E07e Magnetic Fields in Coils

2024-12-04

Author(s)

- **Carla Rotzoll**, 50% contribution
- **Mirzokhid Ganiev**, 50% contribution

Group Number: 03

Abstract

This document explores the behaviour of magnetic fields in three configurations, a long solenoid, a single flat circular coil, and a Helmholtz Configuration of two parallel Coils. For the Long Solenoid, the behaviour of both Measured and Theoretical Data Base followed a long *'Bell-Shaped* curve, with the edges of the curve corresponding to approximately the edges of the solenoid. A Percentage Error Margin between the two sets of 69.5% was mainly due to a shift in the Measured Data set. Similarly, the case of one flat circular coil had a shift in its measured data set, with an Percentage Error Margin of 11.6%. With precision limited by misalignment and sensor positioning. The Helmholtz configuration showed field varying with changing coil separation, where a smaller separation resulted in a greater overlap of individual fields and as such coming closer to behaviours seen in experiments for 'Long Solenoid' and 'One Flat Circular Coil'. While larger separations produced distinct dual peaks corresponding to the coil positions. Experimental deviations ranged from 3.5% to 9.5% for different separations, with noise and mutual inductance identified as key contributors to error.

1 Introduction

Three distinct experiments will be conducted utilising a magnetic field sensor, measuring $B(z)$ at some position. The experiments vary in the shape and positioning of a coil configuration with a non trivial amount of coil *turns*. The set ups of research are:

1.0.1 Case of Long Solenoid

Where the theoretical and experimental analysis of the magnetic field distribution along some axis of a long solenoid will be measured and compared with each other. The Theoretical insights will be derived utilising the Biot-Savart Law. The Experimental setup will be conducted using a long solenoid and a measuring device, a MR-Sensor Adafruit MLX90393, positioned at some varying positions inside and outside of the solenoid. Additionally, the results will be used to find some theoretical insights regarding the behaviour of the magnetic field.

1.0.2 Case of 1 flat Circular Coil

Where the theoretical and experimental analysis of the magnetic field distribution along some axis of one flat circular coil will be measured and compared with each other. The Theoretical insights will be derived utilising the Biot-Savart Law for the case of one coil. The Experimental setup will be conducted using a circular coil and a measuring device, a MR-Sensor Adafruit MLX90393, positioned at some varying positions inside and outside of the coil. Additionally, the results will be used to find some theoretical insights regarding the behaviour of the magnetic field.

1.0.3 Case of 2 flat Parallel Circular Coil

Where the theoretical and experimental analysis of the magnetic field distribution along some axis of two flat parallel circular coil positioned at some varying distances from each other will be measured and compared. The Theoretical insights will be derived utilising the Biot-Savart Law for the case of two coil (using the principle of super positioning of individual coils). The Experimental setup will be conducted using the circular coils and a measuring device, a MR-Sensor Adafruit MLX90393, positioned at some varying positions inside and outside of the coil system, with the coils positioned at varying distance from each other (separation distances of $b = 2R, R, \frac{R}{2}$). Additionally, the results will be used to find some theoretical insights regarding the behaviour of the magnetic field.

For all the following experiments, the axis will be as follows:

Diagram 1 Axis for all Experiments

with the Magnetic Field being measured against the z-axis, as $B(z) = B_z$.

2 Magnetic field distribution in a long solenoid

2.1 Hypothesis

The magnetic induction along the z-axis is expected to be symmetrical about the center of the solenoid, reflecting the inherent geometric symmetry of the solenoid itself. Furthermore, the magnetic field is expected to be significantly stronger inside the solenoid compared to the outside and to remain uniform throughout its interior. This behavior can be attributed to the superposition of magnetic field lines generated by the closely wound, evenly spaced turns of the solenoid, which reinforce the field within and minimize its divergence outside.

2.2 Theoretical Exploration

We take Amperes Maxwell equation for stationary fields in the differential form

$$\nabla \times \vec{H} = \vec{J} \tag{1}$$

with the special solution by the Boit Savart law:

$$d\vec{H} = \frac{I}{4\pi}\frac{d\vec{l} \times \hat{\vec{r}}}{r^2} \tag{2}$$

Now plugging in $d\vec{H} = \frac{dH_z}{\sin\varphi}$ and $\sin\varphi = \frac{R}{r}$ we get:

$$dH_z = \frac{I}{2}\frac{R^2}{\sqrt{R^2+z^2}^3} \tag{3}$$

We approximate absence of magnetic core so that $B = \mu_0 H$ The turn density (number of turns per unit length) of a long, tightly wound solenoid is given by $n = \frac{N}{L}$, where N is the total number of turns and L is the length of the solenoid. Therefore, $n\,dz$ wire loops are located between z and $z + dz$, each carrying the current I, resulting in a total current of $I\,n\,dz$.

$$dB_z = \frac{\mu_0}{2} I R^2 \frac{1}{\sqrt{R^2+(a-z)^2}^3} ndz \tag{4}$$

By integration over the length L of the solenoid we get:

$$B_z = \frac{\mu_0}{2} In\left(\frac{a}{\sqrt{R^2+a^2}} + \frac{L-a}{\sqrt{R^2+(L-a)^2}}\right) \tag{5}$$

where equation 5 will be the final equation used in the calculations for the Theoretical Data Set for this case. All calculations through the help of the provided additional guide (E7e Magnetic Fields in Coils)

2.3 Experimental Exploration

2.3.1 Materials

- Solenoid with these Parameters:
  - Length $L = 160$ mm
  - Radius $R = 13$ mm
  - Number of turns $N = 150$
  - Maximum current $I_{\max} = 4$ A
- Laboratory power supply: GwINSTEK GPD-2303S
- MR-Sensor Adafruit MLX90393
- A 0.49 m long metallic tube to position the sensor.
- Optical bench with scale ± 0.005 m.
- Vertical Stands with fixing screws.
- Supplementary Tools; Ruler (± 0.005), Wires, Laptop

2.3.2 Set Up

A rider with a fixing screw, to which the measurement sensor is attached, is placed at one side of the optical bench with a scale. On the other side, another rider holds the solenoid, positioned parallel to the optical bench so that its tunnel points toward the sensor.

The solenoid is connected to a power supply via wires to maintain a constant current of 0.999 A. Initially, the middle of the sensor's rider is positioned at 19cm on the scale, and the middle of the solenoid's rider is positioned at 79cm. This implies that the middle of the solenoid is located at 79cm on the scale, and the sensor is at 70cm. As a result, the edge of the sensor is 9cm away from the beginning of the solenoid. The sensor is connected to a laptop, where the measurement values are displayed. Refer to Image **1** below.

