E11e: Phase Shift in AC Circuits File

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Abstract

In this experiment, the resistance and inductance of an RLC circuit were determined through non-linear regression and resonant frequency analysis. The resistance obtained from the RC circuit was $R_{RC} = 172.3\Omega$ with an r^2 value of 0.9857, showing a 4.1% difference from the actual resistance of $R_{actual} = 179.1\Omega$. Using this resistance, the inductance of the RL circuit was found to be $L_{RL} = 2.92mH$, with a 32.6% deviation from the known value of $L_{actual} = 4.33mH$. Additionally, the inductance and the resistance calculated from the RLC resonant frequency were $L_{RLC} = 3.78mH$, with a 12.7% difference from the known inductance and $R_{RLC} = 156.43\Omega$, with an $r^2 = 0.9911$, corresponding to a percentage difference $\approx$ 12.6% from the actual resistance R_{actual}. Both values of inductance, L_{RL} and L_{RLC}, were lower than the measured value of $L_{actual} = 4.33mH$. The results indicate reasonable agreement with theoretical predictions, within acceptable error margins.

1 Introduction

The experiment measures the Phase Shift in 3 different AC circuit configuration - namely RLC, RC and RL. RLC series circuit is a series connection of a resistor, inductor coil and a capacitor with an Arbitrary Wave Generator power source. RC and RL are similar series connection but with [Resistor and Capacitor] and [Resistor and Inductor Coil] respectively. The measurement of the phase shift will be done utilising an Digital Oscilloscope. For the RLC circuit, the time shift between the two waves of the current and the voltage against time will be measured around the resonance frequency. The time shift can be used to find the phase shift (further details in the theoretical section). The time shift between the current and voltage will also be further measured for RL and RC circuits.

2 Theoretical Exploration

All derivations done through the help of the provided aid document; E11e Phase Shift in AC Circuits.

The time shifts measured for this experiment can be utilised to find the phase shift. As a full cycle in a sinusoidal function is 2π radians, the time it takes for the said cycle is $T = \frac{1}{f}$, from which it can be said that a time shift Δt corresponds to a fraction of that period - $\frac{\Delta t}{T} = \Delta t f$, from which (after multiplying with 2π to get radians),

$$\phi = 2\pi f \Delta t \tag{1}$$

where Eq. 1 will be utilised for all three circuits in finding the phase shift.

For the RLC circuit, as there is both the inductor coil and capacitance present, a resonance frequency can be observed at the position of zero phase shift, $\phi = 0$. The mentioned frequency corresponding to that point can be utilised to find both either the capacitance or the inductance if one or the other is known. As the condition for resonance frequency can be derived as when the impedance of the capacitor and the coil are equal, one can find that, with X_L reactance of the Inductor and X_C reactance of the Capacitor, $X_L = 2\pi f L \;\; X_C = \frac{1}{2\pi f C}$, and meeting the condition $X_L = X_C$, leads to the following;

$$f = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{LC}} = f_0 \tag{2}$$

where Eq. 2 is the equation to find either the inductance or capacitance from a known resonance frequency.

The phase shift for the RC circuit can further be derived from the following relation;

$$\tan(\phi) = -\frac{1}{2\pi f RC} \tag{3}$$

where with Eq. 3, one can determine the resistance from a curve fit if the capacitor is known.

In the same manner, if with a known resistance, one can determine the inductance, L, for a RL circuit through the following relation;

$$\tan(\phi) = \frac{2\pi f L}{R} \tag{4}$$

And finally, the resistor can further be determined through a curve fit of a phase shift equation for the RLC circuit, as,

$$\tan(\phi) = \frac{2\pi f L - \frac{1}{2\pi f C}}{R} \tag{5}$$

All equations above will be utilised in some manner in the analysis of the three circuit configurations below.

