Consider the workspace file attached as *ws_homework_4.mat*. Within this file, you can find the values of a flight with a quadrotor with the commanded thrust (*thrust*) and torques (*tau*), the measured linear velocity (*linear_vel*), attitude expressed as Euler angles (*attitude*) and the time derivative of such angles (*attitude_vel*). The employed quadrotor has a supposed mass of *1.1 kg*, and an inertia matrix referred to the body frame equal to *diag([1.2416 1.2416 2*1.2416])*. Implement yourself the momentum-based estimator of order *r* to estimate the external disturbances acting on the UAV during the flight. Suppose the sampling time of the estimator is equal to *1 ms*. Compare the obtained estimation with the following disturbances applied during the flight:

- *1 N* along the *x*- and *y*-axis of the world frame;
- *0.1 Nm* around the yaw axis.

Compare the estimation results with different values of *r*. Try to answer the following questions.

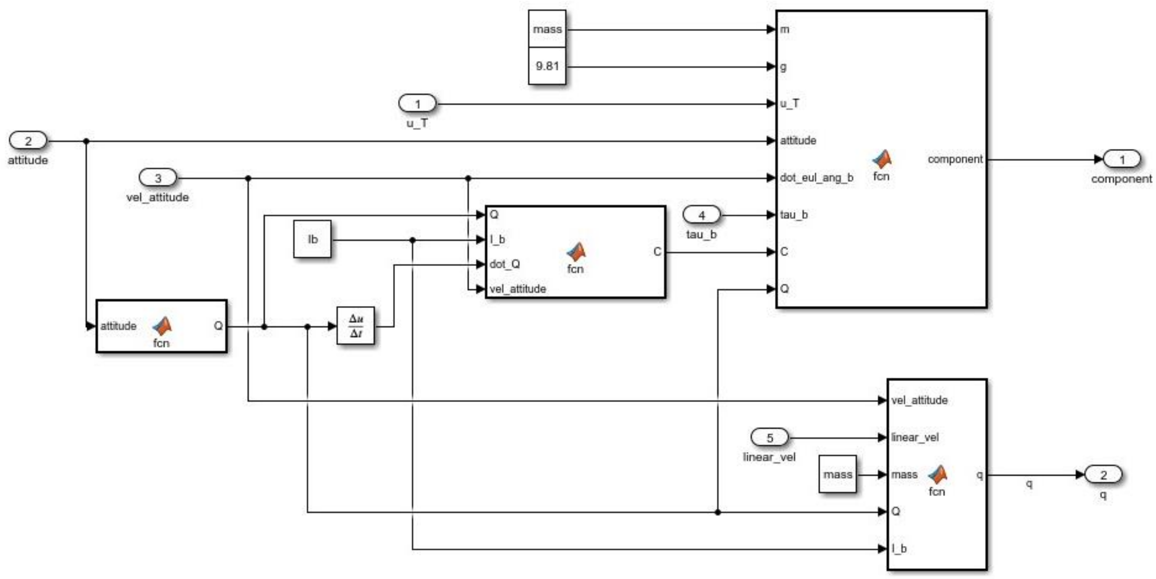
- From which value of *r* the estimation results do not improve too much?
- Are you able to estimate the real mass of the UAV from the estimated disturbance along the *z*-axis?

The equations that regulate the estimation are:

$$\gamma_1(t) = K_1\left(q - \int_0^t \begin{bmatrix}\hat{f}_e \\ \hat{\tau}_e\end{bmatrix} + \begin{bmatrix} mge_3 - u_T R_b e_3 \\ C^T(\eta_b, \dot{\eta}_b)\dot{\eta}_b + Q^T(\eta_b)\tau^b \end{bmatrix} dt\right)$$

$$\gamma_i(t) = K_i \int_0^t -\begin{bmatrix}\hat{f}_e \\ \hat{\tau}_e\end{bmatrix} + \gamma_{i-1} dt \qquad i = 2, \dots r$$

