Тиновешке

N 845

$$y=\frac{2x}{1-x^2}\quad;\quad y'=\frac{2(1-x^2)-2x(-2x^2)}{(1-x^2)^2}=\frac{2(1+x^2)}{(1-x^2)^2}$$

N 850

$$y=\frac{x^p(1-x)^q}{1+x}\quad;\quad y'=\frac{\left(px^{p-1}(1-x)^q-qx^p(1-x)^{q-1}\right)(1+x)-x^p(1-x)^q}{(1+x)^2}$$

$$=\frac{x^{p-1}(1-x)^{q-1}\left(p-(q-1)x-(p-q-1)x^2\right)}{(1+x)^2}$$

N 858

$$y=\sqrt[3]{\frac{1+x^3}{1-x^3}}\quad;\quad y'=\frac{1}{3\sqrt[3]{\left(\frac{1+x^3}{1-x^3}\right)^2}}\cdot\frac{3x^2(1-x^3)+3x^2(1+x^3)}{(1-x^3)^2}$$

N 859

$$y=\frac{1}{\sqrt{1+x^2}\left(x+\sqrt{1+x^2}\right)}$$

$$y'=-\frac{1}{\left(x\sqrt{1+x^2}+1+x^2\right)^2}\cdot\left(\sqrt{1+x^2}+\frac{x^2}{\sqrt{1+x^2}}+2x\right)$$

$$=-\frac{1+2x^2+2x\sqrt{1+x^2}}{\sqrt{1+x^2}\,(1+x^2)\left(x+\sqrt{1+x^2}\right)^2}=-\frac{1}{\sqrt{(1+x^2)^3}}$$

N 863

$y = (2 - x^2)\cos x + 2x\sin x = \cancel{(2\cos x - x^2\cos x + 2x\sin x)}$

$y' = -\cancel{2x\cos x} - \cancel{2\sin x} + x^2\sin x + \cancel{2\sin x} + \cancel{2x\cos x} = x^2\sin x$

N 868

$y' = \frac{\cos x}{2\sin^2 x}$; $y' = \frac{-2\sin 3x - 4\sin x\cos^2 x}{4\sin^4 x} = -\frac{1+\cos^2 x}{2\sin^3 x}$

N 872

$y = tg\,x - \frac{1}{3}tg^3 x + \frac{1}{5}tg^5 x$

$y' = \frac{1}{\cos^2 x} - \frac{tg^2 x}{\cos^2 x} + \frac{tg^4 x}{\cos^2 x}$

N 879

$y = sec^2\frac{x}{a} + cosec^2\frac{x}{a}$

$y' = \frac{2\sec^2\frac{x}{a}\,tg\frac{x}{a} - 2\,cosec^2\frac{x}{a}\,ctg\frac{x}{a}}{a}$

N 880

$y = e^x(1 + ctg\frac{x}{2})$; $y' = e^x(1 + ctg\frac{x}{2}) + e^x(\frac{1}{2}\cdot(-\frac{1}{\sin^2\frac{x}{2}})$

N 885

$y = x^{a^a} + a^{x^a} + a^{a^x}$

$y' = a^a x^{a^a-1} + a x^{a-1} a^{x^a} \ln a + a^x \cdot a^{a^x} \ln^2 a$

N 886

$y = \lg^3 x^2$; $y' = 3\lg^2 x^2 \cdot \frac{1}{x^2} \cdot 2x \cdot \frac{1}{\ln 10} = \frac{6\log^2 x^2}{x \ln 10}$

N 894

$y = \sqrt{x+1} - \ln(1+\sqrt{x+1})$

$y' = \frac{1}{2\sqrt{x+1}} - \frac{1}{2\sqrt{x+1}(1+\sqrt{x+1})} = \frac{1}{2(1+\sqrt{x+1})}$

N 895

$y = \ln(x+\sqrt{x^2+1})$ $y' = \frac{1}{x+\sqrt{x^2+1}}\left(1+\frac{1}{2\sqrt{x^2+1}} \cdot 2x\right) =$