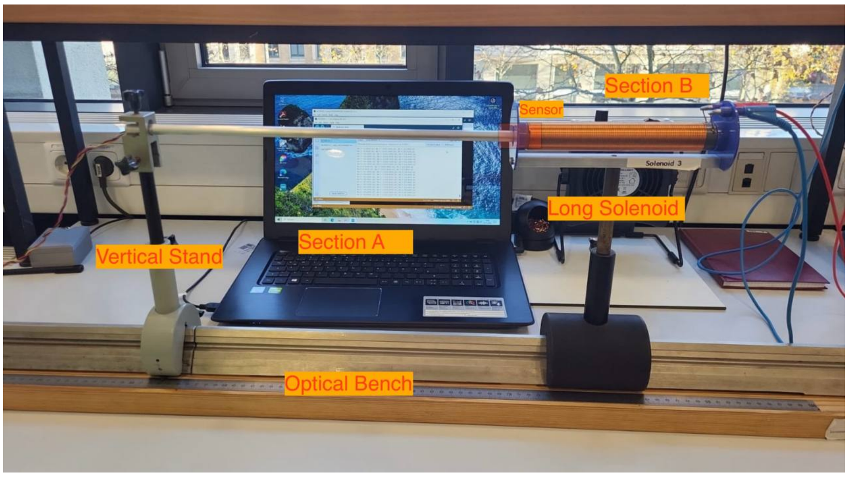

Image 1 Set up for Section 2

2.3.3 Methodology

The magnetic induction is measured using the MR-Sensor Adafruit MLX90393, with data recorded via the serial monitor of the Arduino GUI. The y-component of the sensor's measurements is read out. Starting from the initial position, the sensor is moved 1 cm to the right, and the magnetic induction is measured again. This process is repeated 20 times, allowing the sensor to pass completely through the solenoid and emerge on the other side.

The recorded values of $b(z)$ are plotted against z (the sensor's position). Additionally, calculated values of the magnetic induction are plotted and compared with the experimental results.

2.4 Results

2.4.1 Data and Analysis

From the retried data, the following data presents the Magnetic Field Density μT) (uncertainty on the graph) against the position along the Optical Bench's Scale ± 0.005(m). Where the *Blue* data presents the measured values while the *Red* data presents the corresponding Theoretical Data from running relative positions through equation 5.

```python
def theo_magnetic_long(x):
    b = ((mu0*0.999*(150/0.16))/2)*((x/(np.sqrt(0.013**2+x**2)))+((0.16-x)/
    (np.sqrt(0.013**2+(0.16-x)**2))))*10**6
    return b

distance_values_task_1_theo = pd.Series(np.linspace(-0.035, 0.16+0.035, 24))
distance_to_overlap_task_1 = pd.Series(np.linspace(0.19, 0.42, 24))
task_1_theo_values = theo_magnetic_long(distance_values_task_1_theo)

data_task_1 = r"task_1.csv"
new_headers_task_1 = ["Distance (cm)", "Magnetic Field"]
data_task_1_evaluated = pd.read_csv(data_task_1, header=None)
data_task_1_evaluated.columns = new_headers_task_1
distance_task_1 = (1/100)*data_task_1_evaluated.iloc[:, 0] #distance (cm)
magnetic_field_task_1 = -1*data_task_1_evaluated.iloc[:, 1] #magnetic field
```

Note: The code for plotting is omitted as it is considered general and as to avoid clutter. If needed, can be accessed in the attached Jupyetr Folder

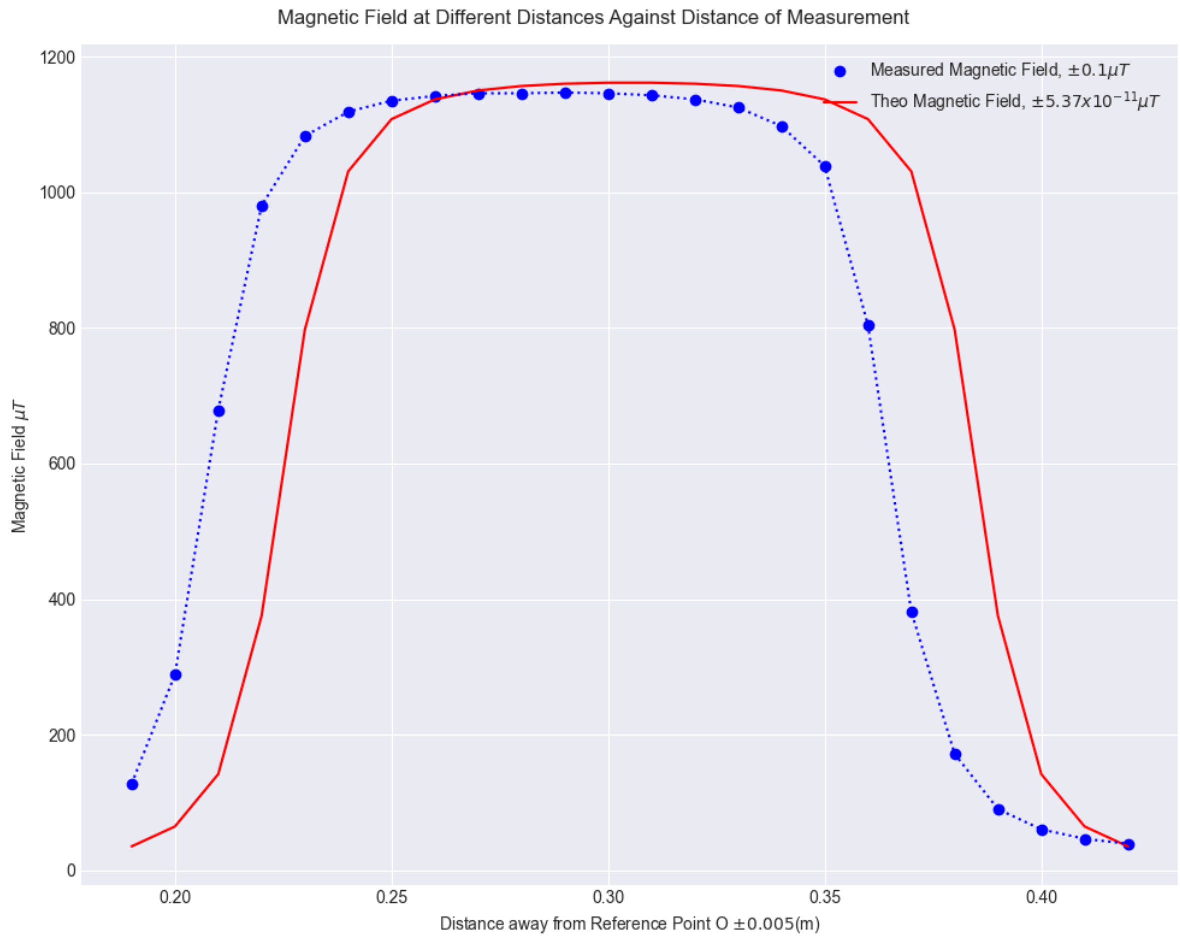

Figure 1: Magnetic Field Density (μT) against the position along the Optical Bench's Scale ± 0.005(m)

In the graph above the x axis represents position or distance, indicating how the magnetic field changes spatially. And the y axis represents the magnitude of the magnetic field (in microtesla). Both data sets present a "bell curve" shape, with a steep rise and an approximately flat plateau at the peak, with a steep decline. The maximum value of the curve for Measured and Theoretical Data Set was observed at **1147** and 1161.509614 $\approx$ **1161** μT respectively, with corresponding positions as 29 and 30-31 cm respectively. With the edges of the plateau for Measured coming out to **23 to 35 cm** for respectively each edge (a length of 12 cm - shorter in comparison to the length of the solenoid of 16 cm). With the edges of the plateau for the Theoretical yielding as **24 to 37 cm** (a length of 13 cm - shorter in comparison to the length of the solenoid of 16 cm). The edges of the peak were taken at the point where the values stabilised, i.e there is less than 100 μT between adjacent values.

