M6e Gyroscope with three Axes

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Abstract

This study explores the parameters of a three-axis gyroscope through experimental and theoretical methods. The third component of the moment of inertia (I_3) was determined using energy conservation and applied force methods, yielding percentage errors of approximately 5034% and -1.15%, respectively, with the latter aligning closely with theoretical values. The damping constant ($\beta = -0.364$) was derived from angular velocity decay, showing a strong linear relationship ($R^2 = 0.98$). Precession frequency exhibited an inverse correlation with rotational speed, but experimental values showed significant deviations, with errors averaging 505% and 615% for different mass systems. Nutation frequency displayed a closer alignment, with an average error margin of 28.8%. Discrepancies due to measurement challenges, especially with rotational speed accuracy.

1 Introduction

For the first part of the exploration of paraments of a Gyroscope, the torque due to some mass attached to a bobbin of the Gyro Disk will be utilised to measure the 3rd component of the Moment of Inertia, I_3. The measurement for the angular velocity will be correlative to the measurement of the rotational speed of the disk. Where, using a specialised instrument, will be measured as *Rotations Per Minute, RPM.* The calculations will be done using the concept of conservation of energy. Additionally, the time for the mass to reach to the ground will be timed and used to calculate I_3 through forces. Both methods will be compared toward the theoretical value determined by empirical calculations.

Following that, the damping constant due to the overall decay of the rotation speed will be determined by letting the Gyro Disk spin and run up to some length of time. The determining of the constant will be through the plot of RPM against the time length, with measurements at even intervals.

For the last parameters, the Gyroscope will be let to Precess and Nutate over some period of length for the measurement of the Precession and Nutation Frequency. For precession, the Gyroscope will be let to run up to a half a period, where the time length of the period and the average rotational speed will both be used for separate calculations of the Precession Frequency. While the Nutation Frequency, in the same manner, will be determined from the values of three periods and the average rotational speed.

2 Determining I_3

2.1 Theoretical Exploration

For a Gyro Disk rotating only against one axis, the measurements of the Angular Velocity can be utilised to calculate the third component of the Moment of Inertia, I_3. As for an oblate system let run in the manner as specified in the Image **1** below, only the third component would have any influence on the rotation of the Disk.

The calculations of the Inertia can be done through 2 different methods experimentally; by energy conservation and understanding forces being applied on the system.

2.1.1 Energy conservation

Using the free fall of an attached mass, an external torque is applied to the Gyro Disk, and as such allowing it to run with some rotational speed. From the assumption that energy is conserved, we can assume that the potential energy from the mass (held at some height h_0) equals to the kinetic energy of a mass as it reaches the ground. The Kinetic Energy can be split into two parts; rotational and translation components

$$mgh = \frac{1}{2}I(\omega)^2 + \frac{1}{2}mv^2 \tag{1}$$

where upon substituting $v = \omega r$, we would get

$$mgh = \frac{1}{2}I(\omega)^2 + \frac{1}{2}m(\omega r)^2 \tag{2}$$

where the component I would be correlative to the third component, I_3, as desired initially. Rearranging for I_3:

$$I_3 = \frac{2(mgh - \frac{1}{2}m\omega r^2)}{\omega^2} \tag{3}$$

,

where now equation 3 will be our I_3 of our Gyroscope System through the means of energy conservation.

2.1.2 Forces

Through the utilisation of forces, inertia is directly related to torque through Newton's second law of rotation, where

$$\tau = I\alpha \tag{4}$$

where τ would be the torque due to the free fall of a mass (for our system), I the inertia I_3 and α the angular acceleration. Taking how $\tau = rF\sin\theta$ (but taking that for our system the force applied is perpendicular to the Disk), we can relate both equations and get,

$$rmg = \alpha I_3 \tag{5}$$

, where the force was equated to the gravitational force for some mass m.

The Angular Acceleration, α, can be determined from from the correlation that $a = \alpha r$ and that the acceleration for a ***frictionless*** system is related to the height of the falling object by $h = h_0 + v_0 t + at^2 \frac{1}{2}$ (taking v_0 and h_0 as zero), we can write our final I_3 as

$$I_3 = \frac{mgr^2t^2}{2h} \tag{6}$$

.

The literary value will be calculated using the standard Moment of Inertia formula for an oblate disk rotating along the 3rd axis (source)

$$I_3 = \frac{1}{2}mr^2 \tag{7}$$

where r is the Radius of the disk. I_3 will have the units $kg \cdot m^2$. All calculations through the help of the provided additional guide (Ziese)

2.2 Physcial Exploration

2.2.1 Material

- 3-Axis Gyroscope
- Revolution Speed Meter (RSM), Class 2 Laser ±0.001
- Timer ±0.001
- Ruler ±0.0005
- String with negligible Mass
- Weight $0.05kg \pm 0.001$

2.2.2 Set up

The Gyroscope, positioned at the edge of the table, was fixed with an additional rod as such to ensure a fixed axis to hold the Gyroscope in place. Attached to the back was a string with negligible mass to a bobbin of also negligible mass. The other end of the string was attached to a mass of mass $0.05kg \pm 0.001$. The string was cut to be longer then the height of the fall distance as to ensure the mass reaches the ground unrestricted. Letting the mass go would rotate the Gyro Disk from the applied torque. Refer to the diagram below. The provided measurements of the parts of the set up:

Component	Measurement
Mass of the Gyroscope	1.5 kg
Diameter Disk	0.25 m
Diameter bobbin	0.065 m
Height of the Table Edge	0.75 m

Table 1: The Standard Deviation and Coefficient of Variation for the two measured value sets.

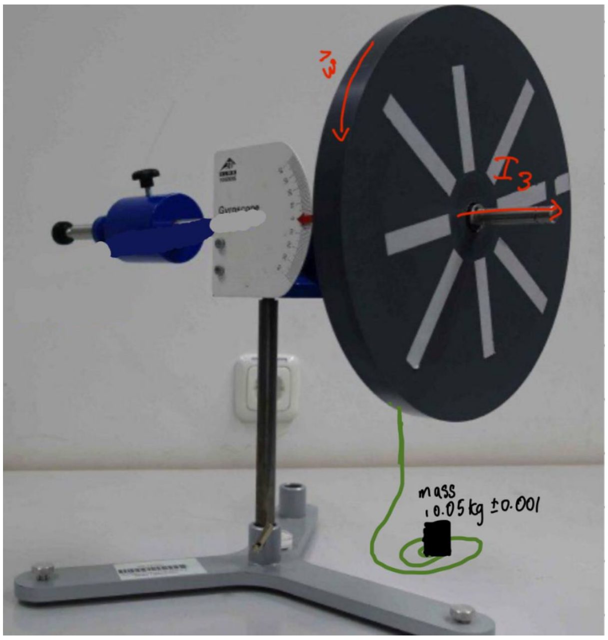


Image 1 Set up for Section 1, direction of I_3 and w as indicated.

2.2.3 Methodology

Initially the mass was held at rest at the height of the table, near the edge. Letting the mass fall, the time to reach the ground and the rotational speed of the Gyroscope was measured for the instance the mass touched the ground. This process was repeated 10 times and averaged out over the 10 values.

2.3 Results

2.3.1 Data and Analysis

Averaging the rotational speed, w as $w = 1.3720882240540153 \approx 1.37$, and utilising equations 3 and 6, we get an experimental value for I_3 as:

Note: As there are 8 stripes on the Gyroscope, the RSM measured the w_3 value with an increased factor of x8. As such, the calculations for the mean divided everything out by 8 and also with an additional 60 for conversion of the rpm to rps (rotations per second).

```
g = 9.81
h = 0.725
m = 0.050
r = 0.0325

data_task_1 = r"task_1_data.csv"
data_task_1_evalauted = pd.read_csv(data_task_1)
t_task_1 = data_task_1_evalauted.iloc[:, 0]   #time
w_r_8 = data_task_1_evalauted.iloc[:, 1]   #w

theo_moment_inertia = (1/2)*(1.5)*(0.125)**2
```

```python
w_mean = np.mean(w_r_8/(8*60))
t_task_1_mean = np.mean(t_task_1)
def method_energy(w_mean):
    return (2*(m*g*h - (0.5*m*w_mean*r**2))/w_mean**2)
def method_forces(t_task_1_mean):
    a = (2*h)/(t_task_1_mean**2)
    return (m*g*r**2*t_task_1_mean**2)/(2*h)
```

Method	Value
Energy Conservation	$0.6017343937052143 \approx 0.62 \pm 6.9 \cdot 10^{-2}$
Applied Force	$0.011584376316387935 \approx 0.012 \pm 6.9 \cdot 10^{-7}$
Literary	$0.01171875 \approx 0.012$

Table 2: The measurement and literary values for I_3, uncertainty in calculation as presented in each row

Table 2 presents the measured values and literary values I_3. Uncertainties were calculated using the uncertainty equations for standard set correlations between parameters (Taylor).

