Дз # 9

Задача 1

Найти жорданову нормальную форму матрицы

$$\begin{vmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ 1 & 2 & 0 & 0 & \dots & 0 \\ 1 & 2 & 3 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 2 & 3 & 4 & \dots & n \end{vmatrix}$$

$$|A - \lambda E| = \begin{vmatrix} (1-\lambda) & 0 & 0 & 0 & \dots & 0 \\ 1 & (2-\lambda) & 0 & 0 & \dots & \\ 1 & 2 & (3-\lambda) & 0 & \dots & \\ \vdots & & \ddots & & & \vdots \\ 1 & 2 & 3 & & & \dots \end{vmatrix}$$

$$\Delta = \prod_{i=1}^{n} (i - \lambda) \qquad \sigma = \{1, 2, \dots, n\}$$

$\lambda = 1$

$$\left|\begin{array}{ccccc|c} 0 & 0 & 0 & 0 & \dots & 0 \\ 1 & 1 & 0 & 0 & \dots & 0 \\ 1 & 2 & 2 & 0 & \dots & \vdots \\ \vdots & \vdots & & \vdots & & \vdots \\ & & & & & 0 \end{array}\right|$$

$$\vartheta_1 = \begin{vmatrix} 1 \\ [x_1] \end{vmatrix}$$

$$\vartheta_i = \begin{vmatrix} 0 \\ 1 \\ [x_i] \end{vmatrix}$$

x_i – до конца от x_i-того числа

$$A^J = \begin{vmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 2 & 0 & \dots & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & \dots & & n \end{vmatrix}$$

$$\begin{vmatrix} 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & 0 & 0 & \dots & 0 \end{vmatrix}$$

$|A - E\lambda| = 0$

$\sigma = \{0^{(n)}\}$

$\lambda = 0 \qquad v_1 = \begin{vmatrix} 1 \\ \vdots \\ 0 \end{vmatrix}$

v_i – присоединен к v_{i-1} $\quad i = 2 \dots n$

$v_i = \begin{vmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{vmatrix}$ – i-тая позиция

$$A^J = \begin{vmatrix} 0 & 1 & 0 & 0 & 0 \dots 0 \\ 0 & 0 & 1 & \dots & \vdots \\ \vdots & 0 & 0 & 1 \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & 1 \\ 0 & 0 & 0 & \dots & 0 \end{vmatrix}$$

Задача 2

$$A = \begin{vmatrix} 3 & 1 & -1 \\ 0 & 2 & 0 \\ 1 & 1 & 1 \end{vmatrix} \qquad |A - E\lambda| = \begin{vmatrix} (3-\lambda) & 1 & -1 \\ 0 & (2-\lambda) & 0 \\ 1 & 1 & (1-\lambda) \end{vmatrix}$$

$$\Delta = ((3-\lambda)(2-\lambda)(1-\lambda)) + (0 \cdot 1 \cdot (-1)) + (1 \cdot 0 \cdot 1)$$

$$- (-1 \cdot 1 \cdot (2-\lambda)) - (0 \cdot 1 \cdot (1-\lambda)) - (1 \cdot 0 \cdot (3-\lambda))$$

$$= ((3-\lambda)(2-\lambda)(1-\lambda)) + 0 + 0 - (-(2-\lambda) - 0 - 0 =$$

$$= ((6 - 3\lambda - 2\lambda + \lambda^2)(1-\lambda)) + 2 - \lambda =$$

$$= 6 - 6\lambda - 3\lambda + 3\lambda^2 - 2\lambda + 2\lambda^2 + \lambda^2 - \lambda^3 + 2 - \lambda =$$

$$= -\lambda^3 + 6\lambda^2 - 12\lambda + 8 = \lambda^3 - 6\lambda^2 + 12\lambda - 8$$

$$\begin{vmatrix} (3-\lambda) & 1 \\ 0 & (2-\lambda) \end{vmatrix} = \lambda^2 - 5\lambda + 6 = (\lambda-3)(\lambda-2) \qquad \begin{vmatrix} (3-\lambda) & 1 \\ 1 & 1 \end{vmatrix} = 2-\lambda \qquad \begin{vmatrix} 0 & (2-\lambda) \\ 1 & 1 \end{vmatrix} = \lambda-2 = -(2-$$

$$\begin{vmatrix} (3-\lambda) & -1 \\ 0 & 0 \end{vmatrix} = 0 \qquad \begin{vmatrix} (3-\lambda) & -1 \\ 1 & (1-\lambda) \end{vmatrix} = \lambda^2 - 4\lambda + 4 = (\lambda-2)^2 \qquad \begin{vmatrix} 0 & 0 \\ 1 & (1-\lambda) \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & -1 \\ (2-\lambda) & 0 \end{vmatrix} = 2-\lambda = -(\lambda-2) \qquad \begin{vmatrix} 1 & -1 \\ 1 & (1-\lambda) \end{vmatrix} = 2-\lambda = -(\lambda-2) \qquad \begin{vmatrix} (2-\lambda) & 0 \\ 1 & (1-\lambda) \end{vmatrix} = \lambda^2 - 3\lambda - 2 = (\lambda-2)(\lambda-1)$$

