

Implement via software the unicycle posture regulator based on polar coordinates, with the state feedback computed through the Runge-Kutta odometric localization method. Starting and final configurations are as in the previous exercise.

The robot must be brought from $q_i = [x_i \ y_i \ \theta_i] = [5 \ 2 \ \frac{\pi}{2}]$ to the origin.

The localization is the procedure of estimating the robot's state. The Runge-Kutta odometric localization method takes in input from the unicycle the constant heading velocity v_k and angular velocity ω_k in a time interval $[t_k, t_k + T_s]$, where the sampling time T_s has been chosen equal to 0.01 and gives back the value

$q(t_{k+1}) = q_{k+1}$ through these equations

$$\begin{cases} x_{k+1} = x_k + v_k T_s \cos\left(\theta_k + \frac{1}{2} \omega_k T_s\right) \\ y_{k+1} = y_k + v_k T_s \sin\left(\theta_k + \frac{1}{2} \omega_k T_s\right) \\ \theta_{k+1} = \theta_k + \omega_k T_s \end{cases}$$

To express the problem in polar coordinates:

$$\begin{aligned} \rho &= \sqrt{x^2 + y^2} \\ \gamma &= \text{atan2}(y, x) - \theta + \pi \\ \delta &= \gamma + \theta \end{aligned}$$

Where:

- $\rho = |e_p|$ is the distance between the unicycle and the origin
- γ is the angle between e_p and the sagittal axis
- δ is the angle between e_p and the x-axis

We design the following controllers

$$\begin{aligned} v &= k_1 \rho \cos \gamma \\ \omega &= k_2 \gamma + k_1 \sin \gamma \cos \gamma \left(1 + k_3 \frac{\delta}{\gamma}\right) \end{aligned}$$

$$\blacksquare \quad k_1, k_2, k_3 > 0$$

The value chosen are $k_1 = 2, k_2 = 7, k_3 = 1$

The implementation is present in the file HW2_5.slx, the results are:

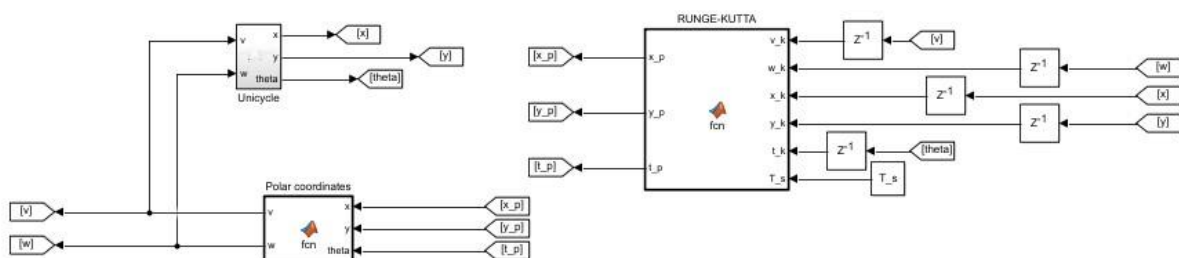


Figure 1:scheme of the fifth exercise

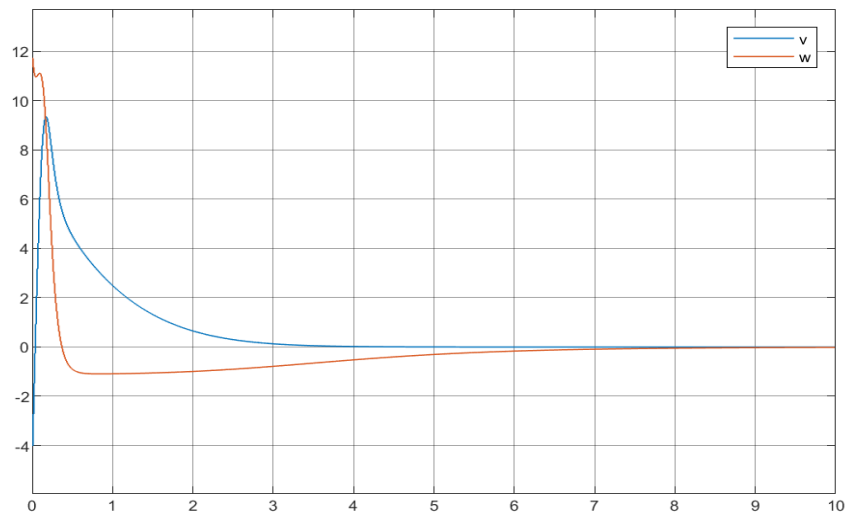


Figure 2: heading and angular velocity

the v starts from a negative value, this means that it moves backwards, while the w takes on a value approximately equal to 12.