The method of energy conservation yielding an Percentage Error of **5034.80015962** $\approx$ **5034%**, relative to the literary value provides a result that is clearly an error of the experiment itself. To some extent, such a large difference in the measured value for energy conservation is due to lose of energy in the system. As, not all of the potential energy is converted to kinetic (as assumed). **However** for a height of 75 cm ± 0.005 that we had, it does not seem large of a height as such to provide a 5 000% error margin. As compared to the I_3 from the force measurements, the Percentage Error comes out to **-1.14665543349** $\approx$ **-1.15%**. Where the average time period of the fall was $\approx$ **5.7 s**. This difference of the I_3 value (absolute difference being $|-0.000134373683612| = 0.000134373683612 \approx 0.001$) provides a more accurate understanding for a system where friction would not have caused more than an error difference of in magnitude of 10 at the most. A possible explanation would be the misusage of the RMS device, i.e not properly measuring the rotation speed of the system. More details in the Discussion.

The negativity of the of error margin between the measured I_3 through forces and the literary I_3 could be attributed to the averaging of the values. The averaging, if there are any values (even if not an outlier) which are not as consistent as others, could lead to the mean leaning towards the inconsistent value. As such, Standard Deviation and the Coefficient of Variation will be conducted in Discussion for further understanding of the results. More details in the Discussion.

2.4 Discussion

The errors for the I_3 calculated through the forces applied is of standard expected external influences; friction in the string as it uncurls from the bobbin, the friction between the mass and the fluid (air) is goes through and the existing possible resistive forces within the Gyroscope - limiting how well it can rotate. These are but some examples of standard expected influences due to not having an isolated and perfect system. The error for I_3 through energy conservation wil be analysed further down.

Furthermore, by utilising the calculated mean for time and w_3, we can calculate the Standard Deviation (SD) both data sets and measure the Coefficient of Variation (CoV). Cov provides a reasonable understanding of how low our SD is relative to our data set. As lower SD relates to a more clustered (and as such more precise) data set.

```python
def standarddeviation(mean, value_set):
    total_squared_sum = []
    for i in range(len(value_set)):
        total_squared_sum = (value_set[i] - mean)**2
    return np.sqrt(total_squared_sum/len(value_set))
CoV_t = standarddeviation(t_task_1_mean, t_task_1) / t_task_1_mean
CoV_w = standarddeviation(w_mean, (w_r_8/(8*60))) / w_mean
```

we get:

Component	Standard Deviation Value	Coefficient of Variation
Time, t	$0.010751744044457243 \approx 0.012$	$0.0018882585255659342 \approx 0.002$
Rotational Speed w	$0.008788496663884619 \approx 0.009$	$0.008107157625138597 \approx 0.008$

Table 3: The Standard Deviation and Coefficient of Variation for the two measured value sets.

The obtained SD values for both parameters are already relatively low, and the even lower CoV further suggest that the SD values are small in relation to their respective datasets. This reinforces the idea that the measured values are closely grouped, indicating a high level of precision. Consequently, there is no clear indication of any outliers or disproportionately skewed values that could introduce significant deviations. The negative percentage error difference discussed in Section 2.3 does not appear to stem from any erroneous data points. The most likely explanation is that the stopwatch was stopped slightly too early, before the object fully reached the ground, as a shorter measured time would lead to a smaller I_3 (i.e., $I_3 \propto t^2$).

The I_3 determined through energy conservation has relatively large error difference because, as we suspect, of not positioning the RSM within the recommended measuring distance. The supplementary guide provided for the experiment outlines how RSM should be within 5cm, a point which we did not take into account. The actual positioning could be between 5-10cm, but the exact value is unknown. As such, an error in the procedure of the experiment led to the percentage error of 5 000%.

For future calculations in this report, the value measured from the Forces Applied will be used, as it is the most accurate of the two.

3 Determining Gyroscope's Friction Coefficient

3.1 Theoretical Exploration

Usually, for a damping system, the change between the two parameters decays exponentially. As such we can make the estimation that the relationship between w_3 and time would be:

$$\omega(t) = \omega_{0_3} e^{-\beta t} \tag{8}$$

,

where β would be the damping constant, ω_0 the initial angular velocity. Taking the log of equation 8, we get:

$$ln(\omega_3(t) = ln(\omega_{0_3} - \beta t) \tag{9}$$

Which is now in the form of a linear function, of which we can take the slope as β, the damping constant. Through dimensional analysis, we can determine that β has no units.

3.2 Physical Exploration

3.2.1 Material

- 3-Axis Gyroscope
- Revolution Speed Meter (RSM), Class 2 Laser
- Timer: ± 0.001
- Large String for running the Disk

3.2.2 Set up

The Gyroscope is positioned on the table, with the same axis fixtures as Section 2. We attach a larger string to let the disk run. Refer to **Image 1** for the set up, but without the Mass.

3.2.3 Methodology

By pulling the attached string, the disk is let to rotate. While simultaneously starting the timer, ω_0 of the Disc is measured. The further ω is measured through 10 second intervals for a total time of 120 seconds. Both w against time, and $\ln(\omega_3)$ against time will be plotted.

3.3 Results

3.3.1 Data and Analysis

From the retrieved data, the following present two relationships between the two parameters, w_3 and time.

```python
data_task_2 = r"task_2_data.csv"
data_task_2_evalauted = pd.read_csv(data_task_2)
t_task_2 = data_task_2_evalauted.iloc[:, 0]   #time
w_r_8_task_2 = data_task_2_evalauted.iloc[:, 1]   #w
def t2_graph_1(t, a):
    return a*np.exp(-t)
def t2_graph_2(t, a, b):
    return a*t + b
```

```
params_t_2_graph_1, __ = curve_fit(t2_graph_1, t_task_2, (w_r_8_task_2/(8*60)))
params_t_2_graph_1_b, __ = curve_fit(t2_graph_2, t_task_2, (w_r_8_task_2/(8*60)))
params_t_2_graph_2, __ = curve_fit(t2_graph_2, t_task_2, np.log(w_r_8_task_2/(8*60)))
a_t2_graph_1 = params_t_2_graph_1[0]
a_t2_graph_2 = params_t_2_graph_2[0]
a_t2_graph_1_b = params_t_2_graph_1_b[0]
b_t2_graph_2 = params_t_2_graph_2[1]
b_t2_graph_1_b = params_t_2_graph_1_b[1]
t2_graph_1 = t2_graph_1(t_task_2, a_t2_graph_1)
t2_graph_1_2 = t2_graph_2(t_task_2, a_t2_graph_1_b, b_t2_graph_1_b)
t2_graph_2 = t2_graph_2(t_task_2, a_t2_graph_2, b_t2_graph_2)
r2_fit_1_b = r2_score((w_r_8_task_2/(8*60)), t2_graph_1_2)
```

