Переписка КR №1

0.2 $\lim\limits_{n\to\infty} \sin\left(\frac{\pi}{n}\right)\left(\frac{1}{3+\cos\left(\frac{\pi}{n}\right)} + \frac{1}{3+\cos\left(\frac{2\pi}{n}\right)} + \ldots + \frac{1}{3+\cos\left(\frac{n\pi}{n}\right)}\right)$

$\sin\frac{\pi}{n} \sim \frac{\pi}{n}$ при $n\to\infty$

$$\lim_{n\to\infty} \sin\left(\frac{\pi}{n}\right)\sum_{i=1}^{n}\frac{1}{3+\cos\left(\frac{\pi i}{n}\right)} =$$

$$= \lim_{n\to\infty} \frac{\pi}{n}\sum_{i=1}^{n}\frac{1}{3+\cos\left(\frac{\pi i}{n}\right)} = \pi\cdot\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{3+\cos\left(\frac{\pi i}{n}\right)}\cdot\frac{1}{n}$$

$$\lim_{n\to\infty}\sum_{i=1}^{n} f(\varepsilon_i)\cdot\Delta x_i = \int_a^b f(x)\,dx$$

$$\left.\begin{aligned} &\Delta x_i = \frac{\cancel{\pi}}{n} \quad \varepsilon_i = \frac{i\pi}{n} \\ &f(\varepsilon_i) = \frac{1}{3+\cos(\varepsilon_i)} \\ &0 \le \frac{i\pi}{n} \le \pi \end{aligned}\right\} \quad \pi \lim \sum_{i=1}^{n}\frac{1}{3+\cos\left(\frac{\pi i}{n}\right)}\cdot\frac{1}{n} =$$

$$= \pi\int_0^{\pi}\frac{1}{3+\cos x}\,dx \qquad * \int\frac{1}{3+\cos x}\,dx = \int\frac{dx}{3+2\cos^2\frac{x}{2}}$$

$$= \int\frac{dx}{2+2\cos^2\frac{x}{2}} = \cancel{\int\frac{1}{\cos^2\frac{x}{2}}\cdot\frac{1}{2\left(\sin^2\frac{x}{2}+\cos^2\frac{x}{2}\right)}}$$

$$= \int \frac{1}{\cos^2\frac{x}{2}} \cdot \frac{1}{\frac{2\left(\sin^2\frac{x}{2}+\cos^2\frac{x}{2}\right)}{\cos^2\frac{x}{2}}+2}\,dx =$$

$$= \int \frac{1}{2tg^2\frac{x}{2}+4} \cdot \frac{1}{\cos^2\frac{x}{2}}\,dx \underset{\frac{1}{2}dx=\cos^2\frac{x}{2}dy}{\overset{y=tg\frac{x}{2}}{=}} \int \frac{dy}{y+2} =$$

$$= \frac{\sqrt{2}}{2}\int \frac{d\left(\frac{y}{\sqrt{2}}\right)}{\left(\frac{y}{\sqrt{2}}\right)^2+1} = \frac{1}{2}\cdot\sqrt{2}\cdot arctg\left(\frac{y}{\sqrt{2}}\right)+C = \frac{\sqrt{2}}{2} arctg\left(\frac{tg\left(\frac{x}{2}\right)}{\sqrt{2}}\right)$$

$$F(x) = \frac{\sqrt{2}}{2}\cdot arctg\left(\frac{tg\left(\frac{x}{2}\right)}{\sqrt{2}}\right) \quad \text{при} \quad x\in[0;\pi]$$

$F(x)$ не имеет точек разрыва $\Rightarrow$

$$\Rightarrow \int_0^{\pi} = F(x)\Big|_0^{\pi} = \frac{\sqrt{2}}{2}\Bigg(\underbrace{arctg\Bigg(\frac{\overbrace{tg\left(\frac{\pi}{2}\right)}^{}}{\sqrt{2}}\Bigg)}_{=\frac{\pi}{2}} - \underbrace{arctg\left(\frac{tg(0)}{\sqrt{2}}\right)}_{=0}\Bigg)$$

$$tg\frac{\pi}{2} = \infty \qquad tg(0) = 0$$

$$\Rightarrow F(x)\Big|_0^{\pi} = \frac{\sqrt{2}}{2}\left(\frac{\pi}{2}-0\right) = \frac{\sqrt{2}\pi}{4}$$

