Дз # 8

Задача 2

$x^2+6x-y^2-8y-1=0$

$(x^2+6x+9)-(y^2+8y+16)+4=0$

$(x+3)^2-(y+4)^2=-4$

$\frac{(x+3)^2}{4}-\frac{(y+4)^2}{4}=-1 \Rightarrow O[-3;-4]$

Задача 3

$-x^2+6x-4y^2+10y+5=0$

$x^2-6x+4y^2-10y-5=0$

$(x^2-6x+9)+(4y^2-10y^2+6,25)-20,25=0$

$(x-3)^2+(2y^2-2,25)^2=20,25$

$\frac{(x-3)^2}{20,25}+\frac{4(y^2-1,75)^2}{20,25}=1 \Rightarrow a=\sqrt{20,25}=4,5$

$b=\sqrt{5,025}=2,25$

Задача 4

$-4x^2-10x+2y^2+2y-5=0$

$4x^2+10x-2y^2-2y+5=0$

$(4x^2+10x+6,25)-(2y^2+2y+0,5)-0,75=0$

$$\frac{(2x+2{,}5)^2}{0{,}75} - \frac{\left(\sqrt{2}x + \frac{\sqrt{2}}{2}\right)^2}{0{,}75} = 1$$

$$\Rightarrow a^2 = 0{,}1875 \quad b^2 = 0{,}375 \Leftrightarrow$$

$$\frac{4(x+1{,}25)^2}{0{,}75} - \frac{2\left(y+\frac{1}{2}\right)^2}{0{,}75} = 1 \qquad c = \sqrt{a^2+b^2} = 0{,}75 \Leftrightarrow \underline{2c = 1{,}5}$$

Задача 5

$$2x - y^2 - 10y + 5 = 0$$

$$y^2 + 10y - 2x - 5 = 0$$

$$(y^2 + 10y + 25) - 2x - 24 = 0$$

$$(y+5)^2 = 2x + 28 \qquad y = -5 \quad x = -14$$

$$(y+5)^2 = y^2$$

$$y^2 = 2p(x+14) \quad \Rightarrow p = 1$$

Задача 6

$$2x^2 + 4\sqrt{3}xy - 2y^2 + 5 = 0$$

$$2(x'^2\cos^2\varphi - 2x'y'\cos\varphi\sin\varphi + y'^2\sin^2\varphi) +$$

$$+ 4\sqrt{3}(x'^2\cos\varphi\sin\varphi + x'y'\cos^2\varphi - x'y'\sin^2\varphi -$$

$$- y'^2\sin\varphi\cos\varphi) - 2(x'^2\sin^2\varphi + 2x'y'\cos\varphi\sin\varphi +$$

$$+ y'^2\cos^2\varphi) + 5 = 0$$

$2x'^2\cos^2\varphi - 4x'y'\cos\varphi\sin\varphi + 2y'^2\sin^2\varphi +$
$+ 4\sqrt{3}\,x'^2\cos\varphi\sin\varphi + 4\sqrt{3}\,x'y'\cos^2\varphi - 4\sqrt{3}\,x'y'\sin^2\varphi -$
$- 4\sqrt{3}\,y'^2\sin\varphi\cos\varphi - 2x'^2\sin^2\varphi - 4x'y'\cos\varphi\sin\varphi$
$- 2y'^2\cos^2\varphi + 5 = 0$

$x'^2(2\cos^2\varphi + 4\sqrt{3}\cos\varphi\sin\varphi - 2\sin^2\varphi) +$
$+ y'^2(2\sin^2\varphi - 4\sqrt{3}\sin\varphi\cos\varphi - 2\cos^2\varphi) +$
$+ x'y'(-8\cos\varphi\sin\varphi + 4\sqrt{3}\cos^2\varphi - 4\sqrt{3}\sin^2\varphi) + 5 = 0$

