O09e Fringes of Equal Thickness

Lab Group 11: Mirzokhid Ganiev (3763884), Calina Burciu (3770859)

1 Introduction

This experiment handles the usage of interference of light under between two transparent glass layers under different path lengths due to (i) the curvature of one of the layers, or (ii) an existing material between two flat surfaces. In the first part of the experiment, the theory of Newton Rings is utilised, (more on the theory in Section 2, Theoretical Exploration), to produce circular interference patterns due to the interaction of light between two surfaces, of which one is curved at some radius. The radius, R_1 of the said layer will be found utilising the change in radius of successive fringed circles. In the second part, split into measurement of two different materials (a metallic slab and a strand of hair), the interference due to the path difference from an existing material wedged between the layers is used to find the thickness of the material. The thickness between each successive vertical fringes will be measured. As such, from the two materials, metallic slab and a strand of hair, two different thicknesses will be found, D_2 and D_3 respectively.

2 Theoretical Exploration

All derivation through the provided Lab Guide (*O09e Fringes of Equal Thickness*).

2.1 Interference under Convex Lense; Newton Rings

For the interface between two surfaces, from which one is curved, the reflected light from the curved surface and the light which has first refracted then reflected back, experience a path difference.

The optical path difference between the two rays is given by:

$$\Delta x = 2(d + d_0) + \frac{\lambda_0}{2}, \tag{1}$$

where Δx is the total optical path difference, d is the vertical distance from the lens to the observation plane, d_0 is the contact distance, and λ_0 is the wavelength of the incident light.

The corresponding phase shift δ between the two rays is:

$$\delta = \frac{2\pi}{\lambda_0}\Delta x = \frac{4\pi}{\lambda_0}(d + d_0) + \pi. \tag{2}$$

The relationship between the radius of the k-th interference ring, r_k, and the radius R of the lens is given by the geometric relation:

$$r_k^2 = d(2R - d), \tag{3}$$

where d is the vertical distance from the lens to the observation plane.

Using Eqs. (1) and (2), which describe the optical path difference and phase shift respectively, and applying the condition for constructive interference (i.e., bright rings), one can derive, under the assumption that $d \ll R$, the following expression for the phase shift:

$$\delta = 2\pi k = \frac{4\pi}{\lambda_0}(d + d_0) + \pi, \tag{4}$$

where k is an integer representing the interference order.

Furthermore, under the approximation $d \ll R$, Eq. (2) simplifies to:

$$r_k^2 \approx 2dR. \tag{5}$$

Combining Eqs. (4) and (5), we can solve for the distance d in terms of k:

$$\frac{2\pi \Delta x}{\lambda_0} = 2\pi k \quad \Rightarrow \quad k = \frac{\Delta x}{\lambda_0}, \tag{6}$$

leading to

$$2d = \lambda_0 \left(k - \frac{1}{2} \right) - 2d_0. \tag{7}$$

Substituting this back into Eq. (5), the radius of the bright (constructive) interference rings is:

$$r_k^2 = \lambda_0 R \left(k - \frac{1}{2} \right) - 2d_0 R. \tag{8}$$

Similarly, for destructive interference (corresponding to dark rings), the condition becomes:

$$\delta = (2k - 1)\pi, \tag{9}$$

which leads to:

$$\frac{4\pi}{\lambda_0}(d + d_0) + \pi = (2k - 1)\pi, \tag{10}$$

and thus

$$2d = \lambda_0 \left(k - \frac{1}{4} \right) - 2d_0. \tag{11}$$

Finally, the corresponding radius for the dark (destructive) interference rings is:

$$r_k^2 = \lambda_0 R \left(k - \frac{1}{4} \right) - 2d_0 R. \tag{12}$$

2.2 Interference under a Wedge-Shaped Layers

The interference from the two rays, from which the path difference arises from how a wedged material causes the light rays to come in at an angle. Which causes each light ray to travel different path lengths before meeting up after reflection from the bottom surface,

Let x_k be the horizontal position of the k-th bright fringe, corresponding to a plate separation d_k. Then, the phase shift is given by

$$\delta_k = \frac{2\pi}{\lambda_0}\Delta x_k = \frac{4\pi}{\lambda_0} d_k + \pi, \tag{13}$$

where Δx_k is the optical path difference for the k-th fringe.

