Universität des Saarlandes
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Lehrstuhl für Signalverarbeitung
NNIA Winter Term 2018/2019

Exercise Sheet 2

PCA and Numerical Computation

Deadline: 12.11.2018, 23:59

Exercise 2.1 - Principal Component Analysis (1.5 + 1.5 + 1 + 1 + 1 + 1 = 7 points)

In this exercise we will exploit the PCA in order to compress a 2-dimensional dataset into a 1-dimensional set. Do not forget to normalize your data such that the mean becomes 0, when taking the eigenvalue approach as described in the lecture for part a) and b).

a) Consider the following dataset consisting of 4 2-dimensional vectors:

$$\mathbf{x}^{(1)} = (1,1)^T,\ \mathbf{x}^{(2)} = (2,2)^T,\ \mathbf{x}^{(3)} = (3,1)^T,\ \mathbf{x}^{(4)} = (4,1)^T.$$

Compress this dataset to a 1-dimensional set using the PCA i.e. derive the encoder function $f(\mathbf{x}) = \mathbf{D}^T \cdot \mathbf{x}$ as defined in the lecture. Then apply f to the dataset in order to compress it.

b) Now consider the set:

$$\mathbf{x}^{(1)} = (-1,1)^T,\ \mathbf{x}^{(2)} = (-2,2)^T,\ \mathbf{x}^{(3)} = (-1,3)^T,\ \mathbf{x}^{(4)} = (-1,4)^T.$$

As in part a) compress this set by deriving the encoder function f and apply it to the set.

c) For both the parts a) and b) sketch the corresponding datasets in a separate figure. Also include the reconstructed vectors into the corresponding figures. Explain the values of the reconstructed vectors.

d) PCA can be used for a lot of applications. One of them is image recognition. Discuss briefly how PCA can be utilized in such task.

e) Is PCA a supervised or unsupervised algorithm? Explain your answer and discuss what the tunable parameter in PCA is. How can we choose this parameter?

f) Why is PCA a linear dimensionality reduction? What are other non-linear dimensionality reduction techniques?

Please provide an analytical solution for tasks (a,b and c).

Exercise 2.2 - Derivatives and Critical Points (1 + 1 = 2 points)

Please provide an analytical solution for all tasks in this exercise.

a) Let $f(x)$ be a twice differentiable function that satisfies the following equation: $f(x) = \sin(\pi e^x)$. What are the values of $f^{'}(0)$ and $f^{''}(0)$? [Hint: Consider Chain rule]

b) Let $f(x) = 9x^2 - 3x^3$ where x defined in $-4 < x < 4$, find all critical point of the function $f(x)$, and indicate if they are saddle, local or global min/max points.

Exercise 2.3 - Matrices (1 points)

Consider a real symmetric matrix $A \in R^{m \times m}$ with the eigenvalues $\lambda_1, \lambda_2, ..., \lambda_m$.
Prove that the eigenvalues of A^k are ${\lambda_1}^k, {\lambda_2}^k, ..., {\lambda_m}^k$ for $k \in N$

Submission instructions

The following instructions are mandatory. If you are not following them, tutors can decide to not correct your exercise.

- You have to submit the solutions of this assignment sheet as a team of 2-3 students.
- Hand in a **single** PDF file with your solutions.
- Therefore make sure to write the student ID and the name of each member of your team on your submission.
- Your assignment solution must be uploaded by only **one** of your team members to the course website.
- If you have any trouble with the submission, contact your tutor **before** the deadline.