$= \frac{1}{\sqrt{x^2+1}}$

N 898

$y = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\ln(x+\sqrt{x^2+a^2})$

$y' = \frac{1}{2}\sqrt{x^2+a^2} + \frac{x}{2} \cdot \frac{1}{2\sqrt{x^2+a^2}} \cdot 2x \cdot \frac{a^2}{2} \cdot \frac{1}{x+\sqrt{x^2+a^2}} \cdot \left(1+\frac{1}{2\sqrt{x^2+a^2}} \cdot 2x\right)$

$= \sqrt{x^2+a^2}$

N 913

$$y = \arcsin\frac{x^2}{2}\ ;\ y' = \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}\cdot\frac{1}{2} = \frac{1}{\sqrt{4-x^2}}$$

N 916

$$y = \frac{1}{\sqrt{2}}\operatorname{arctg}\frac{\sqrt{2}}{x}\ ;\ y' = -\frac{1}{\sqrt{2}}\cdot\frac{1}{1+\left(\frac{\sqrt{2}}{x}\right)^2}\cdot\frac{-\sqrt{2}}{x^2} = \frac{1}{x^2+2}$$

N 893

$$y = \frac{1}{1-k}\ln\frac{1+x}{1-x} + \frac{\sqrt{k}}{1-k}\ln\frac{1+x\sqrt{k}}{1-x\sqrt{k}} =$$

$$= \frac{1}{1-k}\left(\ln(1+x) - \ln(1-x) + \sqrt{k}\ln(1+x\sqrt{k}) - \sqrt{k}\ln(1-x\sqrt{k})\right)$$

$$y' = \frac{1}{1-k}\left(\frac{1}{1+x} - \frac{1}{1-x}\cdot(-1) + \frac{\sqrt{k}}{1+x\sqrt{k}}\cdot\sqrt{k} - \frac{\sqrt{k}}{1-x\sqrt{k}}\cdot(-\sqrt{k})\right)$$

$$= \frac{1}{1-k}\left(\frac{1}{1+x} + \frac{1}{1-x} + \frac{k}{1+\sqrt{k}x} + \frac{k}{1-x\sqrt{k}}\right) =$$

$$= \frac{2}{1-k}\left(\frac{1}{1-x^2} + \frac{1}{1-x^2k}\right)$$

N 929

$$y = \frac{1}{\arccos^2(x^2)}\ ;\ y' = -\frac{2}{\arccos^3(x^2)}\cdot\frac{-2x}{\sqrt{1-x^4}}$$

$\frac{1}{x^2+2}$

№ 938

$$y = \frac{\arccos x}{x} + \frac{1}{2} \ln \frac{1-\sqrt{1-x^2}}{1+\sqrt{1-x^2}}$$

$$y' = \frac{-\frac{1}{\sqrt{1-x^2}} x - \arccos x}{x^2} + \frac{1}{x\sqrt{1-x^2}} = -\frac{\arccos x \, \cancel{\sqrt{1-x^2}}}{x^2 \cancel{\sqrt{1-x^2}}}$$

№ 942

$$y = \frac{x^6}{1+x^{12}} - \operatorname{arcctg} x^6$$

$$y' = \frac{6x^5(1+x^{12}) - 12x^{17}}{(1+x^{12})^2} + \frac{6x^5}{1+x^{12}} = \frac{12x^5}{(1+x^{12})^2}$$

№ 943

1

2)

$$y = \ln \frac{1-\sqrt[3]{x}}{\sqrt{1+\sqrt[3]{x}+\sqrt[3]{x^2}}} + \sqrt{3} \operatorname{arctg} \frac{1+2\sqrt[3]{x}}{\sqrt{3}}$$

$$y' = \frac{1+\sqrt[3]{x}}{2\sqrt[3]{x^2}(x-1)} + \frac{1}{2\sqrt[3]{x^2}(1+\sqrt[3]{x^2}+\sqrt[3]{x^4})} = -\frac{1}{\sqrt[3]{x^{11}}(1-x)}$$