The experimental data aligns well, in terms of behaviour, with the theoretical ones as the theoretical model predicts a flat plateau at the maximum magnetic field, indicating a region of uniform field strength. With the Percentage Error Margin between the two sets as 69.45994081399787% $\approx$ **69.5%** and a Mean Absolute Error of |-15.153302403579803| $\approx$ **15.2** μT, the two sets are similar only through qualitative behaviour. Additionally the experimental data also shows a sharp rise and fall on both sides of the plateau, but the alignment with the theoretical curve is not perfect in these regions. As there is a clear shift to the left of the Measured Data in relation to the Theoretical Data set. The shift, as measured using Cross-Correlation *("Cross Correlation")*, being, the maximum, **0.02 m**, there is a clear systematic error in the experimental procedure. With a Max Difference of 649.2011787404825 $\approx$ **649** μT, the Measured Data set is not within a reasonable close approximation to the Theoretical Data Set. Making the Measured Data Set ***accurate but imprecise*** relative to the Theoretical Data Set. More details in the Discussion.

Note: The Code for MAE, MD, Percentage Error and Data Shift can be found in the Appendix, omitted from here to avoid clutter

2.4.2 Discussion

From the experimental data, we can see that the magnetic induction is indeed symmetrical about the centre of the solenoid and is approximately constant and significantly higher inside the solenoid than outside. Confirming the hypothesis. This occurs because the tightly wound, evenly spaced coils of the solenoid produce a uniform magnetic field along its axis, while the field outside is weaker.

The position of the edges, which should correlate to the length of of the solenoid, were not exact (as briefly mentioned in the Analysis). Here the data repeated,

Data Set	Positions, L & R Edge	Difference from Actual Length
Theoretical	24 cm to 37 cm	16 - 13 = 3 cm
Measured	23 to 35 cm	16 - 12 = 4 cm

Table 1: The Position of the *edges* of the elongated Bell Curves

The difference of 3 and 4 cm relative to the length of the curve can possibly be attributed to the solenoid not having a very consistent *turn* spread. If referred to Image **1**, the coil wraps on the edges of the solenoid are not tightly wounded together but a relatively spread out. As the length of the solenoid is measured including those spread *turns*, the length of tightly held area could be 3-4 cm less than 16 cm.

The existing shift, of a maximum value of 0.02 m, can be attributed to possible experimental errors such as errors in reading the scale. This is because the sensor used for the measurements is not a single point but has a finite length, making it challenging to align the measurements perfectly with the exact theoretical positions along the solenoid. These slight misalignments can lead to discrepancies between the two datasets. A more comprehensive understanding of the shift is further seen in Section 3.

If the shift taken into account, where the Measured Data is shifted by 0.02 to the right, the following result is seen:

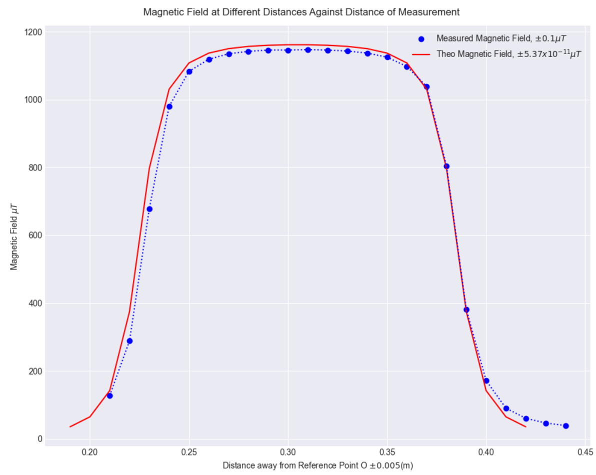

Figure 2: Shifted Magnetic Field Density (μT) against the position along the Optical Bench's Scale ± 0.005(m)

where with this configuration, taking into account the shift, we get an percentage error margin of $22.943449198141913\% \approx$ **23%** which presents a more sustainable percentage error. In redoing the experiment, a more careful configuration of the positions should be taken. *In plotting the graph, the first index of the Theoretical Data Set was removed, as to match the plateau of both sets.*

Note: The code for plotting is omitted as it is considered general and as to avoid clutter. If needed, can be accessed in the attached Jupyetr Folder

2.4.3 Error Analysis

The uncertainty for the Measured Data Set is derived from the smallest value provided by the sensor (as for digital measurement devices, the uncertainty corresponds to the smallest value). While the uncertainty for the Theoretical Data Set is calculated following the equations of John R. Taylor, from An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements (Taylor):

$$\partial B(a) = |\frac{\partial B}{\partial a}| \cdot \sigma_a \tag{6}$$

, where the error is propagated through $B(z)_t$ for the only measured value in our equation 5, a. As such, the uncertainty equation will be:

$$\partial B(a) = -\frac{I R^2 n \mu_0 \left((a^2 + R^2)^{\frac{3}{2}} - \left((L-a)^2 + R^2 \right)^{\frac{3}{2}} \right)}{2 \left((L-a)^2 + R^2 \right)^{\frac{3}{2}} (a^2 + R^2)^{\frac{3}{2}}} \tag{7}$$

Utilising equation 7, propagating the error through the whole set of values of a yields the following Mean and Max values:

Data Set	Mean Error Propagation	Largest Error (Max)
Theoretical	$\pm\ 5.37x10^{-11}$	$\pm\ 2.09x10^{-10}$
Measured	$\pm$ 0.1	$\pm$ 0.1

Table 2: Error Propagation; Mean and Maximum Values for a Long Solenoid

3 Magnetic field distribution of a Flat Circular Coil

3.1 Hypothesis

The case of one flat circular coil should produce a curve of one quick peak and have symmetry against the central axis. Because for one circular coil, all the coils are concentrated into one small region (the circle), $B(z)$ should yield a much faster peak spike near the circle, with a similarly fast descend. This is further supplemented from the idea that in positions closer to the area of the coil, the density of the magnetic field lines are larger and would produce a higher magnetic field density measurement, i.e $B(z)$ would be larger as closer it is to the central axis.

3.2 Theoretical Exploration

We can utilize the Biot-Savart law (from equation **2**) to derive the equation for a single flat circular loop with some radius R and number of turns N. Taking the initial dB in the direction of $\hat{z}$ (as presented in the Diagram 1) we can write equation **5** as:

$$dB\hat{z} = dB\sin(\theta) = \frac{\mu_0}{2}\frac{IR^2 d\phi}{\sqrt{R^2+z^2}}\frac{R}{\sqrt{R^2+z^2}} \tag{8}$$

where, the $\frac{R}{\sqrt{R^2+z^2}}$ component is equal to $\sin(\theta)$. Where now, taking the integral of equation 8 from $[0, 2\pi]$ for $d\phi$, we can derive the total B_z as:

$$B_{z,total} = \frac{\mu_0 IR^2}{2(R^2+z^2)^{2/3}} \tag{9}$$

As for N number of turns,

$$B_{z,total,N} = \frac{\mu_0 INR^2}{2}\frac{1}{(R^2+z^2)^{2/3}} \tag{10}$$

presents the equation for one flat circular coil with some N number of loops. Where now, equation 10 will be the equation used to calculate the Theoretical Magnetic Field Density for the case of one Flat Circular Coil with some N number of loops. All calculations through the help of the provided additional guide (E7e Magnetic Fields in Coils).

