M10e Resonance and Phase Shift in Mechanical Oscillations

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note before the note: I did not have time to finish the lab, too much time was taken up by trying to do the fitting which did not work. All possible applications was tried but to no avail, no filitering or fitting worked enough to provide a data to provide parameters to analyse and do the tasks. I did not start writing the lab not until like 2 hours ago so did not have time to even acknowledge the issue as errors existing in our experiment or data. I apologize for the mess of a report this is

Initial Remark: The following Lab Report does not provide sufficient Data Fitting to determine specific constant values, such as the damping coefficient, moment of inertia, respective constants, etc.. as any fitting model which has been attempted has been unsuccessfully with either proportionally off by a large margin (in the thousands) or the code directly not being able to find any parameters which satisfy the behaviour. Even though the measured sets themselves showed expected behaviour when graphed. All approaches, including but not limited to Gaussian Filtration, Mean Rolling Filtration, Fourier Analysis to identify extra noise, outlier analysis etc... had been implemented to try to smooth out the data, exclude any outliers and try to limit the data sets to sections which would provide sufficient data fitting but to no avail fitting did not work. Fitting model codes have been thoroughly checked, and even tried upon other data sets from other labs (where they have worked), yet proper fitting could not be done. All the data filtration / smoothing and attempt at data modelling are addressed in respective sections. Reasons for such a situation is addressed in the Discussion for each section.

1 Introduction

2 Rotary pendulum without external drive

2.1 Hypothesis

2.2 Theoretical Exploration

The equation of motion for a rotary pendulum without external drive is:

$$J\ddot{\varphi} + D\dot{\varphi} + \gamma\varphi = 0 \tag{1}$$

where J is the Moment of inertia of the pendulum, D the directional moment of the spring and γ the damping coefficient. It can also be expressed as:

$$\ddot{\varphi} + 2\delta\dot{\varphi} + \omega_0^2\varphi = 0 \tag{2}$$

with $\frac{\gamma}{J} = 2\delta$, $\omega_0^2 = \frac{D}{J}$ and $\omega_d = \sqrt{\omega_0^2 - \delta^2}$. In case of weak damping $\delta^2 < \omega_0^2$ the differential equation is solved as:

$$\varphi(t) = Ce^{-\delta t}cos(\omega_d t - \alpha) \tag{3}$$

The damping constant δ or logarithmic decrement Λ can be determined from the observed decrease in angular displacement $\varphi(t)$

$$\Lambda = \delta T_d = ln(\frac{\varphi(t)}{\varphi(t + T_d)}) \tag{4}$$

where $T_d = \frac{2\pi}{\omega_d}$. So that:

$$f_d = \frac{\omega_d}{2\pi} \tag{5}$$

2.3 Experimental Exploration

2.3.1 Materials

- Laboratory power supply: GwINSTEK GPD-2303S
- Pohl's Wheel with inbuilt Sensor, made up of
  - Pair of coil with yoke
  - Magnetic Coils
  - Angle Sensor Wheel
- PAC Software

2.3.2 Set Up

Pohl's Wheel is attached to power supply providing some current with constant voltage. The oscillating pin C1, as seen in Image akdshajkhskad, is held stationary before any measurements are taken. The Wheel is attached to a device to record the deflection angle position from the sensor.

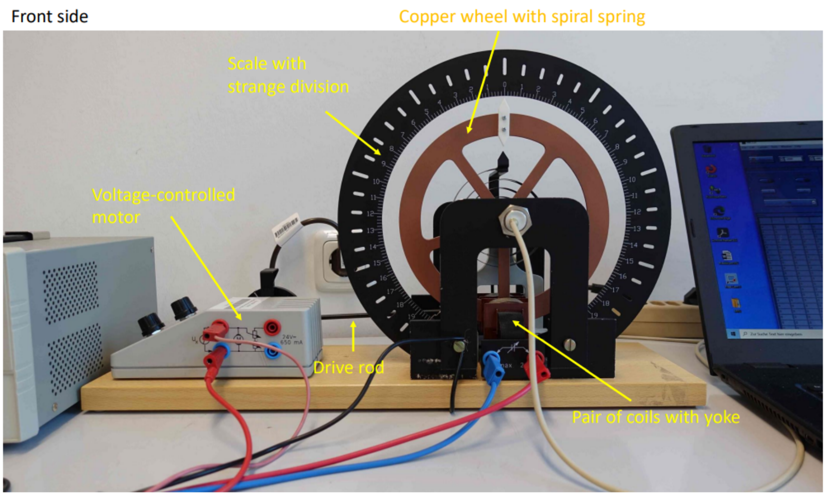


Image 1 Set up for Section 2, (From provided supplementary aid)

2.3.3 Methodology

Varying the current supply, from 400 to 1200 mA at intervals of 100 (8 different currents), the damping effect of the eddy current is measured from an some initial angle deflection.

