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Camera calibration from vanishing points in a vision system

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Abstract

Camera calibration has been studied for many years and there are many methods available to find the parameters precisely. However, most existing methods require information of the known scene points in general three-dimensional positions for the calibration. A simple, geometrically intuitive method is proposed. The intrinsic parameters of the camera are determined by using the vanishing points in each image. The rotation matrix of the projection matrix is computed from the vanishing and image edges and the translation matrix are obtained with additional translation motion between the viewpoints. Our approach does not need any a priori information about the cameras being used. Computer simulations and real data experiments are carried out to validate our method.

Keywords: Geometrically intuitive; Intrinsic parameters; Vanishing and image edges

1. Introduction

In an active vision system, since the sensor needs to move from one place to another for performing a multi-view vision task, a traditional vision sensor with fixed structure is often inadequate for the robot to perceive the object features in an uncertain environment as the object distance and size are unknown before the robot sees it. A dynamically reconfigurable sensor can help the robot to control the configuration and gaze at the object surfaces. However, traditional calibration algorithm requires known three-dimensional (3D) coordinates of the feature points. Auto-calibration only requires the corresponding points of images, and thus provides more flexibility in practical applications. In general, auto-calibration algorithm results in a nonlinear optimization problem using constraints from the intrinsic parameters of the camera. Thus, it requires proper initialization for the nonlinear minimization. Traditional approaches of initialization assume unchanged intrinsic parameters while dealing with the situation where the intrinsic parameters of the camera may actually change.

Faugeras et al. [1] proposed a self-calibration algorithm that uses the Kruppa equation. It enforces that the planes through two camera centers that are tangent to the absolute conic should also be tangent to both of its images. Hartley [2] proposed another method based on the minimization of the difference between the internal camera parameters for the different views. Polleyfeys et al. [3] proposed a stratified approach that first recovers the affine geometry using the modulus constraint and then recover the Euclidean geometry through the absolute conic. Heyden et al. [4], Triggs [5] and Pollefeys and Van Gool [6] use explicit constraints that relate the absolute conic to its images. These formulations are interesting since they can be extended to deal with the varying internal camera parameters.

Recently self-calibration algorithms that can deal with the varying camera's intrinsic parameters were proposed. Heyden et al. [7] proposed a self-calibration algorithm that uses explicit constraints from the assumption of the intrinsic parameters of the camera. They proved that self-calibration is possible under varying cameras with the assumptions that the aspect ratio is known and there is no skew. They solved the problem using the bundle

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adjustment that requires simultaneous minimization on the all reconstructed points and cameras. Moreover, the initialization problem was not properly presented. Bougnoux [8] proposed a practical self-calibration algorithm that used the constraints derived from Heyden et al. [7]. He proposed the linear initialization step in the nonlinear minimization. He used the bundle adjustment in the projective reconstruction step. Similarly, Pollefeys et al. [9] proposed a versatile self-calibration method that can deal with a number of types of constraints about the camera. They showed a specialized version for the case where the focal length varies, possibly also the principal point. In Zhang's work, data for calibration is collected from images of the calibration object, a 1D stick with three or more markers, rotating around a fixed point [10]. Wu et al. [11] proved that the planar motion of the 1D object can be converted to a rotational one, and solve the calibration problem using the constraints of conjugate points with respect to the absolute conic. Most recently, Hammarstedt et al. [12] analyzed the critical motion patterns of 1D object for the calibration purpose and provide simplified closed-form solutions in Zhang's configuration.

Vanishing points have been used to compute the principal point and the rotation [15,16]. Thus, the calibration problem can be solved by two steps. The first step focuses on the computation of the vanishing points and the related calibration parameters. The second step computes the remaining parameters in a system with reduced degrees of freedom. In [17], the calibration equation is simplified by specifically controlled motions. However, there are usually not many vanishing points in images and they are difficult to obtain. Thus, considerable research effort has been directed towards the computation of vanishing points from images [18]. Most of the existing methods for computing vanishing points rely on line pair intersections obtained from parallel lines in the scene. In [14], a method for decoupling translation and rotation for a collection of 3D scene points that are related to 2D image points by a projection. It does not require parallel lines, but points at infinity are computed from arbitrary straight lines in the scene, and a constraint on the location of the corresponding vanishing points is obtained.