НОД$\{\ldots\} = \lambda - 2$

$$\mu_A(\lambda) = \frac{(-1)^3(\lambda^3 - 6\lambda^2 + 12\lambda - 8)}{(\lambda-2)} = \frac{-(\lambda-2)(\lambda^2-4\lambda-4)}{(\lambda-2)}$$

$$= \underline{-\lambda^2 + 4\lambda + 4}$$

$$\begin{vmatrix} 4 & -2 & 2 \\ -5 & 7 & -5 \\ -6 & 6 & -4 \end{vmatrix} = A \qquad |A - E\lambda| = \begin{vmatrix} (4-\lambda) & -2 & 2 \\ -5 & (7-\lambda) & -5 \\ -6 & 6 & (-4-\lambda) \end{vmatrix}$$

$$\Delta = ((4-\lambda)(7-\lambda)(-4-\lambda)) + (-5 \cdot 6 \cdot 2) + (-2 \cdot (-5) \cdot (-6))$$

$$-(2 \cdot (7-\lambda) \cdot (-6)) - (6 \cdot (-5) \cdot (4-\lambda)) - (-5 \cdot (-2) \cdot (-4-\lambda))$$

$$= ((28 - 11\lambda + \lambda^2)(-4-\lambda)) + (-60) + (-60) - (-84 + 12\lambda) -$$

$$-(-120 + 30\lambda) - (-40 - 10\lambda) =$$

$$-112 - 28\lambda + 44\lambda + 11\lambda^2 - 4\lambda^2 - \lambda^3 - 60 - 60 + 84 - 12\lambda + 120 - 30\lambda + 40 + 10\lambda$$

$$= \cancel{+10} - 16\lambda + 7\lambda^2 - \lambda^3 = \lambda^3 - 7\lambda^2 + 16\lambda \cancel{+10} - 12$$

$$\begin{vmatrix} (4-\lambda) & -2 \\ -5 & (7-\lambda) \end{vmatrix} = \lambda^2 - 11\lambda + 18 = (\lambda-2)(\lambda-2) \quad \begin{vmatrix} (4-\lambda) & -2 \\ -6 & 6 \end{vmatrix} = 12 - 6\lambda = -6(\lambda-2) \quad \begin{vmatrix} -5 & (7-\lambda) \\ -6 & 6 \end{vmatrix} = 5 - 6\lambda$$

$$\begin{vmatrix} (4-\lambda) & 2 \\ -5 & -5 \end{vmatrix} = 5\lambda - 10 = 5(\lambda-2) \quad \begin{vmatrix} (4-\lambda) & 2 \\ -6 & (-4-\lambda) \end{vmatrix} = \lambda^2 - 4 = (\lambda-2)(\lambda+2) \quad \begin{vmatrix} -5 & -5 \\ -6 & (-4-\lambda) \end{vmatrix} = (5\lambda - 10) = 5(\lambda-2)$$

$$\begin{vmatrix} -2 & 2 \\ (7-\lambda) & -5 \end{vmatrix} = 2\lambda - 4 = 2(\lambda-2) \quad \begin{vmatrix} -2 & 2 \\ +6 & (-4-\lambda) \end{vmatrix} = 2\lambda - 4 = 2(\lambda-2) \quad \begin{vmatrix} (7-\lambda) & -5 \\ 6 & (-4-\lambda) \end{vmatrix} = \lambda^2 - 3\lambda + 2 = (\lambda-2)(\lambda-1)$$

НОД$\{\ldots\} = (\lambda - 2)$

$$\mu = \frac{(-1)^3(\lambda^3 - 7\lambda^2 + 16\lambda \overset{-12}{\cancel{+10}})}{(\lambda-2)} = \frac{-(\lambda-2)(\lambda^2 - 5\lambda + 6)}{\lambda - 2}$$

$$= \underline{-\lambda^2 + 5\lambda + 6}$$

Задача 3

Вычислить значение $\sin(A)$, если

$$A = \begin{vmatrix} \pi-1 & 1 \\ -1 & \pi+1 \end{vmatrix}$$

$$\sin(A) = T \cdot \sin(A J) \cdot S$$

$$\sin(A J) = \begin{vmatrix} \sin\pi & \cos\pi \\ 0 & \sin\pi \end{vmatrix} = \begin{vmatrix} 0 & -1 \\ 0 & 0 \end{vmatrix}$$

$$T = \begin{vmatrix} -1 & 1 \\ -1 & 0 \end{vmatrix} \qquad S = \begin{vmatrix} 0 & -1 \\ 1 & -1 \end{vmatrix}$$

$$\sin(A) = \begin{vmatrix} 1 & -1 \\ 1 & -1 \end{vmatrix}$$