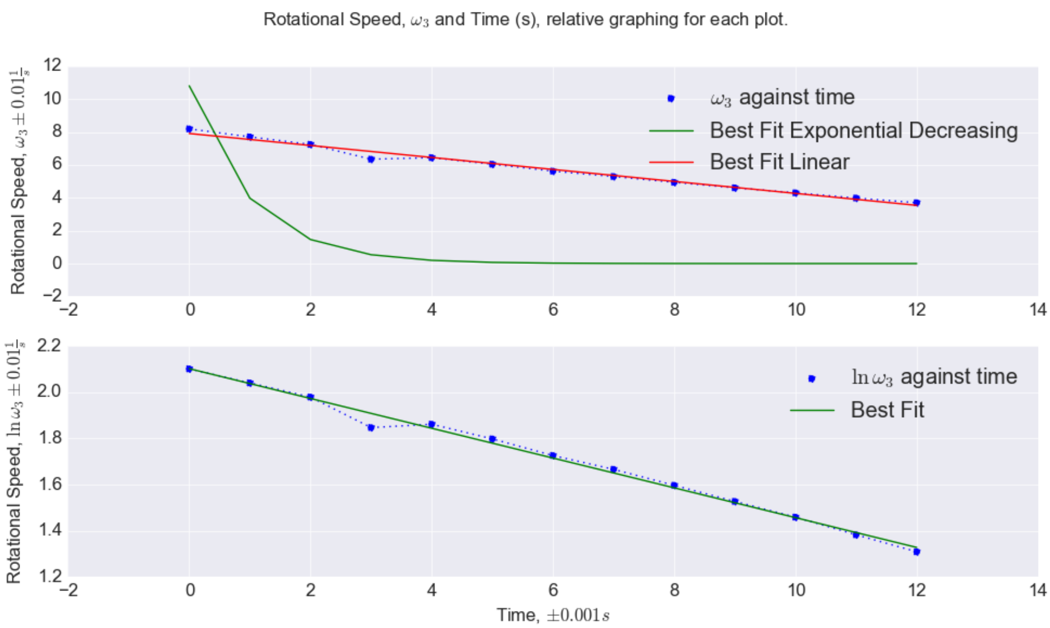


Figure 1: Rotational Speed, $\omega_3 \frac{1}{s}$ and Time (s), relative graphing for each plot

As seen from the Figure 1, the overall decay between our parameters is linear. And as such, as opposed to the prediction in equation 8, the decay of the rotational speed can be modelled using linear relationship. Meaning, the gradient of the best fit for Figure 1 (a) will yield the damping constant. $\beta = -0.3639995421275173 \approx -0.364$, a decreasing and steady relationship. The fitting of the best fit line yield an R^2 of $0.9843091358607999 \approx 0.98$. Where R^2 determines how close is the best fit line to the actual data, where closer it is to 1, higher the accuracy. More details in the Discussion.

3.4 Discussion

With the high accuracy of the R^2 score, the found value for β = -0.364 can be considered to be accurate to the actual change of w_3.

4 Precision Frequency

4.1 Hypothesis

As the mass of the system increases, the precession also increases, leading to a higher precession frequency. This is because the torque exerted by the mass grows in direct proportion to the increase in mass. According to the relationship between angular momentum and torque ($\frac{d}{dt}L = \tau$), an increase in torque results in a faster change in angular momentum. When the change in angular momentum is faster (in terms of direction, not magnitude), it leads to a quicker rotation along the central axis, which manifests as a faster precession.

Furthermore, the change in precession can be estimated as directly proportional to the factor by which the mass increases. Since all the relationships are direct, the factor for the mass change can be isolated, calculated, and then multiplied back into the final result.

The relation between Rotational Speed and the Precession Frequency would be a decreasing relationship, linearly. As an increase in the Rotational Speed leads to a decrease in the Precession Frequency. Where a rotating Gyro Disk will slow down the precession of the Gyroscope along its central axis (as the Disk rotates opposite to the direction of the Precession, refer to Image **2**) And as such a larger Rotation Speed would lead to a smaller Precession Frequency. And such be consistent with different Mass Systems.

4.2 Theoretical Exploration

Precession of a Gyroscope is defined as the rotation of the Gyro Disk along a changing central axis. As seen in the below Image 2, such a motion happens due to some external torque, $\vec{\tau}$, causing a change in the Angular Momentum's, L_z's, direction.

When a small object is placed some distance z_z away from the centre of mass of the system, it creates some external $\vec{\tau}$, caused by the gravitational force produced from the objects mass, as labelled $\vec{F_g} = m\vec{g}$. Such a force produces a motion along the ϕ component of our disk, and as such *precessing* the Gyro Disk. During some infinitesimal time interval dt, the ϕ angle changes by some infinitesimal $d\phi$, with $d\phi = \frac{dL_z}{L_z}$, the Angular Velocity of the precession (the Precession Velocity), ω_p, is given by:

$$\Omega_p = \frac{d\phi}{dt} = \frac{dL/dt}{L} = \frac{\tau}{L} \tag{10}$$

.

If torque is defined as $\tau = F_g z_z = m_z g z_z$ and the Angular Momentum as:

$$|\vec{L}| = |\vec{I}\vec{\omega}| = \sqrt{(I_1\Omega_p)^2 + (I_3\omega_3)^2} \tag{11}$$

and since $\Omega_p >> \omega_3$, equation 10 can be rewritten as:

$$\Omega_p = \frac{m_z g z_Z}{I_3\omega_3} \tag{12}$$

However, as Angular Velocity and Angular Frequency is related by $\omega_i = 2\pi f_i$, the final equation for the Precession Frequency will be:

$$f_p = \frac{m_z g z_Z}{2\pi I_3 \omega_3} \quad (13)$$

Where equation 13 will be the final equation used to calculated the Measured values of f_p, as f_{pm}. Both with units $\frac{1}{s}$. The measured values, ω_3 also has the units of $\frac{\theta}{s}$, which can be approximated to $\frac{1}{s}$ if we are treating mean ω_3 as the Rotational Frequency.

For f_{pt}, f_p will be calculated using the relation $f_3 = \frac{1}{T}$, where $T = 2T_h$, twice of a half a period measured. T_h has the units of seconds, s. The w_3 for f_{pm} will be measured directly from the experiment. All calculations through the help of the provided additional guide (Ziese)

4.3 Physical Exploration

4.3.1 Materials

- 3-Axis Gyroscope
- Counterweights for Stabilisation
- 2 Mass, each $0.049kg \pm 0.0005$
- Revolution Speed Meter (RSM), Class 2 Laser ± 0.01
- Timer ± 0.001
- Ruler ± 0.0005

4.3.2 Set Up

The experiment is set up by adjusting the counterweights of the Gyroscope onto a position of equilibrium along all axis, as to keep it stable for all rotations. For the initial system, only one mass is used and is positioned $z_z = 0.275m \pm 0.005$ away from the Centre of Mass. For the second system, 2 of the masses are used position at the same distance of z_z away from the Centre of Mass.

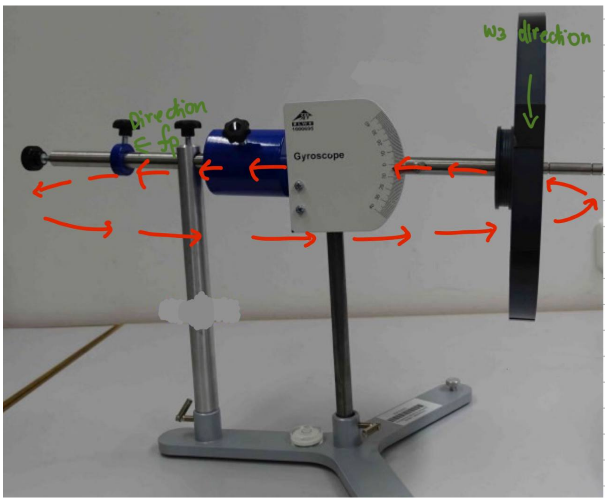


Image 2 Set up for Section 3, direction of f_p and w_3 as indicated.

4.3.3 Methodology

For each mass system, the Gyroscope is let to rotate while holding the Gyro Axis Fixed. The initial Angular Velocity, w_{3i} is measured and the Gyroscope is let to run for a time of half a period, T_h. At the end of T_h, the Final Angular Velocity is measured, w_{3f}, and the experiment for the said mass system is repeated 10 time for initial rotational frequencies between 200 to 600 rotations per minute (rpm).

