop A,

$$30 - 16 - V_1 = 0$$
$$\implies V_1 = 14V$$
$$\implies I_{10\Omega} = \frac{14}{10} = 1.4A$$



Applying KCL at node 1,

$$I_{8\Omega} = I_{2\Omega} + I_{10\Omega}$$
$$\implies I_{2\Omega} = 0.6A$$

Applying KVL in loop B,

$$V_{10\Omega} - V_{2\Omega} - V_x = 0$$
$$\implies V_x = 14 - 1.2 = 12.8V$$

2. $i(t) = \cos\omega_1 t + 2\cos\omega_2 t$

a) $v(t) = 10i(t) = 10\cos\omega_1 t + 20\cos\omega_2 t$
$\therefore \omega_1, \omega_2$ are the frequency components present in $v(t)$.

b) $v(t) = 10i(t) + 0.1i^2(t)$

$$\begin{aligned}
v(t) &= 10(\cos\omega_1 t + 2\cos\omega_2 t) + 0.1(\cos\omega_1 t + 2\cos\omega_2 t)^2 \\
&= 10(\cos\omega_1 t + 2\cos\omega_2 t) + 0.1\Big[\cos^2\omega_1 t + 4\cos^2\omega_2 t + 4\cos\omega_1 t cos\omega_2 t\Big] \\
&= 10(\cos\omega_1 t + 2\cos\omega_2 t) + 0.1\Big[\frac{1+\cos 2\omega_1 t}{2} + 2(1+\cos 2\omega_2 t) + 2\cos(\omega_1+\omega_2)t + 2\cos(\omega_1-\omega_2)t)\Big] \\
&= 0.25 + 10(\cos\omega_1 t + 2\cos\omega_2 t) + 0.1\Big[\frac{\cos 2\omega_1 t}{2} + 2cos\omega_2 t + 2\cos(\omega_1+\omega_2)t + 2\cos(\omega_1-\omega_2)t)\Big]
\end{aligned}$$

$\therefore$ the frequency components in $v(t)$ are: $0, \omega_1, \omega_2, 2\omega_1, 2\omega_2, \omega_1 + \omega_2, \omega_1 - \omega_2$

3. The input, switch and the output waveforms are given below:

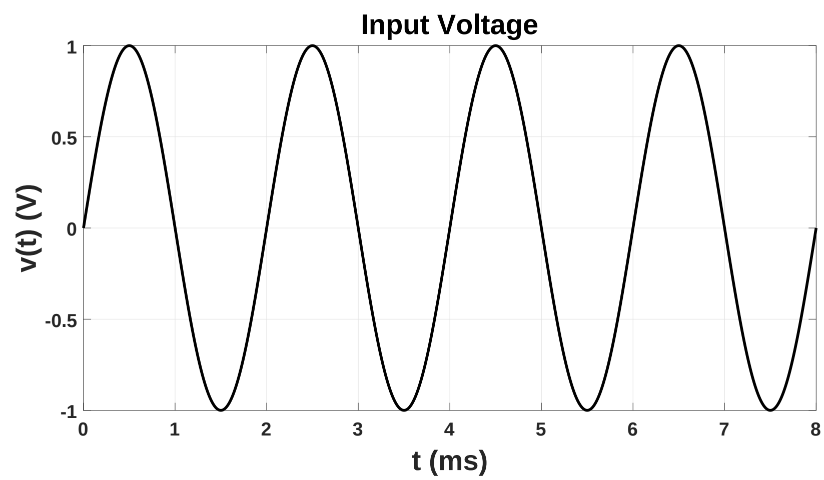


Figure 1

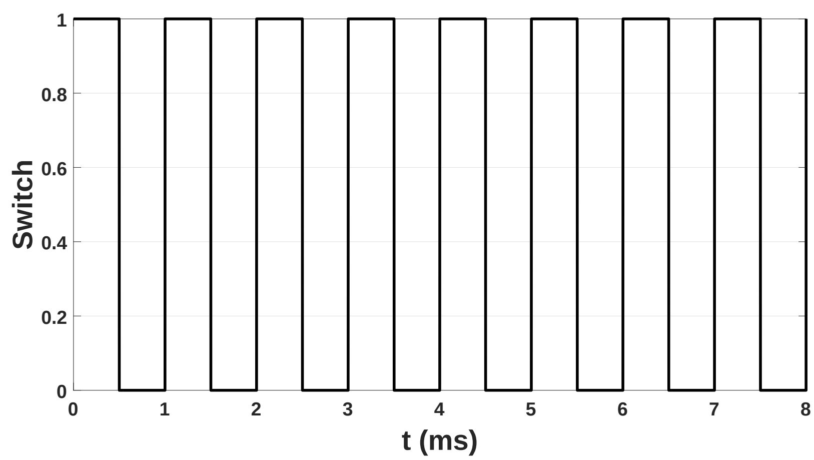


Figure 2

When switch is closed,
$i(t) = \frac{1}{10}\sin(1000\pi t)$

When switch is open,
$i(t) = \frac{1}{20}\sin(1000\pi t)$
Therefore $i(t) = \sin(1000\pi t)f(t)$, where f(t) is a square wave oscillating between 0.05 and 0.1.

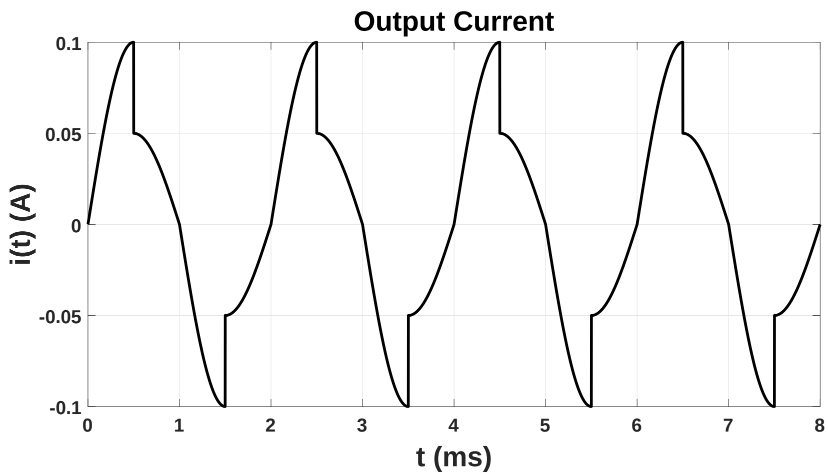


Figure 3

f(t) can be expanded as a Fourier series. Since it is a square wave, it will have only odd harmonics. Therefore the frequencies present in the output are $500 \pm k1000$ Hz and $-500 \pm k1000$ Hz , k odd

The system is linear (output scales with input), but time variant.

