E07e Magnetic Fields in Coils

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Abstract

This report explores fundamental electric circuit concepts, focusing on series and parallel circuits, voltage dividers under load, the Wheatstone bridge, and high-pass filters. Through theoretical analysis, experimental setups, and error evaluation, it investigates key electrical principles, including Ohm's Law, voltage division, and frequency response. Results align closely with theoretical expectations despite minor deviations attributed to real-world imperfections. Final percentage errors were 6.09% for series circuit current, 3.43% for voltage divider plateauing voltage, 28.9% for Wheatstone bridge resistances, and 68% for high-pass filter frequency measurements.

1 Introduction

This report delves into critical electrical principles through a series of experimental studies: current and voltage measurements in series and parallel circuits, voltage dividers under load, the Wheatstone bridge, and high-pass filters. By integrating theoretical frameworks and empirical analysis, the investigation explores the relation between mathematical models and measured value sets.

Four distinct experiments will be conducted utilising basic electrical principles. The set ups of research are:

1.1 Analysing Series and Parallel Basic Circuit

The fundamental relation of how voltage and current is spread in a series and parallel circuit is analysed under the stress of three resistors of varying resistance. The concepts will be analysed utilising Ohm's law and to further develop the theoretical framework of the setup.

1.2 The Wheatsone Bridge

The Wheatstone bridge experiment focuses on precision measurements, showing how changes in resistor values affect output voltage. Where some bridge voltage across two rows of parallel connected resistors will be measured, with one of the resistors being a variable resistor - and as such change the resistor to understand the behaviour of the bridge voltage.

1.3 Voltage Divider under Load

A voltage Divider under load demonstrates the diversion of current into a local parallel system by connecting a variable resistor to a fixed series resistor. The case of small variable resistance and up to theoretical large resistance will be explored, to better understand how voltage across the fixed resistor changed with external parallel stress.

1.4 High Pass Filter

The study of high-pass filters explores frequency-dependent behaviour, illustrating the transition from capacitive to resistive dominance at higher frequencies. This experiment connects the theoretical transfer function to observed data, to understand how a high pass filter manages frequency response in signal processing.

Throughout this report, errors arising from experimental setups and equipment imperfections are critically evaluated to assess their impact on the results. By systematically analysing and comparing measured data with theoretical models, the report provides a comprehensive overview of essential circuit behaviours and the practical challenges in achieving precision.

2 Measurement of Current and Voltage in a Series and Parallel Circuit

2.1 Hypothesis

In series circuits, we expect the current to remain constant while voltage is divided. In parallel circuits, we expect voltage to remain constant while the current splits proportionally to each resistance.

2.2 Theoretical Exploration

The law that relates the voltage (U), current (I) and resistance (R) for each component in the circuit is the Ohm's Law:

$$U = IR \tag{1}$$

The Properties in Series circuit would be $R = \sum R_i, I = \frac{V}{R}, U_i = IR_i$ while the properties of a parallel circuit as $\frac{1}{R} = \sum \frac{1}{R_i}, I = \sum I_i, I_i = \frac{U}{R_i}, U = U_i$. Where the summation is done either through each component for series or each parallel line for parallel circuits.

For the analysis of the data, a linear relationship will be derived for each circuit, series and parallel. For series, as the current is constant, a linear relationship as;

$$V = IR \tag{2}$$

is utilised. With equation 2 referencing a varied value range resistance (R) with the output values of voltage (V). The slope of V v R would yield the constant current, I. For parallel, as the voltage is constant, a linear relationship as;

$$I = \frac{V}{R} \tag{3}$$

is utilised. With equation 3 referencing a varied value range resistance ($\frac{1}{R}$) with the output values of current (I). The slope of I v $\frac{1}{R}$ would yield the constant voltage, V.

2.3 Experimental Exploration

2.3.1 Materials

- 3 Resistors (5000 Ω ohms, 100 Ω ohms, 10000 Ω ohms)
- Wires
- Voltage supply
- Circuit board
- 2 Multimeters

2.3.2 Set Up

First, three resistors with ratings of 5000 Ω (R_1), 100 Ω (R_2), and 10000 Ω (R_3) were connected in series on the circuit board. Wires were used to connect the resistor chain to the voltage supply. The positive terminal of the voltage supply was connected to the free end of the first resistor, while the negative terminal was connected to the free end of the last resistor.

The voltage supply was turned on to apply a known voltage across the series combination of resistors. A multimeter was set up to measure the actual voltage that was applied to the circuit. A second multimeter was used to measure the current flowing through the circuit and the voltage drop across each resistor.

Second, three resistors (R_1, R_2, R_3) were connected in parallel on the circuit board. Wires were used to connect the parallel combination of resistors to the voltage supply. The positive terminal of the voltage supply was connected to one of the common nodes, while the negative terminal was connected to the other node. The voltage supply was turned on to apply a known voltage across the parallel network, which was confirmed by the measurement of one multimeter. A second multimeter was used to measure the current through the circuit as well as the individual currents flowing through each resistor. The circuit schematic is presented in the below image, Image 1

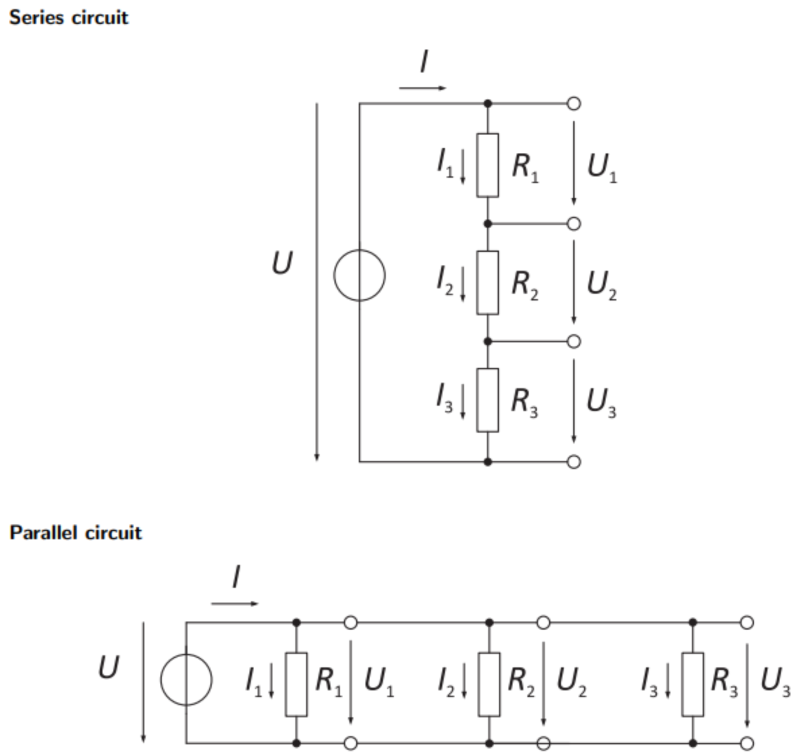

Image 1 Set up for Section 2,(*"E1e Basic Electric Circuits"*)

2.3.3 Methodology

After the power supply was turned on, the circuit was supplied with a constant 5 volts. For the first setup, the current was measured at two different points in the circuit, and the voltage drop across each resistor was recorded. These measurements were then compared to the theoretical predictions based on Ohm's law and circuit analysis.

For the second setup, the voltage was measured at two different points in the circuit, and the current through each resistor was determined. These results were also compared to the predicted values.

2.4 Results

2.4.1 Data and Analysis

From the obtained data, the following Figure presents the graphing for each Series and Parallel circuit as Series: Voltage V against Resistance R. Parallel: Current I against Resistance $\frac{1}{R}$

```python
data_task_1_1 = r"ex6_t_1_1.csv"
new_header_task_1_1 = ["Voltage", "Current"]
data_task_1_1_evaluated = pd.read_csv(data_task_1_1, header=None)
data_task_1_1_evaluated.columns = new_header_task_1_1
voltage_task_1_1 = data_task_1_1_evaluated.iloc[:, 0] #voltage
current_task_1_1 = data_task_1_1_evaluated.iloc[:, 1] #current

data_task_1_2 = r"ex6_t_1_2.csv"
new_header_task_1_2 = ["Voltage", "Current"]
```

```
data_task_1_2_evaluated = pd.read_csv(data_task_1_2, header=None)
data_task_1_2_evaluated.columns = new_header_task_1_2
voltage_task_1_2 = data_task_1_2_evaluated.iloc[:, 0] #voltage
current_task_1_2 = data_task_1_2_evaluated.iloc[:, 1] #current
resistance_task_1 = pd.Series([ 5000, 100, 10.03*10**3])
```

Note: The code for plotting is omitted as it is considered general and as to avoid clutter. If needed, can be accessed in the attached Jupyetr Folder

