Presentación 2

1. **Si** $f = f(r)$ **con** $r = \sqrt{x^2+y^2+z^2}$**, demuestra que**

$$\nabla f(r) = \hat{r}\frac{df(r)}{dr}$$

Por la ec. (6), y usando coordenadas esféricas

$$\nabla f(r) = \sum_{i=1}^{3} \frac{\hat{e}_i}{h_i}\frac{\partial f(r)}{\partial \theta} = \frac{\hat{e}_r}{h_r}\frac{\partial f(r)}{\partial r} + \frac{\hat{e}_\theta}{h_\theta}\frac{\partial f(r)}{\partial \theta} + \frac{\hat{e}_\phi}{h_\phi}\frac{\partial f(r)}{\partial \phi}$$

y como f solo depende de r, entonces

$$\nabla f(r) = \frac{\hat{e}_r}{h_r}\frac{\partial f(r)}{\partial r}$$

ahora, como $\mathbf{r} = \hat{i}rsen\theta cos\phi + \hat{j}rsen\theta sen\phi + \hat{k}rcos\theta$

$$h_r = \|\frac{\partial \mathbf{r}}{\partial r}\| = \sqrt{sen^2\theta cos^2\phi + sen^2\theta sen^2\phi + cos^2\theta} = \sqrt{sen^2\theta\cancel{[cos^2\phi + sen^2\phi]} + cos^2\theta}$$

$$= \sqrt{sen^2\theta + cos^2\theta} = 1$$

así

$$\nabla f(r) = \hat{e}_r\frac{\partial f}{\partial r} = \hat{r}\frac{\partial f}{\partial r}$$

2. **Demuestra que el campo eléctrico de una carga puntual**

$$E = \frac{q\hat{r}}{4\pi\epsilon_o r^2}$$

cumple $\nabla \cdot E = 0$**, para** $r \neq 0$

Por la ec. (8), y usando coordenadas esféricas

$$DivE = \frac{1}{h}\sum_{i=1}^{3}\frac{\partial}{\partial u_i}\left(\frac{E_i h}{h_i}\right) = \frac{1}{h}\left[\frac{\partial}{\partial u_r}\left(\frac{E_r h}{h_r}\right) + \frac{\partial}{\partial u_\theta}\left(\frac{E_\theta h}{h_\theta}\right) + \frac{\partial}{\partial u_\phi}\left(\frac{E_\phi h}{h_\phi}\right)\right]$$

y como E solo tiene componente E_r

$$\nabla \cdot E = \frac{1}{h}\frac{\partial}{\partial r}\left(\frac{E_r h}{h_r}\right)$$

y como ya tenemos h_r, solo fatan h_θ y h_ϕ

$$h_\theta = \|\frac{\partial \mathbf{r}}{\partial \theta}\| = \sqrt{r^2cos^2\theta cos^2\phi + r^2cos^2\theta sen^2\phi + r^2sen^2\theta} = \sqrt{r^2\left[cos^2\theta cos^2\phi + cos^2\theta sen^2\phi + sen^2\theta\right]}$$

$$= r\sqrt{cos^2\theta(\cancel{cos^2\phi + \text{sen}^2\phi}) + sen^2\theta} = r\sqrt{\cancel{cos^2\theta + sen^2\theta}} = r$$

$$h_\phi = \|\frac{\partial \mathbf{r}}{\partial \theta}\| = \sqrt{r^2 sen^2\theta sen^2\phi + r^2 sen^2\theta cos^2\phi} = \sqrt{r^2\left[sen^2\theta sen^2\phi + sen^2\theta cos^2\phi\right]}$$

$$= r\sqrt{sen^2\theta(\cancel{cos^2\phi + sen^2\phi})} = rsen\theta$$

así

$$\nabla \cdot E = \frac{1}{h_r h_\theta h_\phi}\frac{\partial}{\partial r}\left(\frac{q\cancel{h_r}h_\theta h_\phi}{4\pi\epsilon_o r^2 \cancel{h_r}}\right) = \frac{\cancel{sen\theta}}{r^2\cancel{sen\theta}}\cancel{\frac{\partial}{\partial r}\left(\frac{q\cancel{r^2}}{4\pi\epsilon_o \cancel{r^2}}\right)} = 0$$

para $r \neq 0$ (así no se indetermina)

3. **La ley de Gauss para el campo eléctrico tiene la forma :**

$$\oint E \cdot dS = \frac{q}{\epsilon_o}$$

donde $q = \int \rho dV$ es la carga encerada en la superficie y ρ su densidad volumétrica. Demuestra la ley de Gauss en su forma diferencial

$$\nabla \cdot E = \frac{\rho}{\epsilon_o}$$

Por el teorema de la divergencia,

$$\oint E \cdot dS = \int \nabla \cdot E dV = \frac{q}{\epsilon_o} = \frac{1}{\epsilon_o}\int \rho dV$$

como la expresión anterior debe ser cierta para cualquier volumen(incluso uno infinitesimal) se tiene que se pueden igualar los integrandos

$$\nabla \cdot E = \frac{\rho}{\epsilon_o}$$

4. **Demuestra que :** $\nabla \times (\phi A) = \phi \nabla \times A + \nabla\phi \times A$