4. (a) $f(t) = u(t) - 2u(t-2)$

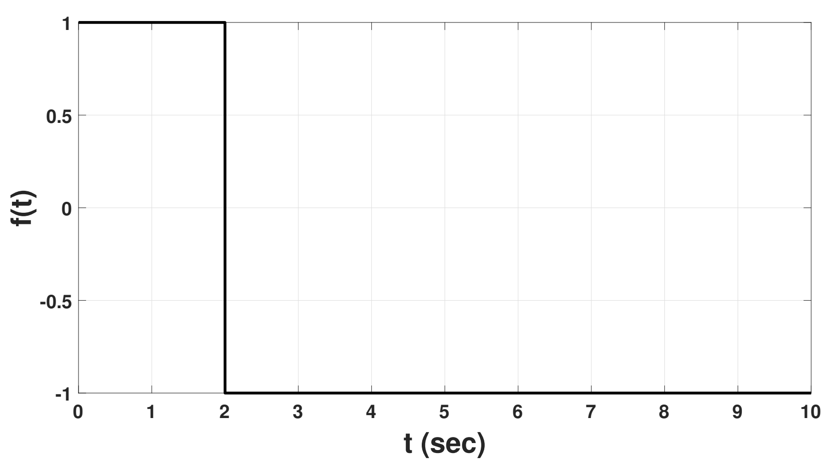


Figure 4

(b) $f(t) = \delta(t) - 2\delta(t-1) + \delta(t-2)$

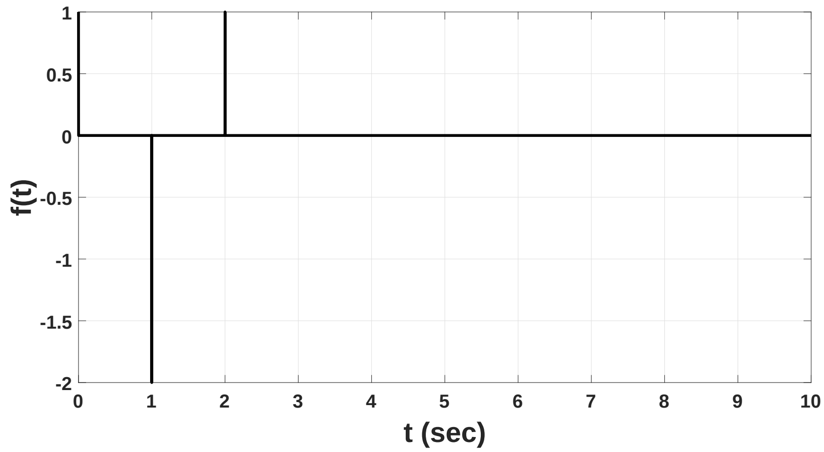


Figure 5

(c) $f(t) = e^{-2t}u(t)$

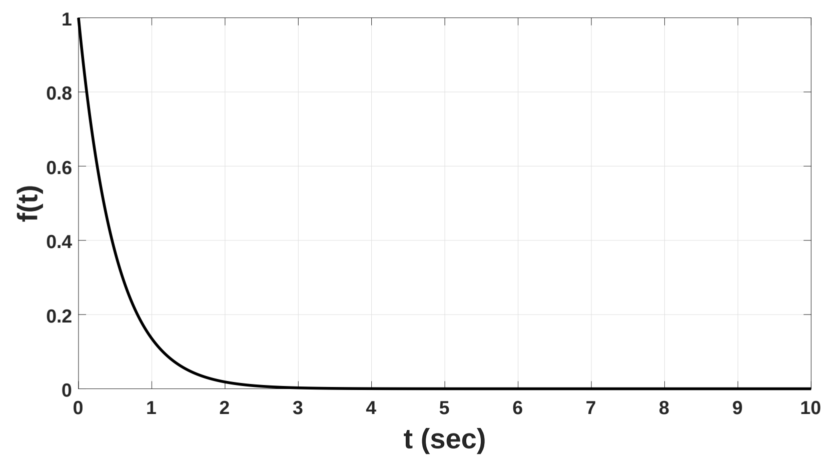


Figure 6

(d) $f(t) = \cos(t)u(t)$

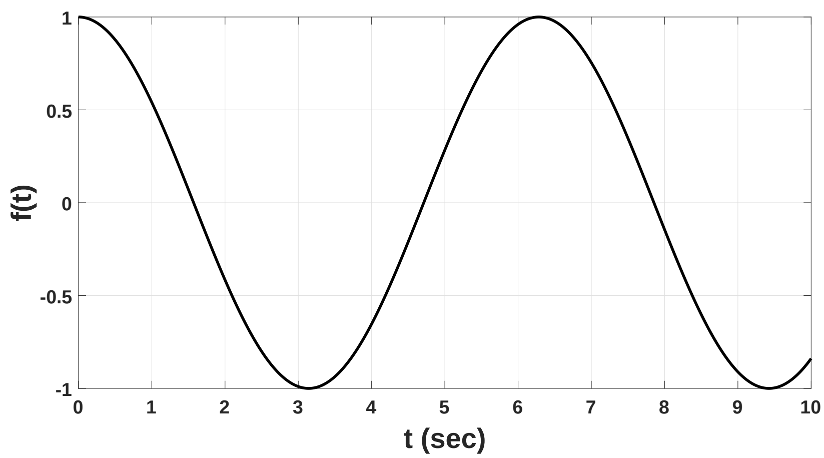


Figure 7

5. For zero initial conditions,

Capacitor voltage, $V_C(t) = \frac{1}{C}\int i(t)dt = \int i(t)dt$ (C=1F)