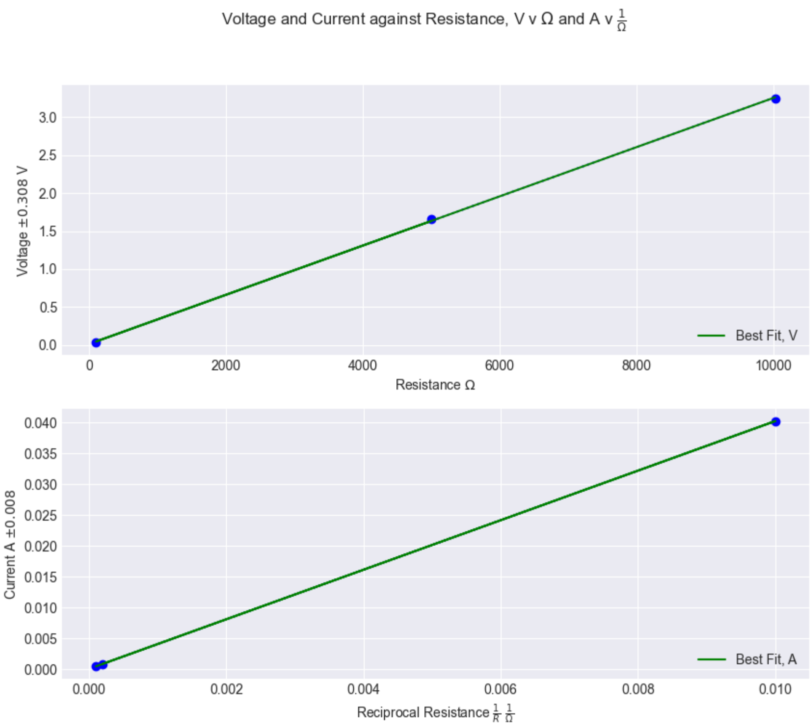

Figure 1: 'Voltage and Current against Resistance, (a) V v Ω for Series and (b) A v $\frac{1}{\Omega}$ for Parallel

The following Table contains the measurements of the Multimeters:

Resistor	Series Circuit			Parallel Circuit		
(in Ω ohm)	Power Supply Voltage (V)	Voltage (V)	Current (mA)	Power Supply Voltage (V)	Voltage (V)	Current (mA)
5000	4.96	1.66	0.32	4.99	4.40	0.87
100	4.96	0.03	0.32	4.99	4.10	40.2
10000	4.96	3.25	0.32	4.99	4.43	0.44

Table 1: Voltage and Current Measurements for Series and Parallel Circuits

From the above Figure 1, the constant values respective to the circuit can be derived utilising the linear fitting done on the values. Note, it is important to acknowledge the accuracy of the values to be limited as the fitting is done utilising 3 data points for all the axis on both graphs. Using equations 2 and 3;

(i) For the Series, the slope of linear fitting yields: 0.00032423378705993545 $\approx$ $0.000324A = 0.32mA$, correlative to the measured values of 0.32 mA across all resistors (as also seen in Table 1 above). This is a percentage error margin of 0%. However, in comparison to the thoeratical value (taking the power supply voltage and dividing it out against the total resistance) of $0.00032847682119 \approx 3.3mA$, there is absolute difference of $0.1mA$ between measured and theoretical.

(ii) For the Series, the slope of linear fitting yields: $4.0146810480299315 \approx 4.01V$, correlative to the mean measured values of 4.27 V (for the range of 4.1 - 4.40). This is a percentage error margin of $|-6.08899297424|\% \approx 6.09\%$ between the mean measured and the slope of the graph. Both values however deviate from the theoretical value of the power supply, 4.99 V, by an absolute difference of 0.72 V between measured and theoretical

```
slope_task_1_1, intercept_task_1_1, r_value_task_1_1, p_value_task_1_1,
    std_err_task_1_1 = linregress(resistance_task_1, voltage_task_1_1)

slope_task_1_2, intercept_task_1_2, r_value_task_1_2, p_value_task_1_2,
    std_err_task_1_2 = linregress(1/resistance_task_1,
    current_task_1_2/1000)
```

Note: The Code for MAE, MD, Percentage Error and Data Shift can be found in the Appendix, omitted from here to avoid clutter

2.4.2 Discussion

For the Series circuit the measured voltage across each resistor matches the expected behavior of a series circuit, where the total voltage (4.86 V) is divided among the resistors based on their resistance values. The voltage drop across the 5000-ohm resistor (1.66 V) and the 10000-ohm resistor (3.25 V) is significantly larger than the drop across the 100-ohm resistor (0.03 V), which is consistent with Ohm's Law (1). Larger resistances result in larger voltage drops for the same current. The current measured in the circuit (0.32 mA) is the same at all points, as expected in a series circuit where current remains constant throughout.

For the parallel circuit, the measured voltage is within the same range across all the measurements, with all values being less than the ideal case of 4.99 V. This is expected as there will be lose of energy due to the materials used to connect the circuit and the existing inaccuracy within the multimeter.

2.4.3 Error analysis

In this experiment, several potential sources of error could have influenced the measurements, including the power supply, circuit board, wires, and multimeter. However, certain sources of error can be reasonably minimized or disregarded.

The power supply may introduce slight fluctuations in the voltage due to internal resistance or instability. However, this does not significantly affect the results since we used the voltage values measured directly from the circuit with the multimeter. This ensures that the exact voltage in the circuit was accounted for, making any error from the power supply negligible.

The circuit board also has a very low resistance, which does not significantly impact the measurements. Its resistance is small compared to the resistors used in the circuit, so any error caused by it can be disregarded.

The connecting wires, on the other hand, do contribute some resistance, which can slightly affect the circuit by introducing small voltage drops. To minimize this source of error, we used as few wires as possible during the experiment.

The multimeter introduces the most significant source of error. Its internal resistance can affect voltage readings, especially when measuring across high-resistance components, and it can slightly alter the circuit when measuring current. Because of this, the multimeterâs effect on the circuit was taken into account, as it represents the largest source of measurement error in this experiment.

3 Voltage Divider under Load

3.1 Hypothesis

For the experiment, it is expected that there will be a $-\exp(x)$ relation between the load voltage and the load resistance, as at a high enough of load resistance, the circuit will start acting as an ideal circuit - case of no load variable resistance. Creating a U_L against R_L graph which plateaus close to the supply voltage at much higher values of R_L. For utilising the linear relation, where $\frac{U}{U_L} = 1 + \frac{R_1}{R_2} + \frac{R_1}{R_L}$ *(derivation seen in the Theoretical Section)*, a more linearly increasing graph is expected with the slope and intercept being within the range of R_1 and $1 + \frac{R_1}{R_2}$. Due to existing uncertainty in the multimeters, the percentage errors between the theoretical (calculated) and measured data sets would be within the ranges of 10-20% (as with the existing lose of energy due to the materials, the errors would add up).

3.2 Theoretical Exploration

The below presented circuit consists, Image 2 of two fixed resistors, of R_1 and R_2, arranged in series in respect to each other. R_L is connected in parallel with R_2. In the case of unloaded condition, where $R_L \to \infty$ (i.e the case of R_L being absent), the load voltage U_L can be simplified to the ideal voltage divider relationship,

$$U_L^\infty = U \cdot \frac{R_2}{R_1 + R_2} \tag{4}$$

This presents the maximum voltage across R_2 without any external load resistance. Where U is the supply voltage. However, introducing R_L with varying resistance modifies the total resistance, as R_L and R_2 form a parallel combination. The effective resistance of this parallel arrangement is given by $R_{\text{eff}} = \frac{R_2 R_L}{R_2 + R_L}$. Where furthermore under these conditions, our U_L becomes,

$$U_L = U \cdot \frac{R_{\text{eff}}}{R_1 + R_{\text{eff}}}. \tag{5}$$

Substituting R_{eff}, the above expression becomes

$$U_L = U \cdot \frac{\frac{R_2 R_L}{R_2 + R_L}}{R_1 + \frac{R_2 R_L}{R_2 + R_L}} \tag{6}$$

simplifying to

$$U_L = U \cdot \frac{R_2 R_L}{R_1(R_2 + R_L) + R_2 R_L} \tag{7}$$

Presenting a non linear relationship between R_L and U_L.

To further analyse the relationship, the supply voltage U is divided by U_L, leading to a linear relationship between U_L and $\frac{1}{R_L}$ as,

$$\frac{U}{U_L} = 1 + \frac{R_1}{R_2} + \frac{R_1}{R_L}. \tag{8}$$

In equation 8, the term $1/R_L$ appears explicitly, with R_1 as the slope and $1 + \frac{R_1}{R_2}$ as the intercept of the linear relationship. To ensure the validity of the analysis, the values of R_1 and R_2 are selected as $R_1 < R_2$. Furthermore, the load resistance R_L is ensured to be varied above the constraint $R_L \geq (R_1 \parallel R_2)$, where $R_1 \parallel R_2$ denotes the parallel resistance of R_1 and R_2. Additionally, $\frac{U}{U_L}$ will be referenced to as the Voltage Ratio.