Because of its simple geometrical structure, a 1D object is easy to construct. This is the main advantage of calibrating with 1D object. In this paper, we proposed a simple approach to the construction of a 3D model by exploiting some constrains presents in the scenes to be models. Particularly, the constraints that can be used are parallelism and orthogonality in the context of architectural environments. These constraints lead to simple and geometrically method to calibrate the four intrinsic parameters of the camera from only two images from arbitrary positions.

2. Methodology

For a pinhole camera, the perspective projection from Euclidean 3-space to an image can be conveniently represented in homogeneous coordinates by a 3*4 camera matrix, P

$$\lambda_i \begin{bmatrix} u_i \\ v_i \\ 1 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}. \tag{1}$$

The projection matrix has 11 degrees of freedom and can be decomposed into the orientation and position of the camera relative to a the world co-ordinate system (a 3×3 rotation matrix R and a 3×1 translation vector T):

$$P = K[R \quad T]. \tag{2}$$

K is called the camera intrinsic matrix,

$$K = \begin{bmatrix} f_u & s & u_0 \\ 0 & f_v & v_0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{3}$$

where f_u and f_v are the scale factors in image u- and v-axes, s the parameter describing the skew of the two images axes, and (u_0, v_0) are the coordinates of the principal point.

2.1. Using vanishing points

From (1) and considering the points at infinity corresponding to the three orthogonal directions, we can derive simple constraints on the elements of the projection matrix:

$$\begin{bmatrix} \lambda_1 u_1 & \lambda_2 u_2 & \lambda_3 u_3 \\ \lambda_1 v_1 & \lambda_2 v_2 & \lambda_3 v_3 \\ \lambda_1 & \lambda_2 & \lambda_3 \end{bmatrix} = P \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \tag{4}$$

where λ_i are the initially unknown scaling factors.

According to (2), (4) can be re-expressed in terms of the camera calibration matrix K and camera orientation (rotation matrix R):

$$\begin{bmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} = KR. \tag{5}$$

2.2. Camera calibration and recovery of orientation

By exploiting the properties of the rotation matrix, R, we can rearrange (5) to recover constraints on the intrinsic parameters of the camera and the unknown scaling parameters λ_i:

$$\begin{bmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} \lambda_1^2 & 0 & 0 \\ 0 & \lambda_2^2 & 0 \\ 0 & 0 & \lambda_3^2 \end{bmatrix} \begin{bmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ 1 & 1 & 1 \end{bmatrix}^{\mathrm{T}} = KK^{\mathrm{T}}, \tag{6}$$

where

$$KK^{\mathrm{T}} = \begin{bmatrix} f_u^2 + u_0^2 & u_0 v_0 & u_0 \\ u_0 v_0 & f_v^2 + v_0^2 & v_0 \\ u_0 & v_0 & 1 \end{bmatrix}. \tag{7}$$

Under the assumption of known zero skew, (6) can be rewritten as 6 linear equations.

In our approach, the vanishing points corresponding to three mutually orthogonal directions can be used to determine the camera parameters for each viewpoint:

1. The camera calibration matrix, K under the assumption of zero skew.
2. The intrinsic parameters of the camera are determined by using the vanishing points in each image.
3. The rotation matrix of the projection matrix is computed from the vanishing points and image edges and the translation matrix are obtained with additional translation motion between the viewpoints.