Inductor voltage, $V_L(t) = L\frac{di(t)}{dt} \quad = \frac{di(t)}{dt}$ (L=1H)

(a) $i(t) = u(t) - 2u(t-2)$

$V_C(t) = tu(t) - 2(t-2)u(t-2)$

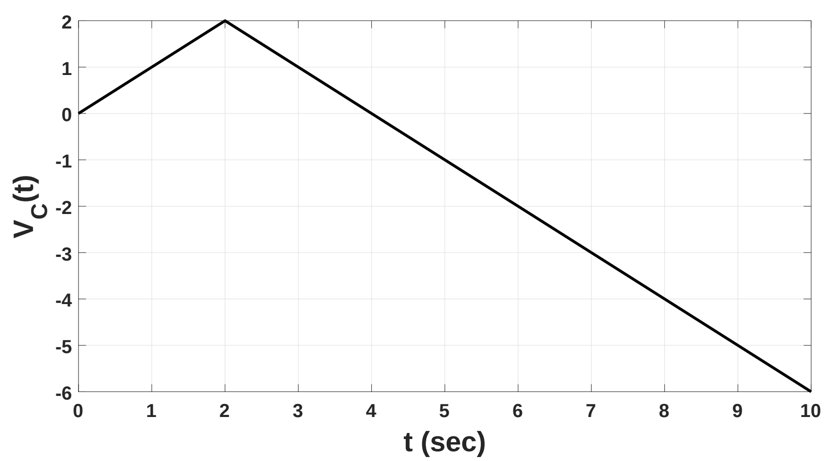


Figure 8

$V_L(t) = \delta(t) - 2\delta(t-2)$

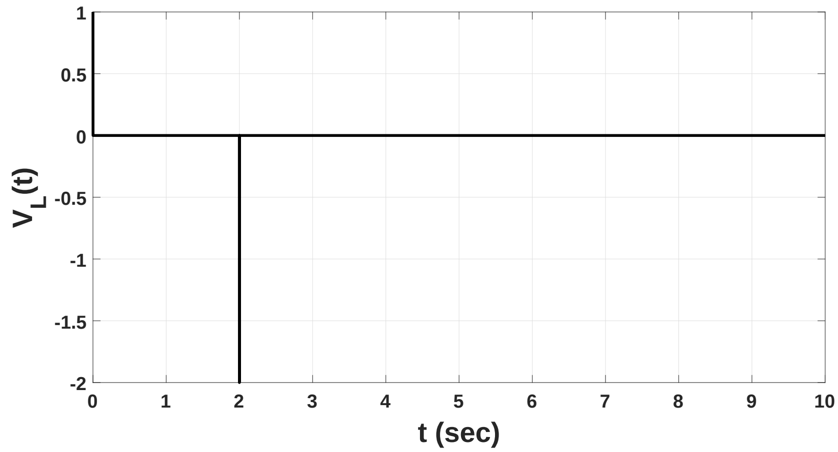


Figure 9

(b) $\quad i(t) = \delta(t) - 2\delta(t-1) + \delta(t-2)$

$V_C(t) = u(t) - 2u(t-1) + u(t-2)$

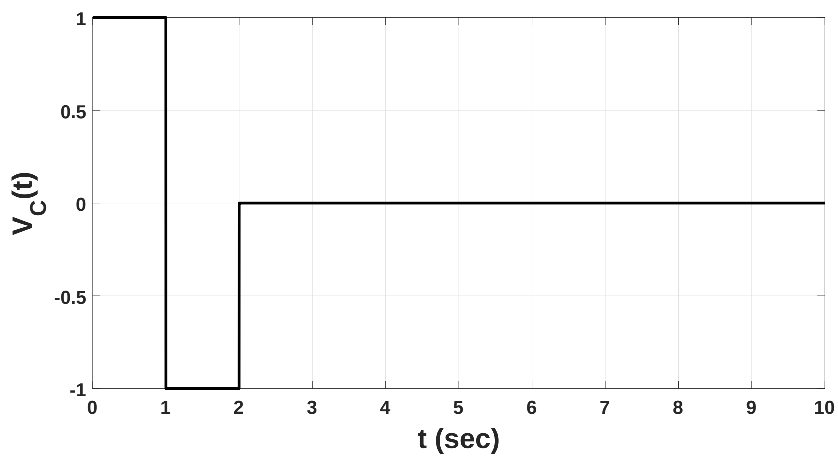


Figure 10

(c) $\quad i(t) = e^{-2t}u(t)$

$V_C(t) = 0.5(1 - e^{-2t})u(t)$
$V_L(t) = -2e^{-2t}u(t) + \delta(t)$