3.3 Experimental Exploration

3.3.1 Materials

- 2 Resistor
  - $R_1 = 100\ \Omega$
  - $R_2 = 5000\ \Omega$
- Wires
- Voltcraft DC Power Supply PS 152 A (DC Power Supply)
- Circuit board
- 2 Volt-craft VC 220 Multimeters
- ELC DR05 Decade Resistor (Variable Resistor, R_L)

3.3.2 Set Up

As presented in the below Image 2, 2 fixed resistors of resistance $R_1 = 100\ \Omega$ and $R_2 = 5000\ \Omega$ are connected in series on a circuit board. The Variable Resistor, R_L is connected in parallel with R_2. One of the multimeter is utilised to measure supply voltage, denoted as U, while the second multimeter is attached to measure the voltage across R_2, denoted as U_L. A picture of the real set up is absent, but a diagrammatic schematic is presented in Image 2.

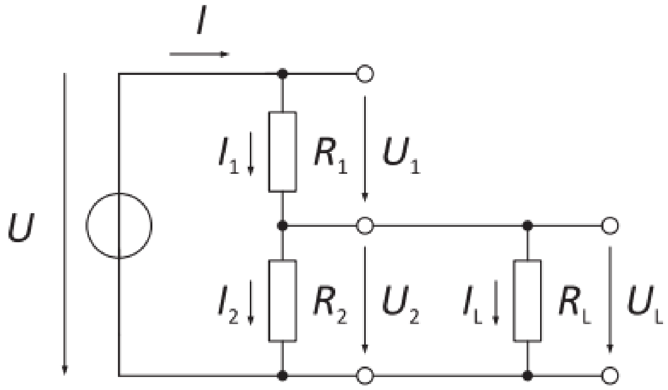

Image 1 Set up for Section 3,(*"E1e Basic Electric Circuits"*)

3.3.3 Methodology

The Variable Resistor's resistance is varied between 100 and 10000 Ω, with intervals starting at 50 Ω for smaller resistance and up to 600 Ω when the resistance is closer to the limiting load voltage (the case of $R_L \to \infty$).

The recorded values of $\frac{U}{U_L}$, U_L are plotted against $\frac{1}{R}\frac{1}{\Omega}$, R_L respectively. Additionally, Theoretical and the Best Fit Data Sets for both graphs are further calculated.

3.4 Results

3.4.1 Data and Analysis

From the retried data, the following graphs presents the Voltage Ratio ($\frac{U}{U_L}$, no units) against Reciprocal Resistance ($\frac{1}{R}\frac{1}{\Omega}$) and the Load Voltage ($U_L$ V) against the Load Resistance (R, Ω). With both Measured and Theoretical Voltages (Theoretical Model using the same range of R_L into equation 8 with predetermined constants of $R_{1,2}$ and U)

```python
data_task_2 = r"ex6_t_2.csv"
new_header_task_2 = ["Resitance (ohms)", "Voltage (V)"]
data_task_2_evaluated = pd.read_csv(data_task_2, header=None)
data_task_2_evaluated.columns = new_header_task_2
resistance_task_2 = data_task_2_evaluated.iloc[:, 0] #resistance
voltage_task_2 = data_task_2_evaluated.iloc[:, 1] #voltage

r_1_t_2 = 10**2
r_2_t_2 = 5*10**3
U_t_2 = 4.96

def voltage_output_task_2(r_l, r1, r_2_t_2, U_t_2):
    return U_t_2 * ((r_2_t_2 * r_l)/(r_2_t_2 * r_l + r_1_t_2 * (r_2_t_2 +
    ↪ r_l)))
```

Note: The code for plotting is omitted as it is considered general and as to avoid clutter. If needed, can be accessed in the attached Jupyetr Folder

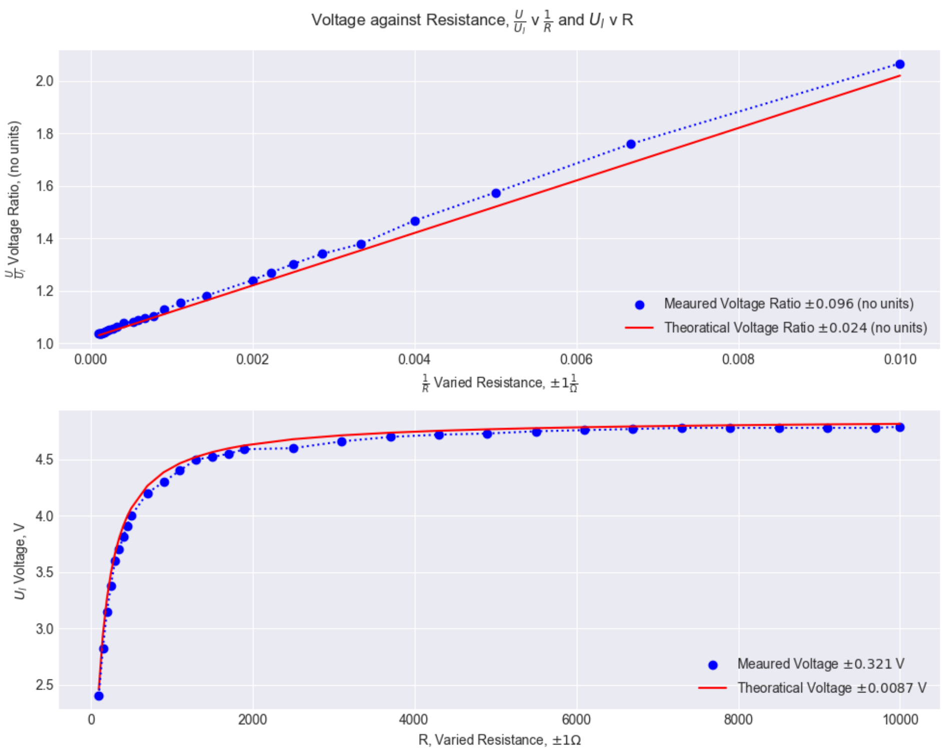

Figure 2: Voltage against Resistance, (a) $\frac{U}{U_l}$ v $\frac{1}{R}$ and (b) U_l v R

with the Measured Data and the Theoretical (Calculated) Data Set as:

Load Resistor, $R_L \Omega$	Measured Voltage, V	Theoretical Voltage, V
100	2.40	2.46
150	2.82	2.94
200	3.15	3.26
250	3.38	3.50
300	3.60	3.66
350	3.70	3.91
400	3.81	3.99
...	...	...
3100	4.66	4.74
3700	4.70	4.75
4300	4.72	4.77
4900	4.73	4.78
5500	4.75	4.79
6100	4.76	4.79
6700	4.77	4.80
7300	4.78	4.80
7900	4.78	4.81
...	...	...
9700	4.78	4.81
10000	4.79	4.82

Table 2: Table of Measured and Theoretical (Calculated) Data

Note: As there 30 values sets, only the noticeable sections (areas of change or consistent values) are presented here as to avoid clutter. With the whole table in the Appendix. 18 value sets shown above.

Figure 2 presents the data in relation with the Load Voltage and the Load Resistance, with 2 (a) presenting the linear relationship as derived from equation 8 and (b) 2 (b) presenting the non-linear relationship as derived from equation 5.

Non-Linear Graph:

The non-linear graph starts plateauing beyond the range of $R_L > 2000\Omega$, with the overall graph tending towards value in the area of 4.79 and 4.82 V for the Measured and Theoretical Data Sets respectively. This is within the range of the supply voltage (which was set to 4.96 V). With the assumption that at a much larger values of R_L, beyond the limitations of the Variable Resistor, the graph would approach closer to the expected voltage of 4.9 V, the percentage error for the plateauing voltage came out to $|-3.42741935484|\% \approx 3.43\%$ and $|-2.82258064516|\% \approx 2.82\%$ for the Measured and Theoretical Data Sets respectively. The error rising due to the non ideal setup from uncertainties and energy lost within wires, multimeter, board, power supply, and other components used. The Max Difference (MD) between the the Theoretical and Measured values came out to $0.12071146245059294 \approx 0.121V$, with the Mean Pair-Wise Difference (MPWD) as $0.12071146245059294 \approx 0.121V$. The Mean Absolute Error (MAE) as $|-0.05500664943017311| \approx 0.055V$, the two data sets had a mean percentage error (MPE) margin of $1.9010562379861429\% \approx 1.90\%$. The existing differences, even if relatively small, being attributed to same reasons as stated above. All accounts to the Measured Data Set as ***accurate and precise***. As such making the further analysis of the linear fitting for the linear graph arguable accurate up to the degree of the uncertainty within the measured data set.

Linear Graph:

The Max Difference (MD) between the the Theoretical and Measured values came out to $0.07219858156028369 \approx 0.072V$, with the Mean Pair-Wise Difference (MPWD) as $0.01852755597366423 \approx 0.019V$. The Mean Absolute Error (MAE) as $|0.01852755597366423| \approx 0.019V$, the two data sets had a mean percentage error (MPE) margin of $1.7647273851664937\% \approx 1.76\%$. The existing differences, even if relatively small, being attributed to same reasons as stated above. All accounts to the Measured Data Set as ***accurate and precise***.

