EE2025: Engineering Electromagnetics

Tutorial 4: Antennas

July-Nov 2024

- Use appropriate approximations and work with reasonable assumptions wherever necessary.
- Good familiarity with the Maxwell's equations helps.
- Visualizing the fields in three-dimensional space also helps.

Calculating Field from Potential

1. Recall the expression for $\mathbf{A}(\mathbf{r})$ for a Hertz dipole antenna from class notes. By converting the direction of $\mathbf{A}$, which is $\hat{z}$, into spherical polar coordinates, write out $\mathbf{A}$ fully in spherical polar coordinates. Next, use the definition of $\mathbf{A}$ to derive the magnetic field $\mathbf{H}$ using the expression for curl in spherical coordinates, i.e. $\nabla \times \mathbf{A} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix}$.

2. The magnetic vector potential (in phasor form) at point $P(r, \theta, \phi)$ due to a small antenna located at the origin is given by

$$\mathbf{A_s} = \frac{50e^{-j\beta r}}{r}\hat{x}$$

where $r^2 = x^2 + y^2 + z^2$. Find $E(r, \theta, \phi, t)$ and $H(r, \theta, \phi, t)$ at the far field.

Power and Radiation Patterns in a Hertz Dipole

3. A magnetic field strength of $5\,\mu$A/m is required at a point on $\theta = \pi/2$, 2 km from an antenna in air. Neglecting ohmic loss, how much power must the antenna transmit if it is a Hertzian dipole of length $\lambda/25$?

4. Consider a Hertzian dipole antenna placed at the origin, two people A and B are communicating over the cellphone and are standing as shown in the figure. How would you orient your dipole antenna such that the users receive maximum signal strength? Calculate the normalized power of the radiated electric field (far field) in the following planes (try to plot also):

   - $\phi = 0°$
   - $\theta = 90°$

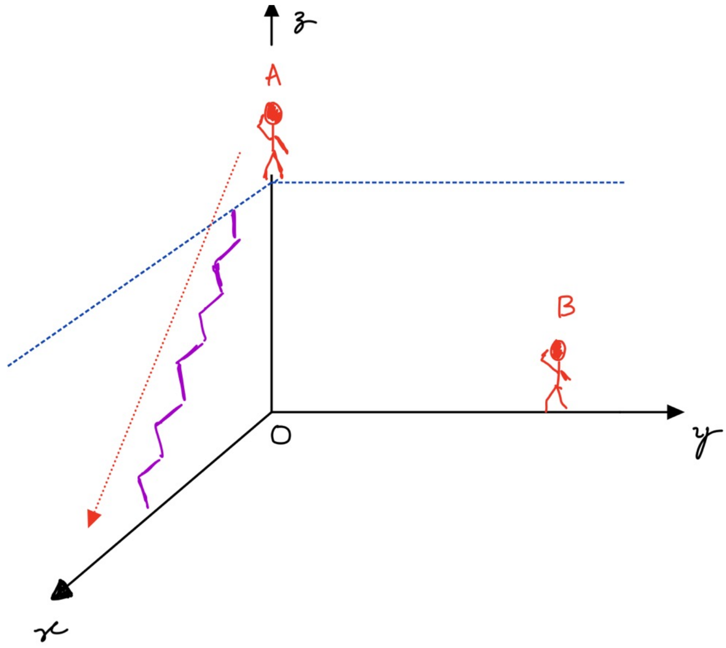
   What happens when one user starts moving towards the z-axis?

5. A Hertzian dipole antenna is 10 mm long and carries a current of 2 A. The dipole is used for cellular telephone communication and radiates at 900 MHz.

   (i) Calculate the total power radiated by the dipole in free space.

   (ii) If the dipole antenna is immersed in water ($\varepsilon_\mathrm{r} = 81, \sigma = 0, \mu = \mu_0$), find out the required dipole current so as to maintain the same radiated power as in free space.

   (iii) Calculate the ratio of the radiation resistance in air and in water.



6. **Lorentz Reciprocity Theorem** Consider a volume containing two sets of sources, J_1 and J_2, which each produce fields E_1, H_1 and E_2, H_2, respectively, as shown in Figure 1. Starting from

$$\boldsymbol{\nabla} \cdot (\mathbf{E_1} \times \mathbf{H_2} - \mathbf{E_2} \times \mathbf{H_1}),$$

show that Equation (1) is true for any antenna.

$$\iiint_V \mathbf{J_1} \cdot \mathbf{E_2} dV = \iiint_V \mathbf{J_2} \cdot \mathbf{E_1} dV \tag{1}$$