To obtain the values q and component= $\begin{bmatrix} mge_3 - u_T R_b e_3 \\ C^T(\eta_b, \dot{\eta}_b)\dot{\eta}_b + Q^T(\eta_b)\tau^b \end{bmatrix}$ this Simulink scheme has been implemented:



The equations used to obtain the scheme are:

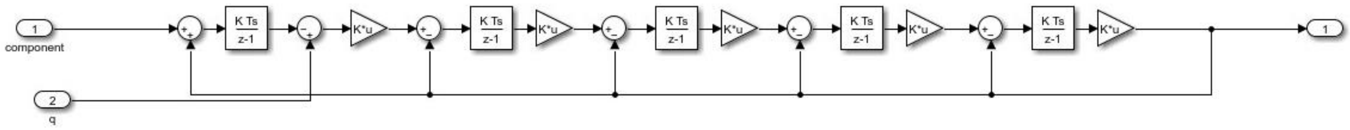
$$q = \begin{bmatrix} mI_3 & O_3 \\ O_3 & M(\eta_b) \end{bmatrix} \begin{bmatrix} \dot{p}_b \\ \dot{\eta}_b \end{bmatrix}$$

$$Q = \begin{bmatrix} 1 & 0 & -s_\theta \\ 0 & c_\phi & c_\theta s_\phi \\ 0 & -s_\phi & c_\theta c_\phi \end{bmatrix}$$

$$M(\eta_b) = Q^T(\eta_b) I_b Q(\eta_b)$$

$$R_b(\eta_b) = \begin{bmatrix} c_\theta c_\psi & s_\phi s_\theta c_\psi - c_\phi s_\psi & c_\phi s_\theta c_\psi + s_\phi s_\psi \\ c_\theta s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & c_\phi s_\theta s_p si - s_\phi c_\psi \\ -s_\theta & s_\phi c_\theta & c_\phi c_\theta \end{bmatrix}$$

The implementation of the estimate was carried out in the following subsystem (here it is presented in the case where r = 5)



The gains have been chosen in order to respect the equations that govern the estimator based control:

$$G_i(s) = \frac{c_0}{s^r + c_{r-1}s^{r-1} + \cdots + c_1 s + c_0}, i = 1, \cdots, 6 \qquad \prod_{i=j+1}^{r} K_i = c_j, \qquad j = 0, \cdots, r-1$$

but also to minimize the error. To do this, Itae method has been implemented which minimizes the quantity $\int_0^{+\infty} t|\varepsilon(t)|dt$ through the choice of the characteristic polynomial as shown in the following image

System Order	Characteristic Polynomial
First	$s + \omega_n$
Second	$s^2 + 1.4\,\omega_n s + \omega_n^2$
Third	$s^3 + 1.75\,\omega_n s^2 + 2.15\,\omega_n^2 s + \omega_n^3$
Fourth	$s^4 + 2.1\,\omega_n s^3 + 3.4\,\omega_n^2 s^2 + 2.7\,\omega_n^3 s + \omega_n^4$
Fifth	$s^5 + 2.8\,\omega_n s^4 + 5.0\,\omega_n^2 s^3 + 5.5\,\omega_n^3 s^2 + 3.4\,\omega_n^4 s + \omega_n^5$
Sixth	$s^6 + 3.25\,\omega_n s^5 + 6.6\,\omega_n^2 s^4 + 8.6\,\omega_n^3 s^3 + 7.45\,\omega_n^4 s^2 + 3.95\,\omega_n^5 s + \omega_n^6$

```
wn1=80;
wn2=80;
wn3=77;
wn4=90;
wn5=103;

s = tf('s')
G1 = wn1/(s+wn1);
%bode(G1)
bandwidth(G1)
k11=wn1;

 G2 = wn2^2/(s^2+1.4*wn2*s+wn2^2)
%  bode(G2)
 bandwidth(G2)

k22=1.4*wn2;
k12=wn2^2/k22;

 G3 = wn3^3/(s^3+1.75*wn3*s^2+2.15*wn3^2*s+wn3^3)
% bode(G3)
 bandwidth(G3)
k33=1.75*wn3;
k23=2.15*wn3^2/k33;
k13=wn3^3/k33/k23;
```

```
G4 = wn4^4/(s^4+2.1*wn4*s^3+3.4*wn4^2*s^2+2.7*wn4^3*s+wn4^4)
 %bode(G4)
 bandwidth(G4)
k44=2.1*wn4;
k34=3.4*wn4^2/k44;
k24=2.7*wn4^3/k34/k44;
k14=wn4^4/k24/k34/k44;

G5 = wn5^5/(s^5+2.8*wn5*s^4+5*wn5^2*s^3+5.5*wn5^3*s^2+3.4*wn5^4*s+wn5^5);
% bode(G5)
 bandwidth(G5)
k55=2.8*wn5;
k45=5*wn5^2/k55;
k35=5.5*wn5^3/k55/k45;
k25=3.4*wn5^4/k35/k45/k55;
k15=wn5^5/k25/k35/k45/k55;
```