The slope of the Linear Graph represents, as mentioned in the Theoretical Section 3.2, the value R_1. While the intercept would be correlative to $1+\frac{R_1}{R_2}$. A linear best fitting was done on the Linear Graph, with a R^2 value (the measuring of how close two data sets are to each other) as: $0.9988578095273815 \approx 0.999$, with 1 being an exact fit. Presented as Figure 3 below, the Linear Fit came out to the Slope: $106.62201982083762 \approx 107\Omega$ and an Intercept of: $1.0281095112766214 \approx 1.03$.

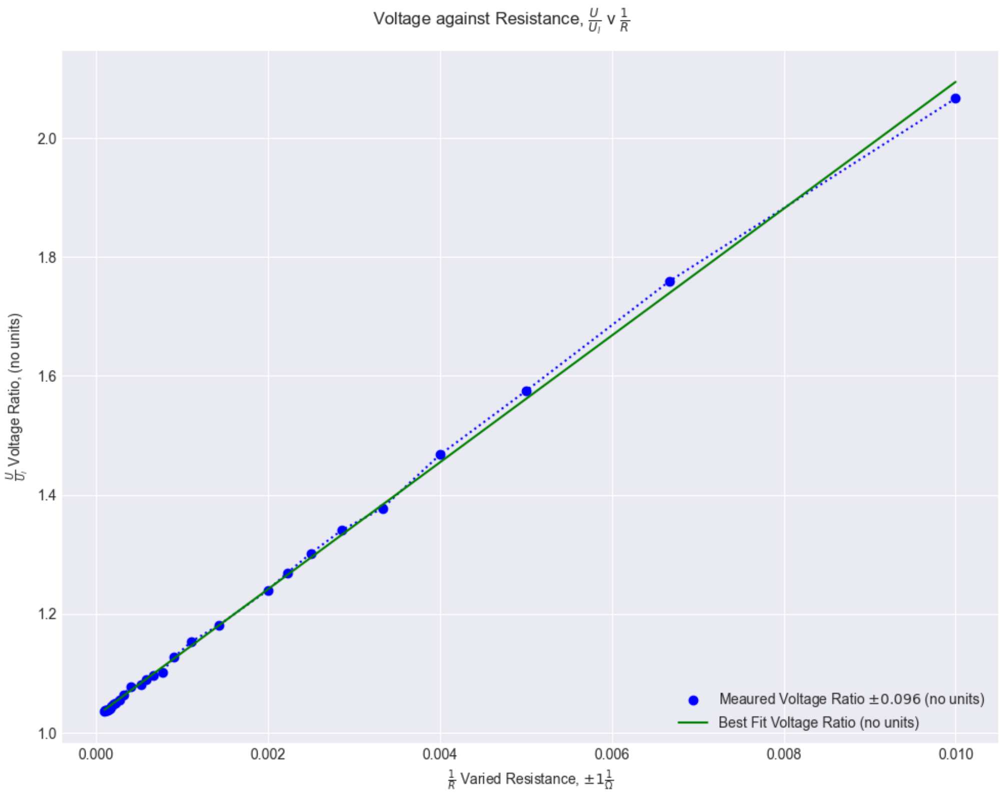

Figure 3: Voltage against Resistance, $\frac{U}{U_l}$ v $\frac{1}{R}$

With this information, the theoretical values of the fixed resistors as $R_1 = 107\Omega$ and $R_2 = 3554.06732736 \approx 3554\Omega$. This is percentage error of $6.62201982084\% \approx 6.62$ and $-28.9186534528\% \approx -28.9\%$ respectively for R_1 and R_2. The larger percentage error for R_2 can be attributed to how for a system of constant current (in our case as R_2 is in series with the R_1, and as such constant current along the connection), the power dissipated increases with resistance (from the relation $P = I^2R$). With an initial fixed resistance of 5000Ω, the measurement of U_L across R_2 would be lower, and as such lead to the calculations of R_2 to be smaller (i.e as equation 8 used U_L which is already subjected to the lose of energy, the R_2 calculated from it would be smaller). Additional reasons possibly due to denominator sensitivity (R_2 is in the denominator, and as such more sensitive to small changes from U_L or R_L) and standard errors within the used materials. Making R_1 ***precise*** and R_2 as ***imprecise***. More details in the Discussion.

```python
def voltage_output_task_2_2(resistance_task_2, U_t_2, r_2_t_2, r_1_t_2):
    return U_t_2 * ((r_2_t_2 * resistance_task_2)/(r_2_t_2 *
    ↪  resistance_task_2 + r_1_t_2 * (r_2_t_2 + resistance_task_2)))
initial_guess_t_2 = [5, 100, 1000]
params_task_2_2, covariance_task_2_2 = curve_fit(voltage_output_task_2_2,
↪  resistance_task_2, voltage_task_2, p0=initial_guess_t_2)
U_t_2_fit, r_2_t_2_fit, r_1_t_2_fit= params_task_2_2
voltage_task_2_2_fit = voltage_output_task_2_2(resistance_task_2,
↪  U_t_2_fit, r_2_t_2_fit, r_1_t_2_fit)

```

Note: The Code for MAE, MD, Percentage Error and Data Shift can be found in the Appendix, omitted from here to avoid clutter

3.4.2 Discussion

The provided data bases provided a correlative understanding of how the load voltage and the load resistance are related. The obtained data follows the expectations as outlined in the Hypothesis. Figure 3 presents a clear linear correlation *($y = bx + c$)*, as expected from equation 8, between $\frac{U}{U_L}$ and $\frac{1}{R_L}$, with Figure 2 (b) presenting the overall $-\exp(x)$ relation between U_L and R_L. An increasing R_L leads the load voltage to converge to the case of an ideal set up - without the load resistance present. This is apparent with the theory that current always chooses the path of least resistance, and in the theory of a $R_{(1,2)} \lll R_L$, the current will (up to negligible approximation) not flow through the parallel connection of R_L. The uncertainties (as seen on the *legend* of both Figures) were calculated using standard uncertainty formalism, taking into account the percentage uncertainties of multimeter, from *An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements*. Expanded more in the Error Analysis.

There were no qualitatively noticeable outliers, and as such the data was not run through an outlier algorithm. The existing deviation between the Measured and Theoretical Data Set can be attributed to:

i. Instrumental Error

The multimeters have a measured uncertainty, due to calibration inaccuracy, of the range of between 0.5% and 1.2 % with an added uncertainty from the number of digits displayed. These uncertainties provide a systematic error in our Measured values, and as such lead to deviation from a perfect system. The resistors further have an existing manufacture acknowledged tolerance error. A fluctuation of a range of (usually for most resistors) ±1 - 5 % of the resistance is expected and as such assumed constant resistance is not possible (actual tolerance error had not been provided and as such the common 1-5% range is taken). However as the resistor fluctuation cannot be consistently mapped, it cannot be taken into account in the uncertainty calculations for the theoretical values.

ii. Power Supply and Internal Resistance Imperfections

Internal resistance of the Power Supply, and the resistance within the wires and the circuit board, lead to small deviations from an ideal system. These small effects can add up to a noticeable deviation.

iii. Contact Resistances

Small energy lose due to the connection between different materials, such as wires, the circuit board, the power supply and the connection to the multimeter.

Any further resistance, deviation or error can be due to standard imperfection of a non ideal system - i.e external noise, energy lose, imperfect behaviour of the components, and more.

3.4.3 Error Analysis

The uncertainty for the Measured Data Set for $\frac{U}{U_L}$ is derived using standard uncertainty relations. Where for $\frac{1}{U_L}$, measured is;

$$\partial U_{L,m} = 4.96 \cdot \left(1 \cdot \frac{(U_L \cdot 0.005) + (0.1 \cdot 3)}{U_l}\right) \cdot U_L^{-1} \tag{9}$$

where $(U_L \cdot 0.005) + (0.1 \cdot 3)$ is from the existing calibration inaccuracy of the multimeter. And the uncertainty for the Theoretical Data Set is calculated following the equations of John R. Taylor, from An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements (Taylor):

$$\partial U_{L,t} = \sqrt{\left(\frac{\partial U_{L,t}}{\partial R_L} \cdot \Delta R_L\right)^2 + \left(\frac{\partial U_{L,t}}{\partial U} \cdot \Delta U\right)^2}. \tag{10}$$

,

for equation 5, leading to

$$\partial U_L = \sqrt{4.96^2 \cdot (\frac{R_2\left[R_2R_L + R_1(R_2+R_L)\right] - R_2R_L(R_2+R_1)}{\left[R_2R_L + R_1(R_2+R_L)\right]^2} \cdot \Delta R)^2 + (\frac{R_2R_L}{R_2R_L + R_1(R_2+R_L)} \cdot \Delta U)^2} \tag{11}$$

leading to the the Mean Error Propagation and the Largest Error as (for the range of R_L) as follows:

Data Set	Mean Error Propagation (no units)	Largest Error (Max) (no units)
Theoretical	± 0.096	$\pm$ 0.041
Measured	± 0.096	$\pm$ 0.268

Table 3: Error Propagation; Mean and Maximum Values for $\frac{U}{U_L}$

while for the U_L itself, utilising the same methods;

Data Set	Mean Error Propagation V	Largest Error (Max) V
Theoretical	± 0.0087	$\pm$ 0.010
Measured	± 0.321	$\pm$ 0.324

Table 4: Error Propagation; Mean and Maximum Values for U_L

4 Wheatstone bridge

4.1 Hypothesis

We expect U_B to be 0 around$R_4 = 5000$ as $R_1 = R_3$ and $R_2 = 5000$, which leads the bridge voltage to be zero.

