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NNIA Winter Term 2018/2019

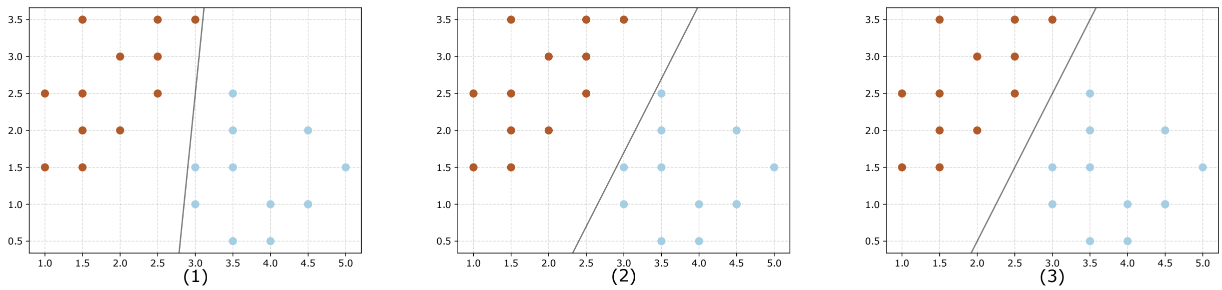
Exercise Sheet 7

Support Vector Machines & Backpropagation

Deadline: 17.12.2018, 23:59

Exercise 7.1 - Support Vector Machines (0.5 + 2.5 + 0.5 + 0.5 = 4 points)

a) Given the following 2D dataset with data from two classes and we want to train a linear classifier to separate samples belonging to different classes. Which of the following classifiers is the most suitable for the problem? Explain your choice!



b) A simple binary Support Vector Machine classifier can be defined by:

$$\hat{y}_i = sgn(w^T x^{(i)} + b)$$

$$\text{with } sgn(z) = \begin{cases} 1, & \text{if } z > 0 \\ -1, & \text{if } z < 0 \end{cases}$$

with the prediction $\hat{y}_i \in \{-1, 1\}$ of a data sample $x^{(i)} \in \mathbb{R}^n$ (*here:* $n = 2$) given the trained weights w and the bias b.

Using a subset of the visible data points from a) as training set X, a linear SVM model learned the following values for α and b (*see chapter 5, slide 35*):

$$X = \begin{bmatrix} 2.5 & 2 & 2.5 & 1.5 & 1.5 & 2.5 & 2 & 3 & 3.5 & 3.5 & 3.5 & 4 & 3 & 4 & 3.5 & 3 \\ 3.5 & 3 & 3 & 1.5 & 2 & 2.5 & 2 & 3.5 & 2 & 1.5 & 0.5 & 1 & 1.5 & 0.5 & 2.5 & 1 \end{bmatrix}^T$$

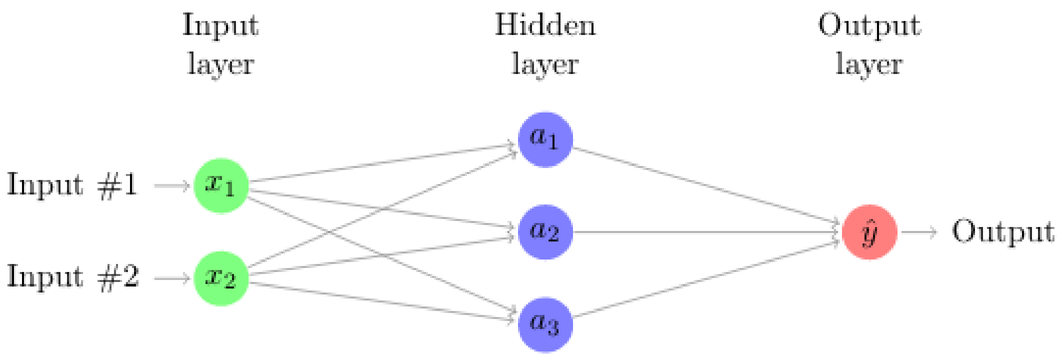
$$\alpha = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 2.48 & 0 & 0.02 & 0 & 0 & 0 & 0 & -0.98 & 0 & -1.52 & 0 \end{bmatrix}^T$$
$$\text{and } b = 3.5$$

Use this SVM to classify the points $p_1 = (4, 1.5)^T$ and $p_2 = (3, 2.5)^T$. State each of your computational steps explicitly and solve this exercise without an SVM framework implementation.

c) Support Vector machines are so called "Maximum Margin Classifiers". Explain this name as well as the term "support vector" w.r.t. the values of α.

d) What can be done with SVMs when data samples are not linear separable? Briefly explain what your approach does.

Exercise 7.2 - Backpropagation (6 points)



We have a Feedforward Neural network with one input layer, one hidden layer and one output layer. The hidden layer and output layer use the sigmoid function σ as activation function. Also note that the network minimizes Binary Cross Entropy loss which is given by,

$$L = \frac{1}{n} \sum_{i=1}^{n} -y_i log(\hat{y}_i) - (1 - y_i) log(1 - \hat{y}_i)$$

We consider the true class labels to be binary, i.e. 0 or 1.
For the purpose of computing the derivatives of the loss/cost function consider the numerical values obtained by the network.

The input layer consists of the two nodes x_1 and x_2. For our problem consider the following input:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

which has the label $y = 1$.

The hidden layer is made up of 3 neurons. The corresponding matrix of weights is given as:

$$W_{hidden} = \begin{bmatrix} w_1^1 & w_2^1 & w_3^1 \\ w_1^2 & w_2^2 & w_3^2 \end{bmatrix} = \begin{bmatrix} 0.15 & -0.25 & 0.05 \\ 0.2 & 0.1 & -0.15 \end{bmatrix}$$

Note: The output of the hidden layer is given by, $a = \sigma(W_{hidden}^T x)$

The output layer consists of one neuron, i.e., the network generates a single output. The weight matrix corresponding to the Output layer is given by:

$$W_{out} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 0.2 \\ -0.35 \\ 0.15 \end{bmatrix}$$

Note: The output from the Output layer is given by, $\hat{y} = \sigma(W_{out}^T a)$

Show that Back-propagation reduces the Binary Cross Entropy loss by performing the following steps:

- Perform a Forward-propagation with the given input x and compute the loss $C^{(1)}$.
- Compute the Back-propagation and apply Gradient descent with a learning rate of 0.1 to update the weights.
- Perform Forward-propagation again with the updated weights and recompute the loss $C^{(2)}$. Briefly explain your findings.

As always: To get full points, you need to give and explain your intermediate steps explicitly! Because this exercise is quite large, make sure to hand in a structured and understandable solution.

Submission instructions

The following instructions are mandatory. If you are not following them, tutors can decide to not correct your exercise.

- You **have to** submit a solution of this assignment sheet as a team of 2-3 members.
- Hand in a **single** PDF file with your solutions to the tasks.
- Therefore, make sure to write the name and matriculation ID of each of the members in your team.
- The solution must be uploaded by only **one** of your team members to the course website.
- If you have any trouble with the submission, contact your tutor **before** the deadline.

Plagiarism of any form is not tolerated. If you refer something from the web, you must give proper credit by citing the source. Lack of this would be considered plagiarism. In such a case, the whole sheet would be awarded zero points and a warning is given. If this act is repeated again, then the whole team is excluded from the course.