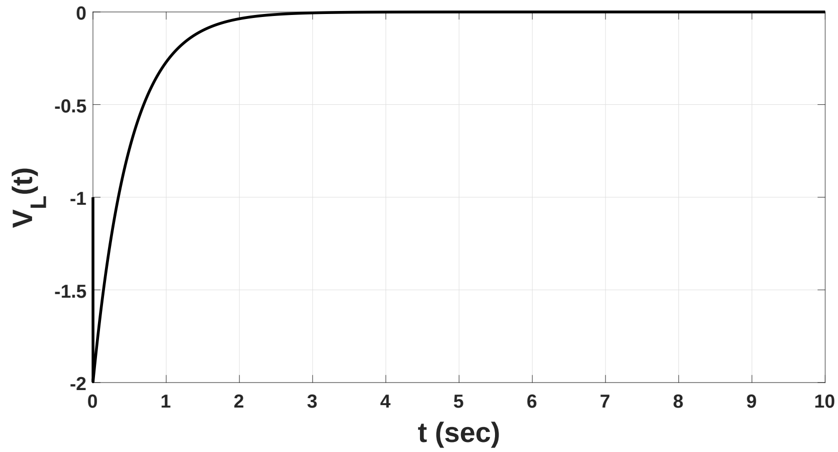


Figure 11

(d) $\cos(t)u(t)$

$V_C(t) = \sin(t)u(t)$

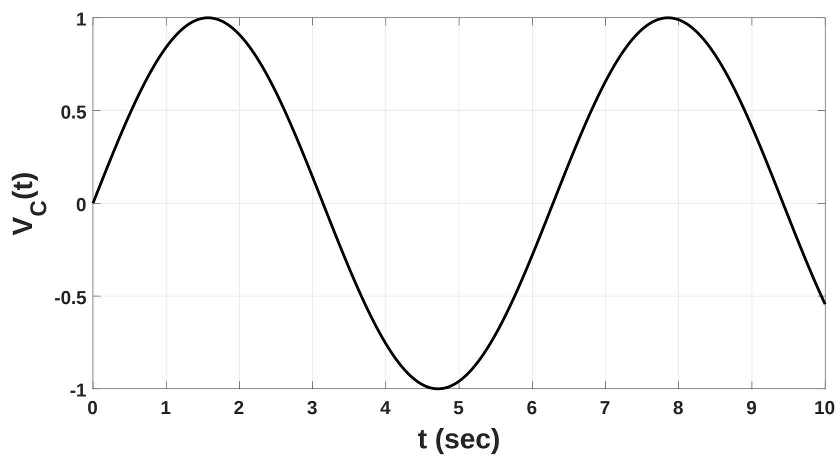


Figure 12

$V_L(t) = -\sin(t)u(t) + \delta(t)$

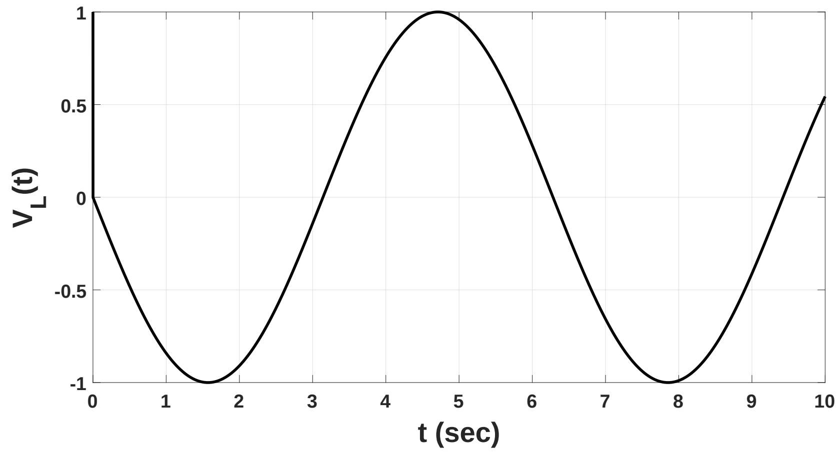
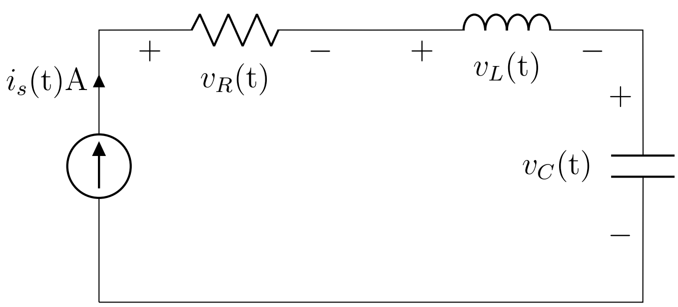
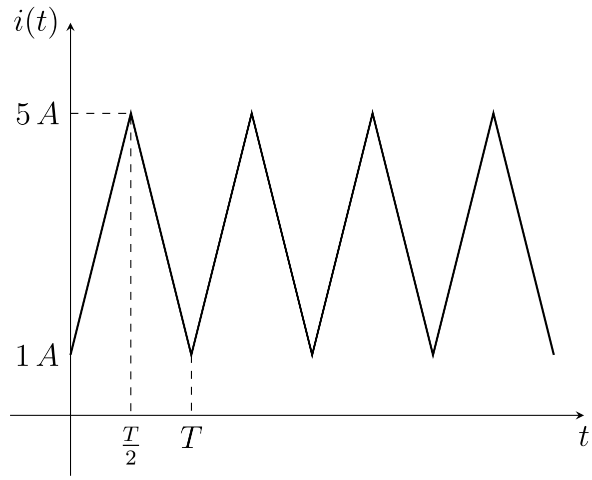


Figure 13

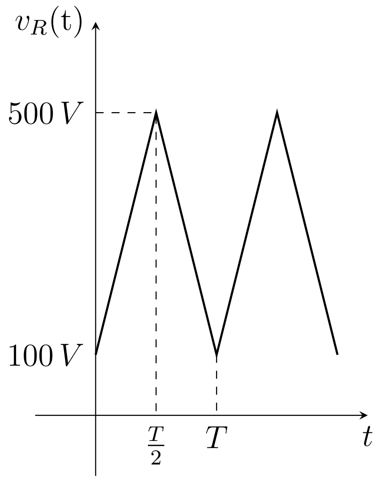
6. In the following circuit R = 100Ω, L = 1mH and C = 1μF. Determine the voltages v_R(t), v_L(t) and v_C(t) if i_s(t) is as shown below .