4.2 Theoretical Exploration

The bridge voltage of a Wheatstone bridge is given by:

$$U_B = U(\frac{R_2}{R_1 + R_2}\frac{R_4}{R_3 + R_4}) \tag{12}$$

derived from standard component relations of circuits (reference to Section 2.2 Theoretical Exploration) which leads to:

$$U_B = 0 \tag{13}$$

when $R_1 = R_3$ and $R_4 = R_2$.

All Resistors are with the unit $R_{1,2,3,4}\Omega$, in Ohms, and U_B, U Voltages in Volts, V.

4.3 Experimental Exploration

4.3.1 Materials

- 3 Resistors (R1 = R3 = 100 Ω, R2 = 5000 Ω)
- Wires
- Voltage supply
- Circuit board
- 2 Multimeters
- Resistor decade (R4)

4.3.2 Set Up

In this experiment, a Wheatstone bridge is set up using a combination of series and parallel resistors. The circuit is configured as follows:

Resistors R_1 and R_2 are connected in series with each other. Resistors R_3 and R_4 are also connected in series, but this combination is placed in parallel with the series combination of $R1$ and R_2. Resistor R_4 is a resistor decade, allowing us to adjust its resistance during the experiment. The power supply is connected to the circuit, providing a constant voltage. One multimeter is used to measure the voltage across the bridge, denoted as U_B and a second multimeter is used to measure the voltage supplied by the power supply. A picture of the real set up is absent, but a diagrammatic schematic is presented here, Image 3.

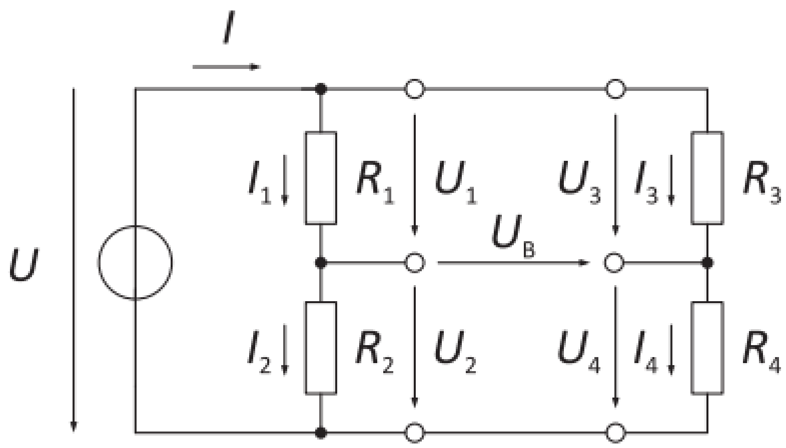

Image 3 Set up for Section 4,(*"E1e Basic Electric Circuits"*)

4.3.3 Methodology

Throughout the experiment,the value of resistor R_4 is adjusted and the corresponding voltage U_B recorded across the bridge. These values are compared to the theoretical data set.

4.4 Results

4.4.1 Data and Analysis

From the obtained data, the following Figure presents the graphing for Bridge Voltage, U_B against Resistance, R_4

```
data_task_3 = r"ex6_t_3.csv"
new_header_task_3 = ["Resistance 4", "Voltage"]
data_task_3_evaluated = pd.read_csv(data_task_3, header=None)
data_task_3_evaluated.columns = new_header_task_3
resistance_4_task_3 = data_task_3_evaluated.iloc[:, 0] #resistance 4
voltage_task_3 = data_task_3_evaluated.iloc[:, 1] / 1000 #voltage u_b

def voltage_theo_task_3_1(R4):
    return V_s * ((R2 / (R1 + R2)) - (R4 / (R3 + R4)))
```

Note: The code for plotting is omitted as it is considered general and as to avoid clutter. If needed, can be accessed in the attached Jupyetr Folder

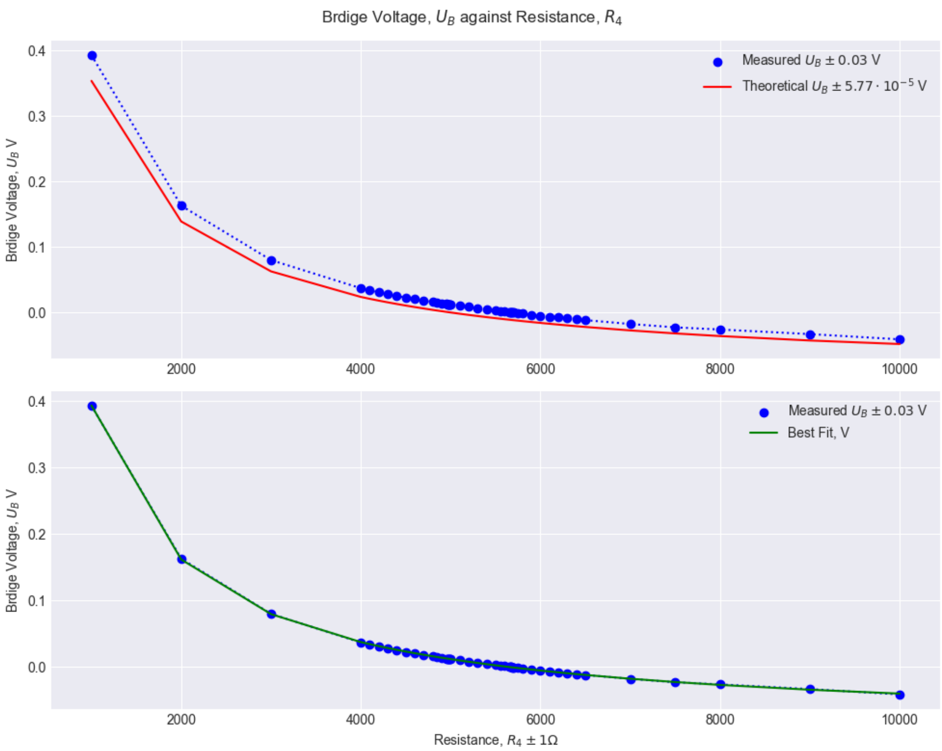

Figure 4: Bridge Voltage, U_B against Resistance, R_4

The Table containing the measurements of the Multimeters can be found in the following page.

The graph shows the relationship between the resistor $R_4\Omega$ and the output voltage U_B V of the Wheatstone bridge circuit based on the experimental data. The curve exhibits the following key features:

For lower values of $R_4(1000 - 4000\Omega)U_B$ is relatively high but decreases rapidly. As R_4 approaches $5000\Omega$the output voltage U_B reaches its lowest point, close to $0V$, which aligns with the theoretical prediction for a balanced bridge.

After 5000Ω, U_B begins to increase again but in the negative voltage range, indicating an unbalanced bridge in the opposite direction. The voltage becomes more negative as R_4 continues to increase.

The Non Linear Fit has a R^2 value of $0.9999434196238158 \approx 0.9999$, where a score of 1 is deemed as a perfect fit. The Non Linear fit produced the following parameters:

```
params_task_3_1, covariance_task_3_1 = curve_fit(voltage_theo_task_3_2, resistance_4_task_3, voltage_task_3, p0=[100, 100, 1000, 4.96])
R_1_task_3_fit, R_2_task_3_fit, R_3_task_3_fit, V_s_task_3_fit = params_task_3_1
voltage_theo_task_3_2_fit = voltage_theo_task_3_2(resistance_4_task_3, R_1_task_3_fit, R_2_task_3_fit, R_3_task_3_fit, V_s_task_3_fit)
```

- Resistance $R_1 = 391.3962254809605 \approx 391\Omega$
- Resistance $R_2 = 22373.40627463241 \approx 22400\Omega$
- Resistance $R_3 = 99.09825546920527 \approx 99.1\Omega$

- Supply Voltage $U = 5.38844226258242 \approx 5.39\Omega$

With the percentage error for each parameter R_1 : 291%, R_2 : 347%, R_3 : −1% and U : 8.67% respectively. The large deviation for R_1 and R_2 can be attributed to errors discussed in Error Analysis. Additional information cannot be retrieved from the Non Linear fit.