the ω_n was chosen to guarantee a bandwidth almost always equal to 80 for all the G. Below are shown the results of the different estimators as r varies

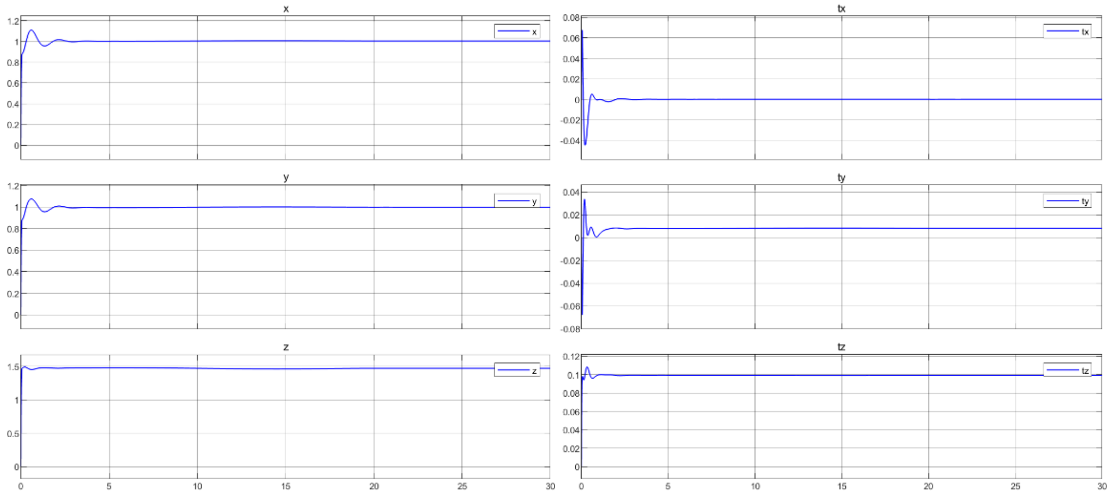


Figure 1:estimator with r=1

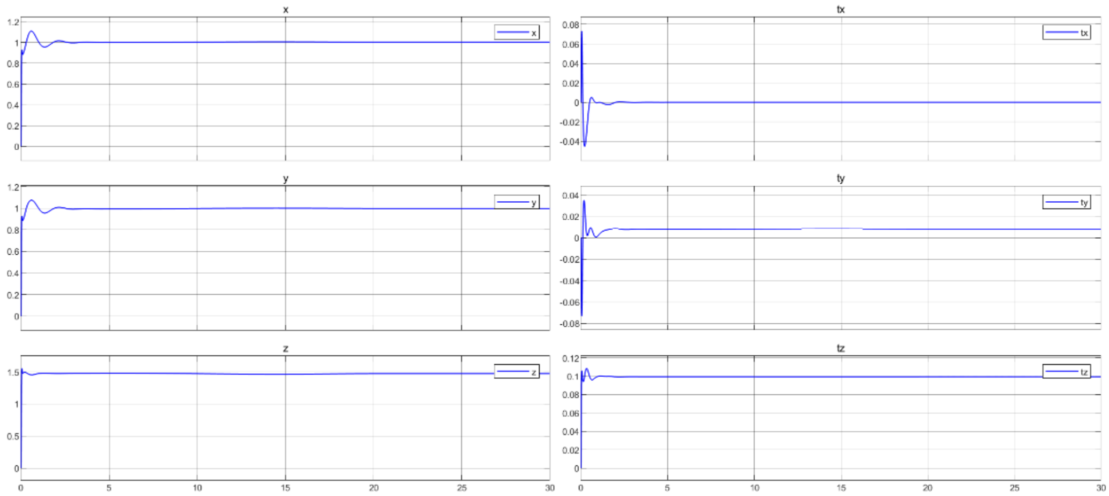


Figure 2: estimator with r=2