$-8\cos\varphi\sin\varphi + 4\sqrt{3}\cos^2\varphi - 4\sqrt{3}\sin^2\varphi = 0 \,/ : -4\sin^2\varphi$
$2\,\mathrm{ctg}\,\varphi - \sqrt{3}\,\mathrm{ctg}^2\varphi + \sqrt{3} = 0 \,/\; \mathrm{ctg}\,\varphi = t$

$2t - \sqrt{3}t^2 + \sqrt{3} = 0 \Rightarrow t_1 = \sqrt{3} \qquad t_2 = -\frac{\sqrt{3}}{3}$
$\mathrm{ctg}\,\varphi = \sqrt{3} \Rightarrow \varphi = \frac{\pi}{6} + \pi n \quad n \in Z \qquad \varphi$

$\mathrm{ctg}\,\varphi = -\frac{1}{\sqrt{3}} \Rightarrow \varphi = \frac{2\pi}{3} + \pi n \qquad n \in Z$

$\varphi = \frac{\pi}{6}$

Задача 7

$7x^2 - 2\sqrt{3}\,xy + x(-42 - 2\sqrt{3}) + 5y^2 + y(10+6\sqrt{3}) + 6\sqrt{3}$

$+70=0$

$\begin{cases} \cancel{x = x' + a} \\ \cancel{y = y' + b} \end{cases}$

Это уравнение имеет вид

$a_{11}x^2 + 2a_{12}xy + 2a_{13}x + a_{22}y^2 + 2a_{23}y + a_{33} = 0$

где $a_{11} = 7$ $a_{12} = \sqrt{3}$ $a_{13} = -21 - \sqrt{3}$ $a_{22} = 5$

$a_{23} = 5 + 3\sqrt{3}$ $a_{33} = 6\sqrt{3} + 70$

$\Delta \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \begin{vmatrix} 7 & -\sqrt{3} \\ -\sqrt{3} & 5 \end{vmatrix} = 32 > 0 \Rightarrow$

$\begin{cases} a_{11}x_0 + a_{12}y_0 + a_{13} = 0 \\ a_{12}x_0 + a_{22}y_0 + a_{23} = 0 \end{cases} \Rightarrow \begin{cases} 7x_0 - \sqrt{3}y_0 - 21 - \sqrt{3} = 0 \\ -\sqrt{3}x_0 + 5y_0 + 5 + 3\sqrt{3} = 0 \end{cases}$

$x_0 = 3$; $y_0 = 1$ Делаем поворот

$7x_1'^2 \cos^2\varphi - 14x'y'\cos\varphi \sin\varphi + 7y'^2 \sin^2\varphi = 7x$

$-2\sqrt{3}xy = (x_1'^2 \cos\varphi \sin\varphi + x'y'\cos^2\varphi - yx'\sin^2\varphi - \varphi^2 \sin\varphi\cos\varphi$

$(-42-2\sqrt{3})x = (-42-2\sqrt{3})x'\cos\varphi - (-42-2\sqrt{3})y'\sin\varphi$

$(10+6\sqrt{3})y = (10+6\sqrt{3})x'\sin\varphi + (10+6\sqrt{3})y'\cos^2\varphi$

$5y^{2'} = 5x'^2 \sin^2\varphi + 10x'y'\sin\varphi\cos\varphi + 5y'^2\cos^2\varphi$

$x_1 y_1(-4\cos\varphi\sin\varphi - 2\sqrt{3}\cos^2\varphi + 2\sqrt{3}\sin^2\varphi) = 0 \;/ 2\cos^2\varphi$

$\sqrt{3}\,tg^2\varphi - 2tg\varphi - \sqrt{3} = 0 \iff t_{1,2} = \sqrt{3};\ -\frac{1}{\sqrt{3}} \Rightarrow \angle = 330^\circ$

Задача 8

$9x^2 - 50\sqrt{3}xy + x(42 - 50\sqrt{3}) + 59y^2 + y(118 - 202\sqrt{3}) - 202\sqrt{3} + 44 = 0$