2.4 Results

2.4.1 Data and Analysis

raw data below everything

3 Rotary Pendulum with External Drive

3.1 Hypothesis

3.2 Theoretical Exploration

If an oscillation is driven by an external torque which varies periodically over time, a forced oscillation takes places. In such a case, the equation of motion for this motion is given by:

$$J\ddot{\phi} + \gamma\dot{\phi} + D\phi = M_0 \sin(\omega t) \tag{6}$$

where the solution is given by

$$\phi(t) = A(\omega)\sin(\omega t + \theta) + Ce^{\delta t}\cos(\omega_d t - \alpha) \tag{7}$$

is provided by finding the particular and general solution for the inhomogeneous and the homogenous equations of Equation 6 respectively. In utilising the particular solution (the component with the sin function of equation 7), the frequency dependent amplitude can be derived as follows,

$$A(\omega) = \frac{M_o/J}{\sqrt{(w_0^2 + w^2)^2 + (\gamma^2\omega^2)/J^2}} \tag{8}$$

where ω is the driving torque's angular frequency, $\omega_0 = \sqrt{\frac{D}{J}}$ is the angular eigenfrequency of the free, un-damped system and γ can be written as 2δ. While the phase shift θ is frequency dependent value which can would be

$$\tan\left(\theta(\omega)\right)) = -\frac{\omega\gamma}{J(\omega_0^2 - \omega^2)} \tag{9}$$

In letting the oscillation to stabilise onto some region of constant amplitude, the term with angular frequency ω_d, in reference to equation 7, becomes very small due to the exponentially decreasing damping factor. The pendulum then settles onto oscillating with some consistent ω, albeit with some phase shift θ relative to the driving torque. Equation 8 will be the equation utilised for the calculation of the theoretical value of the resonance frequency and the corresponding phase shift of the system using equation 9. Any further derivations for analysis of the results and data will be conducted in Section 3.4.2: Discussion, in addition to direct analysis.

3.3 Experimental Exploration

3.3.1 Materials

- Laboratory power supply: GwINSTEK GPD-2303S
- Pohl's Wheel with inbuilt Sensor, made up of
  - Voltage Controlled Motor
  - Drive Rode, Drive Wheel
  - Pair of coil with yoke
  - Magnetic Coils
  - Angle Sensor Wheel
- PAC Software

3.3.2 Set Up

Pohl's Wheel is attached to the external motor and the power supply. The oscillating pins, the corresponding C1 and C2 as seen in Image eajdfhaskjh are held stationary before any measurement. The units of the division on the wheel are unknown.

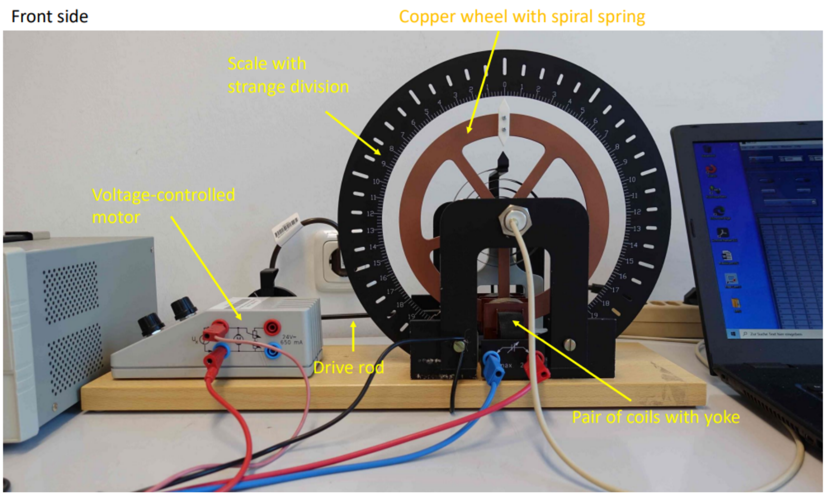


Image 2 Set up for Section 2, (From provided supplementary aid)

