Рубежка

Задача 1

$$x = \begin{vmatrix} -1 \\ -2 \\ 5 \\ 6 \end{vmatrix}, \quad e_1 = \begin{vmatrix} 1 \\ 1 \\ -3 \\ -4 \end{vmatrix}, \quad e_2 = \begin{vmatrix} 0 \\ 1 \\ -1 \\ -2 \end{vmatrix}$$

$$\begin{cases} \langle e_1 x \rangle = \alpha \langle e_1 e_1 \rangle + \beta \langle e_1 e_2 \rangle \\ \langle e_2 x \rangle = \alpha \langle e_1 e_2 \rangle + \beta \langle e_2 e_2 \rangle \end{cases}$$

$\langle e_1 x \rangle = -1 + (-2) + (-15) + (-24) = -42$

$\langle e_2 x \rangle = 0 + (-2) + (-5) + (-12) = -19$

$\langle e_1 e_1 \rangle = 1 + 1 + 9 + 16 = 27$

$\langle e_2 e_2 \rangle = 1 + 1 + 4 = 6$

$\langle e_1 e_2 \rangle = ~~1 + 1 + 4 = 6~~ \; 0 + 1 + 3 + 8 = 12$

$$\begin{cases} -42 = 27\alpha + 12\beta \\ -19 = 12\alpha + 6\beta \end{cases}$$

$\alpha = -4/3$

$\beta = -0,5$

Задача 6

$$A = \begin{vmatrix} 6 & 0 & 2 \\ 0 & 6 & 0 \\ 2 & 0 & 6 \end{vmatrix} \qquad |A - \lambda E| = \begin{vmatrix} (6-\lambda) & 0 & 2 \\ 0 & (6-\lambda) & 0 \\ 2 & 0 & (6-\lambda) \end{vmatrix}$$

$$\Delta = -\lambda^3 + 18\lambda^2 - 104\lambda + 192$$

$$\lambda_1 = 4 \quad \lambda_2 = 6 \quad \lambda_3 = 8$$

Задача 8

$$T = \begin{vmatrix} 1 & -1 \\ -2 & 3 \end{vmatrix} \qquad x = \begin{vmatrix} 4 \\ 3 \end{vmatrix}$$

$$T^{-1} \cdot x = \begin{vmatrix} 1 & -1 \\ -2 & 3 \end{vmatrix}^{-1} \cdot \begin{vmatrix} 4 \\ 3 \end{vmatrix} =$$

$$= \begin{vmatrix} 3 & 1 \\ 2 & 1 \end{vmatrix} \cdot \begin{vmatrix} 4 \\ 3 \end{vmatrix} = \begin{vmatrix} 15 \\ 11 \end{vmatrix}$$

zagara 9

$$a_{ijk} \sim \left\| \begin{array}{cc|cc} 1 & -2 & 1 & 1 \\ 1 & -1 & -2 & -1 \end{array} \right\|$$

Acum $a_{i[jk]}$

$$\left|\begin{array}{cc} 1 & 1 \\ -2 & 1 \end{array}\right| = \left|\begin{array}{cc} 0 & -1,5 \\ 1,5 & 0 \end{array}\right| \qquad \left|\begin{array}{cc} 1 & -2 \\ -1 & -1 \end{array}\right| = \left|\begin{array}{cc} 0 & 0,5 \\ -0,5 & 0 \end{array}\right|$$

$$\left\| \begin{array}{cc|cc} 0 & -1,5 & 1,5 & 0 \\ 0 & 0,5 & -0,5 & 0 \end{array} \right\|$$

Задача 10

Вычислить свёртку a^{ij}_{ij}

$$a \sim \left\| \begin{array}{cc|cc} 1 & -1 & 1 & 0 \\ -2 & 3 & -2 & 1 \\ \hline 1 & -2 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{array} \right\|$$

$$a^{ij}_{ij} = a^{11}_{11} + a^{12}_{12} + a^{21}_{21} + a^{22}_{22} = 2$$

$$\| \quad \| \quad \| \quad \|$$

$$1 + 0 + 0 + 1$$

Задача 7

$$\begin{vmatrix} 1 & -1 & -3 & 0 & 3 \\ -1 & 2 & 4 & 1 & -4 \\ 0 & 0 & 1 & 0 & -2 \\ 2 & -3 & -6 & -1 & 5 \\ -2 & 1 & 2 & -1 & 1 \end{vmatrix} \sim \begin{vmatrix} 1 & -1 & -3 & 0 & 3 \\ 0 & 1 & 1 & 1 & -1 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & -1 & 0 & -1 & -1 \\ 0 & -1 & -4 & -1 & 7 \end{vmatrix}$$

$$\sim \begin{vmatrix} 1 & -1 & -3 & 0 & 3 \\ 0 & 1 & 1 & 1 & -1 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & -3 & 0 & 6 \end{vmatrix} \sim \begin{vmatrix} 1 & -1 & -3 & 0 & 3 \\ 0 & 1 & 1 & 1 & -1 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

$Rank(A) = 3$

Задача 4

$$\begin{vmatrix} -2 & 4 & 2 \\ 0 & 1 & 2 \\ 0 & -10 & -8 \end{vmatrix}$$

$\chi(x) = (x+2)(x+3)(x+4)$

$\begin{vmatrix} 1-\lambda & 2 \\ -10 & -8-\lambda \end{vmatrix} = x^2+7x+12 = (x+4)(x+3)$