$a_{11}x^2 + 2a_{12}xy + 2a_{13}x + a_{22}y^2 + 2a_{23}y + a_{33} = 0$

0,899

$a_{11} = 9 \quad a_{12} = -25\sqrt{3} \quad a_{13} = 21 - 25\sqrt{3} \quad a_{22} = 59$

$a_{23} = 59 - 101\sqrt{3} \quad a_{33} = 44 - 202\sqrt{3}$

$\det A = \begin{vmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{vmatrix} = \begin{vmatrix} 9 & -25\sqrt{3} \\ -25\sqrt{3} & 59 \end{vmatrix} = -1344 \neq 0$

$\begin{cases} a_{11}x_0 + a_{12}y_0 + a_{13} = 0 \\ a_{12}x_0 + a_{22}y_0 + a_{23} = 0 \end{cases} \Rightarrow \begin{cases} 9x_0 - 25\sqrt{3}y_0 - 25\sqrt{3} + 21 = 0 \\ -25\sqrt{3}x_0 + 59y_0 - 101\sqrt{3} + 59 = 0 \end{cases} \quad x_0 = -\frac{33}{4} \quad y_0 = -1 - \frac{2\sqrt{3}}{4}$

$a'_{33} + a_{11}x'^2 + 2a_{12}x'y' + a_{22}y'^2 = 0 \qquad a'_{33} = a_{13}x_0 + a_{23}y_0 + a_{33}$

$a'_{33} = x_0(21 - 25\sqrt{3}) + y_0(59 - 101\sqrt{3}) - 202\sqrt{3} + 44$

$a'_{33} = -\frac{583\sqrt{3}}{4} - 55 + \left(-1 - \frac{2\sqrt{3}}{4}\right)(59 - 101\sqrt{3})$

$9x'^2 - 50\sqrt{3}x'y' + 59y'^2 - \frac{589\sqrt{3}}{4} - 55 + \left(-1 - \frac{2\sqrt{3}}{4}\right)(59 - 101\sqrt{3}) = 0$

$x' = \tilde{x}\cos\varphi - \tilde{y}\sin\varphi \qquad y' = \tilde{x}\sin\varphi + \tilde{y}\cos\varphi$

$\cos(2\varphi = \frac{a_{11} - a_{22}}{2a_{12}} \qquad \cos\varphi = \frac{\sqrt{3}}{2} \qquad \varphi = \frac{\pi}{6} \qquad \sin\varphi = \frac{1}{2}$

$x' = \frac{\sqrt{3}\tilde{x}}{2} - \frac{\tilde{y}}{2} \qquad y' = \frac{\tilde{x}}{2} + \frac{\sqrt{3}\tilde{y}}{2}$

Задача 8

$\Rightarrow 9x'^2 - 50\sqrt{3}x'y' + 59y'^2 - \frac{589\sqrt{3}}{4} - 55 + \left(-1 - \frac{2\sqrt{3}}{2}\right)(59 - 101\sqrt{3}) = 0$

$59\left(\frac{\tilde{x}}{2} + \frac{\sqrt{3}\tilde{y}}{2}\right)^2 - 50\sqrt{3}\left(\frac{\tilde{x}}{2} + \frac{\sqrt{3}\tilde{y}}{2}\right)\left(\frac{\sqrt{3}\tilde{x}}{2} - \frac{\tilde{y}}{2}\right) + 9\left(\frac{\sqrt{3}\tilde{x}}{2} - \frac{\tilde{y}}{2}\right)^2 - \frac{589\sqrt{3}}{2}$

$-55 + \left(-1 - \frac{2\sqrt{3}}{4}\right)(59 - 101\sqrt{3}) = 0 \Leftrightarrow -16\tilde{x}^2 + 84\tilde{y}^2 - \frac{192}{4} = 0$

$\frac{\tilde{x}^2}{\frac{12}{4}} - \frac{\tilde{y}^2}{\frac{16}{59}} = -1$ Повернём гиперболу на $90°$