3. Recovery of intrinsic parameters and projection matrix

3.1. Recovery of the principal point

Under the assumption of known zero skew, Eq. (5) can be rewritten as

$$\begin{bmatrix} \lambda_1 u_1 & \lambda_2 u_2 & \lambda_3 u_3 \\ \lambda_1 v_1 & \lambda_2 v_2 & \lambda_3 v_3 \\ \lambda_1 & \lambda_2 & \lambda_3 \end{bmatrix} = \begin{bmatrix} f_u & 0 & u_0 \\ 0 & f_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} R,$$

thus

$$R = \begin{bmatrix} \lambda_1(u_1 - u_0)/f_u & \lambda_2(u_2 - u_0)/f_u & \lambda_3(u_3 - u_0)/f_u \\ \lambda_1(v_1 - v_0)/f_v & \lambda_2(v_2 - v_0)/f_v & \lambda_3(v_3 - v_0)/f_v \\ \lambda_1 & \lambda_2 & \lambda_3 \end{bmatrix}. \tag{8}$$

The orthonormality of R can be used to provide the following equation:

$$\lambda_1\lambda_2(u_1 - u_0)(u_2 - u_0)/f_u^2 + \lambda_1\lambda_2(v_1 - v_0)(v_2 - v_0)/f_v^2 + \lambda_1\lambda_2 = 0, \tag{9}$$

$$\lambda_1\lambda_3(u_1 - u_0)(u_3 - u_0)/f_u^2 + \lambda_1\lambda_3(v_1 - v_0)(v_3 - v_0)/f_v^2 + \lambda_1\lambda_3 = 0, \tag{10}$$

$$\lambda_2\lambda_3(u_2 - u_0)(u_3 - u_0)/f_u^2 + \lambda_2\lambda_3(v_2 - v_0)(v_3 - v_0)/f_v^2 + \lambda_2\lambda_3 = 0, \tag{11}$$

since $\lambda_i \neq 0$, $f_u \neq 0$ and $f_v \neq 0$, the Eqs. (9)–(11) can be rewritten as

$$f_v^2(u_1 - u_0)(u_2 - u_0) + f_u^2(v_1 - v_0)(v_2 - v_0) + f_u^2 f_v^2 = 0, \tag{12}$$

$$f_v^2(u_1 - u_0)(u_3 - u_0) + f_u^2(v_1 - v_0)(v_3 - v_0) + f_u^2 f_v^2 = 0, \tag{13}$$

$$f_v^2(u_2 - u_0)(u_3 - u_0) + f_u^2(v_2 - v_0)(v_3 - v_0) + f_u^2 f_v^2 = 0. \tag{14}$$

Subtracting (13) from (12) gives:

$$f_v^2(u_1 - u_0)(u_2 - u_3) + f_u^2(v_1 - v_0)(v_2 - v_3) = 0. \tag{15}$$

Subtracting (14) from (12) gives:

$$f_v^2(u_2 - u_0)(u_1 - u_3) + f_u^2(v_2 - v_0)(v_1 - v_3) = 0. \tag{16}$$

Let $s = f_u^2/f_v^2$, we have

$$(u_1 - u_0)(u_2 - u_3) + s(v_1 - v_0)(v_2 - v_3) = 0, \tag{17}$$

$$(u_2 - u_0)(u_1 - u_3) + s(v_2 - v_0)(v_1 - v_3) = 0. \tag{18}$$

There are three unknown parameters in the above two equations, so we must find some more equations in order to obtain the solution.

3.2. Vanishing points constrains

Assume a pair of orthogonal and parallel line in the 3D space, the infinity points of them are $P_{1\infty}$ and $P_{2\infty}$, and the corresponding vanishing points are p_1 and p_2 on the image plane. According to the pinhole camera model, the relationship between the infinity points and their image projection is given by

$$\begin{cases} \alpha_1 p_1 = K[R \quad T]P_{1\infty} \\ \alpha_2 p_2 = K[R \quad T]P_{2\infty} \end{cases}. \tag{19}$$