Using the condition for constructive interference and the relation $\tan\alpha = \frac{d_k}{x_k} = \frac{D}{I}$, one obtains

$$d_k = \frac{D}{I}x_k \implies x_k = \frac{\lambda I}{2D}(k - \frac{1}{2}) \tag{14}$$

The condition for destructive (dark) interference is

$$\delta_k = (2k-1)\pi \implies (2k-1)\pi = \frac{4\pi}{\lambda_0}d_k + \pi \tag{15}$$

leading to

$$x_k = \frac{\lambda I}{2D}(k-1) \tag{16}$$

For adjacent equidistant dark or bright fringes separated by a distance Δ, one has

$$\Delta = \frac{I\lambda}{2D}. \tag{17}$$

3 Radius of a Convex Lens

3.1 Experimental Exploration

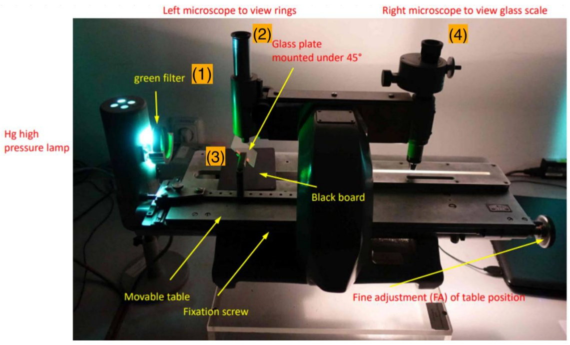
3.1.1 Materials

- Abbe Comparator (See below for more)
- Light Filter; Namely Red ($\lambda_0 = 589 \cdot 10^{-6} mm$)
- A lens of 35 mm Radius
- A plane glass plate
- Residual items: Ruler, Laptop, etc...

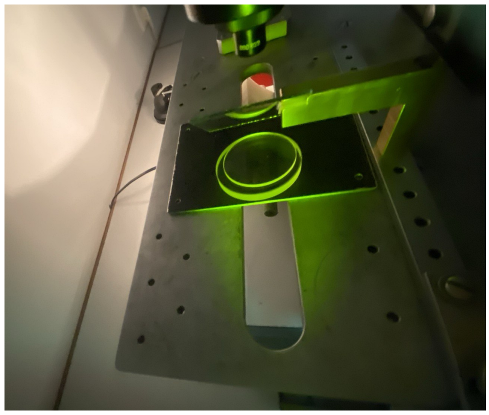
3.1.2 Set up

The setup involves monochromatic light entering horizontally into the Abbe comparator. The light strikes a 45° glass plate and is reflected downwards toward a lens and a horizontal plane glass plate. At each interface, part of the light is reflected. Of particular interest are the reflections from the convex side of the lens and the upper surface of the plane glass plate, as these reflected waves interfere to form Newton's rings.

As further seen in the below image, the Abbe Comparator is set up with a Red Light Filter placed under high mercury (Hg) pressure lamp (1), two glass layers, one flat and another curved as some radius r_1 (2), Position Set from the glass plate (3) and Position Measured from the glass scale (4).



(a) The Overall Used Apparatus (*Guide to Experiment O9e: Fringes of Equal Thickness*)



(b) Two Slabs of Glass Layer, one flat (at the bottom) and another curved with some radius

3.1.3 Methodology

For Task I, the experiment consists of measuring 20 values of the radius corresponding to 10 values of k, which are associated with the dark fringes resulting from destructive interference of two rays with a path length difference and opposite phases.

To obtain a precise value for r_k, half of the absolute value of the difference between the two values obtained for the same magnitude of k (i.e., $k = -5$ and $k = 5$) is used.