3.3 Experimental Exploration

3.3.1 Materials

- Coil with the following parameters:

- Radius, $R = 68 \pm 0.0005$mm.
- Number of loops turns, $N = 320$.
- Maximum current $I_{\max} = 2$ A

- Laboratory power supply: GwINSTEK GPD-2303S
- MR-Sensor Adafruit MLX90393
- A 0.49 m long metallic tube to position the sensor.
- Optical bench with scale ± 0.005 m.
- Vertical Stands with fixing screws.
- Supplementary Tools; Ruler (± 0.005), Wires, Laptop

3.3.2 Set Up

The Coil is attached to the optical bench. The sensor is positioned 11 cm away from the coil. Refer to the Image below, Image **2**.

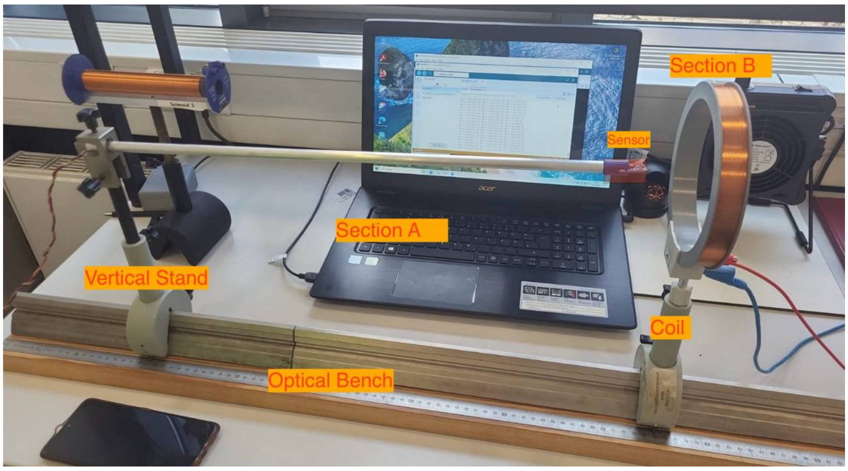

Image 2 Set up for Section 3

3.3.3 Methodology

While the Power Supply is connected to the coil, the sensor is moved at intervals of 1 cm up to at least 20 cm. At each interval, the measured value for the corresponding B is noted. The approximate average of 20 measurements at each interval is taken as final B for that position.

3.4 Results

3.4.1 Data and Analysis

From the retrieved data, the following data presents the Magnetic Field Density (μT against the position along the Optical Bench's Scale (m). Where the *Blue* data presents the measured values while the *Red* data presents the corresponding Theoretical Data from running corresponding positions through equation 10.

```python
def theo_magnetic(x):
    b =  constant_upfront * 10**6 * ((1/(0.068**2 + (x)**2)**(3/2)))
    return b

distance_values_task_2_theo = pd. Series(np.linspace(0.64, 0.84, 21))
task_2_theo_values = theo_magnetic(distance_values_b_2R)

data_task_2 = r"task_2.csv"
new_headers_task_2 = ["Distance (cm)", "Magnetic Field"]
data_task_2_evaluated = pd.read_csv(data_task_2, header=None)
data_task_2_evaluated.columns = new_headers_task_2
distance_task_2 = (1/100)*data_task_2_evaluated.iloc[:, 0] + 0.5 #distance (cm)
magnetic_field_task_2 = -1*data_task_2_evaluated.iloc[:, 1] #magnetic field
```

Note: The code for plotting is omitted as it is considered general and as to avoid clutter. If needed, can be accessed in the attached Jupyetr Folder

Figure 3: Magnetic Field Density against the position along the Optical Bench's Scale(m)

Figure 3 presents both Data Sets of Measured and Theoretical Magnetic Fields. The trend presents one central peak, with a maximum value of **2856** μT and **2954** μT for Measured and Theoretical Magnetic Fields respectively. With each maximum value corresponding to 75 cm and 74 cm respectively. Which is correlative with the position of the flat circular loop, with it being positioned at 75 cm (also seen in Image 2 in Section 3.3.2). Both data sets present the similar trend of an increasing Magnetic Field with ever closeness to the midpoint of the flat coil. With an average difference of 11.618268736244062% $\approx$ **11.6%** between the theoretical and measured, and a Mean Absolute Error (MAE) of |-38.00399222212132| $\approx$ **38** μT, and a Max Difference (MD) of 321.5869297295844 $\approx$ **322** μT, the Measured Data set is within a reasonable close approximation to the Theoretical Data Set. The Measured Data Set is shifted (utilising Cross-Correlation of two data sets) by a max value of -0.01 m through the *x-axis*, Distance, making the Measured Data Set ***accurate and precise*** relative to the Theoretical Data Set. However the value of the Measured Data Set is, on average, smaller then the Theoretical Data Set (if each Data index of the measured values are shifted by one index, i.e to match the peaks with each other). This is counter-intuitive to the understanding that because of background magnetic field noise, the measured values would be amplified. More details in the Discussion.

Note: The Code for MAE, MD, Percentage Error and Data Shift can be found in the Appendix, omitted from here to avoid clutter

3.4.2 Discussion

Figure 3 presents a trend for the Magnetic Field as hypothesised, where from the perspective of Magnetic Field Lines, the strength of the field increases as closer is the measurement point to the centre of the coil. As, at a closer distance, the density per unit area of the magnetic field lines increase - being more *clustered* at smaller distances. This is within the reasoning that the magnetic fields generated by the *turns* of coil pass through the centre of the coil, inducing a greater magnetic field strength in the region through superimposing the field due to each *turn*. The trend also further confirms equation 10, which correlated $B(z) \propto \frac{1}{z}$.

The smaller magnitude of measured $B(z)$ relative to the theoretical values can be attributed to the sensor stand not properly aligning parallel-wise with the central axis. There could, and possible does exist, some deflection of the metallic tube from the central axis. Which produces a measurement of a smaller magnitude (as the some angle deflection of $B(z)$, $B(z)\cos\theta$, produces a smaller value).

The existing shift, of a maximum value of -0.1 m, can be attributed to possible experimental errors such as:

i. Misalignment of the Measurement Set Up

As the shift is, to a qualitative extent, seen to be a consistent to the right, there could be an existing misalignment of the Set Up that was missed in the original experimental procedure. The misalignment could be from the wrong positioning of the coil / sensor (but noting down a different position, which was used for the Theoretical Data Set) or the circular loop not being positioned on the centre of the stand (and as such leading the stand positioning the coil 0.01 m off what was assumed to be the position).

Additionally, as the exact component of the sensor which measured $B(z)$ was not known, the distance to add from the Metal Rod that held the Sensor in place (seen in Image 2 as Section A) was taken up to the centre of the sensor. The actual measuring component could have been off from the centre by some value close to 0.01 m,

ii. Existing Calibration Issues in the Sensor

As the shift for all data points is towards one direction, there could be an existing calibration issue within the Sensor that lead to a consistent mismeasurement of the value (i.e the sensor does not properly measure $B(z)$ relative to its position). However, as how the actual sensor works is unknown, the exact technical reasoning cannot be formulated.