According to the main characteristics of the vanishing points, if the line between the optical center O and the vanishing points is similar to the vector direction of the infinity point, the lines between the optical center O and the infinity point $P_{1\infty}$, $P_{2\infty}$ are orthogonal. The relationship between the lines gives:

$$\begin{aligned} p_1^{\mathrm{T}} K^{-\mathrm{T}} K^{-1} p_2 &= \alpha_1 P_{1\infty}^{\mathrm{T}} [R \quad T]^{\mathrm{T}} \alpha_2 [R \quad T] P_{2\infty} \\ &= \alpha_1 \alpha_2 P_{1\infty}^{\mathrm{T}} P_{2\infty} = 0. \end{aligned} \tag{20}$$

Let

$$C = K^{-\mathrm{T}} K^{-1} = \begin{bmatrix} 1/f_u^2 & 0 & -u_0/f_u^2 \\ 0 & 1/f_v^2 & -v_0/f_v^2 \\ -u_0/f_u^2 & -v_0/f_v^2 & u_0^2/f_u^2 + v_0^2/f_v^2 + 1 \end{bmatrix}.$$

If two pairs of the orthogonal and parallel lines are observed, we have

$$\begin{cases} p_1^{\mathrm{T}} K^{-\mathrm{T}} K^{-1} p_2 = 0 \\ p_3^{\mathrm{T}} K^{-\mathrm{T}} K^{-1} p_4 = 0 \end{cases}. \tag{21}$$

Hence

$$u_1u_2/f_u^2-(u_1+u_2)u_0)/f_u^2+v_1v_2/f_v^2-(v_1+v_2)v_0/f_v^2 + u_0^2/f_u^2+v_0^2/f_v^2+1=0. \quad (22)$$

$$u_3u_4/f_u^2-(u_3+u_4)u_0/f_u^2+v_3v_4/f_v^2 -(v_3+v_4)v_0)/f_v^2+u_0^2/f_u^2+v_0^2/f_v^2+1=0 \quad (23)$$

Subtracting (23) from (22) gives:

$$f_v^2(u_1u_2-u_3u_4)-f_v^2u_0(u_1+u_2-u_3-u_4) +f_u^2(v_1v_2-v_3v_4)-f_u^2v_0(v_1+v_2-v_3-v_4)=0. \quad (24)$$

Substituting $s=f_u^2/f_v^2$ into (24), we obtain (25):

$$(u_1u_2-u_3u_4)-u_0(u_1+u_2-u_3-u_4)+s(v_1v_2-v_3v_4) -sv_0(v_1+v_2-v_3-v_4)=0. \quad (25)$$

Combining (17), (18), with (25), s, u_0, v_0 can be calculated.

3.3. Obtaining λ_i^2

In order to obtain a geometric interpretation of λ_i^2, row normality must be considered. This gives:

$$\lambda_1^2(u_1-u_0)/f_u+\lambda_2^2(u_2-u_0)/f_u+\lambda_3^2(u_3-u_0)/f_u=0, \quad (26)$$

$$\lambda_1^2(v_1-v_0)/f_v+\lambda_2^2(v_2-v_0)/f_v+\lambda_3^2(v_3-v_0)/f_v=0, \quad (27)$$

$$\lambda_1^2+\lambda_2^2+\lambda_3^2=1. \quad (28)$$

Substituting (28) back into (26) and (27) gives:

$$\lambda_1^2(u_1-u_3)+\lambda_2^2(u_2-u_3)=(u_0-u_3), \quad (29)$$

$$\lambda_1^2(v_1-v_3)+\lambda_2^2(v_2-v_3)=(v_0-v_3). \quad (30)$$

Hence

$$\lambda_1^2=\frac{(v_0-v_3)(u_2-u_3)-(u_0-u_3)(u_2-u_3)}{(v_1-v_3)(u_2-u_3)-(u_0-u_3)(v_2-v_3)},$$
$$\lambda_2^2=\frac{(v_1-v_3)(u_0-u_3)-(u_1-u_3)(v_0-v_3)}{(v_1-v_3)(u_2-u_3)-(u_1-u_3)(v_2-v_3)},$$
$$\lambda_3^2=1-\lambda_1^2-\lambda_2^2.$$

Since u_0, v_0 and λ_i^2 have been determined in the previous sections, the left side of (6) is known. So, we have

$$f_u=\sqrt{\lambda_1^2(v_1-v_0)^2+\lambda_2^2(v_2-v_0)^2+\lambda_3^2(v_{13}-v_0)^2-u_0^2}$$
$$f_v=\sqrt{f_u^2/s}.$$

As u_0, v_0, f_u, f_v and λ_i^2 have been obtained, the rotation matrix R can be determined according to (8).