3.2 Results

3.2.1 Data and Analysis

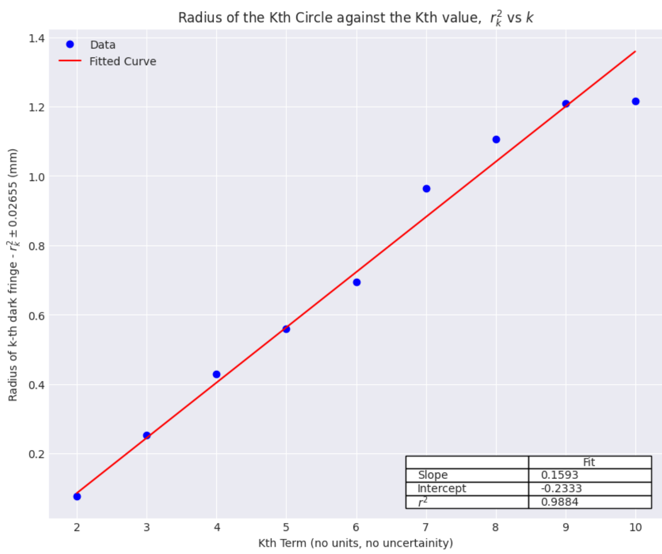


Figure 1: Plot of r_k^2 vs k with a linear fitting

The linear trend observed in Fig. 1 (see below) shows the dependence of the squared radius of the k-th dark fringe on the fringe order k. According to Eq. 12, the slope obtained from the linear fit corresponds to $r_1\lambda_0$, where r_1 is the radius of curvature of the lens and λ_0 is the wavelength of the light used. Thus, the radius of curvature is determined by dividing the slope by the wavelength, yielding $r_1 = 270.4mm$. Additionally, the intercept of the fit, which corresponds to $r_1\left(\frac{\lambda_0}{4}+2d_0\right)$, allows the calculation of the contact distance d_0. Using the intercept value, the contact distance is found to be $d_0 = 3.5\cdot10^{-4}mm$. The R^2 indicates a strong correlation between the measured data and the values predicted by the linear fit, with a value of $R^2 = 0.9884$.

3.2.2 Discussion

Considering the values obtained above, the linear fit appears successful, and the approach is validated. The value of $r = 270.4mm$ is consistent with the visual understanding of how curved the surface is. As the glass lens was seen to be approximately flat on the surface, this signifies the radius of the curvature is much larger (as larger r_k would lead to a more flat surfaces on smaller reference scales). Furthermore, the value of d_0 aligns with theoretical expectations, confirming that the contact distance is not ideal, but still exceedingly small. The R^2 value, which measures the correlation between how close two data sets are (with 1 being an absolute match), indicates a strong fit to the theoretical data, further supporting the validity of the method.

Furthermore, as measured values were around the same expected range and were consistent with each other, it can be said that the data is ***accurate and precise***.

3.2.3 Error Analysis

The uncertainty seen on Figure 1, on the y-axis for the radius of the kth value, was derived through the smallest measurable values from the secondary microscope. The half of the smallest value, for each sub-measurement seen, had been summed. While the effects of uncertainty from subtracting away the -kth and kth value cancelled out with the general dividing the final value by 2 to obtain the radius. The accuracy of the found slope, and as such the found Radius of Curvature, can be further understood through Standard Error (analogous to standard deviation), and the confidence interval (the range of other possible values). As such, those yieled as;

	Found Value	Standard Error	Confidence Interval
Slope, $\lambda_0 \cdot r_1$	≈ 0.1593 (mm)^2	$6.47\cdot10^{-23}$	[0.159256, 0.159256] (mm)^2
Radius, r_1	≈ 270 (mm)	$6.47\cdot10^{-23}$	[270.384, 270.384] (mm)

Table 1: Uncertainty of found values, Slope, $\lambda_0 \cdot r_1$, and r_1

The found standard error signifies that from all the possible found slopes to represent the linear decrease of the data, 68% of the them are within an error of $6.47\cdot10^{-23}$ of the value used for this analysis. As such the interval,the minimum and maximum possible value within that 68%, the below below 2.5% and above 97.5% in the distribution of possible values for the slope, is nearly the same as used values. As the difference is of by a magnitude of 10^{-15} only. Explaining why the lower and upper limits are the same. A marginally ignorable effect of the standard error. The error effects from the slope were further propagated to our final value, r_1, as seen above.

The Kth value index, the x-axis of Figure 1, is unit less and is of no effect to the final found values.

