Ер # 1

Задача 1

Найти косинус угла между двумя векторами x и y, заданными в неортонормальном базисе с матрицей Грама G

$$x = \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \quad y = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad G \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$$

$$\frac{(a,b)}{|a|\,|b|} = \cos\varphi = \frac{-1}{\sqrt{2}}$$

$$|x| = \sqrt{(1,-2)\begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \end{pmatrix}} = \sqrt{(-1,-1)\begin{pmatrix} 1 \\ -2 \end{pmatrix}} = 1$$

$$|y| = \sqrt{(0,1)\begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}} = \sqrt{(3\ 2)\begin{pmatrix} 0 \\ 1 \end{pmatrix}} = \sqrt{2}$$

$$(x,y) = (1,-2)\begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = (-1,-1)\begin{pmatrix} 0 \\ 1 \end{pmatrix} = -1$$

Задача 2

Ортогонализировать систему из трех векторов.

$$x_1 = \begin{pmatrix} 1 \\ 0 \\ -2 \\ 4 \end{pmatrix} \quad x_2 = \begin{pmatrix} -2 \\ 1 \\ 2 \\ -6 \end{pmatrix} \quad x_3 = \begin{pmatrix} 0 \\ -1 \\ 3 \\ -4 \end{pmatrix}$$

$$\bar{y}_1 = \bar{x}_1 \qquad \bar{y}_2 = \bar{x}_2 - \frac{(\bar{x}_2 \cdot \bar{y}_1)}{(\bar{y}_1 \cdot \bar{y}_1)} \cdot \bar{y}_1$$

$$\bar{y}_3 = \bar{x}_3 - \frac{(\bar{x}_3 \cdot \bar{y}_1)}{(\bar{y}_1 \cdot \bar{y}_1)} \bar{y}_1 - \frac{(\bar{x}_3 \cdot \bar{y}_2)}{(\bar{y}_2 \cdot \bar{y}_2)} \cdot \bar{y}_2$$

$$\bar{y}_n = \bar{x}_n - \frac{(\bar{x}_n \cdot \bar{y}_1)}{(\bar{y}_1 \cdot \bar{y}_1)} \cdot \bar{y}_1 - \frac{(\bar{y}_n \cdot \bar{y}_2)}{(y_2 \cdot y_2)} \cdot \bar{y}_2 \dots - \frac{(\bar{x}_n \bar{y}_{n-1})}{(\bar{y}_{n-1})(y_{n-1})} \cdot$$

$$y_1 = \begin{pmatrix} 1 \\ 0 \\ -2 \\ 4 \end{pmatrix} \qquad (x_2 y_1) = (-2\ 1\ 2\ -6) \begin{pmatrix} 1 \\ 0 \\ -2 \\ 4 \end{pmatrix} = (-30)$$

$(y_1 y_1) = 21$ $\qquad x_1 \cdot x_2$

$$y_2 = \begin{vmatrix} -2 \\ 1 \\ 2 \\ -6 \end{vmatrix} - \left(-\frac{30}{21}\right) \cdot \begin{vmatrix} 1 \\ 0 \\ -2 \\ 4 \end{vmatrix} =$$

$$= \begin{vmatrix} -2 \\ 1 \\ 2 \\ -6 \end{vmatrix} + \begin{vmatrix} \frac{10}{7} \\ 0 \\ -20/7 \\ 4/7 \end{vmatrix} = \begin{vmatrix} -4/7 \\ 1 \\ -6/7 \\ -2/7 \end{vmatrix}$$

$(x_3\ y_1) = -22$

$(x_3\ y_2) = -17/7$

$(y_2\ y_2) = 15/7$

$$y_3 = \begin{vmatrix} 0 \\ -1 \\ 3 \\ -4 \end{vmatrix} - \left(-\frac{22}{21}\right) \cdot \begin{vmatrix} 1 \\ 0 \\ -2 \\ 4 \end{vmatrix} -$$

$$- \left(-\frac{17}{7} : \frac{15}{7}\right) \cdot \begin{vmatrix} -4/7 \\ 1 \\ -6/7 \\ -2/7 \end{vmatrix} =$$