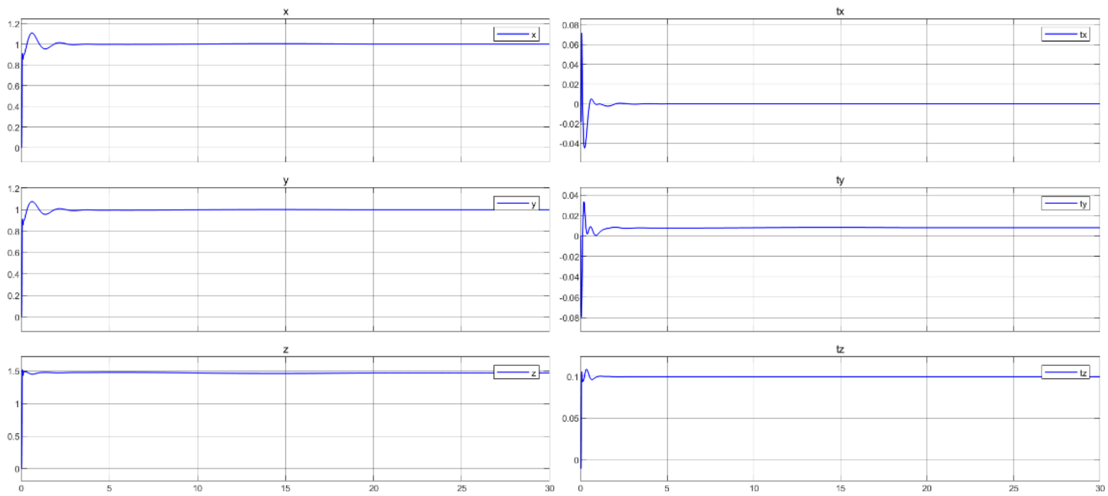


Figure 3: estimator with r=3

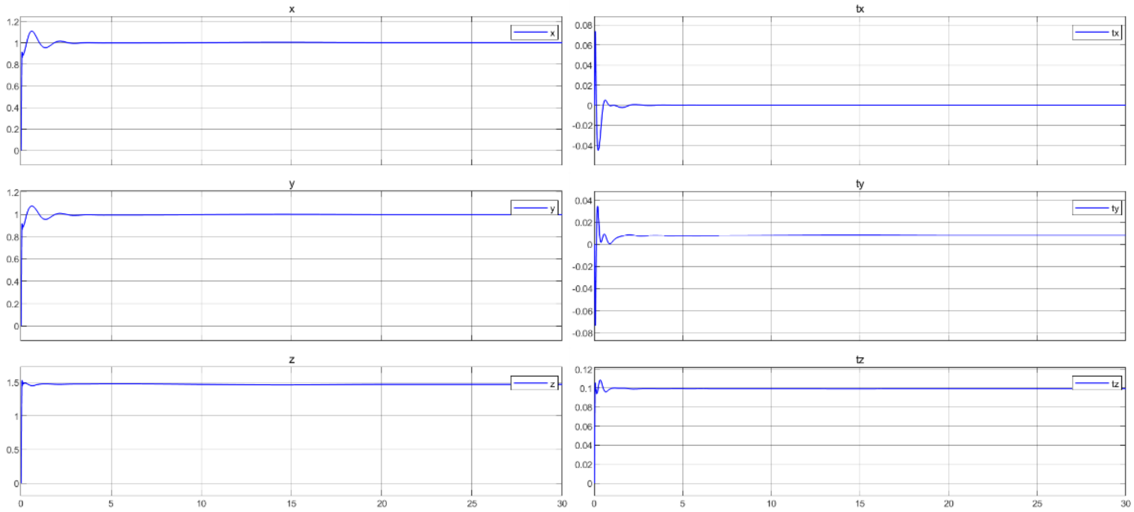


Figure 4: estimator with r=4

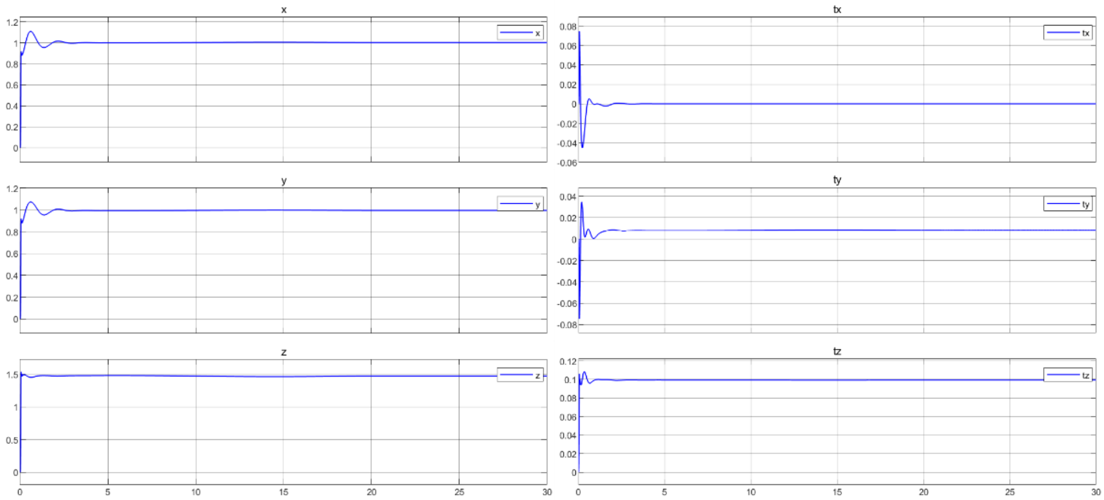


Figure 5: estimator with r=5

As can be seen, all the estimates have very similar and satisfactory results. The value along the x and y axis is 1N and roughly 1.5 N along the z axis. τ_x assumes a value approximately equal to 0 Nm, τ_y almost 0.01Nm and τ_z 0.1Nm.

Below are the results of comparisons of x y and τ_z for the estimators. The time scale was reduced from 30s to 5s to better show the dynamics of the transient