If the shift taken into account, where the Measured Data is shifted by 0.01 m to the left, the following result is seen:

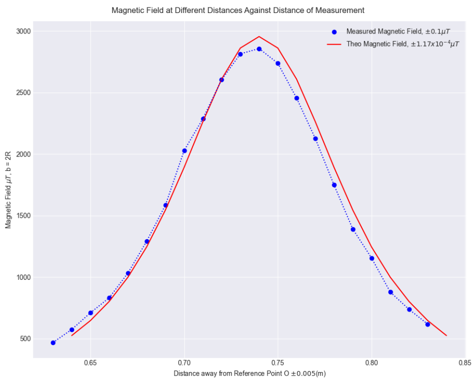

Figure 4: **Shifted** Magnetic Field Density (μT) against the position along the Optical Bench's Scale ± 0.005(m)

where with this configuration, taking into account the shift, we get an percentage error margin of 5.577358833682504% $\approx$ 5.58% which presents a more sustainable percentage error. In redoing the experiment, a more careful configuration of the positions should be taken. *In plotting the graph, the first index of the Measured Data Set was removed, as to match the peak of both sets.*

Note: The code for plotting is omitted as it is considered general and as to avoid clutter. If needed, can be accessed in the attached Jupyetr Folder

A Decrease to Half the Maximum

The half maximum for both Measured and Theoretical Data Set can be used to measure the position correlating to half of the Maximum. Where, when done, leads to the following data:

Data Set	Half Maximum μT	Position m
Theoretical	1428	0.6948, 0.799
Measured	1477	0.6875, 0.7925

Table 3: Half Value Positions

with the values as shown on a graph;

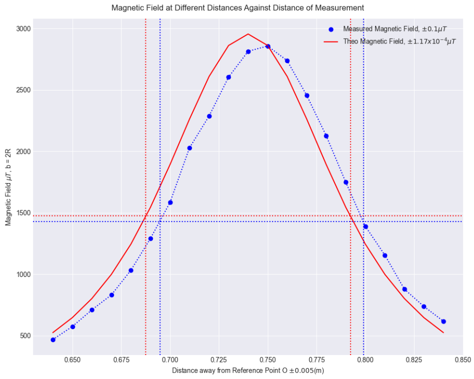

Figure 5: Half Maximum Positions

Note: The code for plotting is omitted as it is considered general and as to avoid clutter. If needed, can be accessed in the attached Jupyetr Folder

The half maximums should present a level of symmetry of the curves, and in comparison of the half maximum positions for both data sets, we get **0.1042** and **0.105** respectively for Measured and Theoretical Sets. Which correspond to $\approx$ 10 cm, about twice the width of the Circular Coil, giving approximation relation between maximum value and the width of the Circular Coils. A helpful relationship that can be used to find the maximum of a similarly shaped coil without positioning the sensor within the coil *turn* boundary. The similarity of both of them further provide proof for the precessions of the measured data set.

3.4.3 Error Analysis

The uncertainty for the Measured Data Set is derived from the smallest value provided by the sensor (as for digital measurement devices, the uncertainty corresponds to the smallest value). While the uncertainty for the Theoretical Data Set is calculated following the equations of John R. Taylor, from An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements (Taylor):

$$\partial B(z) = \left| \frac{\partial B}{\partial z} \right| \cdot \sigma_z, \tag{11}$$

, where the error is propagated through $B(z)_t$ for the only measured value in our equation 10, z. As such, the uncertainty equation will be:

$$\partial B(z) = -\frac{\mu_0 I N R^2}{2} \frac{3z}{(z^2+R^2)^{\frac{5}{2}}} \tag{12}$$

When equation 3.4.3 is utilised, and propagated through all the values of z, the following results are to be seen,

Data Set	Mean Error Propagation	Largest Error (Max)
Theoretical	$\pm\ 1.17x10^{-4}$	$\pm\ 1.84x10^{-4}$
Measured	$\pm\ 0.1$	$\pm\ 0.1$

Table 4: Error Propagation; Mean and Maximum Values

4 Helmholtz Case of Two Flat Circular Coils

4.1 Hypothesis

The case of two parallel Helmholtz flat Circular Coils should produce a result of some magnetic field overlap. At a distance of 2R away from each other, the coils could be treated as two separate single flat circular coils and as such would have two distinct peaks (with each peak similar to of Section 3). Where in moving the coils closer to each other should allow the magnetic field areas to overlap and create some new magnetic field behaviour. With the separation R producing a case similar to Section 2, where one elongated *'bell-curve'* would form. This would be due to only the opposite end poinds of the individual magnetic field lines overlapping, and as such producing a shape of an elongated spike. While the case of separation of $\frac{R}{2}$, the coils would be close enough (relative to the radius) to be treated as one circular coil, and as such yield one peak similar to Section 3.

4.2 Theoretical Exploration

For two loops, equation 10 is added twice for two positions of the two circular loops as

$$B(z) = \frac{\mu_0 I N R^2}{2}\left(\frac{1}{\sqrt{R^2 + (z - \frac{b}{2})^2)}^3} + \frac{1}{\sqrt{R^2 + (z + \frac{b}{2})^2)}^3}\right) \tag{13}$$

with b being the distance away from the centre point O (as additionally seen in the Diagram **1**). Equation 13 will be the equation utilised for the calculation of the Theoretical Values for Helmholtz Case of Two Flat Circular Coils at varying ranges of b. All calculations through the help of the provided additional guide (E7e Magnetic Fields in Coils)

4.3 Experimental Exploration

4.3.1 Materials

- Pair of coils with the following parameters:
  - Radius, $R = 68 \pm 0.0005$mm.
  - Number of loops turns, $N = 320$.
- Laboratory power supply: GwINSTEK GPD-2303S
- MR-Sensor Adafruit MLX90393
- A 0.49 m long metallic tube to position the sensor.
- Optical bench with scale ± 0.005 m.
- Vertical Stands with fixing screws.
- Supplementary Tools; Ruler (± 0.005), Wires, Laptop

4.3.2 Set Up

The Pair of Coils are attached to the optical bench, with a separation distance b, of $2R$, R, and $\frac{1}{R}$ for 3 different cases. The sensor is positioned 5 to 8 cm (depending on b) from the first coil.

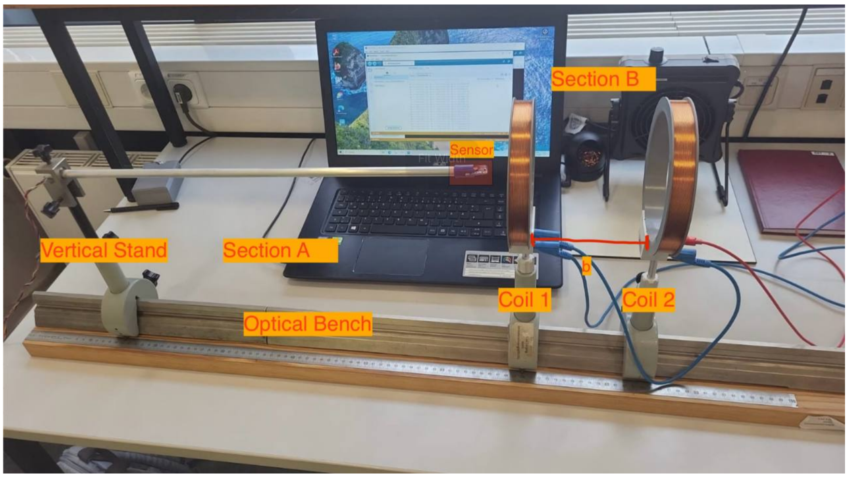

Image 3 Set up for Section 4

4.3.3 Methodology

While the Power Supply is connected to the coils, the sensor is moved at intervals of 1 cm up to at least 20 cm. At each interval, the measured value for the corresponding B is noted. The approximate average of 20 measurements at each interval is taken as final B for that position.