1') $$\frac{\sqrt{1+\sqrt[3]{x}+\sqrt[3]{x^2}}}{1-\sqrt[3]{x}} \cdot \frac{-\frac{\sqrt{1+\sqrt[3]{x}+\sqrt[3]{x^2}}}{3\sqrt[3]{x^2}} - \frac{(1-\sqrt[3]{x})(1+2\sqrt[3]{x})}{6\sqrt[3]{x^2}(1+\sqrt[3]{x}+\sqrt[3]{x^2})^{1/2}}}{1+\sqrt[3]{x}+\sqrt[3]{x^2}}$$

$$= \frac{1+\sqrt[3]{x}}{2\sqrt[3]{x^2}(x-1)}$$

$2')\quad \sqrt{3}\cdot\frac{3\cdot\frac{2}{3\sqrt{3}}x^{-2/3}}{4(1+x^{2/3}+x^{4/3})}=\frac{1}{2x^{2/3}(1+x^{2/3}+x^{4/3})}$

N 947

$$y=\frac{1}{4}\ln\frac{\sqrt[4]{1+x^4}+x}{\sqrt[4]{1+x^4}-x}-\frac{1}{2}\operatorname{arctg}\frac{\sqrt[4]{1+x^4}}{x}$$

$$y'=\frac{1}{2}\cdot\frac{\sqrt[4]{1+x^4}-x}{\sqrt[4]{1+x^4}+x}\cdot$$

$$\cdot\left(\frac{\left(\frac{x^3}{\sqrt[4]{(1+x^4)^3}}+1\right)\left(\sqrt[4]{1+x^4}-x\right)-\left(\sqrt[4]{1+x^4}+x\right)\left(\frac{x^3}{\sqrt[4]{(1+x^4)^3}}-1\right)}{\left(\sqrt[4]{1+x^4}-x\right)^2}\right)$$

$$-\frac{1}{2}\cdot\frac{1}{1+\frac{\sqrt{1+x^4}}{x^2}}\cdot\left(\frac{\frac{1}{4}\cdot\frac{1}{\sqrt[4]{(1+x^4)^3}}\cdot 4x^3\cdot x-\sqrt[4]{1+x^4}}{x^2}\right)$$

N 956

$$y = \frac{x\sqrt{1-x^2}}{1+x^2} - \frac{3}{\sqrt{2}} \operatorname{arcctg} \frac{x\sqrt{2}}{\sqrt{1-x^2}}$$

$$y' = \frac{\left(\sqrt{1-x^2} - \frac{x^2}{\sqrt{1-x^2}}\right)(1+x^2) - 2x^2\sqrt{1-x^2}}{(1+x^2)^2} +$$

$$+ \frac{\cancel{x}\ 3}{\sqrt{2}\left(1+\frac{2x^2}{1-x^2}\right)} \cdot \left(\frac{\sqrt{2}\left(\sqrt{1-x^2}\right) + \frac{\sqrt{2}x^2}{\sqrt{1-x^2}}}{1-x^2}\right)$$

N 957

$$y = \arccos(\sin x^2 - \cos x^2)$$

$$y' = \frac{1}{\sqrt{1-(\sin x^2 - \cos x^2)^2}} \cdot 2x(\cos x^2 + \sin x^2)$$

N 960

$$y = \operatorname{arctg} e^x - \ln\sqrt{\frac{e^{2x}}{e^{2x}+1}}$$

$$y' = \frac{e^x}{1+e^{2x}} - \sqrt{\frac{e^{2x}+1}{e^{2x}}} \cdot \frac{\sqrt{e^{2x}+1}}{2\sqrt{e^{2x}}} \cdot \frac{2e^{2x}\cdot(e^{2x}+1)-e^{2x}\cdot 2e^{2x}}{(e^{2x}+1)^2}$$

N 13.103

$$y = \frac{1}{2\sqrt{6}} \ln\left(\frac{\sqrt{2}+x\sqrt{3}}{\sqrt{2}-x\sqrt{3}}\right)^2 = \frac{1}{\sqrt{6}} \ln\frac{\sqrt{2}+x\sqrt{3}}{\sqrt{2}-x\sqrt{3}}$$

$$y' = \frac{1}{2\sqrt{6}} \cdot 2\cancel{\ln\left(\frac{\sqrt{2}+x\sqrt{3}}{\sqrt{2}-x\sqrt{3}}\right)} \cdot \frac{\sqrt{2}-x\sqrt{3}}{\sqrt{2}+x\sqrt{3}} \cdot$$

$$\cdot \frac{\sqrt{3}(\sqrt{2}-x\sqrt{3})+\sqrt{3}(\sqrt{2}+x\sqrt{3})}{(\sqrt{2}-x\sqrt{3})^2}$$