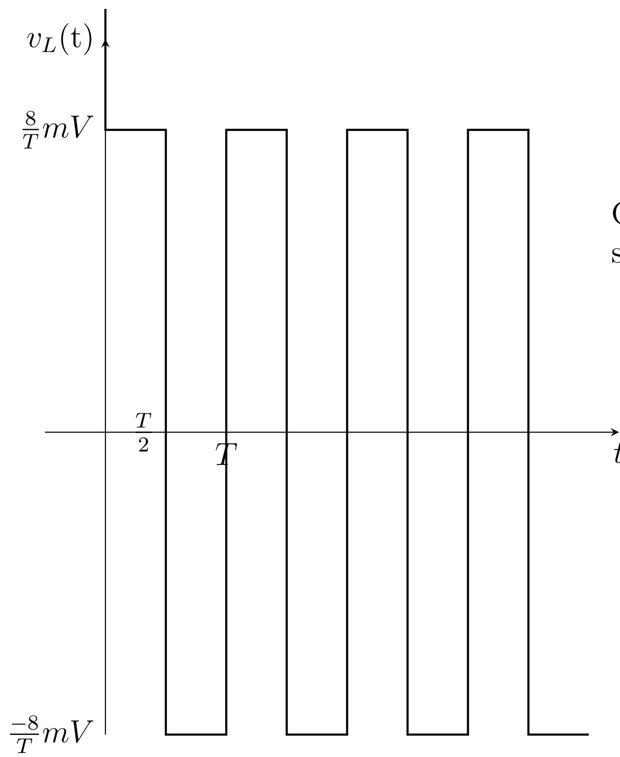
Given waveform is periodic.by considering time period as T this can be represent as

$$
\begin{aligned}
i(t) &= u(t) + \frac{8(t-nT)}{T}, \qquad nT \le t \le nT + \frac{T}{2} \\
&= u(t) + 8 - \frac{8(t-nT)}{T}, \quad nT + \frac{T}{2} \le (n+1)T
\end{aligned}
$$



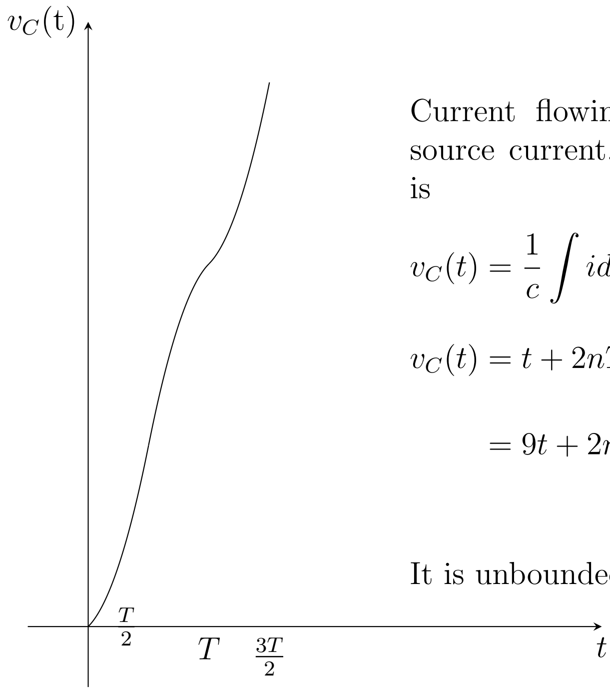
Current flowing through resistor is equal to source current.therefore voltage across resistor is

$$\begin{aligned} v_R(t) &= 100u(t) + \frac{800(t-nT)}{T}, & nT \le t \le nT + \frac{T}{2} \\ &= 100u(t) + 800 - \frac{800(t-nT)}{T}, & nT + \frac{T}{2} \le (n+1)T \end{aligned}$$



Current flowing through inductor is equal to source current.therefore voltage across inductor is

$$\begin{aligned} v_L(t) &= L\frac{di}{dt} \\ &= \delta(t) + \frac{8}{T}, & nT \le t \le nT + \frac{T}{2} \\ &= -\frac{8}{T}, & nT + \frac{T}{2} \le (n+1)T \end{aligned}$$



Current flowing through capacitor is equal to source current.therefore voltage across capacitor is

$$v_C(t) = \frac{1}{c}\int idt$$

$$v_C(t) = t + 2nT + \frac{8(\frac{t^2}{2} - nTt)}{T}, \qquad nT \leq t \leq nT + \frac{T}{2}$$
$$= 9t + 2nT - \frac{8(\frac{t^2}{2} - nTt)}{T}, \quad nT + \frac{T}{2} \leq (n+1)T$$

It is unbounded.

7. The total inductance (L_{eq}) can be calculated in the following way:

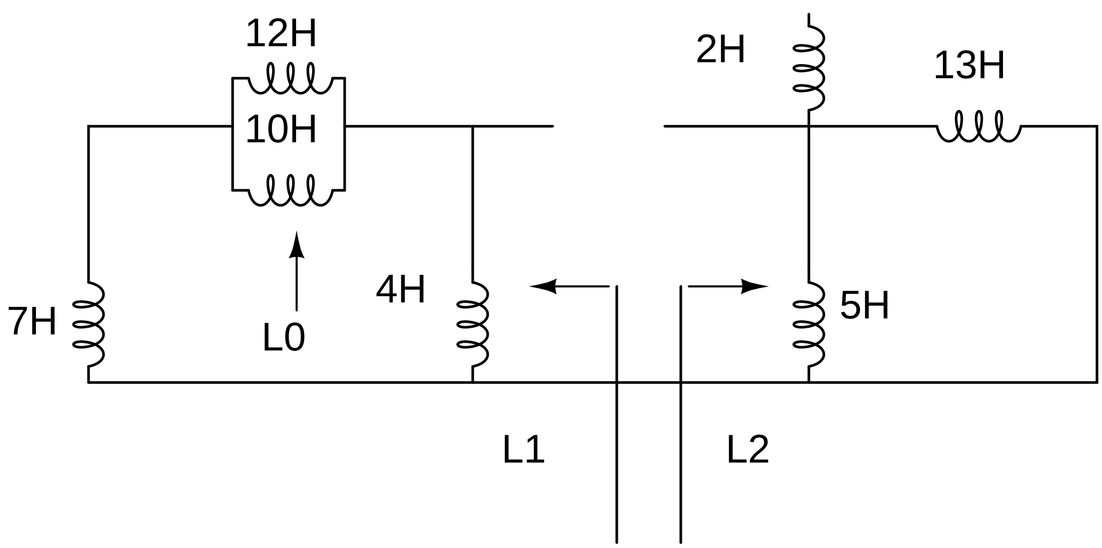
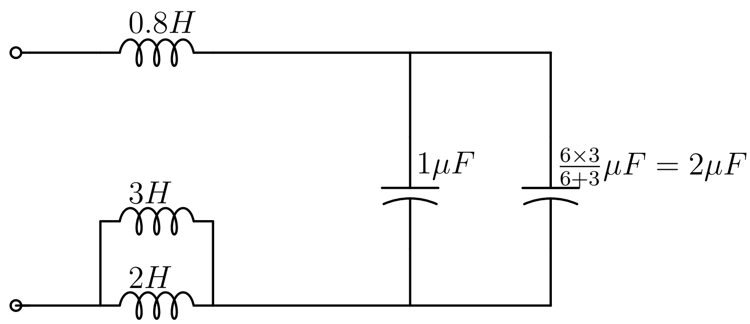