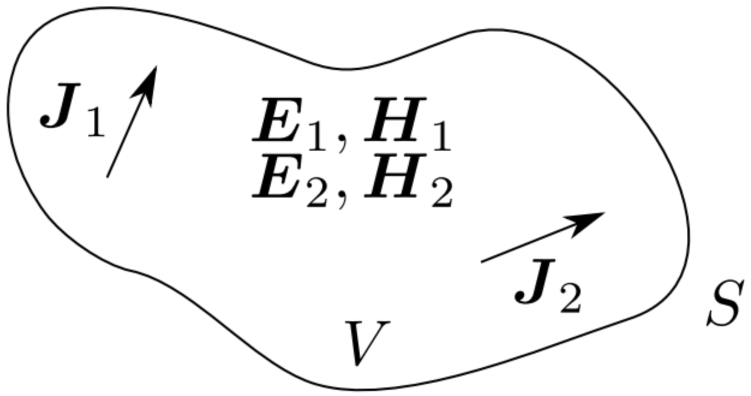


Figure 1: Problem 6

7. A rotating electric dipole can be thought of as the superposition of two oscillating dipoles, one along the x-axis, and the other along the y-axis, with the latter out of phase by 90°.

$$\mathbf{p} = p_0[cos(\omega t)\hat{\mathbf{x}} + sin(\omega t)\hat{\mathbf{y}}]Cm$$

(1) Find the fields of a rotating dipole.

(2) Find the Poynting Vector.

(3) Calculate the intensity and sketch the intensity profile.

Directionality in Antennas

8. A $1m$ long dipole is excited by a $5MHz$ current with an amplitude of $5A$. At a distance of $2km$, what is the power density radiated by the antenna along $\theta = 90°$ (broadside direction)?

9. The power radiated by a lossless antenna is 10 watts. The directional characteristics of the antenna is represented by the radiation intensity of $U(\theta, \phi) = B_0 \cos^3 \theta$ (W/unit solid angle) $0 \leq \theta \leq \pi/2$ $0 \leq \phi \leq 2\pi$ the directivity of the antenna (dimensionless and in dB).

10. Calculate the directivity, total power radiated and radiation resistance of an half-wave dipole (length $= \lambda/2$) antenna with far fields given below:

$$E_\theta \simeq \frac{j\eta I_0 e^{-jkr}}{2\pi r}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]$$

$$H_\phi \simeq \frac{jI_0 e^{-jkr}}{2\pi r}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right]$$

Hint: $\int_0^\pi \frac{\cos^2(\frac{\pi}{2}\cos\theta)}{\sin\theta} d\theta = 1.21883$

11. A thin small wire is placed just above an infinitely large ground plane at z=0 and is fed with an RF source as shown in Fig.2.

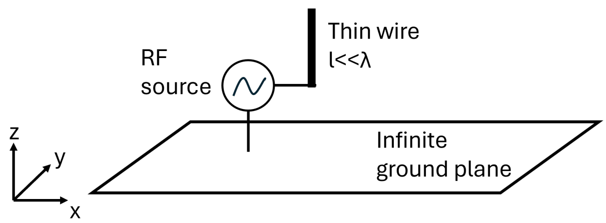


Figure 2: isotropic sources

Can you write the expressions for electric and magnetic fields at any arbitrary far-field point (r, θ, ϕ) for this case using the knowledge of fields derived for hertizian dipole that were derived? (Hint: Use the concept of Image theory). Can you think what would such an antenna be called and mention it's applications?

12. An antenna has been designed as a half wavelength dipole for use with a TV transmitter at 600 MHz (UHF channel 35) and the transmitter supplies 50 kW to the antenna. The antenna is 6 mm thick and made of aluminium with conductivity $\sigma = 3 \times 10^7$ S/m. Consider the radiation resistance of dipole (R_{rad}) is $73.08\,\Omega$ and the current is uniform. Calculate:
(a) The radiated power at 600 MHz
(b) The efficiency of the antenna ($eff = \frac{P_{rad}}{P_{in}} = \frac{R_{rad}}{R_{rad}+R_{in}}$)

13. Four isotropic sources are placed along the z-axis as shown in Fig.3. Assuming that excitations of elements (current fed to the elements) 1 and 2 are $+1$ and the excitations of elements 3 and 4 are -1 (180^0 out of phase with 1 and 2). Find
(a) The array factor in simplified form
(b) All the nulls when $d = \frac{\lambda}{2}$.

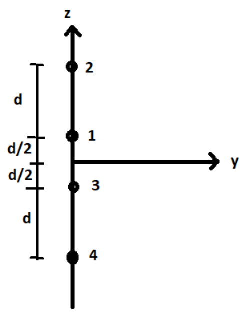


Figure 3: isotropic sources