№ 13.55

$y = (0{,}4\cos(8x+5) - 0{,}6\sin 0{,}8x)^2$

$y' = 2(0{,}4\cos(8x+5) - 0{,}6\sin 0{,}8x)\cdot$

$\cdot(-0{,}4\sin(8x+5)\cdot 8 - 0{,}6\cos 0{,}8x\cdot 0{,}8)$

№ 13.68

$y = \mathrm{ctg}\, x^2 - \frac{1}{3}\mathrm{tg}^3 2x$

$y' = 2\,\mathrm{ctg}\, x\cdot\left(-\frac{1}{\sin^2 x^2}\right) - \mathrm{tg}^2 2x\cdot\frac{1}{\cos^2 2x}\cdot 2$

№ 13.72

$y = \frac{1}{2}\mathrm{arc\,tg}\frac{x}{2} - \frac{1}{3}\mathrm{arc\,tg}\frac{x}{3}$

$y' = \frac{1}{4}\cdot\frac{1}{1+\frac{x^2}{4}} - \frac{1}{9}\cdot\frac{1}{1+\frac{x^2}{9}}$

№ 13.81

$y = 3^{\mathrm{arctg}(2x+\pi)}$; $y' = \frac{2\cdot 3^{\mathrm{arctg}(2x+\pi)}\cdot\ln 3}{(2x+\pi)^2+1}$

N 13.87

$y = \operatorname{arctg}(th(x))$

$y' = \frac{1}{1+th(x)^2} \cdot sech^2(x)$

N 13.97

$y = \sin(\arcsin x) = x$

$y' = 1$

N 13.109

$y = \log_2 \log_3 \log_5 x$; $y' = \frac{1}{\log_3 \log_5 x \ln 2} \cdot$

$\cdot \frac{1}{\log_5 x \ln 3} \cdot \frac{1}{x \cdot \ln 5}$

N13.110

$y = \ln \ln \ln x^2$; $y' = \frac{1}{\ln \ln x^2} \cdot \frac{1}{\ln x^2} \cdot \frac{1}{x^2} \cdot 2x$

N 13.112

$y = \frac{2}{7} \ln\left(\sqrt{x^7} + \sqrt{1+x^7}\right)$

$y' = \frac{2}{7} \cdot \frac{1}{\sqrt{x^7} + \sqrt{1+x^7}} \cdot \left(\frac{1}{2\sqrt{x^7}} \cdot 7x^6 + \frac{1}{2\sqrt{1+x^7}} \cdot 7x^6\right)$

N 13.141

$y = x^x$; $y' = x^x(\ln x + 1)$

N 13.144

$y = \log_2 x \cdot \log_x e + \log_2 x \cdot \ln 2 = \log_2 x \cdot \frac{1}{\ln x} + \log_2 x \ln 2$

$\cancel{y'} = \log_2 x \left(\frac{1}{\ln x} + \ln 2 \right)$

$y' = \frac{1}{x \ln 2} \left(\frac{1}{\ln x} + \ln 2 \right) + \log_2 x \cdot \left(-\frac{1}{x \ln^2(x)} \right)$

N 13.65

$y = \frac{x^2 + 4}{x\sqrt{4 + ((x^2-4)/(2x))^2}} = \frac{x^2+4}{x\sqrt{4 + \left(\frac{x^2-4}{2x}\right)^2}}$

$y' = \left(2x \left(x\sqrt{4 + \left(\frac{x^2-4}{2x}\right)^2} \right) - (x^2+4) \left(\sqrt{4 + \left(\frac{x^2-4}{2x}\right)^2} + \right.\right.$

$\left.\left. + \frac{x}{2\sqrt{4 + \left(\frac{x^2-4}{2x}\right)^2}} \cdot 2 \cdot \left(\frac{x^2-4}{2x}\right) \cdot \frac{4x^2 - (x^2-4) \cdot 2x}{4x^2} \right) \right) /$