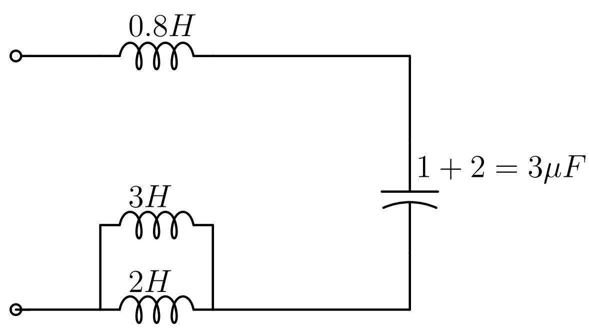
Figure 14: Q 14

$$
\begin{aligned}
L0 &= \frac{1}{\frac{1}{10}+\frac{1}{12}} = \frac{60}{11}\ H \\
L1 &= \frac{1}{\frac{1}{L0+7}+\frac{1}{4}} = \frac{548}{181}\ H \\
L2 &= \frac{1}{\frac{1}{5}+\frac{1}{12+1}} = \frac{65}{18}\ H \\
L_{eq} &= L1 + L2 = 6.64\ H
\end{aligned}
$$

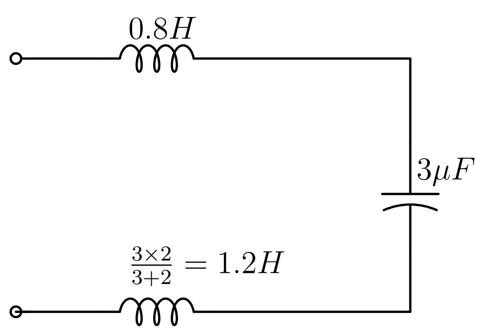
8. $6\mu F$ and $3\mu F$ are in series,



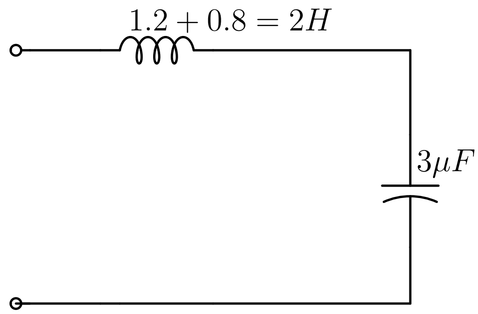
$1\mu F$ and $2\mu F$ are in parallel,



$2H$, $3H$ are in parallel,



$1.2H$, $0.8H$ are in series



9. a) Applying KCL at the left most node, we get

$$I + i_1 = i_1 + 3I = 3A$$

Applying KVL

$$V = 5V$$

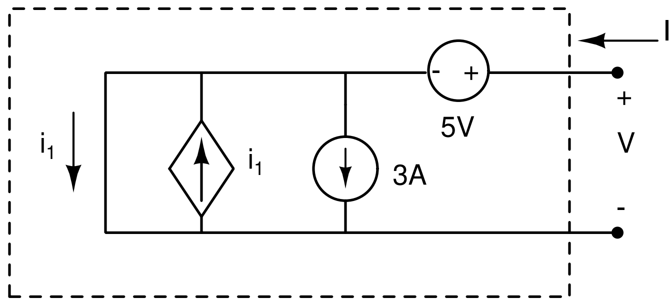


Figure 15: Question 9(a)

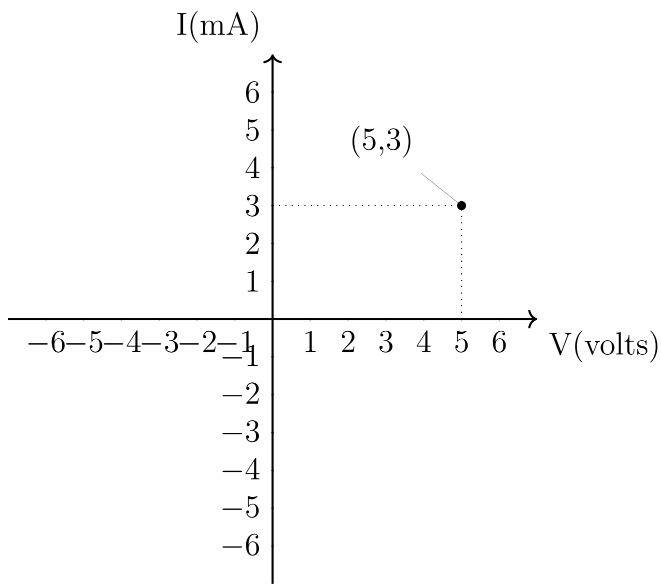


Figure 16: I-V characteristics of first box

b) Using KCL

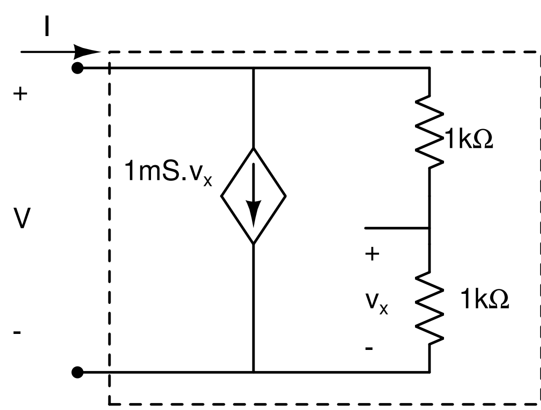


Figure 17: Question 9(b)

$$I = \frac{V_x}{1\,k} + \frac{V}{2\,k}$$

Using Ohm's law

$$\begin{aligned} V_2 = \frac{V}{2\,k} \times 1\,k = \frac{V}{2} \\ I = \frac{V}{2\,k} + \frac{V}{2\,k} \\ I = \frac{V}{1\,k} \end{aligned}$$