3.3.3 Methodology

While the Power Supply is connected to the wheel, the driving motor is switched on at a constant supply for one current value. The voltage output is varied among some value sets to find the area of *visible* resonance effect - dubbed as the resonance neighbourhood. 14 different measured are taken with each corresponding to some voltage around the resonance neighbourhood, ranging from 6.3 to 8.9 with an interval of 0.2 per measurement. Each measurement is let to run for 30 seconds.

3.4 Results

3.4.1 Data and Analysis

From the retried data, the following data presents the Deflection Angle values for both U1 (radians) and U2 (radians) pins from the set up against Time, t (s) for the voltage case of 6.9 V. Similar graphing for all 13 other voltage data sets can be found in the appendix. 6.9 Volts was provided as an arbitrary option.

```
file_path = r"task_2_6.9.csv"
file_path_evaluated = pd.read_csv(file_path, header=0)
c1_un = file_path_evaluated.iloc[:, 0]
c2_un = file_path_evaluated.iloc[:, 1]
c3_un = file_path_evaluated.iloc[:, 2]
c1 = pd.Series([int(num.replace(",", "")) for num in c1_un]) * (1/1000000)
c2 = pd.Series([int(num.replace(",", "")) for num in c2_un]) * (1/10000000)
↪  * (np.pi / 2)
c3 = pd.Series([int(num.replace(",", "")) for num in c3_un]) * (1/10000000)
↪  * (np.pi / 2)
c2_smoothed = gaussian_filter1d(c2, 50)
c3_smoothed = gaussian_filter1d(c3, 50)

c2_dy_dx = np.gradient(c2, c1)
```

```python
c2_dy_dx_smoothed = np.gradient(c2_smoothed, c1)
c3_dy_dx_smoothed = np.gradient(c3_smoothed, c1)

amplitude_set  = np.append(amplitude_set, (max(c2_smoothed) - min(c2_smoothed))/2)
omega_set  = np.append(omega_set, (max(c3_dy_dx_smoothed)))

def amplitude(omega, J, D, gamma):
    numerator = 0.026639847846496978 / J
    denominator = np.sqrt((D / J - omega**2)**2 + (gamma * omega / J)**2)
    return numerator / denominator

points_between = 10
indices = np.arange(len(amplitude_set))
new_indices = np.linspace(indices[0], indices[-1], len(amplitude_set) + (len(amplitude_set) - 1) * points_between)
w_inter = np.linspace(omega_set[0], omega_set[-1], len(amplitude_set) + (len(amplitude_set) - 1) * points_between)

interpolator = interp1d(indices, amplitude_set, kind="quadratic")
interpolated_points = interpolator(new_indices)

initial_guess = [1, 1, 0.1]

params, covariance = curve_fit(amplitude, w_inter, interpolated_points, p0=initial_guess)

J_fit, D_fit, gamma_fit = params

fitted_amplitude = amplitude(w_inter, J_fit, D_fit, gamma_fit)
```

where following, the Amplitude, calculated as the mid-axis to crest distance for each corresponding voltage, against the Angular Frequency, measured as the gradient of the Deflection Graph for each corresponding voltage, provides the Resonance Graph. The scaling with ω_r has been omitted as the required parameters for the determining of ω_r were off proptionally leading to square rooting of a negative. More detials below.

