Universität des Saarlandes
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Lehrstuhl für Signalverarbeitung
NNIA Winter Term 2018/2019

Exercise Sheet 1

Linear Algebra

Deadline: 05.11.2018, 23:59

Exercise 1.1 - Matrix Properties (1 + 1 = 2 points)

Given the matrix A with:

$$A = \begin{bmatrix} 4 & 2 & -1 \\ 2 & -1 & y \\ x & -2 & -8 \end{bmatrix}$$

a) Compute values for x and y so that A

   (i) is symmetric

   (ii) is an orthogonal matrix

   (iii) has rank 2

   (iv) is singular

   or argue why it is not possible.

b) Set $x = 4,\ y = 0$ and compute the Eigendecompostion of the resulting matrix.

Exercise 1.2 - Eigenvalues (1.5 + 1.5 = 3 points)

a) Find a matrix $B \in \mathbb{R}^{2\times 2}$ whose eigenvalues are 1 and 4 with the corresponding eigenvectors (3, 1) and (2, 1). Show your intermediate steps explicitly.

b) Let A and B be two $n \times n$ matrices. Show that if λ is an eigenvalue of AB, then it is also an eigenvalue of BA.

Exercise 1.3 - Covariance Matrix (2 + 1 + 2 = 5 points)

In machine learning, we want to find and learn dependencies of features in given data. From the covariances of the considered features one can obtain how much two features behave similarly. This ability will reappear later in the lecture.

Let $X \in \mathbb{R}^{m \times n}$ be an arbitrary matrix. The covariance matrix $C = X^T X$.

a) Show that C is always a positive semidefinite matrix i.e. $v^T C v \geq 0$ for all non-zero $v \in \mathbb{R}^n$.

   Hint: Try to make use of the fact that the squared norm of a vector is always ≥ 0.

b) Let $U \in \mathbb{R}^{n \times n}$ be an orthogonal matrix with $UU^T = U^T U = \mathbb{1}$, which is the identity matrix. Show that $||U^T x||_2^2 = 1$ for any vector $x \in \mathbb{R}^n$ with $||x||_2 = 1$.

c) In the following, we want to minimize the term $v^T C v$ with $||v||_2 = 1$. The Raleigh-Ritz principle allows us to solve this optimization problem using the Eigendecomposition of C.

   Prove that $min_{||v||_2=1}\ v^T C v = \lambda_{min}$ where λ_{min} is the smallest eigenvalue of C. What can you say about v leading to the minimal value?

   Hint: You can simplify the problem using a variable transformation $w = U^T v$.

Submission instructions

The following instructions are mandatory. If you are not following them, tutors can decide to not correct your exercise.

- You have to submit the solutions of this assignment sheet as a team of 2-3 students.
- Hand in a **single** PDF file with your solutions.
- Therefore Make sure to write the student ID and the name of each member of your team on your submission.
- Your assignment solution must be uploaded by only **one** of your team members to the course website.
- If you have any trouble with the submission, contact your tutor **before** the deadline.