4 Thickness of a Metal Foil; Using wedge-shaped layers

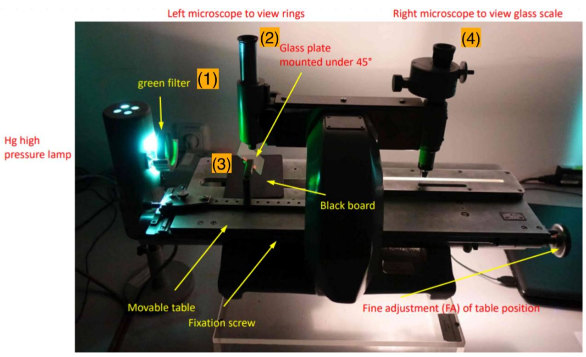
4.1 Experimental Exploration

4.1.1 Materials

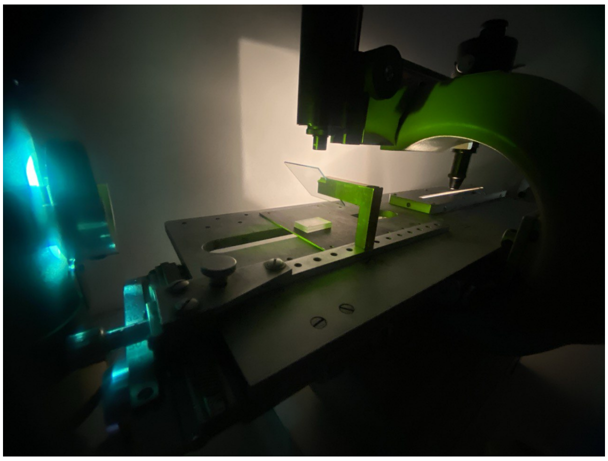
- Abbe Comparator (See below for more) Filter; Namely Red ($\lambda_0 = 589 \cdot 10^{-6} mm$)
- Two Slabs of Transparent Wedge-Shaped Layers
- A Plank of Aluminum Metal; provided by the Lab Technician
- Residual items: Ruler, Laptop, etc...

4.1.2 Set up

As further seen in the below image, the Abbe Comparator is set up with a Red Light Filter placed under high mercury (Hg) pressure lamp (1), the Plank of Aluminum wedged between the slabs of transparent layer and placed on the black board (2), Position Set from the glass plate (3) and Position Measured from the glass scale (4).



(a) The Overall Used Apparatus (*Guide to Experiment O9e: Fringes of Equal Thickness*)



(b) Two Slabs with a aluminium metal wedged in between

4.1.3 Methodology

In reeling the Abbe Comparator to the its far end on the right, the position of a reference vertical line is set on any one of the stead straight lines seen through the main microscope. The measurement of the position is taken from from the the secondary microscope over the glass scale. The measurements are taken following the pattern explained by the lab technician. It accumulates from adding 3 sub-measurements seen through the secondary microscope, one on a horizontal line, one on left on a semi-circle and one on the top as one large measurement reference. The measured value is in reference to an unknown position, but as the experiment deals with absolute differences, the reference point is irrelevant. After each measurement, the stand is reeled to the left, placing the reference vertical line one fringe away from the prevision measurement. A total of 10 measurements taken.

4.2 Results

4.2.1 Data and Analysis

From the retrieved data, the following graph presents the plotting of the measured position, $x_{2,k}$, against the kth value (analogous to the index of measurement). Where the data in Blue is the measured values, and the data in Red is the Best Linear Fit to find the needed slope (as before explained in Section 2.2)