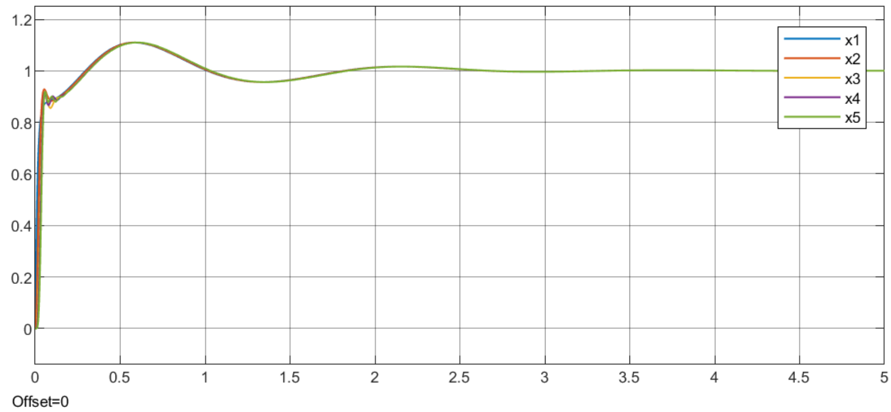


Figure 6:comparison of the disturbances along the x axis

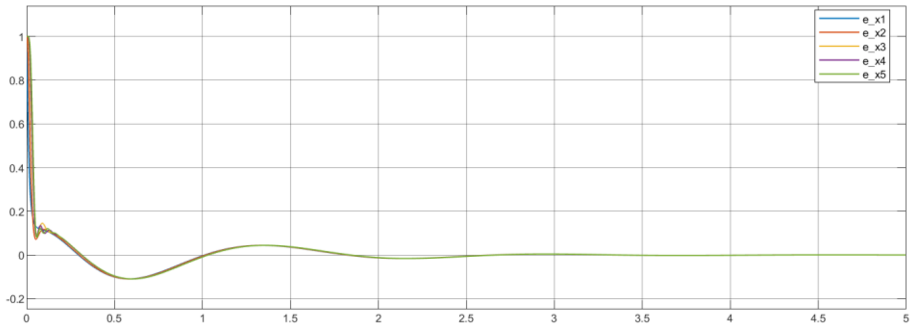


Figure 7: comparisons of the error along the x axis

The steady-state error along the x axis is approximately of $3{\cdot}10^{-3}$N. The settling time is less than 3s.

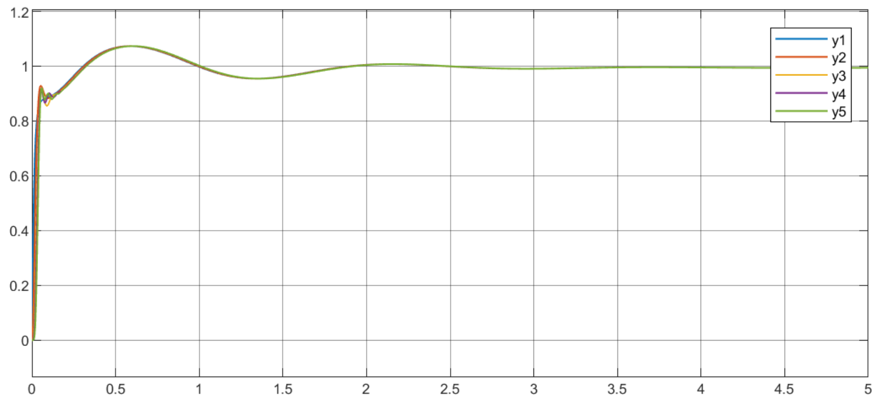


Figure 8: comparison of the disturbances along the y axis

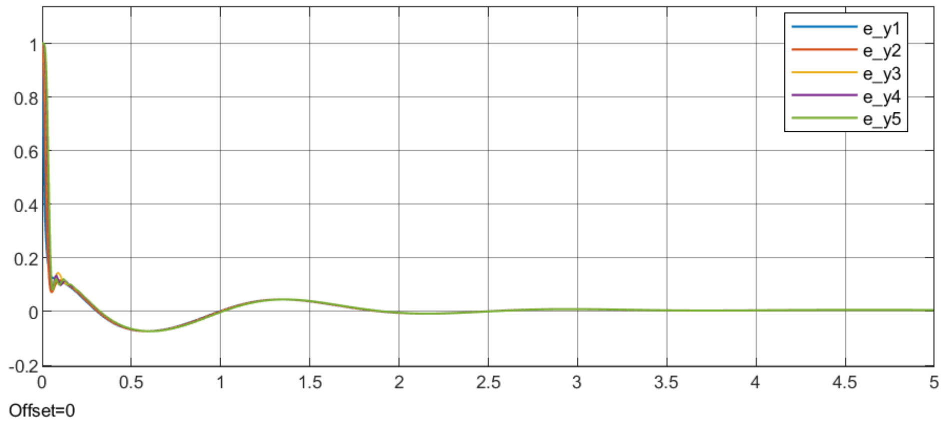


Figure 9: comparisons of the error along the y axis

The steady-state error along the x axis is approximately of $3{\cdot}10^{-3}$N. The settling time is less than 3,5s.