The average of the initial and final w_3 is used as the Angular Velocity w_3, and will be used to solve equation 13 for f_{pm}. While T_h will be utilised to calculated f_{pt}. In the calculation for f_{pm}, the measured I_3 from Section 2 will be used.

4.4 Results

4.4.1 Data and Analysis

From the retrieved data, the following data presents $f_{p(m,t)}(\frac{1}{s})$ against the Average Rotation Frequency ($w_3(\frac{1}{s})$):

```
data_task_3_a = r"task_3_a_data.csv"
data_task_3_b = r"task_3_b_data.csv"
data_task_3_a_evaluated = pd.read_csv(data_task_3_a)
w_r_i_task_3_a = data_task_3_a_evaluated.iloc[:, 0]
w_r_f_task_3_a = data_task_3_a_evaluated.iloc[:, 1]
hal_period_3_a = data_task_3_a_evaluated.iloc[:, 2]
data_task_3_b_evaluated = pd.read_csv(data_task_3_b)
w_r_i_task_3_b = data_task_3_b_evaluated.iloc[:, 0]
w_r_f_task_3_b = data_task_3_b_evaluated.iloc[:, 1]
hal_period_3_b = data_task_3_b_evaluated.iloc[:, 2]
calc_moment_inertia = 0.0115315638164
w_r_task_3_a_mean = np.array([])
w_r_task_3_b_mean = np.array([])
period_3_a = []
for i in hal_period_3_a:
    period_3_a.append(i+i)
w_r_task_3_a_theo = (1)/(np.array(period_3_a))
period_3_b = []
for i in hal_period_3_b:
    period_3_b.append(i+i)
w_r_task_3_b_theo = (1)/(np.array(period_3_b))
for i in range(len(w_r_i_task_3_a)):
     w_r_task_3_a_mean = np.append(w_r_task_3_a_mean,
     ↪   (((w_r_i_task_3_a[i])/(8*60)) + ((w_r_f_task_3_a[i])/(8*60)))/2)
for i in range(len(w_r_i_task_3_b)):
     w_r_task_3_b_mean = np.append(w_r_task_3_b_mean,
     ↪   (((w_r_i_task_3_b[i])/(8*60)) + ((w_r_f_task_3_b[i])/(8*60)))/2)
w_r_task_3_b_mean_evaluated = pd.Series(w_r_task_3_b_mean)
w_r_task_3_b_mean_evaluated = pd.Series(w_r_task_3_b_mean)
def precesion_frequency(w_r_task_3_mean):
    return (m_z*g*r_z)/(2*np.pi*calc_moment_inertia*w_r_task_3_mean)
def precesion_frequency_b(w_r_task_3_b_mean):
    return (2*m_z*g*r_z)/(2*np.pi*calc_moment_inertia*w_r_task_3_b_mean)
```

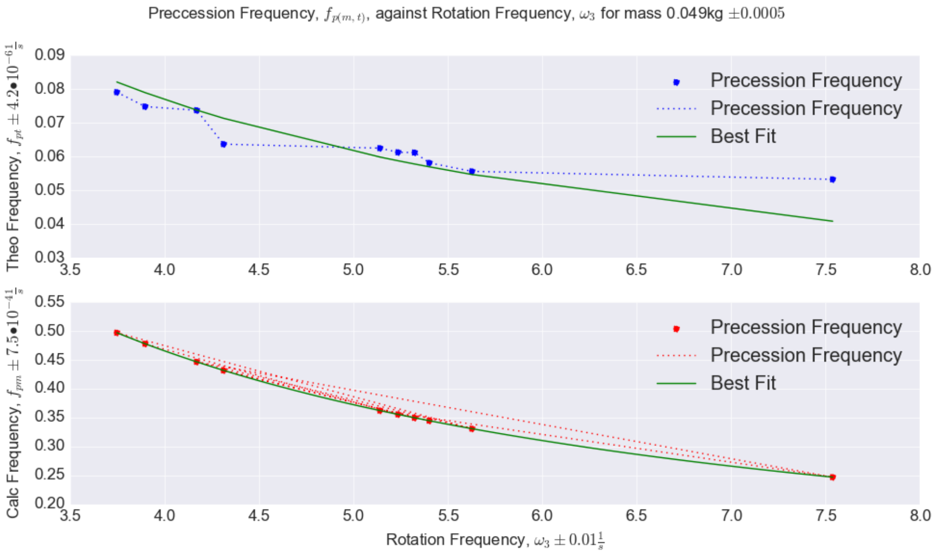


Figure 2: Precession Frequency, $f_{p(m,t)}(\frac{1}{s})$, against Rotation Frequency, $\omega_3(\frac{1}{s})$ for mass 0.049 kg

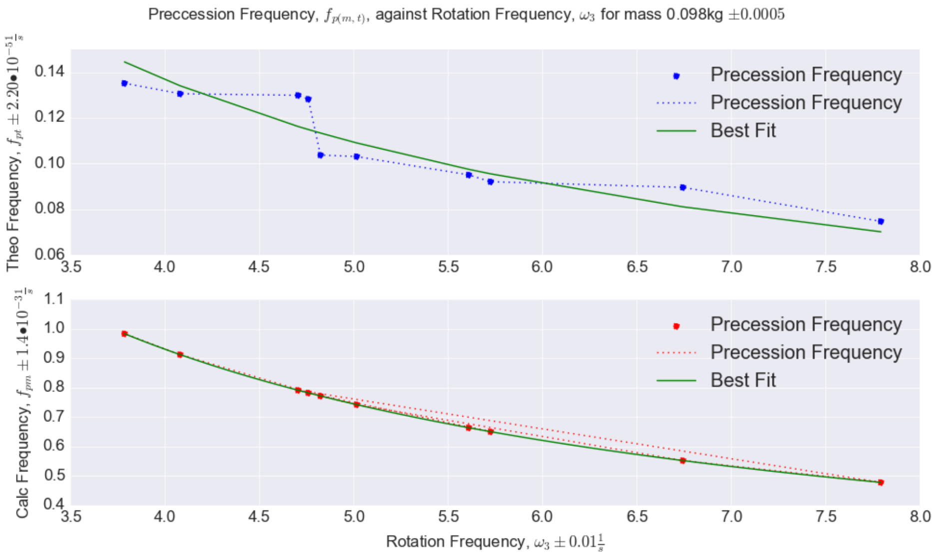


Figure 3: Precession Frequency, $f_{p(m,t)}(\frac{1}{s})$, against Rotation Frequency, $\omega_3(\frac{1}{s})$ for mass 0.098 kg

Note: As there are 8 stripes on the Gyroscope, the RSM measured the w_3 value with an increased factor of x8. As such, the calculations for the mean divided everything out by 8 and also with an additional 60 for conversion of the rpm to rps (rotations per second).

4.4.2 Analysis

4.4.2.1 Analysis of Figure 2

Index	Error Margin
1	336%
2	477%
3	374%
4	458%
5	665%
6	711%
7	441%
8	668%
9	344%
10	580%

Table 4: Error Margin for each respective measurement (i.e, for the same trial, the error margin for $f_p(m,t)$). Rounded to 3 s.f

Figure 2 presents $f_{p(m,t)}$ against w_3 for the first mass system of only one mass block (of 0.049kg). As seem from the Figure, the overall trend for how the $f_{p(m,t)}$ is related to w_3 is consistent with both the measured and theoretical components. With an increasing w_3, there is a decreasing f_p, with the overall curve following a trend of $f_p \propto \frac{1}{w_3}$, which is consistent with the equation implemented in Section 4.2. This however deviates from the expected negative linear relationship predicted in the Hypothesis, yet the overall notion of an increasing w_3 leading to a decrease in f_p is consistent as predicted.

