



---

## Exercise Sheet 9

### Optimization

**Deadline: 21.1.2019, 23:59**

---

#### Exercise 9.1 - Dropout

(3 + 0.5 + 1 + 0.5 + 1 = 6 points)

- a) Given  $n$  random variables  $X_i, i = 1, \dots, n$  which are identically distributed with positive pairwise correlation  $\rho$  and  $\text{var}(X_i) = \sigma^2$ . Show that

$$\text{var}\left(\frac{1}{n} \sum_{i=1}^n X_i\right) = \rho\sigma^2 + \frac{1-\rho}{n}\sigma^2$$

- b) Explain why Bootstrap Aggregating (Bagging) can alleviate the effect of overfitting with the help of the formula above.
- c) Why can dropout be considered as an approximation to Bagging? Explain in two or three sentences.
- d) Do we apply dropout during the inference? Justify your answer.
- e) In the lecture, we have been introduced to Dropout, one common way to implement that is inverted Dropout. Explain the idea behind inverted Dropout. [Hint: think about how dropout creates a mask drawn out of a Bernoulli distribution with probability  $p$ ].

#### Exercise 9.2 - Stochastic Gradient Descent

(0.5 + 1 + 0.5 = 2 points)

In practice, we have 3 common variations for Gradient Descent Method, namely Stochastic Gradient Descent (SGD), Batch Gradient Descent and Mini-Batch Gradient Descent. Stochastic Gradient Descent takes only one data point to update in each step. Batch Gradient Descent takes the whole dataset for updating in each step. Mini-Batch Gradient Descent compromises the two methods above and takes a subset of the whole dataset for updating in each step.

- a) Is it necessary to use learning rate decay for batch gradient descent based learning to converge? Please give your reason.
- b) Why is it important to use learning rate decay when doing stochastic gradient descent?
- c) What is the advantage of using a small batch size instead of the full set of examples in the training data?

### Exercise 9.3 - Vanishing and Exploding Gradient

(1 + 1 = 2 points)

In the lecture we've discussed some of the challenges in Neural Network Optimization. One among them is the so-called Exploding/Vanishing gradient problem. To be more specific, this problem happens only during the backward pass in training (very deep) Neural Networks.

- a) Assume that you have a 100-layer Feed Forward Neural Network with *sigmoid* activation function as non-linearities. Explain why the exploding and vanishing gradient problem occurs only during the backward pass but not the forward pass with formulas.
- b) Explain how can we avoid the problem of gradient explosion based on what you learn in the lecture.

## Submission instructions

The following instructions are mandatory. If you are not following them, tutors can decide to not correct your exercise.

- You **have to** submit the solutions of this assignment sheet as a team of 2-3 students.
- Hand in a **single** PDF file with your solutions.
- Therefore make sure to write the student ID and the name of each member of your team on your submission.
- Your assignment solution must be uploaded by only **one** of your team members to the course website.
- If you have any trouble with the submission, contact your tutor **before** the deadline.