R_4 (Ω ohms)	**Bridge Voltage (V)**
1000	0.393
2000	0.163
...	...
4700	0.018
4800	0.016
4850	$14.7 \cdot 10^{-3}$
4900	$13.8 \cdot 10^{-3}$
4950	$12.9 \cdot 10^{-3}$
4960	$12.7 \cdot 10^{-3}$
...	...
5550	$2.3 \cdot 10^{-3}$
5600	$1.4 \cdot 10^{-3}$
5650	$0.6 \cdot 10^{-3}$
5660	$0.4 \cdot 10^{-3}$
5670	$0.01 \cdot 10^{-3}$
5675	0.000
5680	$-0.1 \cdot 10^{-3}$
5690	$-0.2 \cdot 10^{-3}$
5700	$-0.2 \cdot 10^{-3}$
5750	$-1.0 \cdot 10^{-3}$
5800	$-1.8 \cdot 10^{-3}$
5900	$-3.4 \cdot 10^{-3}$
...	...
7000	$-17.6 \cdot 10^{-3}$
7500	$-22.7 \cdot 10^{-3}$
8000	-0.026
9000	-0.033
10000	-0.041

Table 5: Values of R_4 vs bridge voltage U_B

Note: As there 49 values sets, only the noticeable sections (areas of change or consistent values) are presented here as to avoid clutter. With the whole table in the Appendix. 25 value sets shown above.

The Max Difference (MD) between the the Theoretical and Measured values came out to $0.03934581105169349 \approx 0.039V$, with the Mean Pair-Wise Difference (MPWD) as $0.012423742054777963 \approx 0.012V$. The Mean Absolute Error (MAE) as $|0.0124237..| \approx 0.012V$, the two data sets had a mean percentage error (MPE) margin of $472.811461987\% \approx 473\%$ (where the data point with 0 were excluded). The large MPE is due to small value sensitivity in the values close to the zero Bridge Voltage, where even if the absolute difference is within the range of 0.011805 V, the percentage error jumps up to magnitudes of 1000%. However, taking into account the MPWD, which is correlative to the standard deviation, the measured data set can be said to be ***accurate and precise*** relative to the theoretical data set.

Note: The Code for MAE, MD, Percentage Error and Data Shift can be found in the Appendix, omitted from here to avoid clutter

4.4.2 Discussion

As our hypothesis predicted, the output voltage U_B approached $0V$ when the resistance of R_4 reached 5000ΩThis supports the expectation that when R_4 is equal to$R_2(5000\Omega)$, the Wheatstone bridge reaches a balanced state, leading to no potential difference across the output. This finding is consistent with the theoretical analysis, where U_B is expected to be zero when the ratio of resistances in the bridge is such that the voltage drop across each side is equal. The overall pattern of the relationship being related close to $\frac{1}{x}$ relations.

The experimental values of U_B closely align with the theoretical predictions, confirming the accuracy of our understanding of the Wheatstone bridgeâs behaviour. Both the theoretical and experimental curves exhibit a similar trend, with U_B decreasing as R_4 increases. This indicates that our experimental setup and methodology were sound, and the behaviour of the circuit matched the expected outcomes.

However, some small differences between the theoretical and experimental values were observed, most likely due to measurement errors.

4.4.3 Error analysis

In this experiment, several factors may have contributed to the discrepancies between the experimental and theoretical values of the output voltage U_B As discussed in Section 2, the primary sources of error are the Multimeters. We still neglect the errors from the power supply, the wires and the circuit board.

The uncertainty for the Measured Data Set for U_B is derived using standard uncertainty relations. Where for U_B measured,

$$\Delta U_B = U_B \frac{(U_B \cdot 0.005 + 0.01 \cdot 3}{U_b}) \tag{14}$$

where $(U_B \cdot 0.005 + 0.01 \cdot 3)$ are from the existing calibration inaccuracy of the multimeter. And the uncertainty for the Theoretical Data Set is calculated following the equations of John R. Taylor, from An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements (Taylor):

$$\Delta U_B = \sqrt{\left(\frac{\partial U_B}{\partial R_4} \cdot \Delta R_4\right)^2}. \tag{15}$$

,

for equation 12, leading to

$$\Delta U_B = -U \cdot \frac{R_3}{(R_3 + R_4)^2} \cdot \Delta R_4 \tag{16}$$

leading to the the Mean Error Propagation and the Largest Error as (for the range of R_4) as follows:

Data Set	Mean Error Propagation V	Largest Error (Max) V
Theoretical	$\pm 5.77 \cdot 10^{-5}$	$8.22 \cdot 10^{-5}$
Measured	± 0.03	$\pm$ 0.032

Table 6: Error Propagation; Mean and Maximum Values for U_B

5 High-Pass Filter

5.1 Hypothesis

The expected behaviour would be of the ratio $\frac{U_a}{U_e}$ graph against the frequency, f, to showcase a $-\exp(x)$ relation, with a plateauing around the value of 1. This should happen due to high frequency signals causing the capacitance / resistor dependent state at low frequencies to change to a resistive-dominated of the filter state at higher frequencies (i.e the internal resistance of the filter would overpower and limit the ratio between the input and output voltage to be the same).

5.2 Theoretical Exploration

A High-Pass filter allows signals with higher frequencies than a specific cut off frequency, f_c, to pass while reducing the effects of signals with lower frequencies. The behaviour of the High Pass filter can be analysed by considering an RC circuit where the resistor, R, and the capacitor, C, arranged in series (as further seen in Image 4 below), with an Oscilloscope.

Following derivation from *"Introduction to Electric Circuits"* Chapter 16 by **Richard C. Dorf and James A. Svoboda**; Utilising the principle of voltage division in AC circuits, for an input voltage, U_e applied across the RC circuit, and an output voltage U_a measured across the resistor, R, the output voltage can be express as

$$U_a = U_e \cdot \frac{Z_R}{Z_R + Z_C} \tag{17}$$

where Z_R and Z_C are the impedances of the resistor and capacitor, respectively. With $Z_R = R$ and $Z_C = \frac{1}{j\omega C}$, where j is the imaginary value $\sqrt{-1}$ and $\omega = 2\pi f$ is the angular frequency. Substituting these values into equation 17, while defining a new relationship known as the transfer function $\frac{U_a}{U_e}$, we can get,

$$\frac{U_a}{U_e} = \frac{R}{R + \frac{1}{j\omega C}}. \tag{18}$$

In simplifying the expression and taking the magnitude (to get the real part of the equation), the $\mathrm{Re}(\frac{U_a}{U_e})$ yields,

$$|\frac{U_a}{U_e}| = \frac{\omega RC}{\sqrt{(\omega RC)^2 + 1}}. \tag{19}$$

Equation 19 presents how the magnitude of the output voltage changes with varied frequency, f. At low frequencies equation 19 would tend to zero while at higher frequencies tend to 1 (as in the approximation of $f \to \infty$, the "+ 1" becomes negligible and the expression simplifies to 1). Where equation 19 will be the equation used to calculate the theoretical data set of the experiment.

The cut off frequency f_c (or limiting frequency) is defined as the frequency at which the transfer function, $\frac{U_a}{U_e}$, tends to approximately 70.7% $\approx \frac{1}{\sqrt{2}}$ of its maximum value. Equating our $\frac{U_a}{U_e}$ to $\frac{1}{\sqrt{2}}$ and simplifying, the cut off angular frequency ω_c yields,

$$\omega_c = \frac{1}{RC}. \tag{20}$$

Using equation 20, with the relation $\omega_c = 2\pi f_c$, the limiting frequency yields as,

$$f_c = \frac{1}{2\pi RC}. \tag{21}$$

Equation 19 and 21 will be utilised to compare and analyse the Measured Value Data set.

5.3 Experimental Exploration

5.3.1 Materials

- Resistor, R = 1000Ω
- Capacitor, C = $3.9 \cdot 10^{-9} F$
- Wires
- Circuit board
- PicoScope 2000 Series Oscilloscope
- Laptop with the PicoScope measurement software

Note: We forgot the exact capacitor we used, but vaguely remember it was 3.9 and as using equation 20 we get ≈ 3.9, we assume it is 3.9

5.3.2 Set Up

Following the schematic from Image 4, the Capacitor and the Resistor are connected in series, with the Oscilloscope connected as a power supply and the measuring device simultaneously. The Oscilloscope is connected to a laptop with a pre-installed software to measure both U_a and U_e. A picture of the real set up is absent, but a diagrammatic schematic is presented here, Image 4.

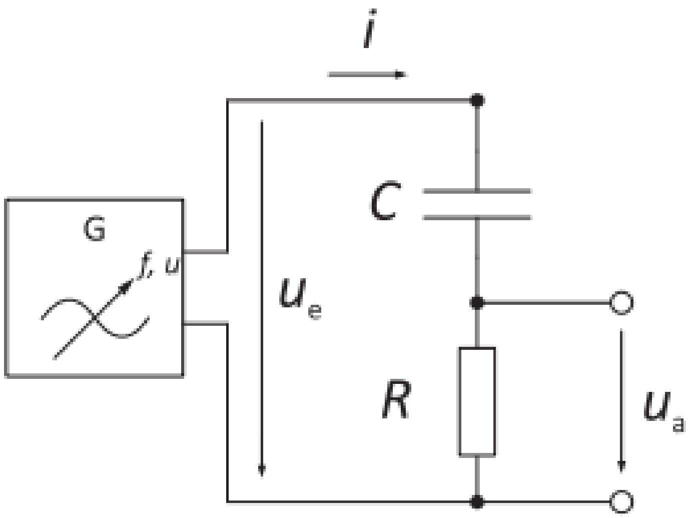

Image 4 Set up for Section 5,(*"E1e Basic Electric Circuits"*)

5.3.3 Methodology

The frequency, f, is varied directly from the software and the resultant initial values for U_a and U_e are taken down. The frequency varied by 200 Hz between 0 Hz up to 1,000 Hz. From 1,000 Hz up to 10,000 Hz it was varied by 2,000 Hz, and from 10,000 Hz up to 100,000 Hz, frequency was varied by 5,000 Hz intervals. The recorded values of $\frac{U_a}{U_e}$ is plotted against the frequency, f. Additionally, Theoretical and the Best Fit Data Sets for the graphing is calculated.