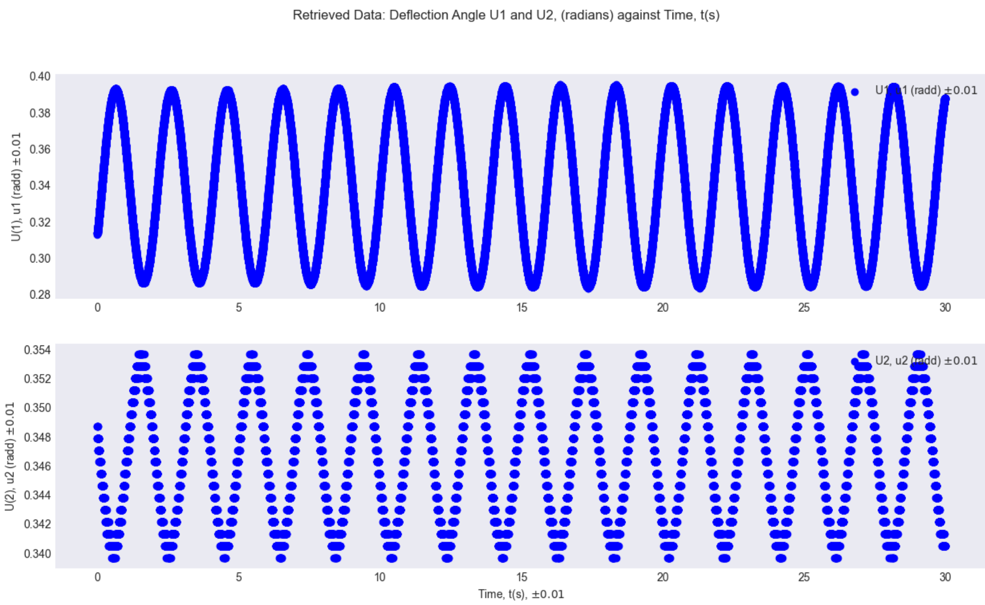


Figure 1: U1 and U2, V against Time, t(s) for the Voltage 2.6 V

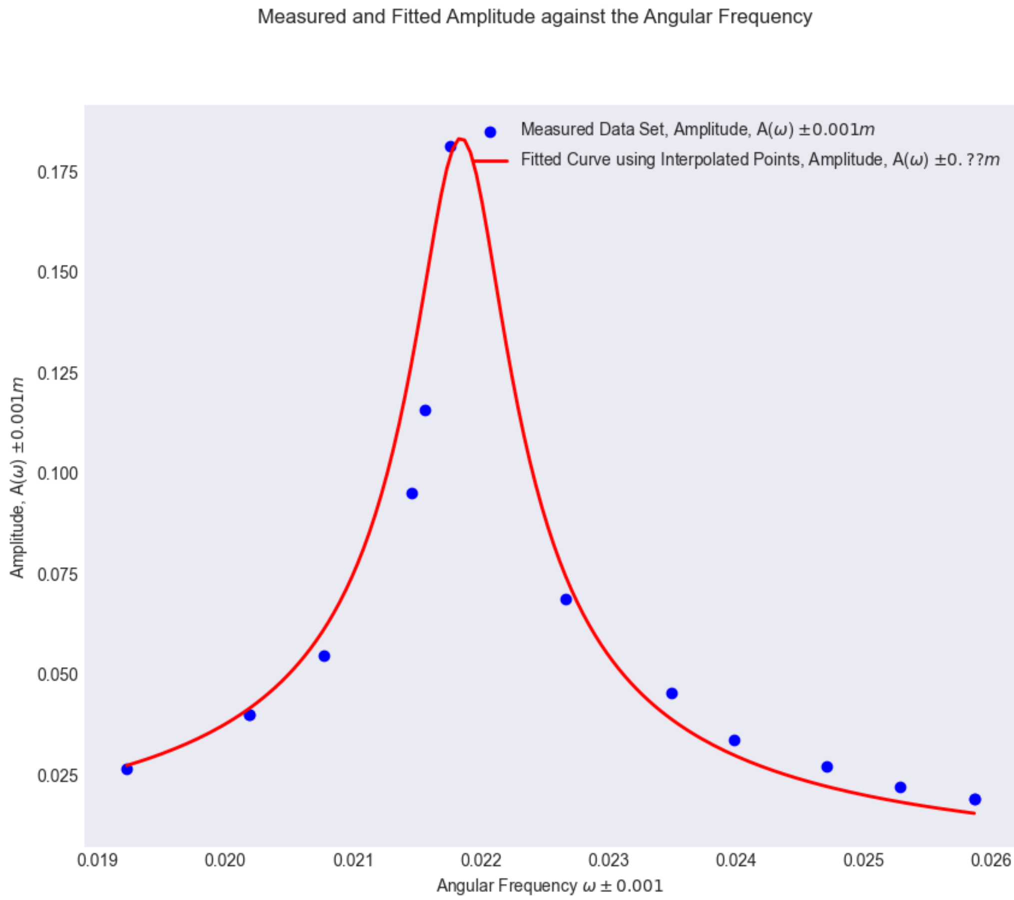


Figure 2: Measured and Fitted Amplitude, A(ω) $\pm 0.001m$, against the Angular Frequency, $\omega \pm 0.001$