3.4. Recovery of the translation matrix T

The fourth column of the projection matrix depends on the position of the world co-ordinate system relative to the camera coordinate system. An arbitrary reference point can be chosen as the origin. Its image coordinates fix the translation, T, up to an arbitrary scale factor λ_4:

$$\lambda_4\begin{bmatrix}u_4\\v_4\\1\end{bmatrix}=\begin{bmatrix}p_{11}&p_{12}&p_{13}&p_{14}\\p_{21}&p_{22}&p_{23}&p_{24}\\p_{31}&p_{32}&p_{33}&p_{34}\end{bmatrix}\begin{bmatrix}0\\0\\0\\1\end{bmatrix}=KT, \quad (31)$$

hence

$$\lambda_4=P_{34}=t_z=Z, \quad (32)$$

$$\lambda_4u_4=P_{14}=f_ut_x+u_0\lambda_4, \quad (33)$$

$$\lambda_4v_4=P_{24}=f_vt_y+v_0\lambda_4. \quad (34)$$

With a single viewpoint without metric information, λ_4 will be an arbitrarily value. So, additional views with the image correspondence of a fifth point are required to fix this scale factor. This is equivalent to recovering pure unknown translation from the translational component of image motion under known rotation. So, only two point correspondences are required to recover the direction of translation.

Let P be a visible point in the scene as in Fig. 1, $X=(X,Y,Z)^{\mathrm{T}}$ and $X'=(X',Y',Z')^{\mathrm{T}}$ be its 3D coordinates with respect to the two viewpoints. $(x,y)^{\mathrm{T}}$ and $(x',y')^{\mathrm{T}}$ are the image coordinates of P on the left and right images, respectively. The general motion equation of the camera is

$$X'=R(X-T), \quad (35)$$

where T is an unknown translation vector $(T_x,T_y,T_z)^{\mathrm{T}}$ and we can assume that it has unit length $\|T\|^2=1$. Therefore there are only two unknowns for the translation vector. R is a rotation matrix. In our case R is known.

A general relationship between the two sets of image coordinates—a relationship which expresses the condition that corresponding rays through the two centers of projection must intersect in space—can be established as in [13]:

$$x'^{\mathrm{T}}Ex=0, \quad (36)$$

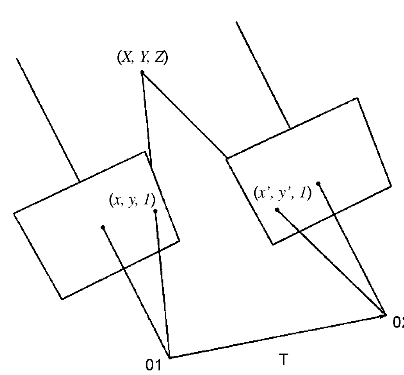

Fig. 1. The imaging geometry (camera undergoes translational motion). O1 and O2 are the camera positions. T is the unknown camera translation vector.

where $x = (x, y, 1)^T$, $x' = (x', y', 1)^T$ are the matched image points in the two views. The essential matrix E is defined as

$$E = RS, \tag{37}$$

where S is a skew-symmetric matrix

$$S = \begin{pmatrix} 0 & T_z & -T_y \\ -T_z & 0 & T_x \\ T_y & -T_x & 0 \end{pmatrix}.$$