$/ \, x^2 \left(4 + \left(\frac{x^2-4}{2x}\right)^2 \right)$

N 13.69

$$y = \frac{1-\cos(8x-3\pi)}{tg 2x - ctg 2x} = \frac{-1+\cos(8x-3\pi)}{2 tg 4x} =$$

$$= \frac{1}{2}(-\cos 8x - 1) tg 4x = -\frac{1}{2}(\cos 8x + 1) \cdot tg 4x$$

$$y' = -\frac{1}{2}\left(-8\sin 8x \, tg 4x + 4(\cos 8x + 1) \cdot \frac{1}{\cos^2 4x}\right)$$

N 13.117

$$y = \frac{x}{(e^{2x}-1)^{1/2}}$$

$$y' = \frac{\sqrt{e^{2x}-1} - x\left(\frac{1}{2\sqrt{e^{2x}-1}} \cdot 2e^{2x}\right)}{e^{2x}-1}$$

N 13.71

$$y = 2^{\sin 2x} \; ; \; y' = 2 \cdot 2^{\sin 2x} \cdot \ln(2) \cdot \cos 2x$$

N 13.96

$$y = 2x \ln(2x + \sqrt{4x^2+1}) - \sqrt{4x^2+1}$$

$$y' = 2\ln(2x + \sqrt{4x^2+1}) + 2x \frac{1}{2x+\sqrt{4x^2+1}} \cdot \left(2 + \frac{8x}{2\sqrt{4x^2+1}}\right)$$

$$- \frac{1}{2\sqrt{4x^2+1}} \cdot 8x$$

N 13.101

$$y = \arccos \frac{x^{2n}-1}{x^{2n}+1}$$

$$y' = -\frac{1}{\sqrt{1-\left(\frac{x^{2n}-1}{x^{2n}+1}\right)^2}} \cdot \left(\frac{2nx^{2n-1}(x^{2n}+1)-(x^{2n}-1)\cdot 2nx^{2n-1}}{(x^{2n}+1)^2} \right)$$

N 13.128

$$y = \ln \sqrt{x^2-2x\cos\alpha+1} + \operatorname{ctg}\alpha \cdot \operatorname{arctg} \frac{x-\cos\alpha}{\sin\alpha}$$

$$y' = \frac{1}{2} \cdot \frac{1}{x^2-2x\cos\alpha+1} \cdot (2x-2\cos\alpha) +$$

$$+ \operatorname{ctg}\alpha \cdot \frac{1}{1+\left(\frac{x-\cos\alpha}{\sin\alpha}\right)^2} \cdot \frac{1}{\sin\alpha}$$

N 13.126

$$y = \arcsin \frac{\sin\alpha \sin x}{1-\cos\alpha \cos x}$$

$$y' = \frac{1}{\sqrt{1-\left(\frac{\sin\alpha \sin x}{1-\cos\alpha\cos x}\right)^2}} \cdot$$

$$\cdot \left(\frac{(\sin\alpha \cos x)(1-\cos\alpha \cos x)-(\sin\alpha \sin x)(\cos\alpha \sin x)}{(1-\cos\alpha\cos x)^2}\right)$$

N 13.118

$$y = \frac{ch\, x^2}{sh^2 x^2} - \ln cth \frac{x^2}{2}$$

$$y' = \frac{2sh\, x \cdot sh^2 x^2 - ch\, x^2 \cdot 2sh\, x^2\, ch\, x^2 \cdot 2}{(sh^2 x^2)^2} -$$

$$- \frac{1}{cth \frac{x^2}{2}} \cdot \left(-\frac{1}{sh^2 \frac{x^2}{2}}\right) \cdot 2x$$

N 13.123

$$y = \frac{1}{\sqrt{b^2-a^2}} \ln \frac{b + a\cos x + \sqrt{b^2-a^2}\sin x}{a + b\cos x}$$

$$y' = \frac{1}{\sqrt{b^2-a^2}} \cdot \frac{a + b\cos x}{b + a\cos x + \sqrt{b^2-a^2}\sin x} \cdot$$

$$\cdot \cancel{-b\sin x\,(b + a\cos x + \sqrt{b^2-a^2}\sin x)}$$

$$\cdot\left(\frac{-a\sin x + \sqrt{b^2-a^2}\cos x)(a + b\cos x) - (b + a\cos x + \sqrt{b^2-a^2}\sin x)(-b\sin x)}{(a+b\cos x)^2}\right)$$