Note: The code for plotting is omitted as it is considered general and as to avoid clutter. If needed, can be accessed in the attached Jupyetr Folder

Figure 4 presents the Amplitude against the Angular Frequency graph, with a resonance position corresponding to the maximum value of the graph. In the initial data fitting, with only 14 points as reference (due to there being only 14 voltage measurements), the data fitting provided a J (Moment of Inertia) of 5.2 million and a D (Restoring Torque Coefficient) of 4 thousand. This mainly was due to the limited sample size, and as such data extrapolation was done as to find in between values of the Measured Data Set. In doing so, the overall values for the parameters went down to more manageable values, but were still limited by their proportionality. The final best fitting curve found provided the following parameters:

- J (Moment of Inertia): 8978.2156
- D (Restoring Torque Coefficient): 4.2869
- γ (Damping Constant): -6.6518

where in utilising these parameters for $w_r = \sqrt{\frac{D}{J} - \frac{\gamma^2}{2}}$ provides an imaginary w_r, -22.12305627682268i ≈ -22.1 i.

Validity of the Angular Frequency Data Set:

The Angular Frequency, determined by taking the gradient of the driving force's deflection relation U2 (as seen in Figure 1 (b)), had been determined through the a Gaussian filtered U2 data set. In verifying the accuracy of the filtered data set, the following presents the Fourier Analysis for both filtered and original U2 Data Set:

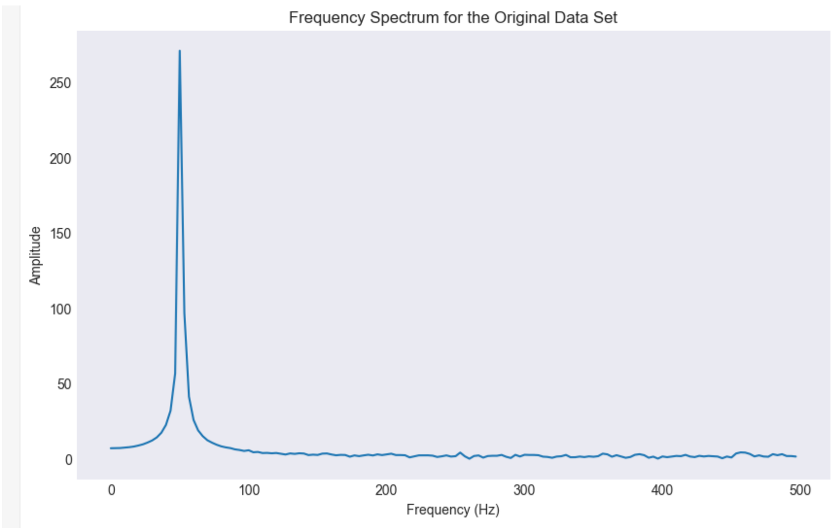


Figure 3: Frequency Spectrum for the Original Data Set

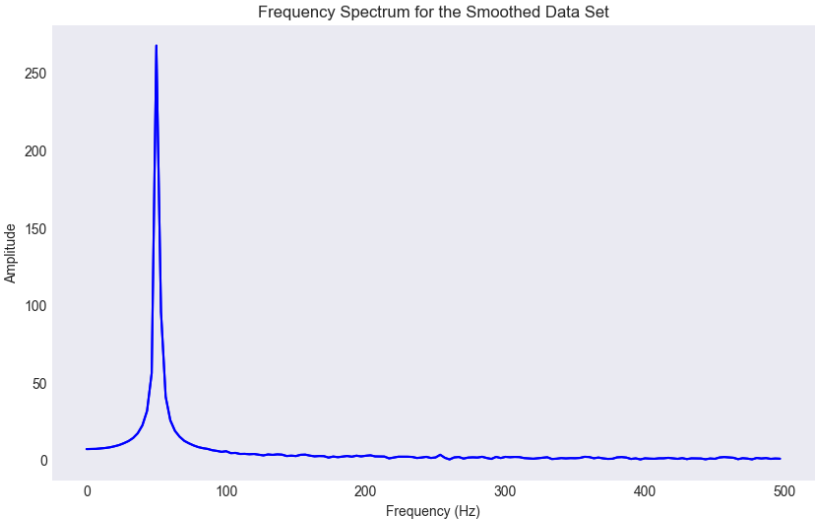


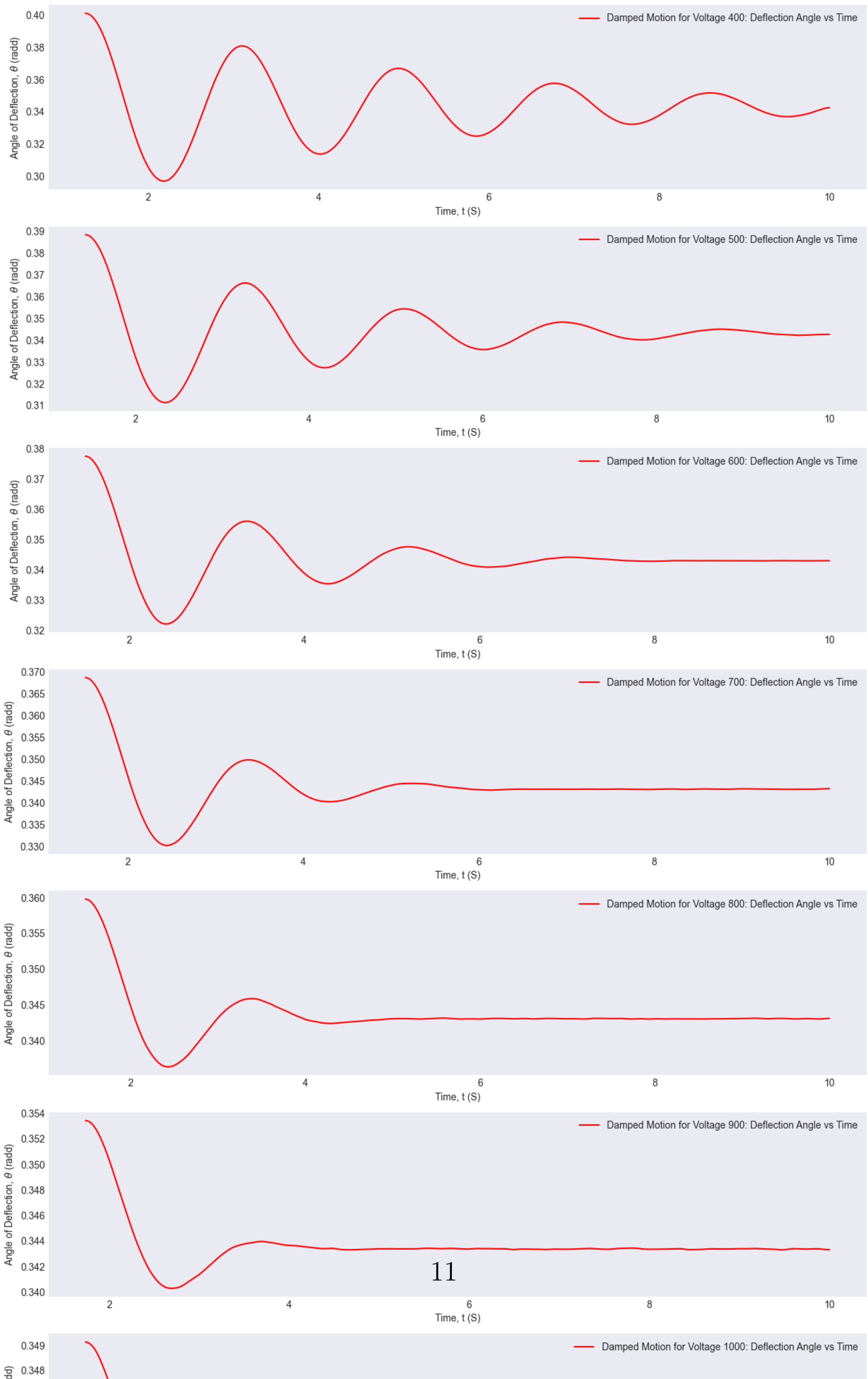
Figure 4: Frequency Spectrum for the Smoothed Data Set