However, the magnitude difference between f_pm and f_pt has an error margin of, on average, **505%**.With an absolute Max Difference of $\mathbf{0.9327227246290515 \approx 0.933}$ and a Mean Absolute Error of $\mathbf{|\text{-}0.70432496040200059|} \approx = \mathbf{0.704}$ the graph of the f_{pm} is overall off by a factor of 6.1 from f_{pt} (f_{pm} grows 6.1 times faster then f_{pt}). The difference in the factor is calculated by comparing the multiplicative difference between the slopes of the lines of best fit. The large difference of the two data sets (relative to the values itself) can be mainly attributed to how our measurement of w_3 was not consistent i.e the distance between the the disk and the RSM were not within the recommended range, leading to a Measurement Data Set off by a large factor. Which leads our Measured Data Set **Imprecise** but **Accurate** (i.e, deviated from Theoretical Data sets but the overall trend of the data set is as expected). More details in Discussion. The Error values on the Margin are the average Propagation values, more details in Error Analysis.

4.4.2.2 Analysis of Figure 3

Index	Error Margin
1	391%
2	398%
3	502%
4	517%
5	361%
6	436%
7	681%
8	738%
9	918%
10	1220%

Table 5: Error Margin for each respective measurement (i.e, for the same trial, the error margin for $f_p(m,t)$). Rounded to 3 s.f

Figure 3 presents $f_{p(m,t)}$ against w_3 for the second mass system of only two mass blocks (of 0.097kg). As seem from the Figure, the overall trend is consistent with what was explained in for Mass System 1. Which further supports the prediction from the Section 4.1 Hypothesis. As such, similar to Mass System 1, the overall trend is consistent with equation 13.

However, similar to Section 4.4.2, the magnitude difference between f_pm and f_pt has an error margin of, on average, **615%** . With an absolute Max Difference of **0.9088647916698667** $\approx$ **0.910** and a Mean Absolute Error of **|-0.6245968043496056|** $\approx$ **0.625**,and the graph of the f_{pm} is overall off by a factor of 6.8 from f_{pt} (f_{pm} grows 6.8 times faster then f_{pt}). The large difference of the two data sets (relative to the values itself) can be mainly attributed to how our measurement of w_3 was not consistent, i.e similar to what was said in Mass System 1. Making our Measured Data Set Imprecise but Accurate (deviated from Theoretical Data sets but the overall trend of the data set is as expected).More details in Discussion. The Error values on the Margin are the average Propagation values, more details in Error Analysis.

4.4.2.3 In comparison between the Two Mass Systems

Further the two Mass Systems is seen to be different from each other by some factor, but the exact factor cannot be directly compared from the data points. As the average w_3 used for both Mass Systems vary, a one-to-one comparison fro each w_3 cannot be made. A comparison using the Best Fit lines will be conducted in the discussions. More details in the Discussion.

4.4.3 Discussion

Overall, for all Data Sets, the relation between the Precession and Rotational Frequency being related as $f_p \propto \frac{1}{w_3}$ is consistent with equation 13. Further, as predicted, an increase in the Mass (from 0.049 Kg to 0.098 Kg) leads to an increase in the f_p.

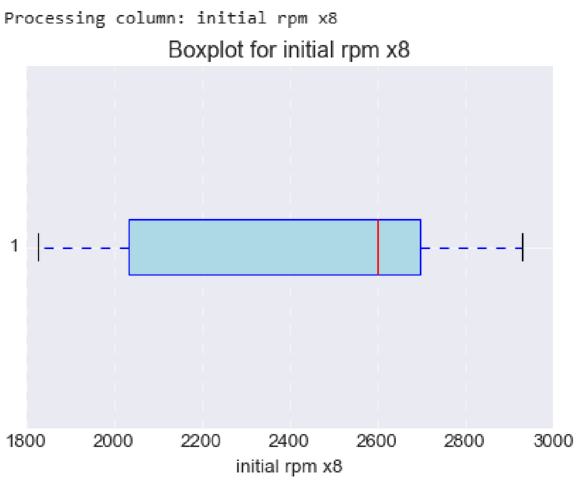
Mass System	Theo Frequency	Calc Frequency
1	0.308	1.86
2	0.547	3.72

Table 6: Slopes, a, i.e the Best Fit Line, for each Data Set. (3 s.f). Overall equation; $f_{p(m,t)} = \frac{a}{\omega_3}$

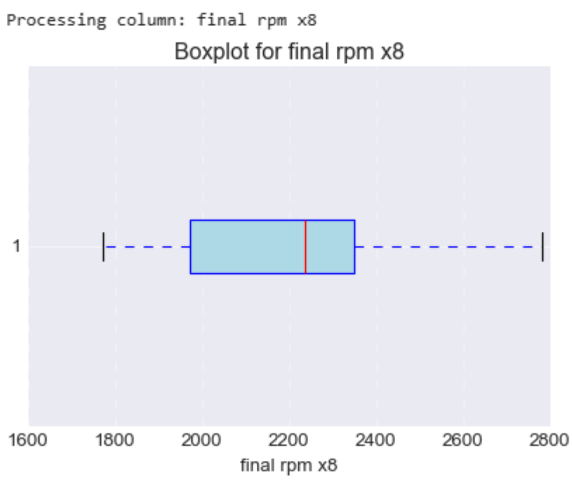
As there is no same average w_3 for each trial for both mass systems, it will be insufficient to compare Max Absolute Difference between the two sets. To understand how an increase in mass changed the resultant f_{pm} and f_{pt}, the comparison of the Best Fit Lines will have to be utilised. Looking at the theoretical data set, the Best Fit Approximation are different by a factor of **1.78** between the two masses, while the f_{pm} is different by a factor of exactly **2**. The exact value of 2 is expected as the equation utilised (equation 13) changes only by a factor of 2 between the two mass systems for f_{pm}. This leads to an approximation that an increase of the mass by some factor, a, with the distance z_z being kept constant, increases the resultant f_p pattern by the same $\approx a$ amount. As a change of 1.7 can be considered nearly of a change by a factor of 2. Which stays consistent with equation 13. However, important to emphasis that this is a limited conclusion as it is derived from just two data difference data sets with only one change in mass.

Deviation between the Theoretical and Calculated Data Sets:

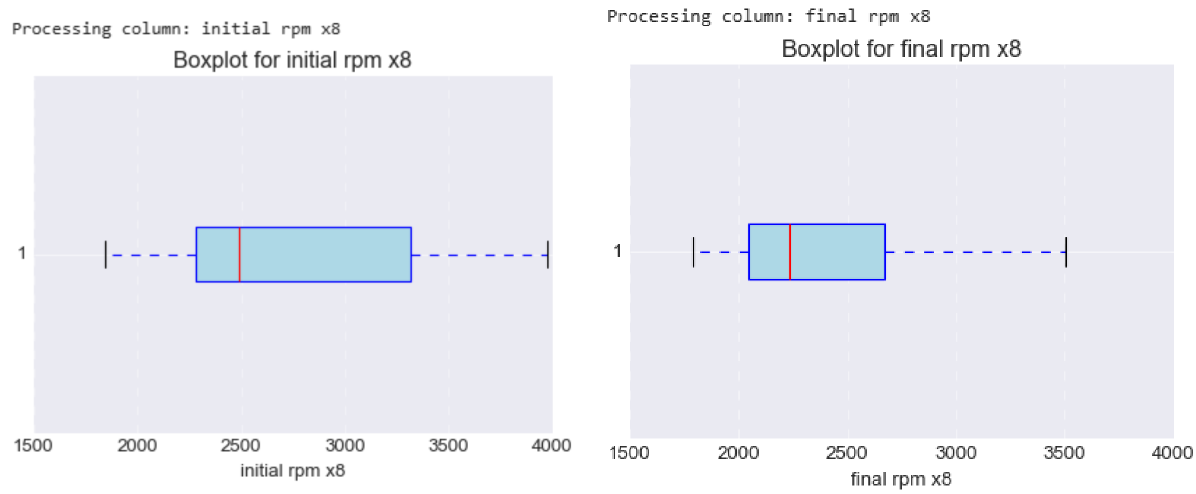
For Both Mass Systems, the deviation of magnitude from the two data sets is largely due to an incorrect measurement of w_3, as recommend by the guide provided for the Experiment. A distance of 5 cm between the disk and the RSM was not kept consistent for all measurements, and furthermore the actual distance that was kept varied from trial to trial. The RMS, as stated in the guide, is the most accurate when it is within the range recommended. Not keeping within that boundary made the actual measurements be off by a large factor, as such creating a large error margin deviation. The effects of friction are negligible as the measurements for a half period is small enough to where the decrease in the w_3 value would not have created an average w_3 with error margin as large as seen here. The original data set was further ran through a **Box Plot**, to see if the data set is consistent with each other and no outliers emerged. Refer to Figure **4** below.