3 Experimental Exploration

3.1 Materials

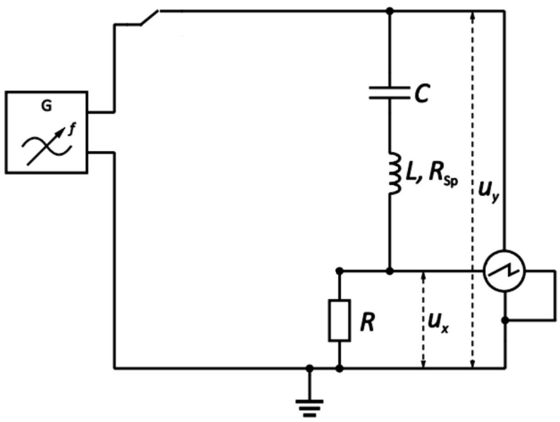
- Arbitrary Wave Generator (GW INTEK AFG-2005)
- 2 Channel Digital Oscilloscope (Tektronix TBS1000c Series)
- Capacitor ($C_{actual} = 1.072 \mu F$)
- Inductor Coil ($L_{actual} = 4.33 mH$)
- Resistor ($R_{actual} = 179.1 \Omega$)
- Multimeter (Voltcraft VC175)

- LC Meter HM8018
- Residual Items; Circuit board, Wires, Laptop, etc...

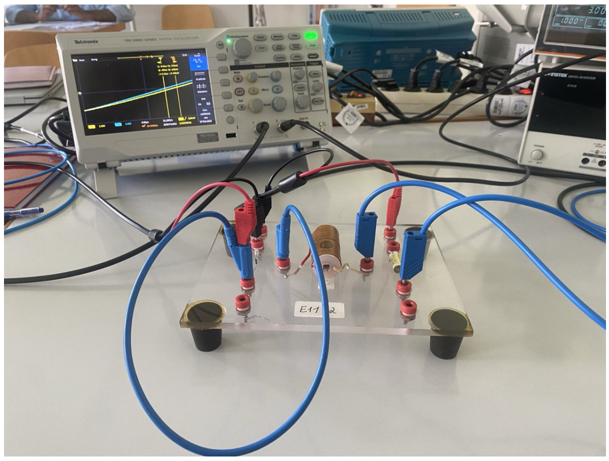
3.2 Methodology and Set Up

As previously stated, this experiment has the measurement for 3 lab configuration.

RLC Circuit: Connect the Capacitor, Inductor and the Resistor in series. With Channel 1 of the Oscilloscope measuring across the resistor and Channel 2 measuring across the whole circuit. Additional with the Wave Generator connected across the whole circuit (see below circuit diagram for further illustration). Utilising the Oscilloscope, find the Frequency which corresponds to the Lissajous figures as a straight line - at a $\frac{\pi}{4}$ angle against the horizontal / vertical axis in the first quadrant. The resonance frequency corresponds to the point of zero phase shift. With the resonance frequency, denoted as $f_{o,RLC}$, measuring the time shift for values on both ends. As for this experiment $f_{o,RLC}$ came out to about 2.4 kHz, a total of 47 data points were taken - [0.5, 2.4], [2.5, 8] kHz. With 0.1 kHz interval from [0.5, 4.5] and 0.5 kHz intervals for [4.5, 8]. The time shift will be utilised to find the phase shift, using Eq. 1 and the corresponding measured frequency.

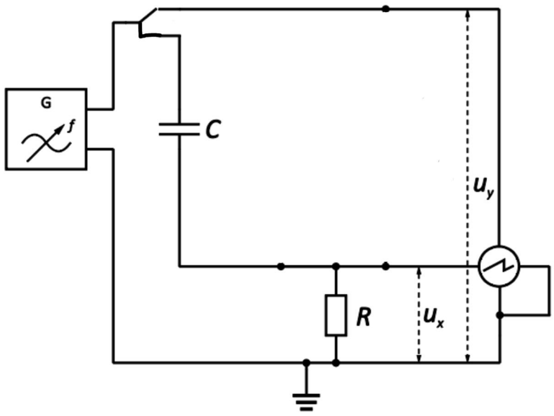


1. (a) Circuit Diagram for the RLC Circuit (E11e Phase Shift in AC Circuits)