Вернёмся к построению:

$$\pi\cdot\int_0^{\pi} f(x)\,dx = \pi\cdot\frac{\sqrt{2}\pi}{4} = \underline{\frac{\sqrt{2}\pi^2}{4}}$$

0.5.1

$$\lim_{x\to\infty} \frac{1}{x^2e^x} \int_0^x e^t \sin\left(\frac{t}{x}\right) \sqrt{t^2+1}\, dt$$

$$= \lim_{x\to+\infty} \frac{1}{x^3e^x} \int_0^x e^t t \sqrt{t^2+1} \le \lim_{x\to+\infty} \frac{1}{x^3e^x} \int_0^x e^t (t^2+1)\, dt$$

$$= \cancel{\int t^2+1} \left[\begin{array}{l} \int fg' = fg - \int f'g \quad f' = 2t \quad g = e^t \\ f = t^2+1 \qquad g' = e^t \end{array} \right] =$$

$$= (t^2+1)\, e^t - \int 2t e^t dt = (t^2+1) e^t - 2 \int t e^t dt$$

$$\int_0^x t e^t dt = \left[\begin{array}{l} \int fg' = fg - \int f'g \quad f = t \quad g' = e^t \\ \qquad f' = 1 \qquad g = e^t \end{array} \right] = te^t - \int e^t dt$$

$$= te^t - e^t$$

$$(t^2+1) e^t - 2te^t + 2e^t = (t^2 - 2t + 3)\, e^t + \cancel{c} \Big|_0^x =$$

$$= (x^2 - 2x + 3) e^x - 3$$

$$\cancel{\lim_{x\to\infty}}$$

$$\lim_{x\to\infty} \frac{1}{x^3 e^x} \cdot (x^2 - 2x + 3) e^x - 3 = \lim_{x\to\infty} \frac{(x^2 - 2x + 3)}{x^3} - \frac{3}{x^3 e^x} =$$

$$= 0$$

0.5.2

$$\lim_{n\to\infty} \frac{1}{\sqrt{n}} \int_1^n \log\left(1+\sin\left(\frac{1}{\sqrt{x}}\right)\right) dx, \quad n \in \mathbb{N}$$

$$\int_1^n \log\left(1+\sin\left(\frac{1}{\sqrt{x}}\right)\right) dx = \int_1^M \log\left(1+\sin\left(\frac{1}{\sqrt{x}}\right)\right) dx$$

$$+ \int_M^n \log\left(1+\frac{1}{\sqrt{x}} + O\left(\frac{1}{x}\right)\right) dx =$$

$$= C + \int_M^n \frac{1}{\sqrt{x}} + O\left(\frac{1}{x}\right)) dx = C + 2\sqrt{x}\Big|_M^n +$$

$$+ \int_M^n O\left(\frac{1}{x}\right) dx = C + 2\sqrt{n} + \int_M^n O\left(\frac{1}{x}\right) dx$$

Пусть M выбрано так, что $\forall x \geq M$

$$c_1 \cdot \frac{1}{x} \leq O\left(\frac{1}{x}\right) \leq c_2 \cdot \frac{1}{x}) \Rightarrow c_1(\log n - \log M)$$

$$\leq \int_M^n O\left(\frac{1}{x}\right) dx \leq c_2(\log n - \log M) \Rightarrow \int_M^n O\left(\frac{1}{x}\right) dx$$

$$= O(\log x)$$

$$\lim_{n\to\infty} \frac{1}{\sqrt{n}} \left(C' + 2\sqrt{n} + O(\log x)\right) = 2$$

при $c_1, c_2, C, M \in \mathbb{R}$