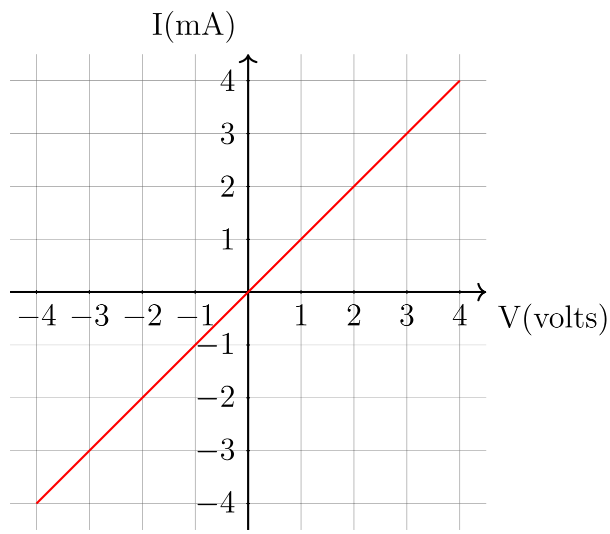
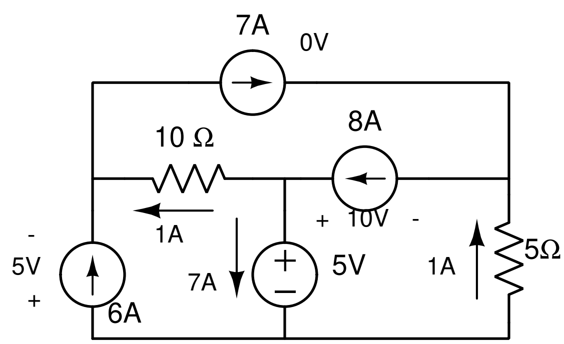


Figure 18: I-V characteristics of 2nd box

10. Using KCL and KVL, we get



$$
\begin{aligned}
P_{5\,\Omega} &= 1^2 \times 5 = 5\,W; \text{Dissipated}\\
P_{10\,\Omega} &= 1^2 \times 10 = 10\,W; \text{Dissipated}\\
P_{6\,A} &= 6 \times 5 = 30\,W; \text{Absorbs}\\
P_{7\,A} &= 7 \times 0 = 0\,W;\\
P_{8\,A} &= (-8) \times 10 = -80\,W; \text{Delivers}\\
P_{5\,V} &= 5 \times 7 = 35\,W; \text{Absorbs}
\end{aligned}
$$

For power calculations, the cuurent is positive if it is in the direction of the voltage drop.

11. Let V_1 be the voltage drop across the resistor as shown in the figure. Using Kirchoff Current

law at the node in the centre

$$\frac{V_1}{4.7} + \frac{V_1}{2.8} + \frac{V_1}{1} + 5 - 3 = 0$$
$$\implies V_1 = -1.27V$$

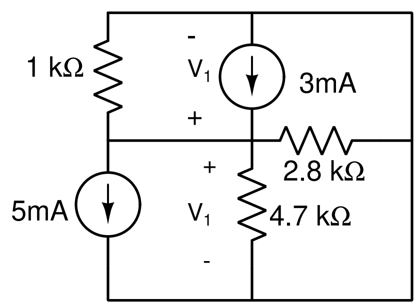
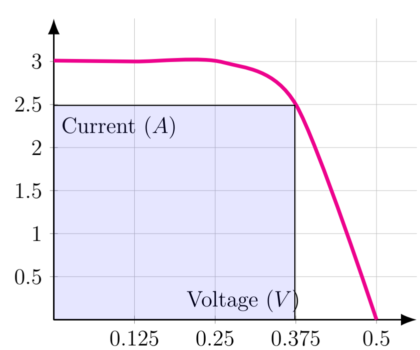


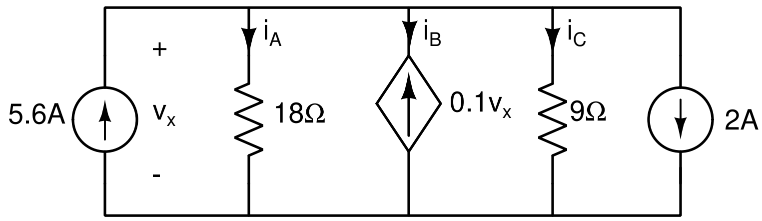
Figure 19: Question 3

$$P_{1\,k\Omega} = \frac{(1.27)^2}{1} = 1.61\,mW; \text{Dissipated}$$
$$P_{2.8\,k\Omega} = \frac{(1.27)^2}{2.8} = 0.57\,mW; \text{Dissipated}$$
$$P_{4.7\,k\Omega} = \frac{(1.27)^2}{4.7} = 0.34\,mW; \text{Dissipated}$$
$$P_{3mA} = V_1 \times (-3) = (-1.27) \times (-3) = 3.81\,mW; \text{Absorbed}$$
$$\implies \text{Power delivered is } -3.81\text{mW}$$
$$P_{5mA} = V_1 \times 5 = -6.35\,mW; \text{Generated}$$
$$\implies \text{Power delivered is } 6.35\text{mW}$$