получим $a^2 = 0,82$ $b^2 = 1,71$

$\Rightarrow \varepsilon = c/a = 2,5$ $p = b^2/a = 3$

Задача 10

$16\sqrt{3}L^2xy - 288L^2 + 144\sqrt{3}L^2 + x^2(8 - 8L^2) + x(-48L^2 + 48\sqrt{3}L^2 + 32)$
$+ y^2(8 - 24L^2) + y(-144L^2 + 48\sqrt{3}L^2 + 16\sqrt{3} + 48) + 48\sqrt{3} + 96 = 0$

$L = 6$

$(a - d)\sin 2\varphi = 2b\cos 2\varphi$

$16L^2 \sin 2\varphi = 16\sqrt{3}L^2 \cos 2\varphi \quad | : 16\sqrt{3}L^2 \sin 2\varphi$

$1 = \sqrt{3}\,\mathrm{ctg}\,2\varphi$

$\varphi = \frac{\pi}{6}$ $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ $\sin\frac{\pi}{6} = \frac{1}{2}$

$(x'^2 \cos^2\varphi - 2x'y'\cos\varphi\sin\varphi + y'^2\sin^2\varphi)(8-8L^2) +$

$+(x'^2\sin^2\varphi + 2x'y'\cos\varphi\sin\varphi + y'^2\cos^2\varphi)(8-24L^2) +$

$+(x'^2\cos\varphi\sin\varphi + x'y'\cos^2\varphi - y'x'\sin^2\varphi - y'^2\sin\varphi\cos\varphi)\cdot$

$\cdot 16\sqrt{3}L^2 + (x'\cos\varphi - y'\sin\varphi)(-48L^2 + 48\sqrt{3}L^2 + 32) +$

$+(x'\sin\varphi + y'\cos\varphi)(-144L^2 + 48\sqrt{3}L^2 + 16\sqrt{3} + 48) - 288L^2 +$

$+144\sqrt{3}L^2 + 48\sqrt{3} + 96 = 0$

Выразим коэф при x'^2 и y'^2

$x'^2(8\cos^2\varphi - 8L^2\cos^2\varphi + 8\sin^2\varphi - 24L^2\sin^2\varphi + 16\sqrt{3}L^2\cos\varphi\sin\varphi)$

$y'^2(8\sin^2\varphi - 8L^2\sin^2\varphi + 8\cos^2\varphi - 24L^2\cos^2\varphi - 16\sqrt{3}L^2\cos\varphi\sin\varphi)$

Поставим значение $\sin\frac{\pi}{6}$ и $\cos\frac{\pi}{6}$

получим:

$x'^2(6 - 6L^2 + 2 - 6L^2 + 12L^2) = 8x'^2$

$y'^2(2 - 2L^2 + 6 - 18L^2 - 12L^2) = y'^2(8 - 32L^2)$

Мы знаем коэф при x'^2 и y'^2 =>

$$\Rightarrow e = \sqrt{\frac{b^2 - a^2}{a}} = \sqrt{\frac{k}{8-32L^2} \div \frac{k}{\cancel{8-32L^2}}} \cdot \sqrt{\frac{8-32L^2}{k}}$$

$$= \sqrt{\frac{k(1+4L^2+1)}{\cancel{8-32L^2}}} \cdot \frac{\cancel{8-32L^2}}{\cancel{k}} \qquad \sqrt{4L^2} = 6$$

$4L^2 = 36$

$L^2 = 9 \qquad \underline{L = \pm 3}$

задача 1

$21x^2 + 6\sqrt{3}\,xy + x(-84 - 6\sqrt{3}) + 15y^2 + y(-30-12\sqrt{3}) +$
$+12\sqrt{3} + 119 = 0$

Приводим к квадратичному виду

$B = 21x^2 + 6\sqrt{3}xy + 15y^2$ $\qquad \begin{vmatrix} 21 & 6\sqrt{3}/2 \\ 6\sqrt{3}/2 & 15 \end{vmatrix}$

$\begin{vmatrix} 21-\lambda & 3\sqrt{3} \\ 3\sqrt{3} & 15-\lambda \end{vmatrix} =$