4.4 Results

4.4.1 Data and Analysis

From the retried data, the following data presents the Magnetic Field Density $\pm 0.1(\mu T)$ against the position along the Optical Bench's Scale ± 0.005(m). Where the *Blue* data presents the measured values while the *Red* data presents the corresponding Theoretical Data from running relative positions through equation 13.

```python
mu = 4*np.pi*10**(-7)
constant_upfront = (mu * 0.999 * 320 * 0.068**2) / 2
def theo_magnetic(x, d):
    b =  constant_upfront * 10**6 * ((1/(0.068**2 + (x - d)**2)**(3/2)) + (1/(0.068**2 + (x + d)**2)**(3/2)))
    return b

#theo b=2R
distance_values_b_2R = np.linspace(-0.068-0.05, 0.068+0.05, 24)
distance_to_overlap_b_2R = np.linspace(0.15 + 0.5, 0.88, 24)
b_2R = theo_magnetic(distance_values_b_2R, 0.068)

```

```python
#theo b=R
distance_values_b_R = np.linspace(-0.034-0.08, 0.034+0.08, 22)
distance_to_overlap_b_R = np.linspace(0.62, 0.84, 22)
b_R = theo_magnetic(distance_values_b_R, 0.034)

#theo b=R/2
distance_values_b_R_2 = np.linspace(-0.017-0.08, 0.017+0.08, 20)
distance_to_overlap_b_R_2 = np.linspace(0.12, 0.30, 20)
b_R_2 = theo_magnetic(distance_values_b_R_2, 0.017)
```

Note: The code for plotting is omitted as it is considered general and as to avoid clutter. If needed, can be accessed in the attached Jupyetr Folder

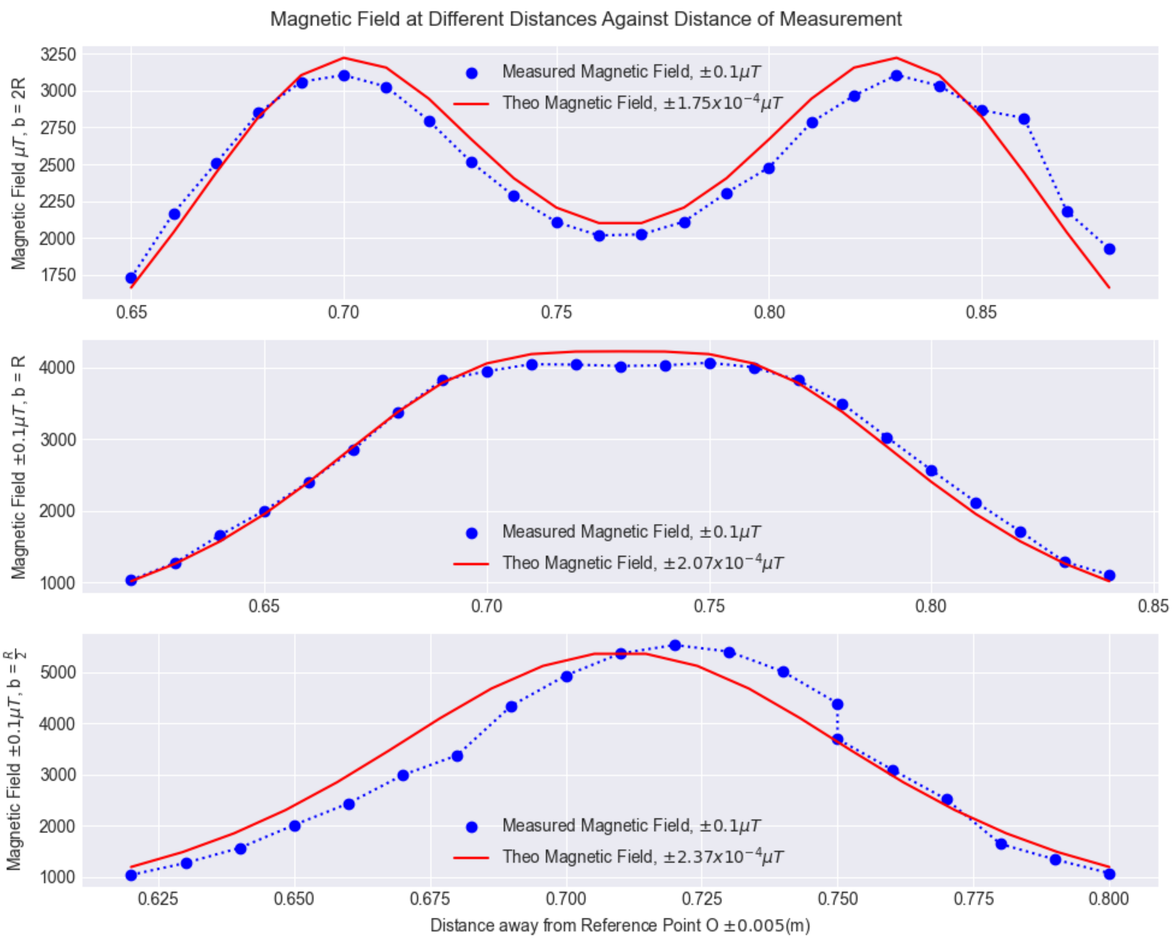

Figure 6: Magnetic Field Density $\pm 0.1 (\mu T)$ against the position along the Optical Bench's Scale ± 0.005(m) with (a)$b = 2R$ (b)$b = R$ (c)$b = \frac{R}{2}$

Note: The uncertainty for each data set is presented directly on the legend

Figure 6 presents the data set for all 3 different positioning of the pair of coils, $2R$, R, $\frac{R}{2}$, for both the measured and theoretical data sets. The trend for each measured data set follows through with the theoretical expected values, where:

For (a) b = 2R, 2 peaks are observed with each peak corresponding to the positions of the coils, with the local minimum (within Section B, Diagram **??**) at the half the length of b, $\frac{b}{2}$.The maximum value of $B(z)$ were observed as **3105** and **3106** μT at positions 70 and 83 cm respectively (which correspond to the positions of the coils) for the Measured Values. With 3222.57574567 $\approx$ **3223** μT for both peaks at positions 70 and 83 cm for the Theoretical Values. With the local minimum for Measured and Theoretical coming out to $B(z) =$ **2018** and 102.02935601 $\approx$ **2102** μT at the position of 76 cm for both. With the mean Percentage Error Margin (PEM) between the two sets, Measured and Theoretical, coming out to 5.156707374970321% $\approx$ **5.17%**, the Mean Absolute Error (MAE) as |-32.86702457754951| $\approx$ **32.9**μT can be attributed towards the Background Magnetic Field Noise. Where, the value 32.9 is off by 2.9 from the background magnetic field of 29.9191478511 $\approx$ 29.9191478511 μT. The backgruond magnetic field is further explained in the discussion. As the maximum difference (MD) between the value sets were 370.39637169901016 $\approx$ **370**μT, all accounts to the Measured Data Set as ***accurate and precise***. More details in the Discussion.

