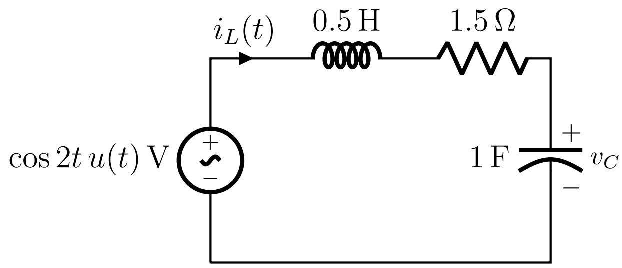
EE2015: Electric Circuits and Networks

Tutorial 8

(4^{th} and 11^{th} October 2024)

1. For the circuit shown on the right find $v_C(t)$ for $t \geq 0$. The initial conditions are: (i) $v_C(0^-) = 1\,\text{V}$, and (ii) $i_L(0^-) = 2\,\text{A}$.



Solution:

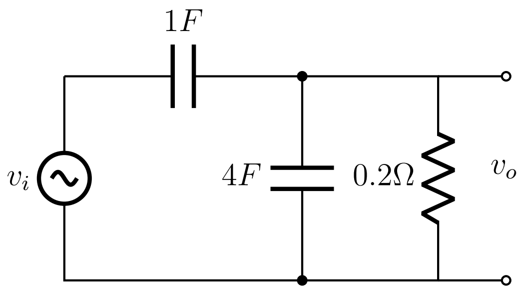
Solving the circuit in frequency domain, the initial conditions can be neglected.

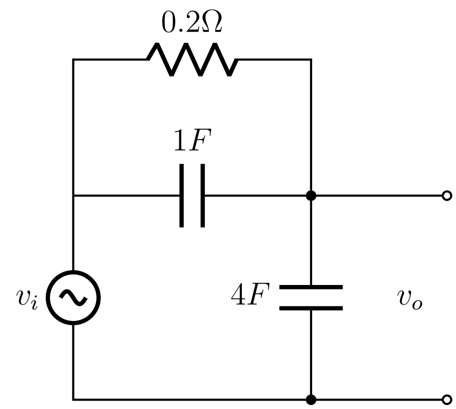
If the current flowing in the circuit is i, the voltage across the inductor $v_L = i\measuredangle 90°$, the resistor $v_R = 1.5i$ and the capacitor $v_C = 0.5\measuredangle{-90°}$. The input voltage can be represented as $1\measuredangle 0$ V, and $v_L + v_R + v_C = 1\measuredangle 0$. Solving this, we get $v_C = 0.316\measuredangle{-108.43°}$. In time domain, $v_C(t) = 0.316sin(2t - 8.43°)$.

2. The circuit (same as Q6 in Tutorial-6) shown below is excited by $v_i(t)$, where $v_i(t) = \sum_{k=0}^{\infty} x(t - kT_0)$. The waveform $x(t)$ is defined as follows:

$$x(t) = \begin{cases} 10\,V & 0 < t < T_0/2 \\ -5\,V & T_0/2 < t < T_0 \end{cases}$$

where $T_0 = 2s$.
a) Sketch $v_o(t)$ after steady-state has been reached in both circuits and mark the important points. b) Sketch $v_o(t)$, if T_0 is increased or decreased by a factor of 10, keeping the component values the same.





Solution:

a) Recall the differential equation we obtained for this (from tutorial 6):

$\frac{d(V_o)}{dt} + \frac{V_0}{R(C_1+C_2)} = \frac{C_1}{(C_1+C_2)}\frac{d(V_{in})}{dt}$

$\frac{d(V_o)}{dt} + \frac{V_0}{1} = \frac{1}{5}\frac{d(V_{in})}{dt}$

This gives us the time constant, $\tau = R(C_1 + C_2) = 1s$.

Now, assume that the circuit has reached steady state at $t = 0$. *Note: this is just for convenience, actually steady state will be reached only at $t = \infty$, however we assume this happens at some initial time say $t = 0$.*

Assume that steady state has been achieved at $t = 0^-$ and let the voltage at 0^- be aV. Now, at $t = 0^+$, due to the impulse current, the voltage increases to $a + 3$V.

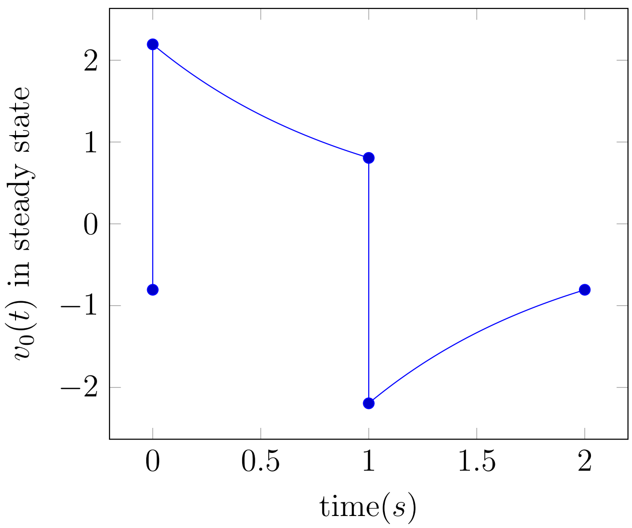
Now, since there is a 0.2Ω resistor in parallel with the $4F$ capacitor, v_0 is going to decay exponentially to 0. Now, at $t = \frac{T_0}{2}^-$, the voltage $v_0(\frac{T_0}{2}^-) = (a+3)e^{-\frac{T_0}{2\tau}}$.