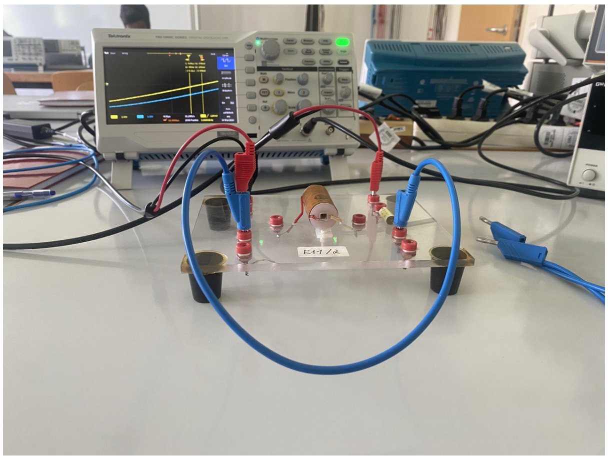


1. (b) Real Connections for the RLC Circuit

RC Circuit: Connect the Capacitor and the Resistor in series, the channels of the oscilloscope in the same manner as for RLC. A RC circuit has no resonance frequency as a reference point. Measure the time shift for frequency range of [0.5,8] kHz at intervals of 0.1 kHz for [0.5,1.5] kHz, and 0.5 kHz for [1.5,8], a total of 23 points.

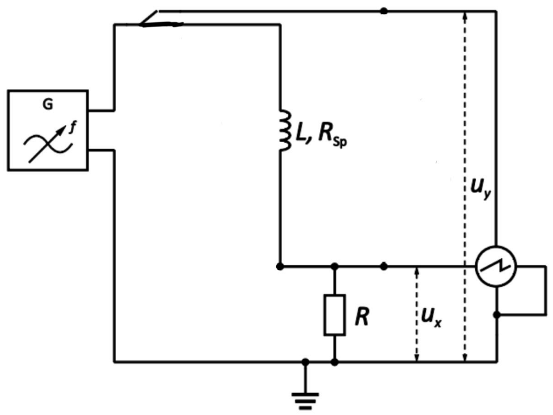


2. (a) Circuit Diagram for the RC Circuit (E11e Phase Shift in AC Circuits)

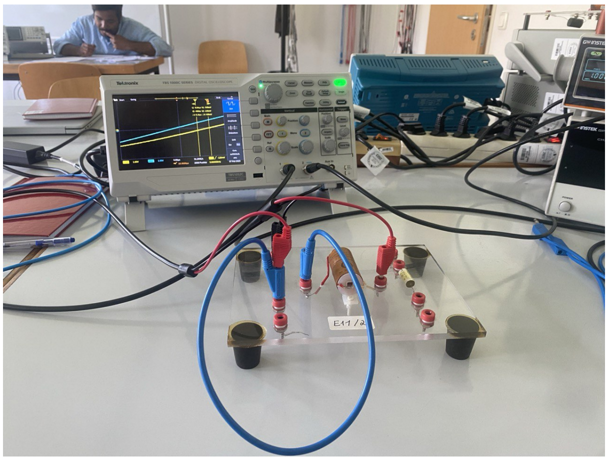


2. (b) Real Connections for the RC Circuit

RL circuit: Connect the Inductor Coil and the Resistor in series, the channels of the oscilloscope in the same manner as for RLC. A RL circuit has no resonance frequency as a reference point. Measure the time shift for frequency range of [1,24] kHz at an interval of 1 kHz, a total of 24 points.



3. (a) Circuit Diagram for the RL Circuit (E11e Phase Shift in AC Circuits)



3. (b) Real Connections for the RL Circuit

3.3 Data and Analysis

The below figure, Figure 1, presents all three configurations' phase shift against frequency data in one graph (as per Task 2 (a)).

Figure 1: Phase Shifts across Frequency

The RLC and the RL circuits present an increasing trend, with positive correlation. And the RC cirucit presents a decreasing trend, a negative correlation. This is true to the equations for the phase angle, ϕ, from the theoretical section. As the equation of $\tan(\phi_{RLC})$, $\tan(\phi_{RC})$ were related to a positive frequency, $\propto f$, while $\tan(\phi_{RC}) \propto -\frac{1}{f}$. Each circuit is further analysed below.

Using Eq. 3, a non-linear regression is performed to find the resistance R, as shown in Fig. 2. The value obtained is $R_{RC} = 172.3\Omega$ (as per Task 2 (b)), with an $r^2 = 0.9857$.