5.4 Results

5.4.1 Data and Analysis

From the retried data, the following graphs presents the Transfer Function ($\frac{U_a}{U_e}$, no units) against Frequency, f (Hz). With both Measured and Theoretical Voltages (Theoretical Model using the same range of f into equation 19 with predetermined constants of R and C)

```
data_task_4 = r"ex6_t_4.csv"
new_header_task_4 = ["Frequency", "U_a", "U_e"]
data_task_4_evaluated = pd.read_csv(data_task_4, header=None)
data_task_4_evaluated.columns = new_header_task_4
frequency = data_task_4_evaluated.iloc[:, 0] #resistance
u_a = data_task_4_evaluated.iloc[:, 1] / 1000 #voltage u_a
u_e = data_task_4_evaluated.iloc[:, 2] #voltage u_e

def h(f, R, C):
    return (2*np.pi*f*R*C)/(np.sqrt(1 + (2*np.pi*f*R*C)))
```

Note: The code for plotting is omitted as it is considered general and as to avoid clutter. If needed, can be accessed in the attached Jupyetr Folder

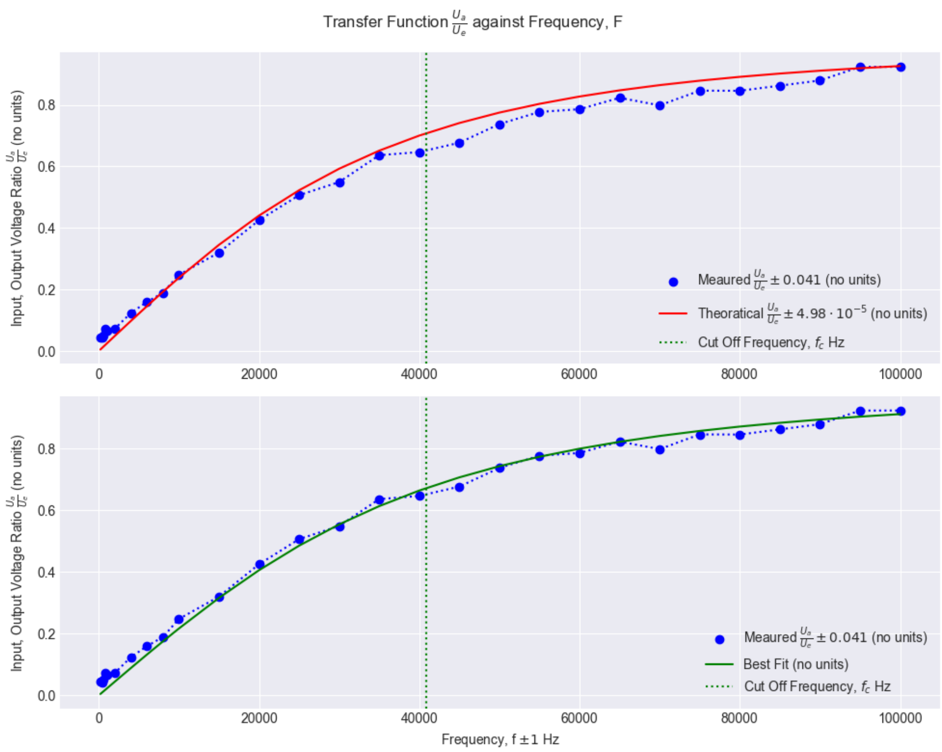

Figure 5: Transfer Function ($\frac{U_a}{U_e}$, no units) against Frequency, f (Hz) (a) Measured and Theoratical (b) Measured and Non Linear Fit

Frequency, f Hz	Measured $\frac{U_a}{U_e}$ (no units)	Theoretical $\frac{U_a}{U_e}$ (no units)
200	0.044	0.005
400	0.043	0.010
600	0.050	0.015
800	0.073	0.020
1000	0.067	0.024
2000	0.073	0.049
4000	0.124	0.98
6000	0.159	0.145
...	...	...
50000	0.737	0.775
55000	0.777	0.803
60000	0.786	0.827
65000	0.823	0.847
70000	0.798	0.878
75000	0.846	0.891
80000	0.846	0.901
85000	0.862	0.912
90000	0.879	0.919
95000	0.923	0.919
100000	0.923	0.926

Table 7: Table of Measured and Theoretical (Calculated) Data

Note: As there 28 values sets, only the noticeable sections (areas of change or consistent values) are presented here as to avoid clutter. With the whole table in the Appendix. 16 value sets shown above.

Figure 5 presents the data in relation between the transfer function, the ratio of $\frac{U_a}{U_e}$ and the frequency, f. With 5 (a) presenting the Theoretical (Calculated) Data set as derived utilising equation 19. T

In comparison between Measured and Theoretical Sets:

The Measured Data Set, as expected from the the theoretical exploration, is seen to start plateauing close to the value of 1 as the frequency, f, increases. The values beyond the limiting frequency, f_c, (which is calculated using equation 20 to be at $f_c = 40808.9597672 \approx 41000Hz$, the green dotted line), can qualitatively be seen to have started *slowing down* and is converging to some area around the value of 1 of $\frac{U_a}{U_e}$. The same expected behaviour is seen in the theoretical data set, as expected utilising equation 19 to derive the calculated values.

The Max Difference (MD) between the the Theoretical and Measured values came out to $0.06574465575809374 \approx 0.066$, with the Mean Pair-Wise Difference (MPWD) as $0.030736832124250015 \approx 0.031$. Mean Absolute Error (MAE) as $|-0.010731604981285 84| \approx 0.011$, the two data sets had a mean percentage error (MPE) margin of $67.69679508589228\% \approx 68\%$. The large percentage error can be attributed to the large percentage error difference for the initial couple values, for example the values correlating to frequency $f = 200$ yielding 0.005 and 0.044 for the theoretical and measured data sets respectively. Even though the values end up closer on higher frequencies, the early large deviation leads to such a large MPE. And with some amount attributed towards existing uncertainties, errors and energy lose within the system. As such, taking into account the MD and the MPWD, with MPWD being correlative to the standard deviation of the measured set in respect to the theoretical set, the Measured Data set can be said to be ***accurate and precise***

In comparison between Measured Set and Non Linear Fit:

The Non Linear Fit lead to a fitting with an R^2 score value of $0.9934150934744883 \approx 0.993$, with 1 being an exact fit. The behaviour of the Non Linear Fit is the same as of the theoretical data set. There are no additional informations that can be derived from the Non Linear fitting.

```python
params_task_4_1, covariance_task_4_1 = curve_fit(h, frequency, u_a/u_e,
    p0=[1000, 3.9577*10**(-9)])
R_fit, C_fit = params_task_4_1
h_fit = h(frequency, R_fit, C_fit)
```

Note: The Code for MAE, MD, Percentage Error and Data Shift can be found in the Appendix, omitted from here to avoid clutter

5.4.2 Discussion

The provided data bases provided a correlative understanding of how the transfer function $\frac{U_a}{Ue}$ and the frequency f are related. The obtained data follows the expectations as outlined in the Hypothesis. Figure 5 presents a clear $-\exp(x)$ relation between the components, for both measured and theoretical data sets. The observed behaviour of the Measured Data Set aligns well with the theoretical particularly at higher frequencies, in terms of magnitude (as opposed to large deviations at the initial values). Overall, as the frequency increases, the systemâs input-output voltage ratio, $\frac{U_a}{U_e}$, begins to plateau toward its theoretical maximum value. This plateauing behaviour can be attributed to the diminishing influence of capacitance and resistance. In this state, the system operates with dominated resistive effects, leading to a steady-state response where further increases in frequency produce minimal changes in the transfer function.

At lower frequencies, the deviations between the Measured Data Set and the Theoretical Model are more pronounced and can be attributed to;

(i) Reactive Effects:

At lower frequencies, the impedance contributions of capacitive or inductive elements are more significant, influencing the system's transfer function more strongly.

(ii) Measurement Uncertainty:

Experimental errors, instrument precision, and noise in the measured data may lead to deviations, particularly where the voltage ratios are small as they are more sensitive. Especially U_e as it is in the denominator.

(iii) Energy Losses:

Heat lose due to wire connection, current movement and from difference components being operative can introduce real-world losses not accounted for in the idealized theoretical model.