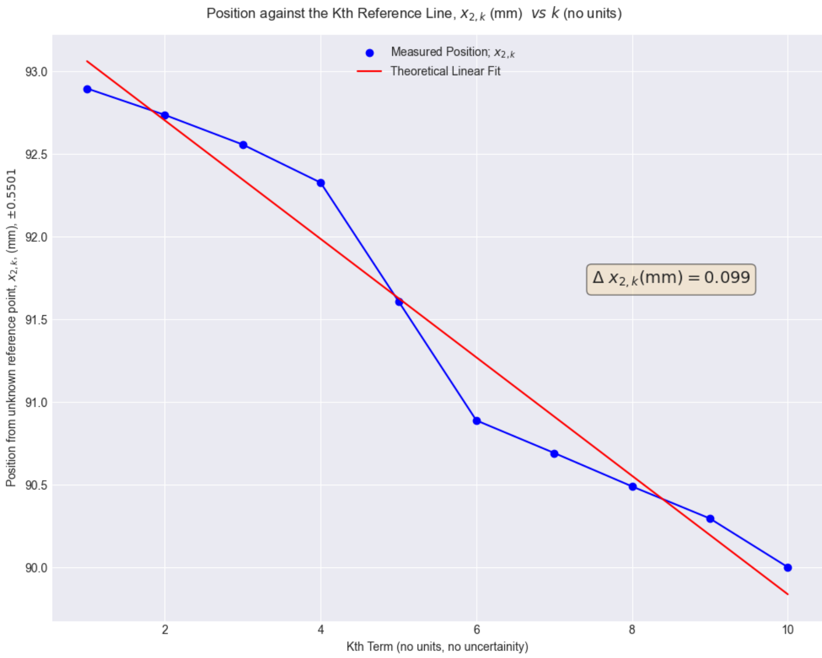


Figure 2: Position against the Kth Reference Line, $x_{2,k}$ (mm) vs k (no units)

Figure 2 presents the relationship between the change in the kth value (the measured index) and the change in relative position in regards to an unknown reference point as the measuring tool was slide from left to right for a slab of thin aluminium metal. There is a clear decreasing relationship, with a R^2 value of $0.9614558010095101 \approx 0.961$ in regards to a linear best fit. The existing inconsistency of the data, in regards to the linearity, is mainly due to the overall imperfection of the system. Further explanation in the details. To be able to utilise equation 16, to find the needed thickness of the material, the slope of the red graph had to be taken (as there is no one possible graph to represent the original data in blue). The overall accuracy of the found slope can be said to be within reasonable estimation, as the two data sets (theoretical data fit and the measured values) had a percentage difference of $0.18562899122939092 \approx 0.186\%$, a Max Difference of $0.38102424260804924 \approx 0.381$ mm and a Mean Absolute Error of $1.852939135460474 \cdot 10^{-10} \approx 1.85 \cdot 10^{-10}$ mm, which all are within in relatively small difference. Meaning the slope of the red graph can be taken as the slope of the blue original measured data.

Utilising equation 16, with the known wavelength λ, known (measured to prior conducting the experiment, $=$ 2.3 cm) length of the slab below where the metal has been jammed, the following results were found to be found;

Parameter	Value
Slope, $\|\frac{I\lambda}{2D_2}\|$	$0.3579515... \approx 0.36$ (mm)
Thickness, $\|D_2\|$	$1.8922953..\cdot 10^{-5} \approx 0.019$ (mm)

Table 2: Found values for an aluminium metal; Slope and Δ_2.

In comparison to the physical slab that was used for the experiment, the thickness of 0.019 mm seems to, at least to the extent of visual comparison, of reasonable magnitude. As the exact real value as not provided, there can be no comparison to any literary value.

4.2.2 Discussion

The result for the thickness of the metal seems to be within a reasonable approximation, if taken solely through a visual comparison, as within the experiment the provided slabs with a metal in between seemed to have no wedge. Hinting on the fact that the thickness is small within the magnitude of the found value, 0.19 mm.

The jumps and the non-linear behaviour seen for the measured data can be attributed to the existing difficulties in positioning the reference vertical line in the main microscope properly along each fringe. As the distance between each successive fringe is too small, and the limitation of the magnification and how the used light source was not great at focusing the lines, the human error had a larger effect than would under more idealised situations. The average Δ_2, difference between successive lines, came out to 0.18599999999999994 ≈ 0.186 mm, is of similar magnitude in comparison to what was seen through the main microscope.

With how all the measured data are within the same range, and consistent linear decrease, and visual confirmation of the literary value, the data can be inferred to be both ***accurate and precise***

Nonetheless, the experiment can be improved by using better light source, more powerful microscope. Further research is limited, other then measuring the thickness of different materials with more accessible knowledge of its thickness prior to the experiment - as to compare the validity of this method in determining the thickness.