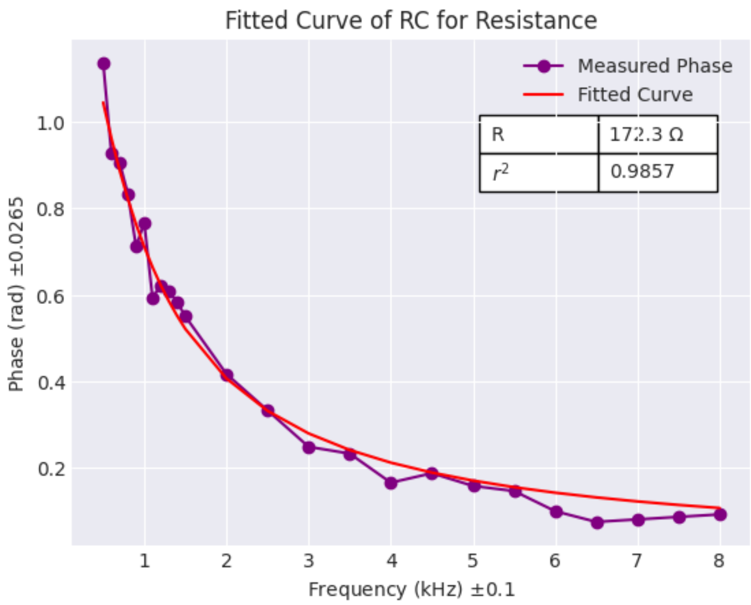


Figure 2: RC; Phase shift against Frequency, Measured Data and the Fitted curve

The overall trend of the RC circuit is in correlation to the expected trend. As the voltage lags behind the current, in a low frequency range the capacitor dominates and creates a large phase shift. While for high frequencies, the resistor dominates, creating small to no phase shift - short circuiting the configuration. As such, with an increasing frequency, the circuit becomes more resistor dominant and leads to a decreasing phase shift, leading to $\approx 0°$. Additionally, the found resistance of $R_{RC} = 172.5\Omega$ and the actual measured resistance of the resistor, $R_{actual} = 179.1$, are at percentage difference of $4.05797101449 \approx 4.1\%$, within a reasonable error margin. Making the data ***precise and accurate***. More details in the Discussion.

The below figure, Figure 3, presents the phase shift of RL circuit, with its non-linear regression line utilising equation 4. From the below figure, utilising the resistance found from the RC circuit, $R_{RC} = 172.3\Omega$, the found Inductance of the Inductor Coil came out to $L_{RL} = 2.92mH$ (as per Task 2(c)(ii)), a percentage difference of 32.5635103926 $\approx 32.6\%$ in regards to the known inductance value of $L_{\text{actual}} = 4.33mH$. Making the data ***precise*** but somewhat not accurate up to a good extent. The large deviation is analysed further in Discussions, and in comparison to other methods for finding inductance. The overall trend of the RL circuit, with an increasing phase shift, can be attributed to how with an increasing frequency, the inductor impedance increases and leads to larger phase shift. Making it approach closer to $\frac{\pi}{2} \approx 1.571$, a full $90°$ out of shift. As also seen as the plateauing in the graph.