When $R = I$, i.e. there is no rotation involved during the camera motion, we have $E = S$. Therefore Eq. (36) becomes:

$$x'^T S x = 0, \tag{38}$$

thus

$$(x', y', 1) \begin{pmatrix} 0 & T_z & -T_y \\ -T_z & 0 & T_x \\ T_y & -T_x & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = 0. \tag{39}$$

We can obtain a linear equation from (39) via the elements of the translation vector **T**:

$$(y' - y)T_x - (x' - x)T_y - (x'y - xy')T_z = 0. \tag{40}$$

For each pair of matched points, there will be one homogeneous equation showing the relationship among the variables T_x, T_y, T_z, and the corresponding points on the images. If a set of homogeneous equations as in (40) is found, the ratios of the three unknowns of **T** can therefore be obtained.

Then, from $x' - u_0 = f_u X'/Z'$ and $y' - v_0 = f_v Y'/Z'$, it follows that:

$$\begin{aligned} x' - u_0 = f_u X'/Z' &= (X - T_z)/(Z - T_z) \\ &= (x - u_0 - T_x/Z)/(1 - T_z/Z). \end{aligned} \tag{41}$$

Rearranging the above equation, Z can therefore be derived as

$$Z = (T_x - (x' - u_0)T_z)/(x - x'). \tag{42}$$

According to (32), we know $Z = \lambda_4 = t_z$. Substituting λ_4 into (32), (33), t_x and t_y are calculated following (43), (44)

$$t_x = \lambda_4 (u_4 - u_0)/f_u, \tag{43}$$

$$t_y = \lambda_4 (v_4 - v_0)/f_v. \tag{44}$$

So, the translation matrix **T** is recovered.

4. Experiments and results

4.1. Finding the vanishing points

The first step in the algorithm to recover the projection matrices requires finding the vanishing points of the parallel lines with known orientations. A vanishing point corresponds to the projection of the intersection of parallel lines at infinity.

4.2. Projection matrices

Having found the vanishing points, the second step to recover the parameters u_0, v_0, f_u, f_v and λ_i scale factors. Then, the rotation matrix can be determined from (8). Finally, when λ_4 is obtained, the translation matrix is obtained by (32), (43), and (44).

4.3. Experiments

Simulation studies are conducted first. The simulated camera has the image resolution of 1280×1024. The 3D points were generated randomly inside a patter of size 100×100 mm that was 230 mm away from the center of the

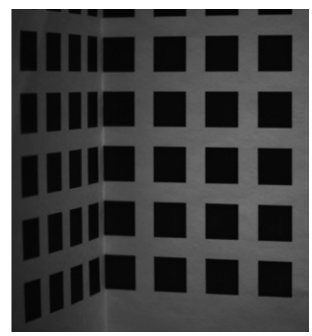

Fig. 2. Original un-calibrated pattern graph.

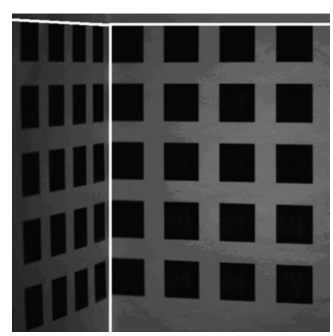

Fig. 3. Primitive definition and localization of the coordinate.

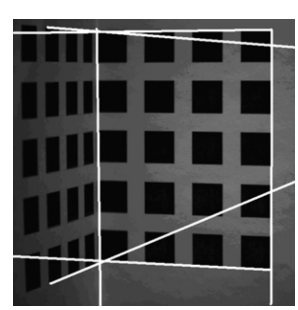

Fig. 4. Finding vanishing points in the camera calibration.

camera. About 40 square edge points are used to calibrate the camera in the patter, as shown in Fig. 2. The primitive definition of the coordinate is defined in Fig. 3. The process of the finding vanishing points is shown in Fig. 4.

The calibration results are given in Tables 1 and 2.