For (b) b = R, one elongated peak is observed, with the edges corresponding to approximately the positions of the two coils. Where the maximum value of the peak is observed approximately at the half the length of b, $\frac{b}{2}$ (Section B, Image **3**). The maximum value of $B(z)$ was observed as = 4048, 4227.18639665 $\approx$ **4227**, at the position of 71 cm and 73 cm respectively, where the position half the length of b was 73.4 cm. With the edges of the peak at values of $B(z)$ as = 3**950** and **4005** for Measured Data set and 4059.27539675 $\approx$ **4059** for both peaks for Theoretical Data. With both sets corresponding to positions 70 and 76 cm respectively. The edges of the peak were taken at the point where the values stabilised, i.e there is less than 100 μT between adjacent values. This is similar to what was derived in Section 2: Magnetic Field Distribution along a Long solenoid. With the PEM between the two sets as 3.5298376769351734% $\approx$ **3.53%** and a MAE of 1.4089965698396514 $\approx$ **1.41**μT, the measured case of b = R was a more accurate data set compared to case (a) of b = 2R. which all accounts to the measured data set being ***accurate and precise***. The MD came out to 204.1863966477158 $\approx$ **204**μT. More details in the Discussion.

For (c) b = $\frac{R}{2}$, a relatively shorter peak is observed approximately at the half the length of b, $\frac{b}{2}$ (Section B, Image **3**). The maximum value of $B(z)$ came out to = **5536**, 5363.92171107 $\approx$ **5364** μT at positions 0.72 and 0.71 cm respectively. Which corresponds to the mid way point of b, where the point of $\frac{b}{2}$. There is no clear local minimum, and the positions of the coils correlate to nothing specific on Figure 6 (c). With the PEM of 9.495566217567% $\approx$ **9.5%** and an MAE of |-88.60827900382675| $\approx$ **88.6**μT, this case of the set up deviates the most from the theoretical values. However, the error is within a 10% error margin, making it ***accurate but relatively low precise***. The MD came out to 723.2649409941414 $\approx$ **723**μT. More details in the Discussion.

Additionally, the maximum of the peaks increase with a decreasing b, as seen the peaks for the measured data set decrease on the order of 3105 , 3106 $\rightarrow$ 4048 $\rightarrow$ 5536 μT. Same trend is similarly seen for the Theoretical Data Set. More details in the Discussion.

Note: The Code for MAE, MD, Percentage Error and Data Shift can be found in the Appendix, omitted from here to avoid clutter

4.4.2 Discussion

The provided data bases provided a correlative understanding of how $B(z)$ changed with changing set up of the value b. The obtained data set follows the Hypothesis proposed, and further confirms the theory from equation 13. The correlation, as following through with Figure 6, is that as the b becomes smaller than the radius of the coil ($b \leq R$), the magnetic field density's region of maximum become sharper (as seen in correlation between $b = R$ and $b = \frac{R}{2}$). While, expanding out b to beyond R relates to two different peaks at the position of the coils. This is apparent with the theory that as the coils are closer to each other, each individual magnetic field created by each coil start to overlap and superimpose. With, at a relatively smaller distances, each individual peak start overlapping into one much larger peak. Which further supplements to the pattern mentioned in the analysis, where an decreasing b led to an increasing maximum (i.e the more overlap between the coils' individual $B(z)$ leads to a resultant peak with a higher magnetic field density). However, such an increase can also be formulated from equation 13, where as the value of b is in the denominator, and as such any increasing value of b leads to a smaller variation of the magnetic field density. When, b is let to go to zero, the resultant system is a one flat circular coil with 2N number of loops / turns (and as such twice the magnetic field density, and twice the values seen in Section 3).

The Background Magnetic Field, which was calculated to be **29.9** μT, was seen for case (a), yet not apparent in the next two cases. For the case of (b), a 3.53% error margin can be attributed to the case of this data set being just more accurate. There is no clear theoretical reasoning why the set of b=R would produce a more accurate result, but in theory should provide a larger PEM deviation from case (a) (as seen for (c)). The reason for the deviation in (c), and what would in theory cause (b) to deviate would be:

i. Mutual Inductance

When two coils are close, their magnetic field would interact more, leading to any fluctuations from one coil to induce additional noise in the other coil. This can amplify the effects of background magnetic field noise.

ii. Superposition of Local Background Noise

The noise from each coli's environment (including thermal or electronic noise) will superimpose more directly with the other coil, leading to additional background noise.

iii. Enhanced Sensitivity to Local Variations

As the two fields becomes closer to each other, the superimposed magnetic field becomes smaller and sharper. Which limits the magnetic field area, and as such makes it more sensitive to changes. Meaning, the background noise is amplified compared to if the coils were further away.

Which further supplements the reasoning why the Measured Data set for (c) is more inconsistent (relative to the theoretical data). Other minor errors in the measurements come from existing uncertainties within the sensor, any minor vibration or movement of the sensor which were overlooked, and the fact the sensor was not exactly straight along the z axis for each measurement. As the tightness of the fixing of the screws of the metallic pole for the sensor varied (however ensured to keep within reasonable similarity), the sensor would not have been exactly parallel to the z-axis (as would the theoretical data assumes), as seen in Section **3**.

The background magnetic radiation is calculated by finding the difference between an assumed perfect $B(z)$ of an ideal Solenoid ($B(z) = \mu_0 I \frac{N}{L}$), a value of $1176.91914785 \approx 1177$ μT, from max $B(z)$ (the value at the centre) of a long Solenoid, 1147 μT. Yielding a background magnetic noise as 29.9 μT.

4.4.2.1 Region of less than 5% Deviation from its Central Value

```
#The below patterns repeated for (a) for both Measured and Theoratical Data set
percent5p_regeion_3_a_1_theo = []
percent5p_regeion_3_a_2_theo = []
for i in range(len(b_2R)):
    if distance_to_overlap_b_2R[i] <= 0.76:
        if abs((b_2R[i] - max(b_2R))/max(b_2R))*100 <= 5:
            percent5p_regeion_3_a_1_theo = np.append(percent5p_regeion_3_a_1_theo, i)
    if distance_task_3_a[i] >= 0.76:
        if abs((b_2R[i] - max(b_2R))/max(b_2R))*100 <= 5:
            percent5p_regeion_3_a_2_theo = np.append(percent5p_regeion_3_a_2_theo, i)
percent5p_regeion_3_b_theo = []
for i in range(len(b_R)):
    if abs((b_R[i] - max(b_R))/max(b_R))*100 <= 5:
        percent5p_regeion_3_b_theo = np.append(percent5p_regeion_3_b_theo, i)
```

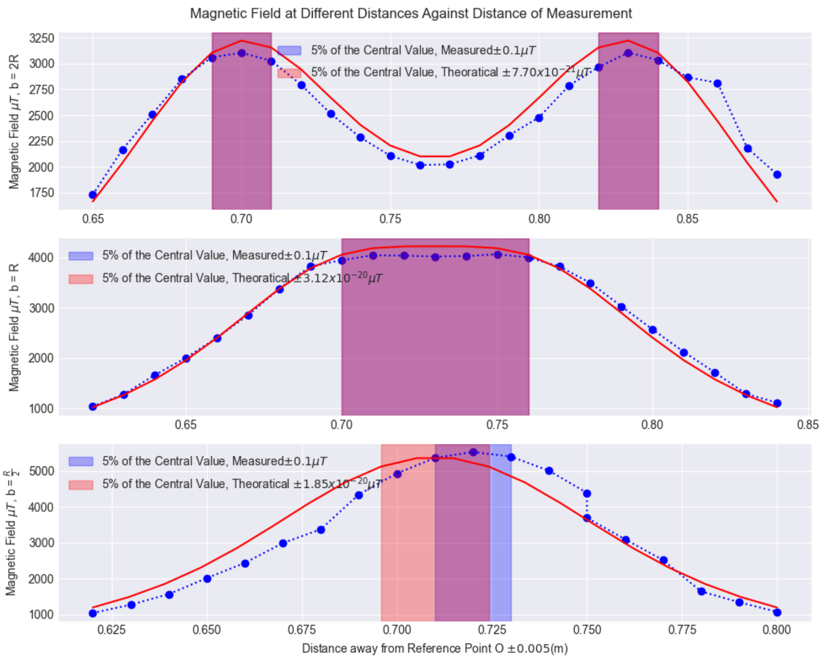

Figure 7: Region where the values within 5% of the Central Values for Figure 6
for the same cases of b (a) 2R (b) R (c) $\frac{R}{2}$