Ответ $y_1\ y_2\ y_3$

$A = \begin{vmatrix} 0 & 4 & 0 \\ -2 & 0 & 6 \end{vmatrix}$ $U_1 = \begin{vmatrix} 0 \\ 0 \end{vmatrix}$

Zadatak 5

$A = \begin{vmatrix} 10 & -2 & 0 \\ -2 & 10 & 0 \\ 0 & 0 & 2 \end{vmatrix}$ $(A - \xi\lambda) \begin{vmatrix} (10-\lambda) & -2 & 0 \\ -2 & (10-\lambda) & 0 \\ 0 & 0 & (2-\lambda) \end{vmatrix}$

$\Delta = -\lambda^3 + 22\lambda^2 - 156\lambda + 192$

$\lambda = \{12 ; 8 ; 2\}$

Задача 4

$$A = \begin{vmatrix} 6 & 0 & -2 \\ 0 & 4 & 0 \\ -2 & 0 & 6 \end{vmatrix} \qquad \Delta = (-\lambda - 8)(\lambda - 4)^2 = 0$$

$\lambda_1 = 8 \qquad \lambda_2 = 4$

при $\lambda = 8$

$$\left|\begin{array}{ccc|c} -2 & 0 & -2 & 0 \\ 0 & -4 & 0 & 0 \\ -2 & 0 & -2 & 0 \end{array}\right|$$

$x_1 = -x_3$

$x_2 = 0$

$x_3 = x_3$

$$\underline{v_1} = \begin{vmatrix} -1 \\ 0 \\ 1 \end{vmatrix}$$

$\lambda = 4$

$$\left|\begin{array}{ccc|c} 2 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ -2 & 0 & 2 & 0 \end{array}\right|$$

$x_1 = x_3$

$x_2 = x_2$

$x_3 = x_3$

при $x_2 = 1 \quad x_3 = 0$

$$\underline{v_2} = \begin{vmatrix} 0 \\ 1 \\ 0 \end{vmatrix}$$

~~$v_3 =$~~

при $x_2 = 0 \quad x_3 = 1$

$$\underline{v_3} = \begin{vmatrix} 1 \\ 0 \\ 1 \end{vmatrix}$$

Zadaca 3

$$x = \begin{vmatrix} -1 \\ -2 \\ 4 \\ 8 \end{vmatrix} \qquad e_1 = \begin{vmatrix} 1 \\ 1 \\ -1 \\ -3 \end{vmatrix} \qquad e_2 = \begin{vmatrix} 0 \\ 1 \\ -2 \\ -4 \end{vmatrix}$$

$$x = \alpha e_1 + \beta e_2 + \varepsilon$$

$$\begin{cases} (e_1, x) = \alpha (e_1, e_1) + \beta (e_1, e_2) \\ (e_2, x) = \alpha (e_1, e_2) + \beta (e_2, e_2) \end{cases}$$

$$\begin{cases} -31 = \alpha \cdot 12 + \beta \cdot 15 \\ -42 = \alpha \cdot 15 + \beta \cdot 21 \end{cases}$$

$\alpha = -7/9$

$\beta = -13/9$

Задача 6

полное - 4 $(1-x)^4$

алгебр - 3

спектр - 2

$$\left|\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right|$$

Задача 7

$$A = \begin{vmatrix} -2 & 0 & -1 \\ 0 & -2 & 1 \\ 0 & 0 & -2 \end{vmatrix} \qquad \chi(x) = (x+2)^3$$

$(A+2E)^{\wedge m} = 0$ надо найти m

$$\begin{vmatrix} -2 & 0 & -1 \\ 0 & -2 & 1 \\ 0 & 0 & -2 \end{vmatrix} + \begin{vmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{vmatrix}^{\wedge m} = 0$$

$\min(m) = 2 \Rightarrow$ Ответ

$(x+3)2$