At $t = \frac{T_0}{2}$, an impulse current flows again, and the voltage at $\frac{T_0}{2}^+$ becomes $v_0(\frac{T_0}{2}^+) = (a+3)e^{-\frac{T_0}{2\tau}} - 3$.

Again, this will decay exponentially to 0, and hence $v_0(T_0^-) = ((a+3)e^{-\frac{T_0}{2\tau}} - 3)e^{-\frac{T_0}{2\tau}}$.

Now, since the input is periodic, the output in the steady state will be periodic with period T_0, and hence $v_0(T_0^-) = v_0(0^-)$. Thus, $a = ((a+3)e^{-\frac{T_0}{2\tau}} - 3)e^{-\frac{T_0}{2\tau}}$.

For the first part, $T_0 = 2s$. Using this, we get $a = \frac{3e^{-2}-3e^{-1}}{1-e^{-2}}V = -0.806V$.



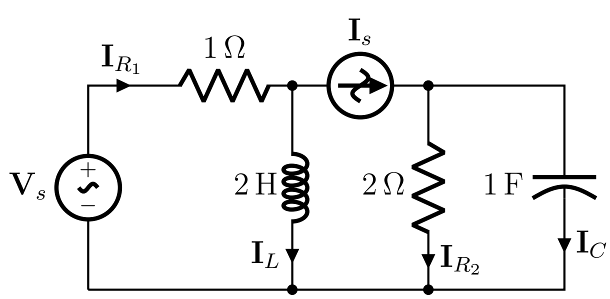
b) **Increasing T_0 by a factor of 10:**

Now, $T_0 = 20s$, we notice that the corresponding value of $a = -0.00013V$, or $-0.13mV$. This is expected because as we increase the time period, the signal has more time to decay and settles at a lower value.

Decreasing T_0 by a factor of 10:

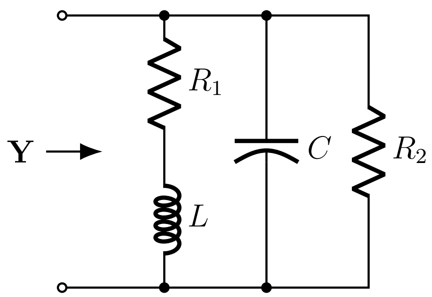
Now, $T_0 = 0.2s$, we notice that the corresponding value of $a = -1.42V$. This is expected because as we decrease the time period, the signal has lesser time to decay and settles at a higher value.

3. a) For the given RLC circuit, determine I_s and $i_s(t)$ if both sources are operating at $\omega = 2\,\text{rad/s}$, and $I_C = 2\measuredangle 28°$ A. b) Now let $\omega = 1\,\text{rad/s}$, $I_C = 2\measuredangle 28°$ A and $I_L = 3\measuredangle 53°$ A. Calculate (i) I_s, (ii) V_s, and (iii) $i_{R_1}(t)$. c) Find the Thevenin impedance as seen from the terminals of the current source. What should you assume before proceeding to calculate Z_{Th}?

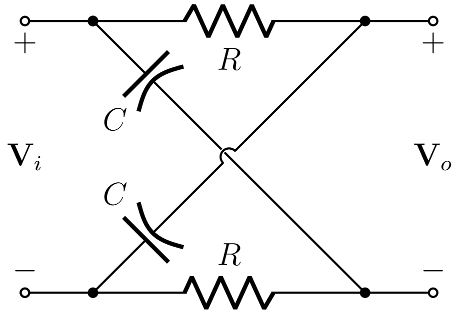


Solution:
a) For $\omega = 2\text{rad/s}$ and Ic $= 1\measuredangle 28°$, I(s) $= 2\measuredangle 14°$
b) For $\omega = 21\text{rad/s}$ and $I_C = 2\measuredangle 28°$ A and $I_L = 3\measuredangle 53°$ A ,I(s) $= 2.23\measuredangle 1.44°$, V(s)=$6.11\measuredangle 97°, I_{R1} = 4.73\measuredangle 31.25°$
c) Zth $= \omega j||1 + \frac{-j}{c\omega}||2$

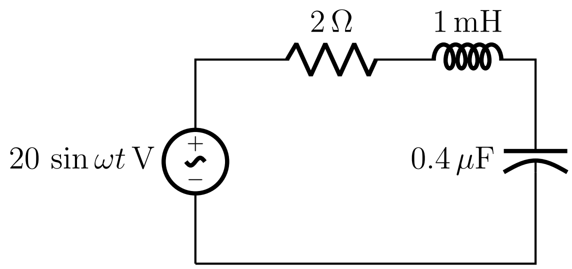
4. In the circuit given on the right, $R_1 = 1\,\text{k}\Omega$ and $C = 2.533\,\text{pF}$. Determine the inductance value that will make the circuit's resonance frequency as $1\,\text{MHz}$.



5. In the circuit shown on the left, $R = 1\,\text{k}\Omega$ and $C = 1\,\text{nF}$. Plot the magnitude and phase of $\dfrac{\mathbf{V}_o}{\mathbf{V}_i}$ versus frequency. For the magnitude response use a log-log plot, whereas, for the phase plot use a linear scale for y-axis and the log scale for the x-axis.



6. Consider the circuit shown on the right. a) Find the resonant frequency (ω_0) and the half-power frequencies (ω_1, ω_2). b) Calculate the Q-factor and bandwidth. c) Determine the amplitude of the current at ω_0, ω_1, and ω_2.



7. a) Derive the transfer function of the network whose magnitude Bode plot is shown below. Assume that the poles and zeros are either on the imaginary axis or in the left-half plane. b) Draw the phase Bode plot of the network