The plateauing of the transfer function at higher frequencies highlights the system's tendency to stabilize as the frequency increases. This behaviour is consistent with the expected transition from a capacitance / resistor dependent state at low frequencies to a resistive-dominated of the filter state at higher frequencies. The measured deviations, though noticeable in certain regions, are within reasonable bounds considering experimental uncertainties and real-world imperfections.

5.4.3 Error Analysis

The uncertainty for the Measured Data Set for $\frac{U_a}{U_e}$ is derived using standard uncertainty relations. Where for $\frac{U_a}{U_e}$, measured is;

$$\partial\frac{U_a}{U_e} = \frac{U_a}{U_e}\left(\left(\frac{U_a \cdot 0.005 + 0.01 \cdot 3}{U_a}\right) + \left(\frac{U_e \cdot 0.005 + 0.01 \cdot 3}{U_e}\right)\right) \tag{22}$$

where $(\frac{U_a \cdot 0.005+0.01\cdot 3}{U_a})$ and $(\frac{U_e \cdot 0.005+0.01\cdot 3}{U_e})$ are from the existing calibration inaccuracy of the multimeter. And the uncertainty for the Theoretical Data Set is calculated following the equations of John R. Taylor, from An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements (Taylor):

$$\partial\frac{U_a}{U_e} = \sqrt{\left(\frac{\partial U_a/U_e}{\partial f} \cdot \Delta f\right)^2}. \tag{23}$$

,

for equation 19, leading to

$$\partial \frac{U_a}{U_e} = \frac{2\pi RC}{(1+(2\pi fRC)^2)^{3/2}} \cdot \Delta f \tag{24}$$

leading to the the Mean Error Propagation and the Largest Error as (for the range of f) as follows:

Data Set	Mean Error Propagation (no units)	Largest Error (Max) (no units)
Theoretical	$4.98 \cdot 10^{-5}$	$5.2 \cdot 10^{-5}$
Measured	± 0.041	$\pm$ 0.067

Table 8: Error Propagation; Mean and Maximum Values for $\frac{U_a}{U_e}$

6 Conclusion

This study provides a comprehensive exploration of fundamental electric circuit principles, validated through a combination of theoretical analysis and experimental implementation. The investigation into series and parallel circuits confirmed expected behaviors, including constant current and voltage division, reinforcing the foundational applications of Ohm's Law. The voltage divider experiment further highlighted the practical effects of load resistance on circuit performance, establishing the importance of external resistance to the sensitivity of voltage across a fixed resistor.

The Wheatstone bridge experiment demonstrated the precision and sensitivity of this configuration in detecting resistance variations, underscoring its utility in instrumentation and calibration. Similarly, the high-pass filter analysis connected frequency-dependent behaviors to theoretical transfer functions, illustrating the transition from capacitive to resistive dominance and its relevance in signal processing.

Despite minor discrepancies attributed to measurement uncertainties, energy losses, and equipment tolerances, the results closely aligned with theoretical predictions. These findings emphasize the critical role of error analysis in experimental validation and highlight the inherent challenges in achieving precision within practical setups.

7 Appendix

```
def largest_pairwise_difference(data_1, data_2):
    differences = [abs(a - b) for a, b in zip(data_1, data_2)]
    max_difference = max(differences)
    return max_difference
resultA2 = largest_pairwise_difference(data_1, data_1)
def quantify_data_similarity(data_1, data_2):
    arr1 = np.array(data_1)
    arr2 = np.array(data_2)
    mae = np.mean((arr2 - arr1))
    return mae
resultB2 = quantify_data_similarity(data_1, data_2)
def accuracy_rate(pf_theo_b, pf_calc_b):
    return ((pf_calc_b-pf_theo_b)/pf_theo_b)*100
mean_accuracy = []
for i in range(20):
     mean_accuracy = np.append(mean_accuracy, accuracy_rate(data_1, data_2[i+1])[i])
print(np.mean(abs(mean_accuracy)))
print("Max Difference: ", resultA2, "Mean Absolute Error: ", resultB2)


def find_shift(x, data1, data2):
    if len(data1) != len(data2):
        raise ValueError("Data1 and Data2 must have the same length.")
    correlation = correlate(data1, data2, mode='full')
    max_corr_index = np.argmax(correlation)
    shift_indices = max_corr_index - (len(data1) - 1)
    x_spacing = x[1] - x[0]
    shift_x_units = shift_indices * x_spacing

    return shift_x_units

estimated_shift = find_shift(distance_task_2, task_2_theo_values, magnetic_field_task_2)
print(estimated_shift)

```

8 References

Author Unknown. *E1e Basic Electric Circuits* , 2024

Taylor, John R. *An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements.* University Science Books, 2022.

Richard Carl Dorf, and James A Svoboda. *Introduction to Electric Circuits.* Hoboken, New Jersey, John Wiley Sons, 2010. â

Load Resistor, $R_L\Omega$	Measured Voltage, V	Theoretical (Calculated) Voltage, V
100	2.40	2.46
150	2.82	2.94
200	3.15	3.26
250	3.38	3.50
300	3.60	3.66
350	3.70	3.91
400	3.81	3.99
450	3.91	4.07
500	4.00	4.27
700	4.20	4.39
900	4.30	4.46
1100	4.40	4.52
1300	4.50	4.56
1500	4.52	4.60
1700	4.55	4.62
1900	4.59	4.68
2500	4.60	4.71
3100	4.66	4.74
3700	4.70	4.75
4300	4.72	4.77
4900	4.73	4.78
5500	4.75	4.79
6100	4.76	4.79
6700	4.77	4.80
7300	4.78	4.80
7900	4.78	4.81
8500	4.78	4.81
9100	4.78	4.81
9700	4.78	4.81
10000	4.79	4.82

Table 9

R_4 (Ω **ohms**)	**Bridge Voltage (V)**
1000	0.393
2000	0.163
3000	0.080
4000	0.037
4100	0.034
4200	0.031
4300	0.028
4400	0.025
4500	0.000
4600	0.021
4700	0.018
4800	0.016
4850	$14.7 \cdot 10^{-3}$
4900	$13.8 \cdot 10^{-3}$
4950	$12.9 \cdot 10^{-3}$
4960	$12.7 \cdot 10^{-3}$
4970	$12.5 \cdot 10^{-3}$
4980	$12.3 \cdot 10^{-3}$
4990	$12.0 \cdot 10^{-3}$
4995	$11.9 \cdot 10^{-3}$
5000	$12.0 \cdot 10^{-3}$
5100	$10.4 \cdot 10^{-3}$
5200	$8.5 \cdot 10^{-3}$

Table 10: Section 4 extended Table, Part (1)

R_4 (Ω ohms)	Bridge Voltage (V)
5300	$6.6 \cdot 10^{-3}$
5400	$4.8 \cdot 10^{-3}$
5500	$3.0 \cdot 10^{-3}$
5550	$2.3 \cdot 10^{-3}$
5600	$1.4 \cdot 10^{-3}$
5650	$0.6 \cdot 10^{-3}$
5660	$0.4 \cdot 10^{-3}$
5670	$0.01 \cdot 10^{-3}$
5675	0.000
5680	$-0.1 \cdot 10^{-3}$
5690	$-0.2 \cdot 10^{-3}$
5700	$-0.2 \cdot 10^{-3}$
5750	$-1.0 \cdot 10^{-3}$
5800	$-1.8 \cdot 10^{-3}$
5900	$-3.4 \cdot 10^{-3}$
6000	$-4.9 \cdot 10^{-3}$
6100	$-6.3 \cdot 10^{-3}$
6200	$-7.7 \cdot 10^{-3}$
6300	$-9.1 \cdot 10^{-3}$
6400	$-10.4 \cdot 10^{-3}$
6500	$-11.7 \cdot 10^{-3}$
7000	$-17.6 \cdot 10^{-3}$
7500	$-22.7 \cdot 10^{-3}$
8000	-0.026
9000	-0.033
10000	-0.041

Table 11: Section 4 extended Table, Part (2)

Table split into two as to be able to present, as otherwise the pdf compiled a badly formated table

Frequency, f Hz	Measured $\frac{U_a}{U_e}$ (no units)	Theoretical $\frac{U_a}{U_e}$ (no units)
200	0.044	0.005
400	0.043	0.010
600	0.050	0.015
800	0.073	0.020
1000	0.067	0.024
2000	0.073	0.049
4000	0.124	0.98
6000	0.159	0.145
8000	0.189	0.192
10000	0.246	0.238
15000	0.320	0.345
20000	0.425	0.440
25000	0.506	0.522
30000	0.549	0.592
35000	0.636	0.651
40000	0.646	0.700
45000	0.677	0.741
50000	0.737	0.775
55000	0.777	0.803
60000	0.786	0.827
65000	0.823	0.847
70000	0.798	0.878
75000	0.846	0.891
80000	0.846	0.901
85000	0.862	0.912
90000	0.879	0.919
95000	0.923	0.919
100000	0.923	0.926

Table 12: Section 5 Extended Table