Table 1
Calibration results with synthetic data

Parameter	Camera
Intrinsic parameters	
f_u	1127.61
f_v	1093.8
f_u/f_v	0.97
(u_0, v_0)	(556.43, 559.29)
Outer parameters	
Rotation matrix	$\begin{bmatrix} 0.976 & -0.016 & 0.217 \\ -0.017 & 0.989 & 0.145 \\ -0.217 & 0.146 & 0.964 \end{bmatrix}$

Table 2
The exact and obtained values of **T**

Parameter	Camera
Translation matrix	Exact $(0.965, 0.000, 0.259)^T$ Obtained $(0.959, 0.003, 0.243)^T$

A series of four images of a grid (shown in Fig. 5) has been acquired with a Marlin F080B, 1032 × 778 pixel, and 8 bit camera. The lens used in the vision system is M0814-MP, with a focal length of 8 mm. The calibration results are given in Table 3.

From the experiments, it can be seen that the calibration method is easy to implement and has high accuracy.

5. Conclusions

We have presented a novel method for recovering the projection matrix from 3D space to the CCD image plane that exploits the use of vanishing points of three orthogonal directions. The simple but powerful constraints of parallelism and orthogonality in images can be used to recover very precise projection matrices with only a few point and line correspondences. The technique presented has been successfully used for camera calibration.

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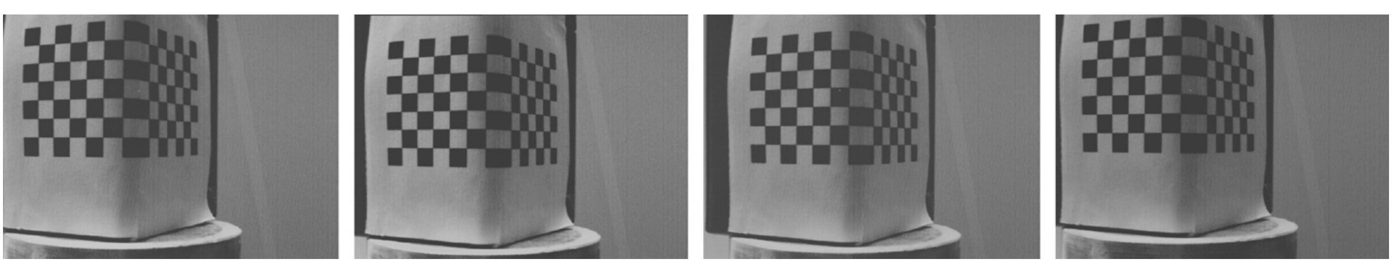

Fig. 5. Four frames from the real image series.

Table 3
Calibration results with real data

Parameters	One frame	Two frame	Three frame	Four frame
Intrinsic parameters				
f_u	907.39	904.93	904.14	905.89
f_v	843.78	849.72	849.89	842.47
f_u/f_v	0.93	0.939	0.94	0.93
(u_0, v_0)	(532.25, 401.39)	(526.43, 396.03)	(536.78, 405.85)	(528.58, 393.29)
Outer parameters				
Rotation matrix	$\begin{bmatrix} 0.955 & 0.051 & -0.317 \\ -0.007 & 0.985 & 0.135 \\ -0.312 & -0.138 & 0.949 \end{bmatrix}$	$\begin{bmatrix} 0.979 & 0.023 & 0.180 \\ -0.017 & 0.964 & 0.258 \\ -0.175 & -0.246 & 0.952 \end{bmatrix}$	$\begin{bmatrix} 0.969 & 0.021 & 0.246 \\ -0.028 & 0.959 & 0.283 \\ -0.237 & -0.254 & 0.941 \end{bmatrix}$	$\begin{bmatrix} 0.975 & 0.017 & 0.221 \\ -0.015 & 0.989 & 0.146 \\ -0.218 & -0.146 & 0.964 \end{bmatrix}$
Translation matrix	$(-3.489, 5.341, 6.215)^T$	$(-4.955, 4.345, 7.021)^T$	$(-9.266, 8.313, 5.529)^T$	$(-5.432, 6.712, 7.057)^T$

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