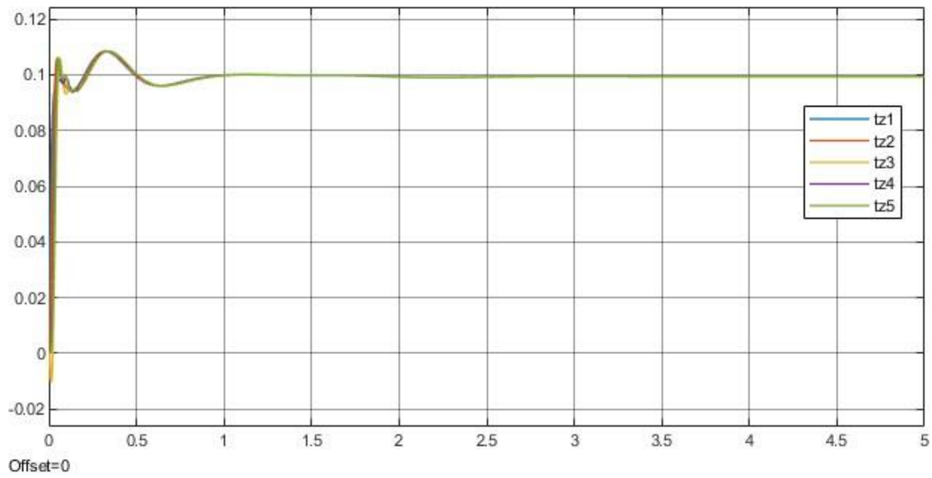


Figure 10:comparison between τ_z.

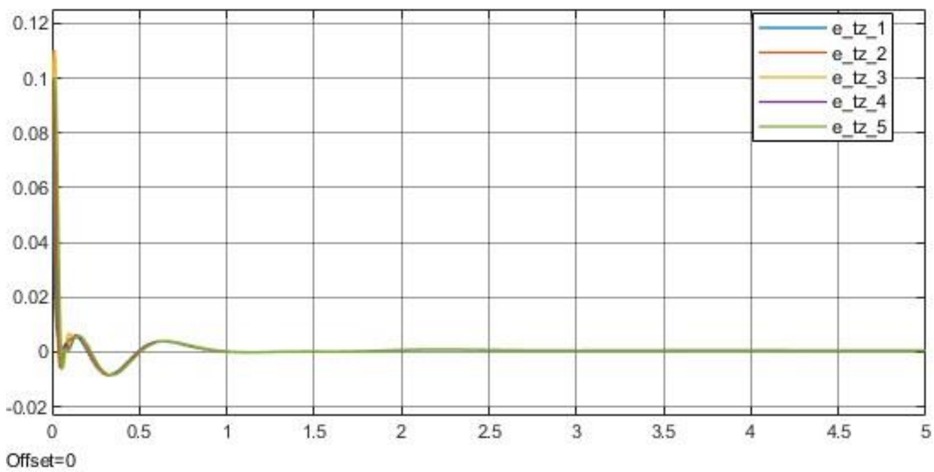


Figure 11:comparisons of the error of τ_z

The steady-state error of τ_z is approximately of $6{\cdot}10^{-4}$Nm. The settling time is less than 3s.