(a) Initial ω_3 for Mass System 1



(b) Final ω_3 for Mass System 1

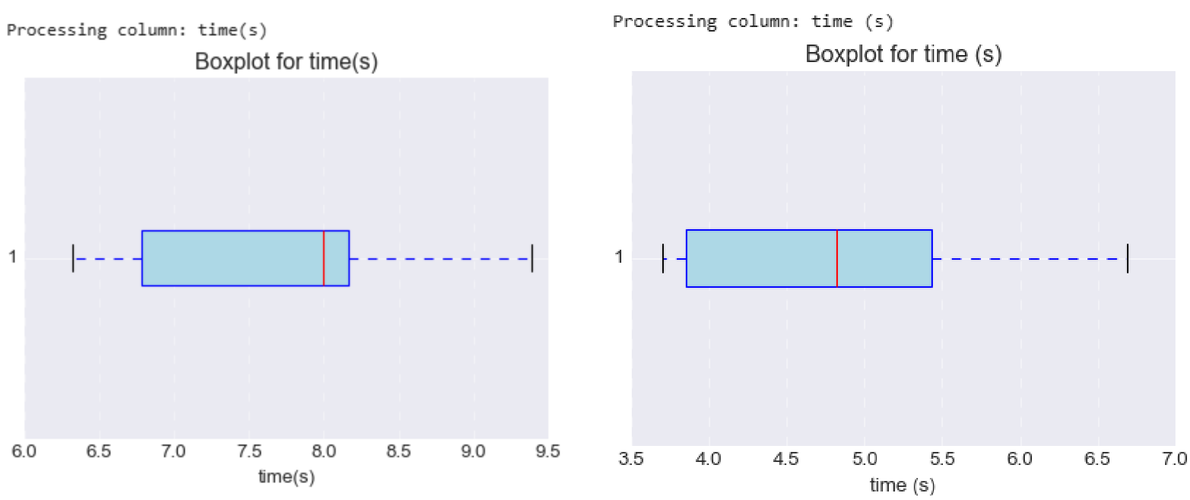


(c) Initial ω_3 for Mass System 2

(d) Final ω_3 for Mass System 2

Figure 4: Box Plot for the original Data Set

The inconsistency of the Theoretical Precession Frequency, as seen in Figure 2 or 3, is due to insufficient amount of data points. In theory, the Time Period (not taking into account the effects of friction in slowing down the disk as it precesses, as amount it decreases is negligible for one period length) would yield a more consistent and *theoretical* data set. As the measurement of a period is straightforward, has smaller uncertainty and would be easier to track. However, this inconsistent measurements can be attributed to the absence of enough data points. As $f_p << \omega_3$, we are measuring for a large range of w_3 with only 10 data points. This creates sudden changes between data points on graphing the set, and further creates a seemingly inconsistent plot. In the same line, the data point *4* for Figure 2 and points *3*and *4* for Figure 3 seem to be outliers that are creating sudden **jumps** that cause inconsistencies (refer to Figure 6). However no outliers were found when the original data set for T_h for both Mass Systems were ran through a **Box Plot** (As seen below in Figure 5).



(a) T_h for Mass System 1

(b) T_h for Mass System 2

Figure 5: Box Plot for the original Data Set of T_h

Note: Only the Box Plot for the original data set were taken, as any further data set is just a derivative of the original, and will show similar Box Plots. Code for all the Box Plots can be found in the Appendix. Omitted here to avoid clutter.

Overall, the inconsistent graphing and large deviation in magnitude is due to wrong set up of the experiment and absence of enough data points. Future repetition of the experiment will be required if a more accurate data sample is needed.

Additionally, the task requires to find I_3 from the precession, but we do not see how. The relation that $L = I \cdot \omega$ applies to relative L and ω. As in, for I_3, it would be $I_3 = \frac{L_3}{w_3}$, and there is no clear way to find L_3 except knowing that it is constant. We tried to use initial conditions to find w_0, and possibly find the resultant $\vec{L_p}$, yet calculations are stuck in the manner that there is always one parameter that we cannot find from our available data.

4.4.4 Error Analysis

The Propagation of error for the each data set was done utilising the standard relations between paraments of some uncertificated. Following the equations of John R. Taylor, from An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements (Taylor),

$$\partial f_{pm,1} = f_{pm_1} \cdot \frac{1}{2\pi}\left(\frac{0.0005}{0.049} + \frac{0.0005}{0.275} + \frac{0.001}{\omega_{3_1}} + \frac{6.9 \times 10^{-7}}{I_3}\right) \tag{14}$$

$$\partial f_{pm,2} = f_{pm_2} \cdot \frac{1}{2\pi}\left(2\frac{0.0005}{0.049} + \frac{0.0005}{0.275} + \frac{0.001}{\omega_{3_2}} + \frac{6.9 \times 10^{-7}}{I_3}\right) \tag{15}$$

$$\partial f_{pt,1} = 2 f_{pt_1} \frac{0.001}{T_{h1}} \tag{16}$$

$$\partial f_{pt,2} = 2 f_{pt_2} \frac{0.001}{T_{h2}} \tag{17}$$

,we get the following uncertainties:

Data Set	Mean Error Propagation	Largest Error (Max)
MS1: Theo	± 0.0000042	± 0.0000063
MS1: Calc	± 0.00075	± 0.00098
MS2: Theo	± 0.000022	± 0.0000046
MS2: Calc	± 0.0014	± 0.0019

Table 7: Error Propagation; Mean and Maximum Values

5 Nutation Frequency

5.1 Hypothesis

As the direction of Nutation and Rotational Speed of the Disk would point in the same direction (refer to the below Image **3**), an increasing Rotational Speed would be proportional to an increasing Nutational Frequency. The correlation between the two sets would have to be linear. As nutation, by definition, would be the resultant $\vec{\omega}$ in the direction of the the Angular Momentum, the Rotational Speed would be related through some factor relative to the moment of inertias of the system (As $L \propto \vec{\omega} \propto \omega_3$, and as $f_N \propto L, f_N \propto \omega_3$). The factor by which they will be related will depend on the system. All calculations through the help of the provided additional guide (Ziese)

5.2 Theoretical Exploration

Considering a symmetric Gyroscope with no external torque, τ, and as creating a Gyro Disk that does not precess around some central axis. Leading to the conservation of the Angular Momentum, L, it would point along a fixed direction. If the Angular Velocity, w, is parallel to the Figure Axis, both w and L would be parallel to the Figure Axis and rotate around a fixed L axis. In the case where the figure axis was deflected by some external sudden impulse, w and L deem not to be parallel any more. In this special case perturbation of the Gyro Disk is called the Nutation, a periodic variation of the Gyro Disk off some rotation axis consistent with the Figure Axis. Refer to the diagram below. The Nutational Frequency, f_N, would be defined as the frequency due to an angular velocity, ω_N, in the direction of the L.

This case leads the Euler Equations as:

$$I_1 \frac{d\omega_1}{dt} + (I_3 - I_1)\omega_3\omega_2 = 0 \tag{18}$$

$$I_1 \frac{d\omega_2}{dt} + (I_3 - I_1)\omega_3\omega_1 = 0 \tag{19}$$

$$I_3 \frac{d\omega_3}{dt} = 0 \tag{20}$$

,

with defining:

$$\omega_p = \frac{I_3 - I_1}{I_1}\omega_3 \tag{21}$$

,

the resultant $\vec{\omega}$ and $\vec{L_p}$ leads to:

$$\vec{\omega} = \begin{pmatrix} \omega_0 \cos(\omega_p t) \\ \omega_0 \sin(\omega_p t) \\ \omega_3 \end{pmatrix} \tag{22}$$

$$\vec{L_p} = \begin{pmatrix} I_1\omega_0 \cos(\omega_p t) \\ I_1\omega_0 \sin(\omega_p t) \\ I_3\omega_3 \end{pmatrix} \tag{23}$$

.

