Group Assignment-3

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Q1

Q1.1

From the given fds, it is clear that i(1-1)/2 + i = n which implies that n = i(i+1)/2 . Hence, n is sum of k natural numbers where k > 1 , which makes the given functional dependencies possible.

So n = k(k+1)/2 for k>1.
When k =1 there are no functional dependencies except A_1 -> A_1 (a trivial functional dependency).

Q1.2

On appying some inference rules on given functional dependencies, we get the following

A_1 -> $A_2A_3A_4$ ..... A_n => A_1 -> $A_1A_2A_3A_4$ ..... A_n

A_2A_3 -> $A_4A_5A_6$ ..... A_nA_1 => A_2A_3 -> $A_1A_2A_3A_4$ ..... A_n

$A_4A_5A_6$ -> $A_7A_8A_9A_{10}$ ..... $A_nA_1A_2A_3$ => $A_4A_5A_6$ -> $A_1A_2A_3A_4A_5A_6A_7$ ..... A_n

: : : : :

and so on

so set of super keys is
{ A_1 , A_2A_3 , $A_4A_5A_6$, ... , $A_{t+1}A_{t+2}....A_{t+i}$ } where t = (i-1)i/2

The key (the minimal super key) is A_1 .

Q1.3

It is in first normal form since it has only simple attributes.
It is in second normal form since there are no partial functional dependencies. i.e., there is only one key and it has no non null subsets.

It is in third normal form since all attributes functionally depend on the primary key A_1
and finally it is in BCNF since for every non trivial X-> Y functional dependency in the given list , X is a superkey.

Q1.4

A_1 -> $A_2A_3A_4$ ..... A_n can be minimised to A_1 -> A_2A_3 since A_2A_3 -> $A_4A_5A_6$ ..... A_nA_1

similarly ,
A_2A_3 -> $A_4A_5A_6$ ..... A_nA_1 can be minimised to A_2A_3 -> $A_4A_5A_6$
and so on...

Hence the minimal cover is

A_1 -> A_2A_3
A_2A_3 -> $A_4A_5A_6$
$A_4A_5A_6$ -> $A_7A_8A_9A_{10}$
: : :
: : :
" " " -> $A_{t+1}A_{t+2}....A_{t+i}$

Q2

Q2.1

Given R(A_1, A_2 , A_3 , ... A_n) is a relation R with functional dependencies as follows

A_i -> A_j for all $1 \le i < j \le n$

and

A_j -> A_i for all $1 \le i > j \le n$

on expanding the given fds we get the following,

A_1 -> A_2A_3 ..... A_n
A_2 -> A_3A_4 ..... A_n
A_3 -> A_4A_5 ..... A_n
: : :
: : :
A_{n-1} -> A_n

and

A_2 -> A_1
A_3 -> A_1A_2
A_4 -> $A_1A_2A_3$
: :
: :
A_n -> A_1A_2 ..... A_{n-1}

using inference rules

A_1 -> A_2A_3 ..... A_n and A_1 -> A_1 => A_1 -> A_1A_2 ..... A_n

A_2 -> A_3A_4 ..... A_n and A_2 -> A_1 and A_2 -> A_2 => A_2 -> A_1A_2 ..... A_n
: : : :
: : : :
A_{n-1} -> A_n and A_{n-1} -> A_1A_2 ..... A_{n-2} and A_{n-1} -> A_{n--1} => A_{n-1} -> A_1A_2 ..... A_n

we infer the following fds after expanding given fds.

Since A_i -> A_j for all $i < j \le n$ and A_i -> A_j for all $1 \le j < i$

A_i -> A_j for all $i \ne j$ and $1 \le j \le n$

Since A_i -> A_i we can infer that A_i -> $A_1A_2A_3$ .... A_n

Hence A_i is a super key for all $1 \le i \le n$

Therefore we have A_1 , A_2 , A_3 , ... A_n each one is an individual key (minimal superkey) for R
So R has n keys.

Q2.2

R (A_1, A_2, A_3, ... A_n) is in first normal form since it has only simple attributes A_1 , A_2 , ... A_n .
R (A_1, A_2, A_3, ... A_n) is in second normal form since there are no non-prime attributes.
R (A_1, A_2, A_3, ... A_n) is in 3^{rd} normal form every key is primary there are no transitive dependecy is present with non prime attributes and since in every fd X -> Y , X is a superkey, it is in BCNF.

Q2.3

$A_1 \rightarrow A_2A_3 \ldots\ldots A_n$ minimizes to $A_1 \rightarrow A_2$ since $A_2 \rightarrow A_3A_4 \ldots\ldots A_n$
$A_2 \rightarrow A_3A_4 \ldots\ldots A_n$ minimizes to $A_2 \rightarrow A_3$ since $A_3 \rightarrow A_4A_5 \ldots\ldots A_n$
: : : : :
: : : : :
$A_{n-1} \rightarrow A_n$ remains as it is.

Hence the final functional dependencies are

$A_i \rightarrow A_{i+1}$ for $1 <= i < n$

There exists other minimal covers based on the way people minimize functional dependencies.
We don't have to normalize the relation to BCNF since it is already present in BCNF.