4.2.3 Error Analysis

The uncertainty seen on Figure 2, on the y-axis for the position, is calculated through the smallest measurable values from the secondary microscope. The half of the smallest measurable value of each sub-measurement was summed together to get the shown final uncertainty of ± 0.5501 mm. The accuracy of the found slope can be further understood through the Standard Error, and the confidence interval. From which, the D_2 is calculated under these effects of the error;

	Found Value	Standard Error	Confidence Interval
Slope, $\|\frac{I\lambda}{2D_2}\|$	≈ 0.36 (mm)	$2.28 \cdot 10^{-15}$	[0.357952, 0.357952] (mm)
Thickness, $\|D_2\|$	≈ 0.019 (mm)	$2.28 \cdot 10^{-15}$	[0.018923, 0.018923] (mm)

Table 3: Uncertainty of found values, Slope, $\|\frac{I\lambda}{2D_2}\|$, and D_2

The found standard error signifies that from all the possible found slopes to represent the linear decrease of the data, 68% of the them are within an error of $2.28 \cdot 10^{-15}$ of the value used for this analysis. As such the interval,the minimum and maximum possible value within that 68%, the below below 2.5% and above 97.5% in the distribution of possible values for the slope, is the nearly the same as the difference is of by a magnitude of 10^{-15}. Explaining why the lower and upper limits are the same. A marginally ignorable effect of the standard error. The error effects from the slope were further propagated to our final value, D_2, as seen above.

The Kth value index, the x-axis of Figure 2, is unit less and is of no effect to the final found values.

5 Thickness of a Strand of Hair; Using wedge-shaped layers

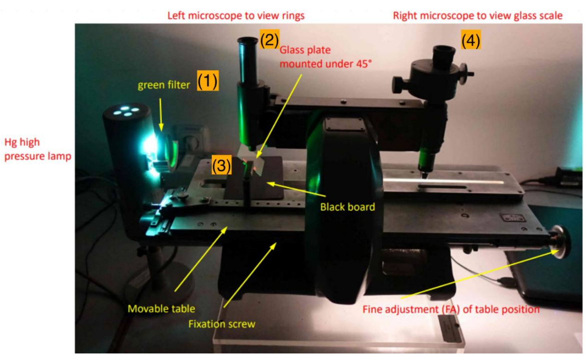
5.1 Experimental Exploration

5.1.1 Materials

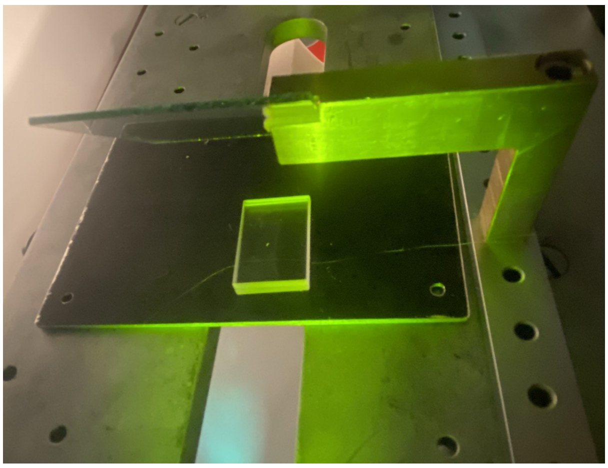
- Abbe Comparator (See below for more)
- Light Filter; Namely Red ($\lambda_0 = 589 \cdot 10^{-6} mm$)
- Two Slabs of Transparent Wedge-Shaped Layers
- Strand of Hair; Taken from Member Calina Burciu (3770859)
- Residual items: Ruler, Laptop, etc...

5.1.2 Set up

As further seen in the below image, the Abbe Comparator is set up with a Red Light Filter placed under high mercury (Hg) pressure lamp (1), the Strand of Hair wedged between the slabs of transparent layer and placed on the black board (2), Position Set from the glass plate (3) and Position Measured from the glass scale (4).



(a) The Overall Used Apparatus (*Guide to Experiment O9e: Fringes of Equal Thickness*)



(b) Two Slabs with a hair wedged in between

5.1.3 Methodology

In reeling the Abbe Comparator to the its far end on the right, the position of a reference vertical line is set on any one of the stead straight lines seen through the main microscope. The measurement of the position is taken from from the the secondary microscope over the glass scale. The measurements are taken following the pattern explained by the lab technician. The measured value is in reference to an unknown position, but as the experiment deals with absolute differences, the reference point is irrelevant. After each measurement, the stand is reeled to the left, placing the reference vertical line one dark fringe away from the prevision measurement. A total of 10 measurements taken.