12. (a) Short-circuit $\implies$ voltage = 0V, $I(V=0) = 3A$
(b) Open-circuit $\implies$ current = 0A, $V(I=0) = 0.5V$
(c) $P_{\max}$ = maximum area under I-V curve $= 2.5 \times 0.375 = 0.9375W$



13. Use KCL with V_x as the branch voltage.



$$
\begin{aligned}
-5.6 + \frac{V_x}{18} - 0.1V_x + \frac{V_x}{9} + 2 = 0; \\
\implies \frac{V_x}{15} = 3.6; \\
\implies V_x = 54V.
\end{aligned}
$$

$$
\begin{aligned}
\text{Now, } i_a &= \frac{V_x}{18} = 3A; \\
\text{and, } i_b &= -0.1V_x = -5.4A; \\
\text{and, } i_c &= \frac{V_x}{9} = 6A.
\end{aligned}
$$

14. Across V_π, resistors 3 $k\Omega$ and 15 $k\Omega$ are in parallel, $3||15 = 2.5\ k\Omega$.

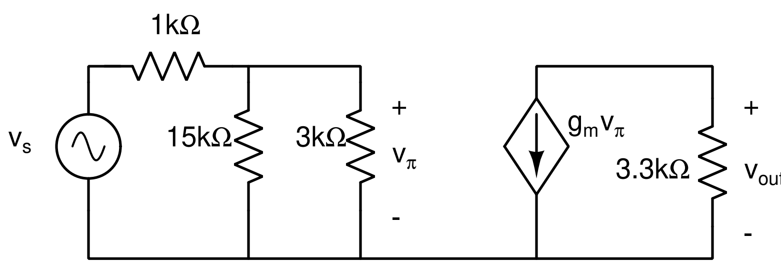


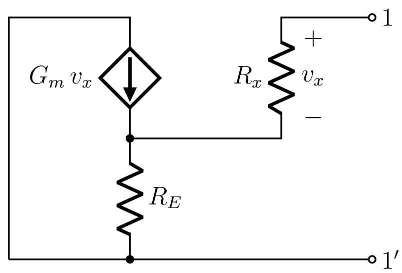
Figure 20: Question 6

Using Voltage divider rule:

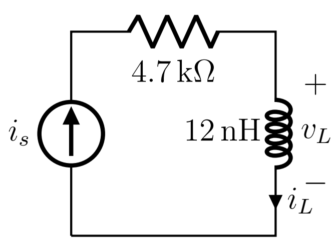
$$V_\pi = V_s \times \frac{2.5}{2.5 + 1} = \frac{5}{7} V_s$$

$$\text{Now, } V_{\text{out}} = -g_m V_\pi \times 3.3k\Omega$$
$$\implies V_{\text{out}} = (-322mS) \times \frac{5}{7} V_s \times 3.3k\Omega$$
$$\implies V_{\text{out}} = -759 \times V_s$$
$$\implies V_{\text{out}} = -759 \times (6\cos 2300t\ \mu V)$$
$$\implies V_{\text{out}} = -4554\cos 2300t\ \mu V = -4.554\cos 2300t\ mV$$

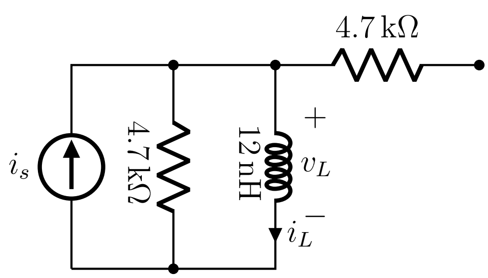
15.



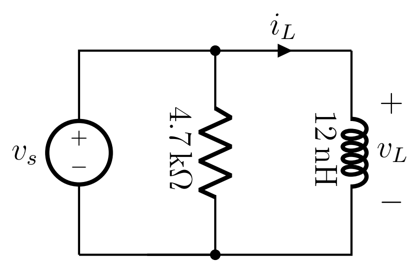
If we connect, a current source with $I = 1A$ between terminals 1 and 1′,
$v_x = R_x \quad (1)$
Current through R_E, $\quad I_{R_E} = G_m v_x + \frac{v_x}{R_x}$
Voltage across R_E, $\quad V_{R_E} = \left(G_m + \frac{1}{R_x}\right) R_E v_x$
$V_{11'} = V_{R_E} + v_x = R_x + (1 + G_m R_x) R_E \quad (\because$ From (1))
$R_{\text{eq}} = V_{11'} = 230\,\text{k}\Omega \quad (\because I_{11'} = 1A)$



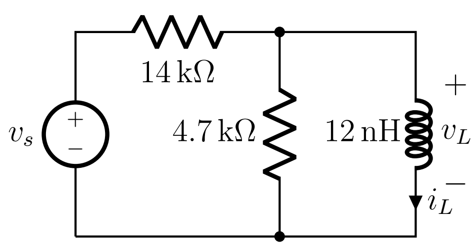
(i)



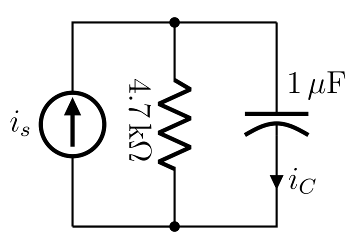
(ii)



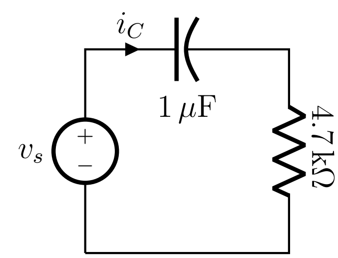
(iii)



(iv)



(v)



(vi)

16. (a) In steady state, $v_L = 0$ and $i_C = 0$
(i) $i_s = i_L = 1\text{mA}$
(ii) $i_L = 1mA$
(iii) Circuit does not reach steady-state, i_L is indeterminate
(iv) $i_L = \frac{2}{14000} = \frac{1}{7}$ mA
(v) and (vi) $i_C = 0$