N 13.135

$$y = \ln \frac{1-\sqrt[3]{x^2}}{\sqrt{1+\sqrt[3]{x^2}+\sqrt[3]{x^4}}} + \sqrt{3}\,\mathrm{arctg}\,\frac{1+2\sqrt[3]{x^2}}{\sqrt{3}}$$

$$y' = \frac{\sqrt{1+\sqrt[3]{x^2}+\sqrt[3]{x^4}}}{1-\sqrt[3]{x^2}} \cdot \left(\frac{-\frac{2}{3\sqrt{x}}\left(\sqrt{1+\sqrt[3]{x^2}+\sqrt[3]{x^4}}\right)}{1+\sqrt[3]{x^2}+\sqrt[3]{x^4}} -\right.$$

$$\left.- \frac{(1-\sqrt[3]{x^2})\left(\frac{1}{2\sqrt{1+\sqrt[3]{x}+\sqrt[3]{x^4}}} \cdot \left(\frac{2}{3\sqrt{x}} + \frac{4}{3}\cdot\sqrt[3]{x}\right)\right)}{1+\sqrt[3]{x^2}+\sqrt[3]{x^4}}\right) +$$

$$+\left(\sqrt{3}\; \frac{1}{1+\left(\frac{1+2\sqrt[3]{x^2}}{\sqrt{3}}\right)^2} \cdot \left(\frac{2}{\sqrt{3}} \cdot \frac{2}{3\sqrt[3]{x}}\right)\right)$$

N 13.136

$$y = \ln\sqrt{\frac{\sqrt{x^4+1} - \sqrt{2}\,x}{\sqrt{x^2+1} + \sqrt{2}\,x}} - \operatorname{arctg}\frac{\sqrt{2}\,x}{\sqrt{x^4+1}} -$$

$$y' = \frac{1}{2}\,\frac{\sqrt{x^4+1} + \sqrt{2}\,x}{\sqrt{x^4+1} - \sqrt{2}\,x} \cdot \left(\frac{\left(\frac{2x^3}{\sqrt{x^4+1}} - \sqrt{2}\right)(\sqrt{x^4+1} + \sqrt{2}x)}{(\sqrt{x^4+1} + \sqrt{2}x)^2}\right.$$

$$\left. - \frac{\left(\frac{2x^3}{\sqrt{x^4+1}} + \sqrt{2}\right)(\sqrt{x^4+1} - \sqrt{2}\,x)}{(\sqrt{x^4+1} + \sqrt{2}x)^2}\right) -$$

$$- \frac{1}{1 + \left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)^2} \cdot \left(\frac{\sqrt{2}\,(\sqrt{x^4+1} - \sqrt{2}\,x\left(\frac{4x^3}{2\sqrt{x^4+1}}\right)}{(\sqrt{x^4+1})^2}\right)$$

N 949

$$y = \sqrt{1-x^2} \cdot \ln\sqrt{\frac{1-x}{1+x}} + \frac{1}{2}\ln\frac{1-\sqrt{1-x^2}}{1+\sqrt{1-x^2}} +$$

$$+ \sqrt{1-x^2} + \arcsin x$$

$$y' = -\frac{x}{\sqrt{1-x^2}}\ln\sqrt{\frac{1-x}{1+x}} + \frac{1}{2}\sqrt{1-x^2}\left(-\frac{1}{1-x} - \frac{1}{1+x}\right)$$

$$+ \frac{1}{2}\left(\frac{x}{(1-\sqrt{1-x^2})\sqrt{1-x^2}} + \frac{x}{(1+\sqrt{1-x^2})(\sqrt{1-x^2})}\right)$$

$$- \frac{x}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-x^2}} = \frac{\sqrt{1-x^2}}{x} - \frac{x}{\sqrt{1-x^2}}\ln\sqrt{\frac{1-x}{1+x}}$$