The disturbance along the z axis is indicative of the fact that the real mass is different from the supposed mass. To obtain the real mass of the UAV, it is sufficient to add to the supposed mass a quantity Δ_m equal to the ratio between the component along the z axis divided by the gravitational acceleration

```
function m1 = fcn(z,m,g)

m1=m+(z/g);
```

Figure 12:code to calculate the real mass for r = 1

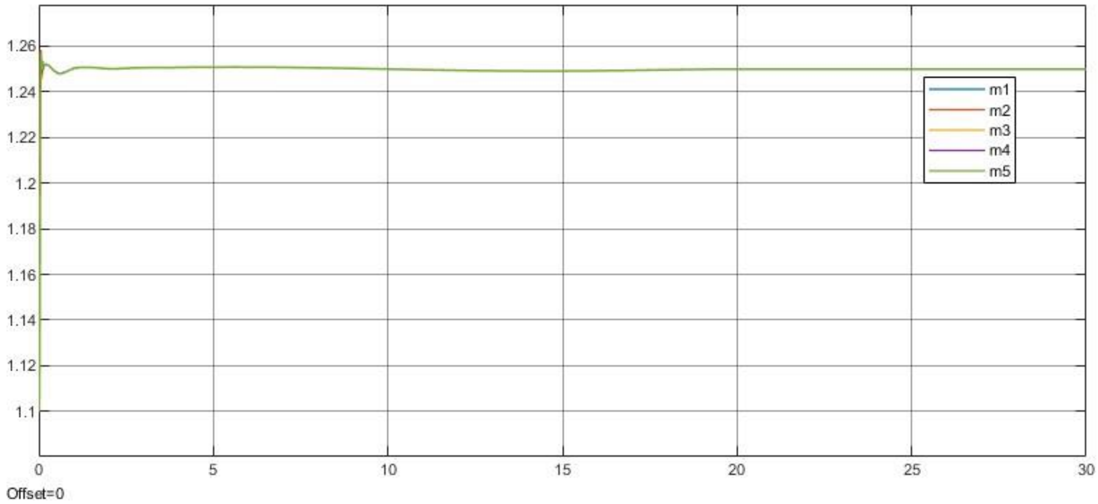


Figure 13:real mass

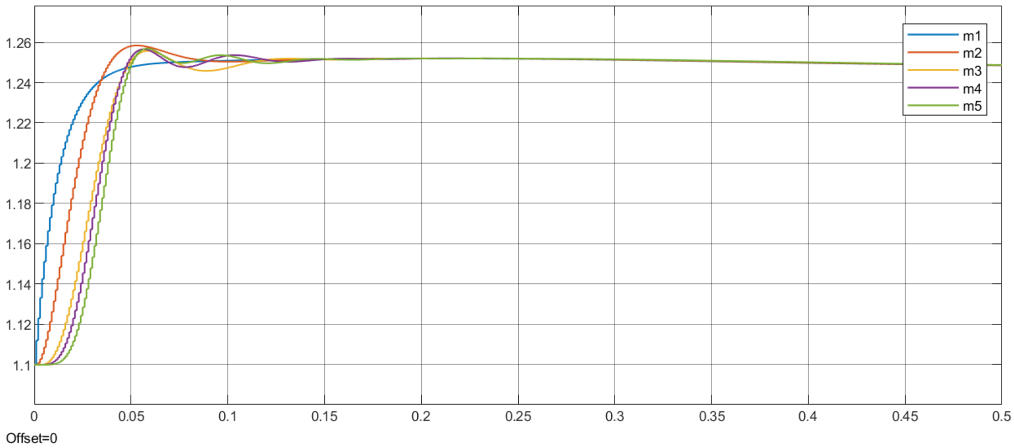


Figure 14:transient real masses

As can be seen, all the masses converge at steady state to a value equal to 1.25 kg, the estimator with r = 1 demonstrates the least overshoot. Therefore, it is possible to conclude that as the order r varies all the estimators have given similar results, and it can be concluded that there are no significant differences in the estimators.

The files containing the algorithm are called: “momentum based estimator.slx” and "momentum based estimator.m".