5.2 Results

5.2.1 Data and Analysis

From the retrieved data, the following graph presents the plotting of the measured position, $x_{3,k}$, against the kth value (analogous to the index of measurement). Where the data in Blue is the measured values, and the data in Red is the Best Linear Fit to find the needed slope (as before explained in Section 2.2)

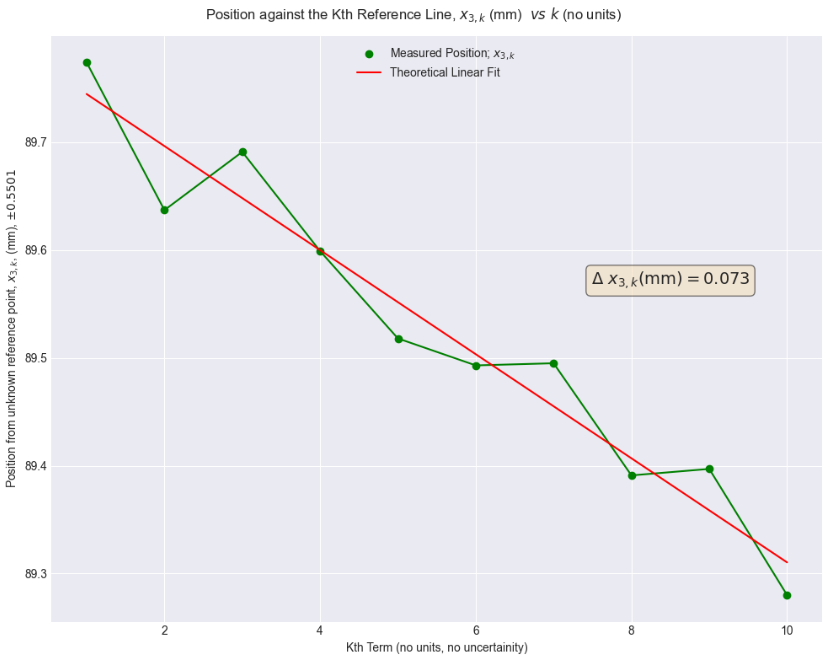


Figure 3: Position against the Kth Reference Line, $x_{2,k}$ (mm) vs k (no units)

Figure 3 presents the relationship between the change in the kth value (the measured index) and the change in relative position in regards to an unknown reference point as the measuring tool was slide from left to right for a strand of hair. There is visible linearly decreasing correlation, yet somewhat weak as there is consistent jumps and cuts in the overall decrease of the data set. As the slope of the decreasing relationship is needed, utilised in reference to equation 16 from section 2.2, a linear best fit is taken (as seen in red). Even though it seems to be a very inconsistent similarity between the theoretical and the measured data set, between both sets there is a R^2 value of

0.94259134928588520≈ **0.943**. More details regarding the visually inconsistent comparison between the theoretical and measured values are seen in the Discussion. However for this analysis, as the average percentage difference between the them is 0.03361091235988796% ≈ 0.03%, a Max Difference of 0.05941212140122332 ≈ 0.06 mm and a Mean Absolute Error of $1.810946059777052 \cdot 10^{-6} \approx 1.81 \cdot 10^{-6}$ mm, it can be confidently said that the slope found through the (Red) linear fit can be used in finding the thickness of the hair.

Utilizing equation 16, with the known wavelength λ, known (measured prior to conducting the experiment) length of the length of the slab below where the hair is jammed, the following results were found;

Parameter	Value
Slope, $\|\frac{I\lambda}{2D_3}\|$	0.0482606.. ≈ 0.048 (mm)
Thickness, $\|D_3\|$	$7.3227426..\cdot 10^{-5} \approx 0.073$ (mm)

Table 4: Found values for a strand of hair; Slope and Δ_3.

With an average thickness of a hair of strand approximated to be between $0.03 \text{ mm} \leq D_3 \leq 0.1 \text{ mm}$ (*Student, LAURA ARNOLD MU Graduate*), the found Δ_3 is of the same magnitude, and the value the falls within the literary range. This produces a possible percentage difference of $-143\% \leq D_3 \leq 25\%$ respective to each boundary of the literary value.