And as such, if the resultant $\vec{\omega}$ is taken in the direction of $\vec{L}$, we get:

$$\frac{\vec{\omega}\vec{L_p}}{\vec{L_p}} = \frac{\omega_0^2 I_1 + \omega_3^2 I_3}{\sqrt{I_1^2\omega_0^2 + I_3^2\omega_3^2}} = \vec{\omega_N} \tag{24}$$

.

where our final f_N would be,

$$f_N = \frac{\omega_N}{2\pi} \tag{25}$$

Where now the final equation 25 will be the final equation used to calculate the measured f_{Nm}. The units for f_{Nm} and w_3, similar to other frequencies, will be $\frac{1}{s}$. The calculations for the theoretical Nutation Frequency, f_{Nt} will be done through the relation that for a three periods,T_3, of a nutation, $f_{Nt} = \frac{3}{T_3}$. The value for I_1 is calculated purely theoretically as $I_1 = \frac{1}{2} \cdot 1.5 \cdot 0.125^2 = 0.01171875 \approx 0.012(kg \cdot m^2)$. All calculations through the help of the provided additional guide (Ziese)

5.3 Physical Exploration

5.3.1 Materials

- 3-Axis Gyroscope
- Counterweights for Stabilisation
- Revolution Speed Meter (RSM), Class 2 Laser
- Timer ± 0.001

5.3.2 Set Up

The Experiment is set up by stabilising the Gyroscope in place, as such it does not sway precess around a central axis. Refer to the image below.

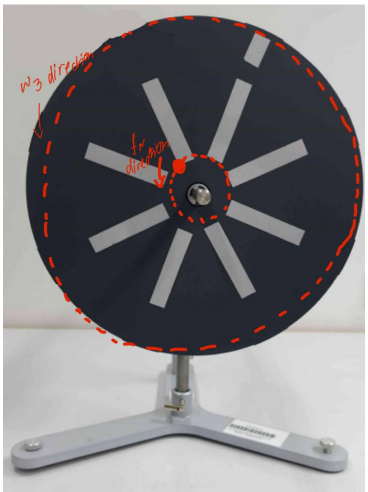


Image 3 Set up for Section 5, direction of f_N and w_3 as indicated.

5.3.3 Methodology

The Gyroscope is let to rotate while holding the Gyro Axis Fixed, and then a brief vertical impulse is imparted onto the Gyro Disk, and let to run for a duration of 3 periods. Measure the time for the 3 periods, T_3, and take down the rotational speed, w_3, for before and after the T_3 elapses. The Average of the initial and final $w_{3(i,f)}$ is used as the Angular Velocity w_3 and the T_3 is divided by 3 to find the period, T, of one nutation. Repeat for 10 trials, with rotation frequencies in the range of 150 to 500 rpm.

5.3.4 Hypothesis

5.4 Results

5.4.1 Data and Analysis

From the retried data, the following data presents the $f_{N(m,t}(\frac{1}{s})$ against the average Rotation Frequency $(\omega_3(\frac{1}{s})$

```python
data_task_4 = r"task_4_data.csv"
data_task_4_evaluated = pd.read_csv(data_task_4)
w_r_i_task_4 = data_task_4_evaluated.iloc[:, 0]
w_r_f_task_4 = data_task_4_evaluated.iloc[:, 1]
third_period_4 = data_task_4_evaluated.iloc[:, 2]
calc_moment_inertia = 0.0115315638164
w_r_task_4_mean = []
period_task_4 = third_period_4 /3
I_1 = (1/4)*(1.5)*(0.125)**2
w_r_task4_theo = 1/period_task_4
for i in range(len(w_r_i_task_4)):
     w_r_task_4_mean = np.append(w_r_task_4_mean, ((((w_r_i_task_4[i])/(8*60*2*np.pi)) + ((w_r_f_task_4[i])/(8*60*2*np.pi)))/2))
w_r_task_4_mean_evaluated = pd.Series(w_r_task_4_mean)
def nutation_frequency(w_r_task_4_mean, w_r_i_task_4):
    return (I_1*(w_r_i_task_4/(8*60))**2 + calc_moment_inertia*w_r_task_4_mean**2)/(np.sqrt(I_1*
    (w_r_i_task_4/(8*60))**2 + calc_moment_inertia*w_r_task_4_mean**2))
```

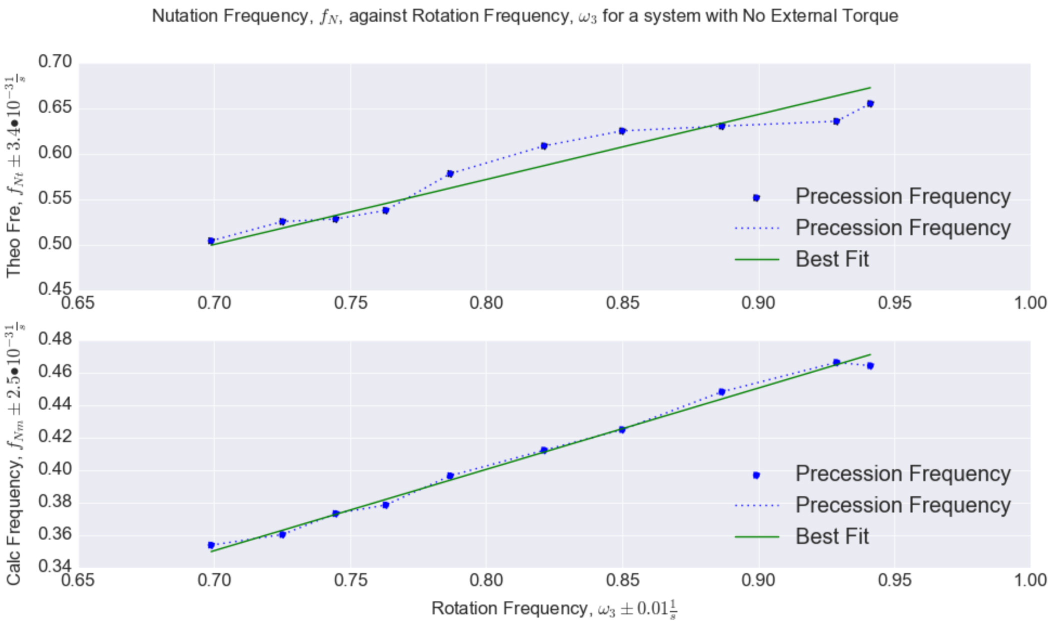


Figure 6: Nutation Frequency, $f_{N(m,t)}(\frac{1}{s})$, against Rotation Frequency, $\omega_3(\frac{1}{s})$ for a system with no external τ

Index	Error Margin
1	7.93%
2	11.3%
3	15.1%
4	21.0%
5	28.7%
6	34.8%
7	39.4%
8	40.8%
9	43.3%
10	46.0%

Table 8: Error Margin for between f_{Nt} and f_{Nm}. Rounded to 3 s.f

Figure 6 presents $f_{N(t,m)}$ against its respective w_3. As seen from the figure, the overall trend of the f_N increasing with an increasing w_3 is consistent with both the theoretical and measured data set. Which satisfies equation 25. Both data sets exhibit a nearly linear relationship, with the graph of the f_{Nm} overall off by a factor of 1.4 from f_{Nt}, i.e f_{Nm} grows 1.4 times faster then f_{Nt}. Moreover, compared to the data set from Section 4, the values for Measured Nutational Frequency is aligned more with the Theoretical Frequencies. With an average Error Margin of **28.837999031701077 $\approx$ 28.8%**, the magnitude of both data sets are more consistent with each other. Such a consistency can be attributed for how for the Nutation we had kept our RSM within the recommended range of 5 cm. More details in the Discussion. The Error values on the Margin are the average Propagation values, more details in Error Analysis.