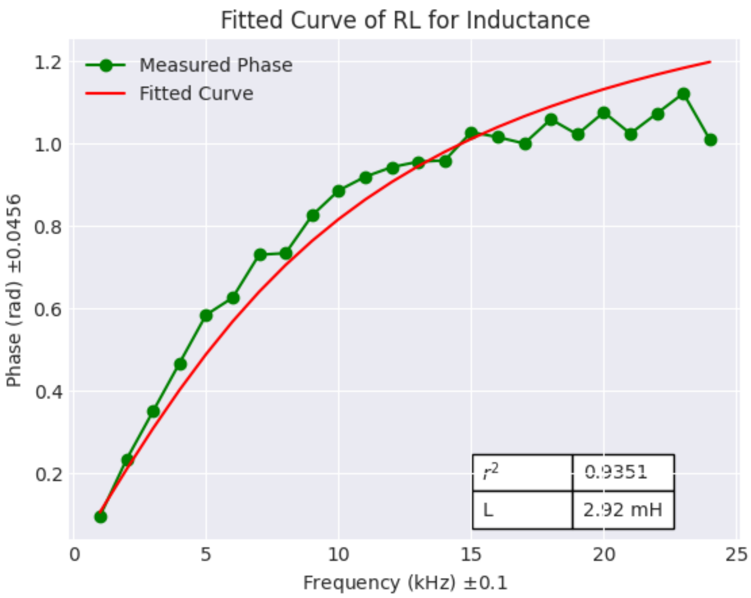


Figure 3: RL; Phase shift against Frequency, Measured Data and the Fitted curve

Furthermore, the inductance can be determined from the resonance frequency of the RLC circuit. The below graph presents the phase shift against frequency for the RLC circuit, with its fitting.

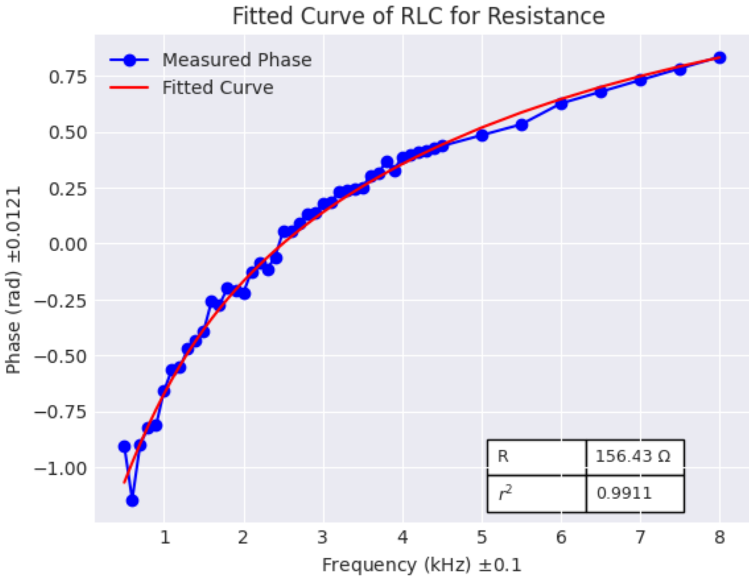


Figure 4: Fitted Curve of RLC

Using Eq. 2, the inductance of the coil is calculated using the measurement of the capacitor with an LC meter of $C = 1.072\mu F$ and the resonant frequency of the RLC circuit of $f_0 = 2.4kHz$. The value of the inductance is found to be $L_{RLC} = 3.78mH$ (as per Task 2(c)(i)). With the known L_{actual}, it is percentage difference of 12.7020785219 $\approx$ 12.7%. Making the data ***precise and weakly accurate***. A value of more moderate error compared to R_{RL}. More details, in comparison, in the Discussion.

The above value of the inductor, L_{RLC}, is introduced into Eq. 3 and plotted as a non-linear regression in order to find the corresponding value of the resistor R. From Fig. 4, the resistor R is obtained as $R_{RLC} = 156.43\Omega$ (as per Task 2 (d)), with an $r^2 = 0.9911$, corresponding well with the measured values. A percentage difference of 12.5628140704 $\approx$ 12.6%. Making the data ***precise and weakly accurate***. The overall trend of the RLC circuit is further as expected, where it is going from (in terms of sheer magnitude) a decreasing phase shift to an increasing phase shift. This is due to how, in the first part the RLC circuit behaves like a RC circuit (with voltage lagging behind the current) but after the point of resonance frequency, starts behaving in the manner of an RL circuit. Where the current lags behind the voltage instead. The point of resonance frequency corresponds to a phase shift of zero, as such similar to the plateau of Figure 2, and would follow to come closer to $\frac{\pi}{2}$ if more data points were taken beyond 8 kHz. Figure 4 parallels Figures 2 and 3 in combination.

Summary of all the found values:

Parameter	Value
R_{RC}	172.5 Ω
R_{RLC}	156.43 Ω
L_{RLC}	3.78 mH
L_{LC}	2.92 mH

Table 1: Found values for task III; C_G, C_0, d_0

All the uncertainties seen on the graphs are taken from as follows; The Frequency's as the smallest measurable increment from the digital device and Phase's extrapolated using uncertainty equation (see Error Analysis for further explanation)

Logarithmic spacing of the Frequency was omitted as it did not present the proper plateauing relationship of the circuit configurations.