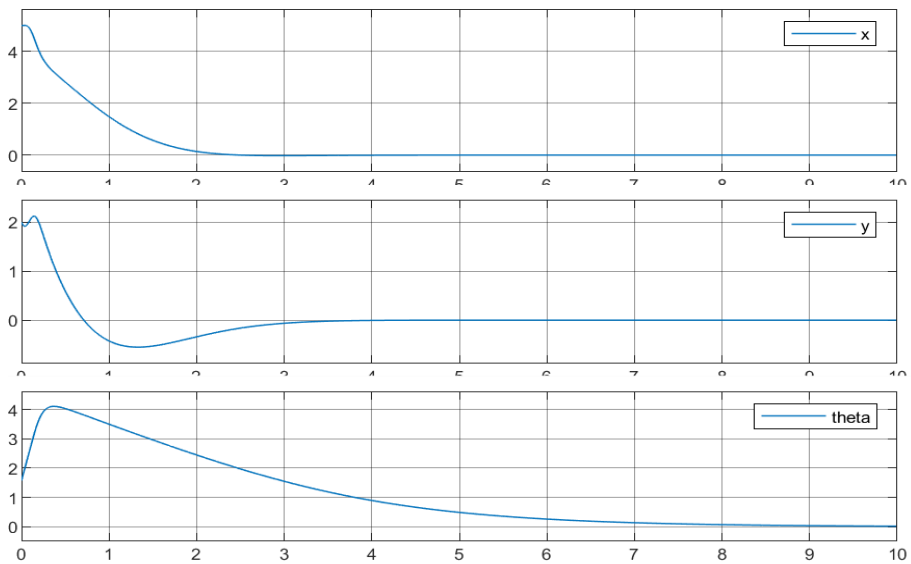


Figure 3: x y theta of the unicycle

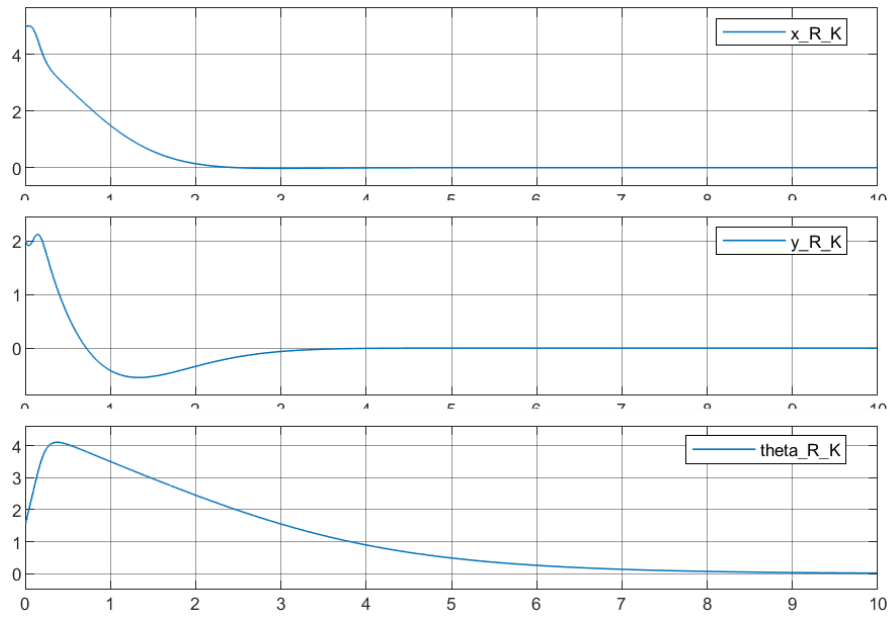


Figure 4: x y θ in output from the RUNGE-KUTTA

Signals x , y and θ tend to 0 asymptotically.