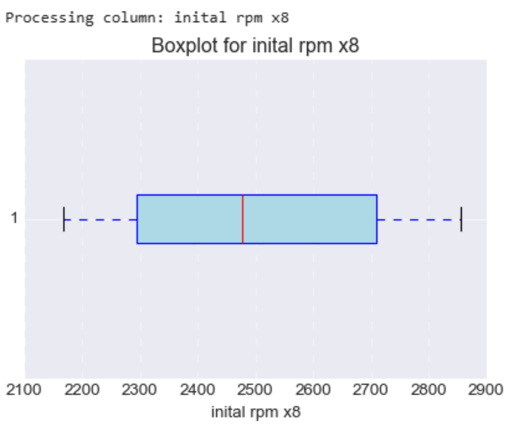
5.4.2 Discussion

Note: The code for the maximum difference and the Mean Absolute Error is in the Appendix. Omitted from here to avoid clutter

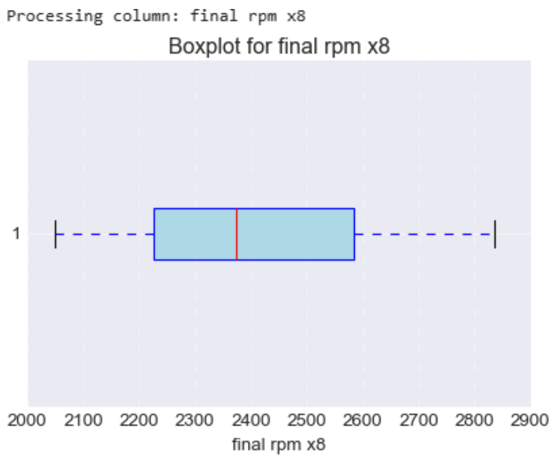
Overall, the results showcase a pattern which is expected from the derived equations and concepts explored in Section 5.3.4. Both data sets, $f_{N(m,t)}$, if looked at figure 6, have a steady slope, with the change in w_3 relative to f_N for both data sets being consistent with each other. For f_{Nm}, for a change of 0.15 $\frac{1}{s}$, w_3 changes by 0.23 $\frac{1}{s}$. While for f_{Nt}, for a change of 0.12 $\frac{1}{s}$, w_3 changes by 0.24 $\frac{1}{s}$. Which would be considered a, up to an approximation, a steady change. Coinciding with the Hypothesis provided, there is a linear relationship between $f_{N(m,t)}$ and some measured w_3.

The existing deviation, with the error margin, are mainly due to possible unsteady Nutation of the Gyroscope. As even for a steady Gyroscope, the exhibited nutation causes some precession. Additionally, the nutation over a time length of 3 periods is prone to decay due to existing friction from the Gyroscope's components interacting with each other. And, to a lesser extent, the friction due to the fluid (air) around the Gyroscope.

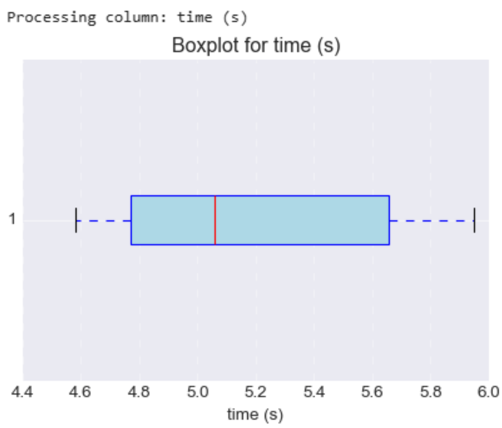
To keep in consistency with Section 4, the original data was run through a Box Plot but, similar to the previous Section, no outliers were found:



(a) w_{3i} for Nutation



(c) w_{3f} for Nutation



(b) T_3 for Nutation

Figure 7: Box Plot for the original Data Sets of Nutation Measurements

The Maximum Difference between f_{Nt} and f_{Nm} being $0.30123306490821533 \approx 0.30$, with the Mean Absolute Error being $0.1749900004639497 \approx 0.18$. Which is a small difference, on average, between the two sets relative to the values - confirming that to an extent the Disparity in Section 4 is due to the positioning of our RSM in measuring ω_3.

5.4.3 Error Analysis

The Propagation of error for the each data set was done utilising the standard relations between paraments of some uncertificated. Following the equations of John R. Taylor, from An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements (Taylor),

$$\partial f_{Nt} = 3 f_N \frac{0.001}{T_h} \tag{26}$$

$$\begin{aligned}\partial f_{Nt} = f_{Nm} \cdot \Bigg(I_1 \cdot \frac{0.001}{w_3} + \frac{1}{2} \cdot \Bigg(\Bigg(I_1 \cdot \frac{0.001}{w_{3i}} \cdot \\ 2 \cdot w_{3i}^2 + I_3^2 \cdot \left(2 \cdot \frac{0.001}{w_3} \cdot w_3^2 \right) \cdot \sqrt{I_1 \cdot \left(\frac{w_{3i}}{8 \cdot 60} \right)^2 + I_3 \cdot w_3^2}\end{aligned} \tag{27}$$

,we get the following uncertainties:

Data Set	Mean Error Propagation	Largest Error (Max)
Theo	$\pm$ 0.0034	$\pm$ 0.0043
Calc	$\pm$ 0.0025	$\pm$ 0.0036

Table 9: Error Propagation; Mean and Maximum Values

6 Conclusion

In conclusion, all the experiments provided, either directly or indirectly, a satisfactory results. With Section 2 providing a nearly accurate value for the Moment of Inertia I_3, by using the method of Applied Forces, at an absolute error difference of $|0.0001|$. While the wrong measurements of w_3 were acknowledged and made notice of the wrong data collection method we utilised.

For Section 3, we were able to successfully find the damping constant, and find that the relationship between w_3 and time in a damped state is linear. With a $\beta = -0.364$, there is a clear negative linear relationship.

For Section 4, similar to Section 2, the wrong measurements of w_3 lead to an average error margin of 505% and 615% for respective Mass Systems, yet the data provided an expected relationship between the Precession and Rotational Frequency ($f_p \propto \frac{1}{w_3}$), resulting in a satisfactory discussion of possible errors of the results.

For Section 5, as compared to Section 4 and 2, provided a much clearer error margin of only 28.8%. Which allowed us to better compare the theoretical and measured data more directly with the actual values from the plots.

7 Appendix

```python
def largest_pairwise_difference(data_1, data_2):
    differences = [abs(a - b) for a, b in zip(data_1, data_2)]
    max_difference = max(differences)
    return max_difference
resultA2 = largest_pairwise_difference(data_1, data_2)
resultA1 = largest_pairwise_difference(data_1, data_2)
def quantify_data_similarity(data_1, data_2):
    arr1 = np.array(data_1)
    arr2 = np.array(data_2)
    mae = np.mean((arr2 - arr1))
    return mae
resultB2 = quantify_data_similarity(data_1, data_2)
resultB1 = quantify_data_similarity(data_1, data_2)


file_path = r'data_file.csv'
data = pd.read_csv(file_path)
def remove_outliers_and_plot(data, column):
    Q1 = data[column].quantile(0.25)
    Q3 = data[column].quantile(0.75)
    IQR = Q3 - Q1
    lower_bound = Q1 - 1.5 * IQR
    upper_bound = Q3 + 1.5 * IQR

    plt.figure(figsize=(6, 4))
    plt.boxplot(data[column], vert=False, patch_artist=True, boxprops=dict(facecolor='lightblue'))
    plt.title(f'Boxplot for {column}')
    plt.xlabel(column)
    plt.grid(axis='x', linestyle='--', alpha=0.7)
    plt.show()
    filtered_data = data[(data[column] >= lower_bound) & (data[column] <= upper_bound)][column]
    fig.savefig('box+plot_3_b.png')
    return filtered_data

columns_to_process = [col for col in data.columns if col != 'index']
cleaned_data = {}

for column in columns_to_process:
    print(f"Processing column: {column}")
    cleaned_data[column] = remove_outliers_and_plot(data, column)
```

References

Michael Ziese. *M6e Gyroscope with three Axes*, 2024 Taylor, John R. *An Introduction* *to Error Analysis: The Study of Uncertainties in Physical Measurements*. University Science Books, 2022.