3.4 Discussion

The value obtained for the inductor, $L_{RLC} = 3.78mH$ and $L_{RL} = 2.92mH$, from the resonant frequency and the RL fitting is smaller than the value measured with the LC meter at the end of the experiment, $L_{measured} = 4.33mH$. This difference can be explained due to how when measuring the inductor at resonance, the inductance value is affected by extra resistive losses and energy dissipation in the inductor, which aren't considered by the LC meter. The LC meter typically gives a measurement at a fixed frequency, while resonance involves the interaction between the inductor and capacitor, which can change the measured inductance. Essentially, at resonance, the circuit behaves differently due to the frequency-dependent effects, leading to a lower value for the inductance compared to the static measurement from the LC meter.

Furthermore, a difference is observed between the values of R_{RLC} and R_{RC}, although they correspond to the same physical parameter. The difference between R_{RLC} and R_{RC} is approximately $\approx 15.87\Omega$. Despite both having high r^2 values, several factors can attribute to this difference. First, the two values are obtained through different measurement methods. R_{RLC} is determined using the RLC resonance method, where the total impedance is influenced by both inductive and capacitive reactance, while R_{RC} is found using the RC phase shift method. This difference in circuit configurations means that circuit elements, such as the series resistance of the inductor and the equivalent series resistance of the capacitor, affect the measurements differently. In the RLC method, the inductor's resistance is included in the data measurement, whereas the RC method primarily reflects the capacitor and the only resistance. The RLC circuit is measured at its resonance frequency and the RC circuit typically analyzed over a broader range. Also, non-linear regression fits, despite high r^2 values, can still exhibit differences due to noise and the distribution of phase data during fitting.

If the values are compared to the measured value obtained with the multimeter, $R_{measured} = 179.1\Omega$, the R_{RC} value is much more accurate, as it is closer to the actual measurement. This suggests that the RC method provides a more representative value of the resistor's value in the circuit, due to the absence of the inductor's resistance on the circuit.

3.5 Error Analysis

The uncertainty in Frequency, as discussed before, is the smallest measurable increment from the digital device. The uncertainty in Phase Shift is following the below formula, the general uncertainty equation for equation is the partial derivative of each variable in the equation squared;

$$(\Delta\phi)^2 = \left(\frac{\partial\phi}{\partial f}\Delta f\right)^2 + \left(\frac{\partial\phi}{\partial t}\Delta t\right)^2 \tag{6}$$

where the above equation comes out to

$$\Delta\phi = 2\pi \cdot \sqrt{(t\Delta f)^2 + (f\Delta t)^2} \tag{7}$$

For each circuit configuration, the uncertainty was taken as the average from the propagation of the respective time and frequency data points through the above equation, Equation 7. Which comes out to as:

Parameter	Mean Uncertainty
$\Delta\phi_{RLC}$	0.026453711711291475 $\approx \pm$ 0.0265 (rad)
$\Delta\phi_{RC}$	0.04558570331362973 $\approx \pm$ 0.0456 (rad)
$\Delta\phi_{RL}$	0.012108076441570187 $\approx \pm$ 0.0121 (rad)
$\Delta f_{\forall}$	$\pm 0.1\ kHz$

Table 2: Uncertainty for frequency, $f_{\forall}$, and the phase shift for each, ϕ

where the uncertainty was taken as the mean of all the uncertainties for the said circuit configuration.

4 Conclusion

In this experiment, the concepts of phase shift in relation to frequency were explored. Using the measured values of time shift and frequency, along with the resonant frequency of the RLC circuit, it was possible to determine the unknown parameters of the circuit and confidently predict how and why the system behaves and evolves. The existing deviations from the literary values are due to the measurement difference between different configurations, existing resistance from other components not present when measuring the literary values and errors due to residual items (energy lose, non ideal systems, etc...). Further improvement of the experiment would be creating a more idealised environment, finding the phase shift using different method and comparing and comparing parallel and other band pass / stop set ups of the RLC, RL and RC circuits. Overall, it was a successful laboratory work.

5 Citation

Unknown Author. *E11e Phase Shift in AC Circuits.* Course General Physics Laboratory 2, 2025.