5.2.2 Discussion

The results for the thickness of this material came out to within reasonable values, and further confirms the found equation of the dark fringes for the interference in a two layered glass slab configuration. The existing jumps and inconsistencies seen in the data, figure 3, even if small in regards to the measurement scale, is mainly due to the difficulty in properly placing the vertical reference line of the main microscope in aligning with the vertical fringes. As not only the light source does not highlight enough of the area, the overall distance between each fringe is small enough to make it hard to centre under the microscopes magnification limit. The averages Δ_3, the difference between successive points came out to 0.04712499999999942 ≈ 0.047 mm, a difference which is hard to properly take into account in moving the vertical reference point. In the same line of thought, the experiment could be improved by using green tinted light sources, which as explained by the lab technician, to be better at producing more clearer images of the fringes - as such making it easier to distinguish between different lines of interferences. Furthermore, there is the human error of being able to properly distinguish the lines, which is a factor that is difficult to eliminate.

The existing percentage large percentage difference between the literary and calculated value can be attributed to how a person hair thickness is subjective to the individual and is not an objective identity across the board. And as the found value, 0.073 mm, is within the reasonable range, it can be said the experiment and the calculations were of reasonable accuracy. With additionally how all the measured data points are within the same reasonable range, the data can be inferred to be both ***accurate and precise***.

Nonetheless, further improvements and research with this experiment can be done with more repeated measurements of the same hair strand, the measurement of a different hair from the same person (to compare consistency of hair thickness) and try the technique of thickness measurement of bigger objects. If there is an object with physically measurable thickness (in the range of 2 - 3 cm), the measured and calculated value both can be compared to and easily formalise the validity of all other much thinner materials.

5.2.3 Error Analysis

As the experimental method of measuring the thickness of hair is same as the measurement for the slab of metal (see error analysis in Section 4.2.3), the reasoning behind the derivation of the uncertainties are the same. As such, the uncertainty in the found position is of the same value, ± 0.5501 mm. Below is the Standard Error and Confidence Interval, with the same effect as discussed before for Section 4

	Found Value	Standard Error	Confidence Interval
Slope, $\|\frac{I\lambda}{2D_3}\|$	≈ 0.048 (mm)	$1.92 \cdot 10^{-15}$	[0.048261, 0.048261] (mm)
Thickness, $\|D_3\|$	≈ 0.073 (mm)	$1.92 \cdot 10^{-15}$	[0.732268, 0.732268] (mm)

Table 5: Uncertainty of found values, Slope, $\|\frac{I\lambda}{2D_3}\|$, and D_3

The found standard error signifies that from all the possible found slopes to represent the linear decrease of the data, 68% of the them are within an error of $1.92 \cdot 10^{-15}$ of the value used for this analysis. As such the interval,the minimum and maximum possible value within that 68%, the below below 2.5% and above 97.5% in the distribution of possible values for the slope, is the nearly the same as the difference is of by a magnitude of 10^{-15}. Explaining why the lower and upper limits are the same. A marginally ignorable effect of the standard error. The error effects from the slope were further propagated to our final value, D_3, as seen above.

The Kth value index, the x-axis of Figure 3, is unit less and is of no effect to the final found values.

6 Conclusion

In measuring the radius of curvature for the curved lens, from Section 3, we were able to find a value of approximately 270 mm. In the similar sense, we were successful in measuring the thickness of our two materials, where the metal slab came out to 0.019 mm and the strand of hair came out to 0.073 mm. As these values make sense in the context from which they were derived from (either comparing to a literary value or from the visual comparison to how the slabs of glass looked while doing the experiment), there was a successful implementation and analysis of the measurements. The accuracy of each found value was argued through its standard deviation, the difference between the best fit and the measured values.

7 Citation

Unknown Author. *Guide to Experiment O9e: Fringes of Equal Thickness*. Course General Physics Laboratory 2, 2025.

Unknown Author. *O09e Fringes of Equal Thickness*. Course General Physics Laboratory 2, 2025.

Student, LAURA ARNOLD MU Graduate. *"Q: How Thin Is a Human Hair?"* Columbia Daily Tribune, eu.columbiatribune.com/story/lifestyle/family/2016/08/10/q-how-thin-is-human/21830395007/.