$\begin{vmatrix} 0 & 1-\lambda \\ 0 & -10 \end{vmatrix} = 0$

$\begin{vmatrix} 4 & 2 \\ -10 & -8-\lambda \end{vmatrix} = -4x-12 = -4(x-3)$

$\begin{vmatrix} -2-\lambda & 4 \\ 0 & -10 \end{vmatrix} = 10x+20 = 10(x+2)$

$\begin{vmatrix} 4 & 2 \\ 10-\lambda & 2 \end{vmatrix} = 2x-10 = 2(x-5)$

$\begin{vmatrix} -2-\lambda & 4 \\ 0 & 1-\lambda \end{vmatrix} = x^2+x-2 = (x+2)(x-1)$

$\begin{vmatrix} 0 & 2 \\ 0 & -8-\lambda \end{vmatrix} = 0$

$\chi(x) = (x+2)(x+3)(x+4)$

$\begin{vmatrix} -2-\lambda & 2 \\ 0 & -8-\lambda \end{vmatrix} = x^2+10x+16 = (x-2)(x-8)$

$\begin{vmatrix} -2-\lambda & 2 \\ 0 & 2 \end{vmatrix} = -2x-4 = -2(x+2)$

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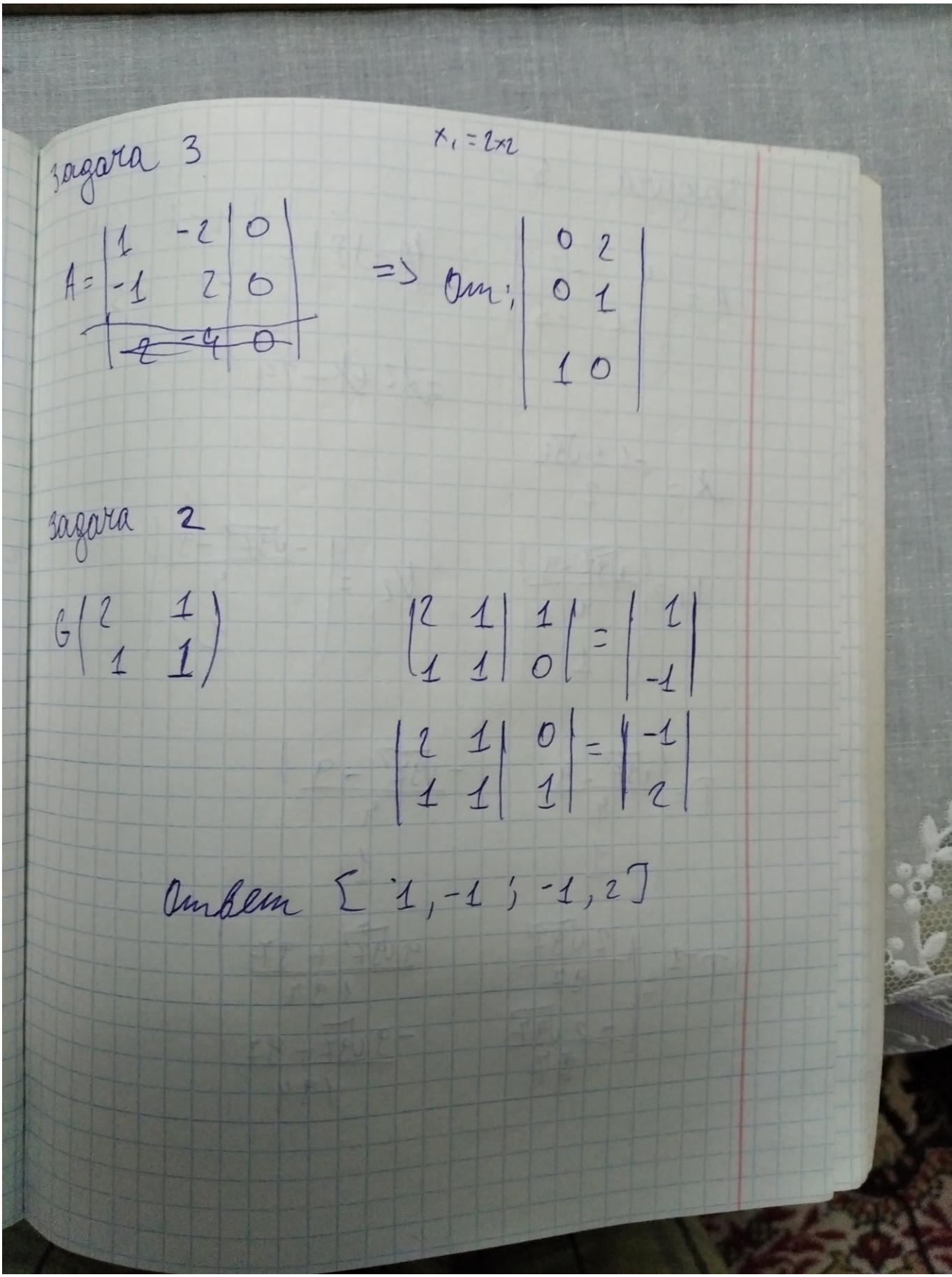

задача 3

$x_1 = 2x_2$

$$A = \left|\begin{array}{cc|c} 1 & -2 & 0 \\ -1 & 2 & 0 \end{array}\right| \Rightarrow \text{Отн.} \left|\begin{array}{cc} 0 & 2 \\ 0 & 1 \\ 1 & 0 \end{array}\right|$$

~~$2 \quad -4 \quad 0$~~

задача 2

$$G\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\left|\begin{array}{cc|c} 2 & 1 & 1 \\ 1 & 1 & 0 \end{array}\right| = \left|\begin{array}{c} 1 \\ -1 \end{array}\right|$$

$$\left|\begin{array}{cc|c} 2 & 1 & 0 \\ 1 & 1 & 1 \end{array}\right| = \left|\begin{array}{c} -1 \\ 2 \end{array}\right|$$

Ответ [1, -1 ; -1, 2]