$= (-\lambda+15)\cdot(-\lambda+21) - 27 = 0$ $\qquad \lambda_1 = 12 \quad \lambda_2 = 24$

=> эллипс

Вид кв. формы $\quad 12x_1^2 + 24y_1^2 \quad \lambda_1 = 12$

$(21-12)x_1 + 3\sqrt{3}y_1 = 0$ $\quad$ собственный вектор отв. числу

$3\sqrt{3}x_1 + (15-12)y_1 = 0$ $\quad$ $\lambda_1 = 12$ при $x_1 = -\sqrt{3}/3$
$\bar{x}_1 = (-\sqrt{3}/3, 1)$

$\bar{i} = \left(\frac{-\sqrt{3}/3}{2\sqrt{3}/3}; \frac{1}{2\sqrt{3}/3}\right)$ $\qquad \sqrt{-\sqrt{3}/3^2 + 1^2} = 2\sqrt{3}/3$

при $\lambda_2 = 24$ $\quad (21-24)x_1 + 3\sqrt{3}y_1 = 0$ $\qquad \bar{x}_2(\sqrt{3}, 1)$
$3\sqrt{3}x_1 + (15-24)y_1 = 0$ $\qquad \bar{j} = \left(\frac{\sqrt{3}}{2}; \frac{1}{2}\right)$

$x = \frac{-\sqrt{3}/3}{2\sqrt{3}/3}x_1 + \frac{\sqrt{3}}{2}y_1$

$\Rightarrow 12x_1^2 - 12\sqrt{3}x_1 + 24x_1 + 24y_1^2 - 48\sqrt{3}y_1 - 24y_1 + 12\sqrt{3} + 119$

$y = \frac{1}{2\sqrt{3}/3}x_1 + \frac{\sqrt{3}}{2}y_1$

$0{,}6\left(x_1 + \sqrt{3}\,\frac{-30-12\sqrt{3}}{48} + \frac{\sqrt{3}}{8} + \frac{7}{4}\right)^2 + 1{,}2\left(y_1 + \sqrt{3}\cdot\frac{-84-6\sqrt{3}}{96} - \frac{5}{16} - \frac{\sqrt{3}}{8}\right)^2 = 1$

мнимый э.

Задача 9

$\frac{123L^2x^2}{2}+\frac{123L^2y^2}{2}+75\sqrt{2}L^2+\frac{507L^2}{2}+xy(27L^2+6)$

$+x(25\sqrt{2}L^2+69L^2-12+\sqrt{2})+y(-219L^2-25\sqrt{2}L^2-2\sqrt{2}+6)$

$-12+3\sqrt{2}=0$

$A=\frac{123L^2}{2}$; $B=\frac{27L^2+6}{2}$; $C=\frac{123L^2}{2}$

$I(\lambda)\begin{vmatrix} A-\lambda & B \\ B & C-\lambda \end{vmatrix}=\begin{vmatrix} \frac{123L^2}{2}-\lambda & \frac{27L^2+6}{2} \\ \frac{27L^2+6}{2} & \frac{123L^2}{2}-\lambda \end{vmatrix}=0$

$\left(\frac{123L^2}{2}-\lambda\right)^2-\left(\frac{27L^2+6}{2}\right)^2=0$

$\frac{15129L^4}{4}-123\lambda L^2+\lambda^2-\left(\frac{729L^4+324L^2+36}{4}\right)=0$

Для того, чтобы кривая описывала параболу или прямую $\lambda_1\lambda_2=0$, т.е. коэффициент при $\lambda^0=1$, должен быть $=0$

$15129L^4-729L^4-324L^2-36=0$

$400L^4-9L^2-1=0$ $R=L^2$

$400R^2-9R-1=0$ $D=41^2$ $R_1=\frac{9+41}{800}=\frac{1}{16}$

$L^2=R$ $L=\sqrt{R}$ $L=\pm\frac{1}{4}$

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