Figure 7 presents, in shaded, the regions where the values are within 5% of the Central Value (the maximum value for each Data Set). For the cases of (a) and (b) came out to be the regions of stabilised peak, i.e there is less than 100 μT between adjacent values. Which, similar to what was explored in the Analysis, are the regions of maximum. For the case of (c), the shaded regions represent the same idea, region of stabilised peaks, but there is a clear difference between the theoretical and measured data sets. This shift can be attributed to the existing PEM between the sets, with possible errors due to already discussed reasons.

4.4.3 Error Analysis

The uncertainty for the Measured Data Set is derived from the smallest value provided by the sensor (as for digital measurement devices, the uncertainty corresponds to the smallest value). While the uncertainty for the Theoretical Data Set is calculated following the equations of John R. Taylor, from An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements (Taylor):

$$\partial B(z)_t = \sqrt{(|\frac{\partial B(z)_t}{b}|\Delta b)^2 + (|\frac{\partial B(z)_t}{z}|\Delta z)^2} \tag{14}$$

, where the error is propagated through $B(z)_t$ for the only measured value in our equation 13, b and z. For all the cases of b, the uncertainty equation will be:

$$\frac{\partial B}{\partial z} = \frac{\mu_0 INR^2}{2}\left[-\frac{3\cdot\left(z-\frac{b}{2}\right)}{\left(R^2+\left(z-\frac{b}{2}\right)^2\right)^{5/2}} - \frac{3\cdot\left(z+\frac{b}{2}\right)}{\left(R^2+\left(z+\frac{b}{2}\right)^2\right)^{5/2}}\right], \tag{15}$$

$$\frac{\partial B}{\partial b} = \frac{\mu_0 INR^2}{2}\left[\frac{3\cdot\left(z-\frac{b}{2}\right)\cdot\left(-\frac{1}{2}\right)}{\left(R^2+\left(z-\frac{b}{2}\right)^2\right)^{5/2}} + \frac{3\cdot\left(z+\frac{b}{2}\right)\cdot\left(\frac{1}{2}\right)}{\left(R^2+\left(z+\frac{b}{2}\right)^2\right)^{5/2}}\right]. \tag{16}$$

$$\partial B(z) = \sqrt{\left[\frac{\mu_0 INR^2}{2}\cdot\left(-\frac{3\cdot\left(z-\frac{b}{2}\right)}{\left(R^2+\left(z-\frac{b}{2}\right)^2\right)^{5/2}} - \frac{3\cdot\left(z+\frac{b}{2}\right)}{\left(R^2+\left(z+\frac{b}{2}\right)^2\right)^{5/2}}\right)\cdot\sigma_z\right]^2 + \left[\frac{\mu_0 INR^2}{2}\cdot\left(\frac{3\cdot\left(z-\frac{b}{2}\right)\cdot\left(-\frac{1}{2}\right)}{\left(R^2+\left(z-\frac{b}{2}\right)^2\right)^{5/2}} + \frac{3\cdot\left(z+\frac{b}{2}\right)\cdot\left(\frac{1}{2}\right)}{\left(R^2+\left(z+\frac{b}{2}\right)^2\right)^{5/2}}\right)\cdot\sigma_b\right]^2}. \tag{17}$$

leading to the the Mean Error Propagation and the Largest Error as (for the range of z):

Theo Data Set	Mean Error Propagation	Largest Error (Max)
b = 2R	$\pm\ 1.75x10^{-4}$	$\pm\ 2.48x10^{-4}$
b = R	$\pm\ 2.07x10^{-4}$	$\pm\ 3.14x10^{-4}$
b = $\frac{R}{2}$	$\pm\ 2.37x10^{-4}$	$\pm\ 3.55x10^{-4}$

Table 5: Error Propagation; Mean and Maximum Values

where the above mean uncertainties were seen in Figure 6 and 7.

5 Conclusion

This report was able to implement the different coil configurations, and provide an exploration on the resultant magnetic field distribution. While experimental results qualitatively align with predictions, there were disparities in the magnitude (especially in terms of shifts) in the actual values of the experiment. However, if shifts taken into account, each experiment provided a magnitude data set with reasonably close data sets. Possible errors were explored and an explanations on how they affected the corresponding system was provided. Future work could focus on improving sensor alignment and calibration to enhance accuracy and precision.

In the case of a Long Solenoid, the error margin of 69.5% was mainly due to the shift of the data set. where, when corrected, provided a more reasonable close data set that further supplemented the idea of a systematic error. The obtained result was consistent with the hypothesis.

In the case of one flat Circular Coil, the relatively smaller shift was assumed to be from the overall thinner width of the coil set up. Taking into account the shift, the error margin reduced to a reasonable value, similar to what was seen in the last case. The obtained result was consistent with the hypothesis.

In the case of two parallel flat Circular Coils, the results provided the most accurate and lower error difference relative to the other cases. For each condition of b, the obtained result was consistent with the hypothesis.

6 Appendix

```python
def largest_pairwise_difference(data_1, data_2):
    differences = [abs(a - b) for a, b in zip(data_1, data_2)]
    max_difference = max(differences)
    return max_difference
resultA2 = largest_pairwise_difference(data_1, data_1)
def quantify_data_similarity(data_1, data_2):
    arr1 = np.array(data_1)
    arr2 = np.array(data_2)
    mae = np.mean((arr2 - arr1))
    return mae
resultB2 = quantify_data_similarity(data_1, data_2)
def accuracy_rate(pf_theo_b, pf_calc_b):
    return ((pf_calc_b-pf_theo_b)/pf_theo_b)*100
mean_accuracy = []
for i in range(20):
     mean_accuracy = np.append(mean_accuracy, accuracy_rate(data_1,
     ↪  data_2[i+1])[i])
print(np.mean(abs(mean_accuracy)))
print("Max Difference: ", resultA2, "Mean Absolute Error: ", resultB2)
```

```python
def find_shift(x, data1, data2):
    if len(data1) != len(data2):
        raise ValueError("Data1 and Data2 must have the same length.")
    correlation = correlate(data1, data2, mode='full')
    max_corr_index = np.argmax(correlation)
    shift_indices = max_corr_index - (len(data1) - 1)
    x_spacing = x[1] - x[0]
    shift_x_units = shift_indices * x_spacing

    return shift_x_units

estimated_shift = find_shift(distance_task_2, task_2_theo_values,
↪  magnetic_field_task_2)
print(estimated_shift)

```

7 References

Author Unknown. *E7e Magnetic Fields in Coils* , 2024

Taylor, John R. *An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements.* University Science Books, 2022.

"Cross-Correlation." Wikipedia, 29 May 2020, en.wikepdia.org/wiki/Cross-Correlation