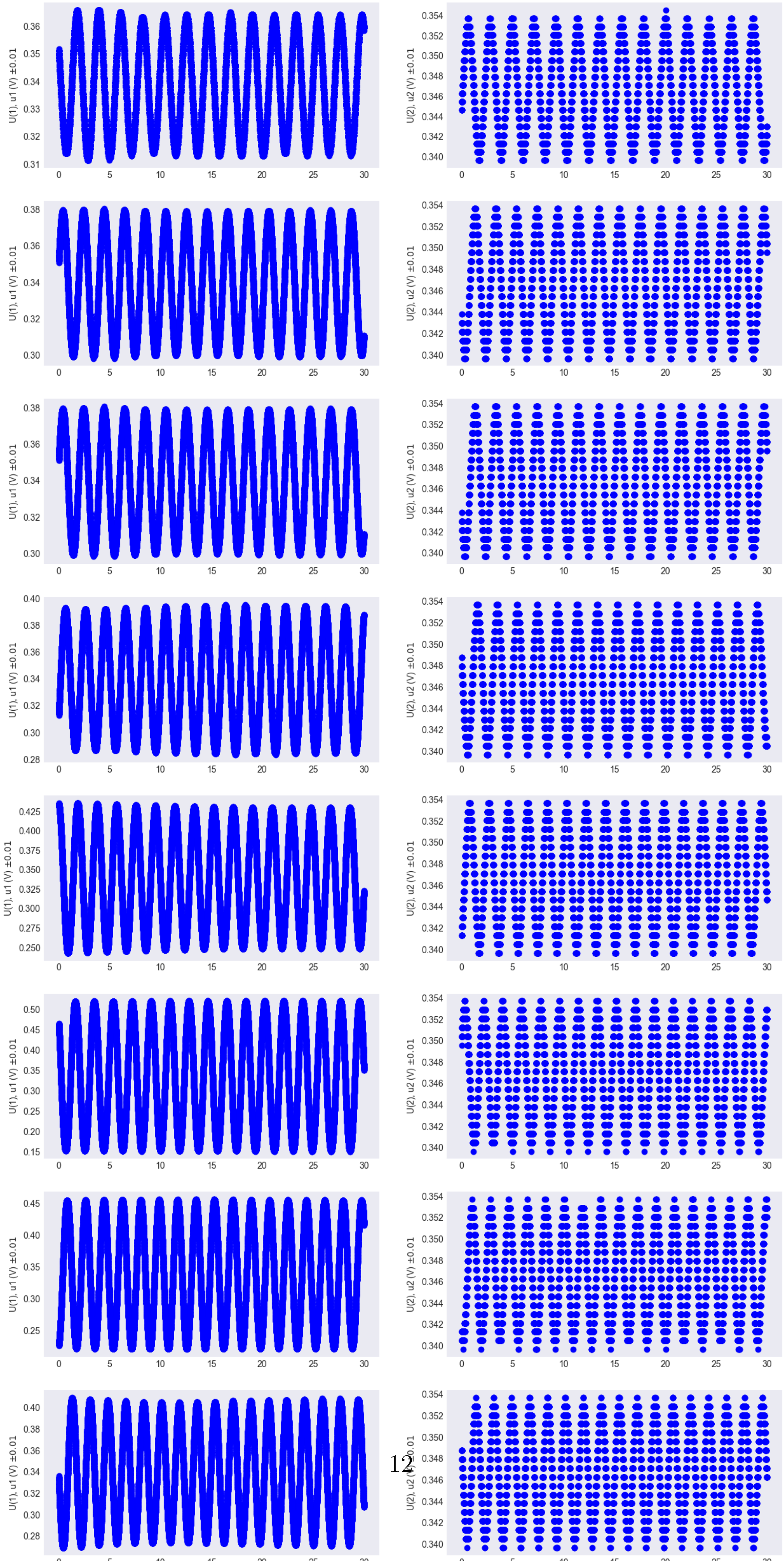
,
with both presenting a common peak at the same spot.

3.4.2 Discussion

3.4.3 Error Analysis

4 Conclusion




U(1), u1 (V) ±0.01
U(2), u2 (V) ±0.01