Usando la ec. (10) en su forma matricial:

$$\nabla \times (\phi \mathbf{A}) = \frac{1}{h}\begin{vmatrix} h_1\hat{e}_1 & h_2\hat{e}_2 & h_3\hat{e}_3 \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \\ \phi A_1 h_1 & \phi A_2 h_2 & \phi A_3 h_3 \end{vmatrix}$$

$$= \frac{1}{h}[h_1\left(\frac{\partial}{\partial u_2}(\phi A_3 h_3) - \frac{\partial}{\partial u_3}(\phi A_2 h_2)\right)\hat{e}_1 - h_2\left(\frac{\partial}{\partial u_1}(\phi A_3 h_3) - \frac{\partial}{\partial u_3}(\phi A_1 h_1)\right)\hat{e}_2$$

$$+h_3\left(\frac{\partial}{\partial u_1}(\phi A_2 h_2)-\frac{\partial}{\partial u_2}(\phi A_1 h_1)\right)\hat{e}_3]$$

o más compacto, usando notación de índices:

$$\nabla\times(\phi\mathbf{A})_i=\frac{h_i}{h}\epsilon_{ijk}\frac{\partial}{\partial u_j}(\phi A_k h_k)$$

ahora metamos el diferencial al paréntesis y reordenemos

$$\nabla\times(\phi\mathbf{A})_i=\frac{h_i}{h}\epsilon_{ijk}\left[\frac{\partial}{\partial u_j}(\phi)A_k h_k+\phi\frac{\partial}{\partial u_j}(A_k h_k)\right]$$

cambiemos a la notación presentada en el material adicional de índices

$$=\frac{h_i}{h}\epsilon_{ijk}\left[\phi\partial_j(A_k h_k)+\partial_j(\phi)A_k h_k\right]$$

$$\frac{h_i}{h}\phi\epsilon_{ijk}\partial_j(A_k h_k)+\frac{h_i}{h}\epsilon_{ijk}\partial_j(\phi)A_k h_K$$

$$=\phi(\frac{h_i}{h}\epsilon_{ijk}\partial_j A_k h_k)+(\frac{h_i}{h}\epsilon_{ijk}\partial_j(\phi)A_k h_k)$$

$$=\phi(\nabla\times\mathbf{A})_i+(\nabla\phi\times\mathbf{A})_i$$

$$\therefore\nabla\times(\phi\mathbf{A})=\phi\nabla\times\mathbf{A}+\nabla\phi\times\mathbf{A}$$

5. **El campo electroestático de un dipolo eléctrico** $p=p_o\hat{e}_z$ **es**

$$E=\frac{p_o(2\hat{e}_r cos\theta+\hat{e}_\theta sen\theta)}{r^3}$$

Demuestra que:

a) $\nabla\times E=0$

b) para $r\neq 0$, se tiene $\nabla\cdot E=0$

Suponiendo que E está en esféricas, entonces:(y usando los factores de escala calculados en los problemas 1 y 2)

$$\nabla\times E=\frac{1}{h}\begin{vmatrix}h_1\hat{e}_r & h_2\hat{e}_\theta & h_3\hat{e}_\phi\\ \frac{\partial}{\partial u_r} & \frac{\partial}{\partial u_\theta} & \frac{\partial}{\partial u_\phi}\\ E_r h_r & E_\theta h_\theta & E_\phi h_\phi\end{vmatrix}$$

$$=\frac{1}{r^2 sen\theta}[\left(\frac{\partial}{\partial\theta}\cancel{(E_\phi r sen\theta)}-\frac{\partial}{\partial\phi}(E_\theta r)\right)\hat{e}_r+r\left(\frac{\partial}{\partial\phi}(E_r)-\frac{\partial}{\partial r}\cancel{(E_\phi r sen\theta)}\right)\hat{e}_\theta+$$

$$r sen\theta \left(\frac{\partial}{\partial r}(E_\theta r) - \frac{\partial}{\partial \theta}(E_r) \right) \hat{e}_\phi]$$

$$= \frac{1}{r^{\cancel{2}}\cancel{sen\theta}} \left[-\cancel{\frac{\partial}{\partial \phi}(\frac{\rho_0 sen\theta r}{r^3})}\hat{e}_r + r\cancel{\frac{\partial}{\partial \phi}(\frac{2\rho_0 cos\theta}{r^3})}\hat{e}_\theta + \cancel{rsen\theta}(\frac{\partial}{\partial r}(\frac{\rho_0 sen\theta r}{r^2}) - \frac{\partial}{\partial \theta}(\frac{2\rho_0 cos\theta}{r^3}))\hat{e}_\phi \right]$$

$$= \frac{1}{r}\left((0)\hat{e}_r + (0)\hat{e}_\theta + (\cancel{\frac{-2\rho_0 sen\theta}{r^3} - (-\frac{2\rho_0 sen\theta}{r^3})})\hat{e}_\phi \right)$$

$$= (0,0,0)\forall r \neq 0$$

$$\nabla \cdot E = \frac{1}{h}\left[\frac{\partial}{\partial r}(\frac{E_r h}{h_r}) + \frac{\partial}{\partial \theta}(\frac{E_\theta h}{h_\theta}) + \frac{\partial}{\partial \phi}(\frac{E_\phi h}{h_\phi}) \right]$$

$$= \frac{1}{r^2 sen\theta}\left[\frac{\partial}{\partial r}(E_r r^2 sen\theta) + \frac{\partial}{\partial \theta}(\frac{E_\theta r^{\cancel{2}} sen\theta}{\cancel{r}}) + \frac{\partial}{\partial \phi}(\frac{\cancel{E_\phi} r^{\cancel{2}}\cancel{sen\theta}}{\cancel{rsen\theta}}) \right]$$

$$= \frac{1}{r^2 sen\theta}\left[\frac{\partial}{\partial r}(\frac{2\rho_0 cos\theta \cancel{r^2} sen\theta}{r^{\cancel{3}}}) + \frac{\partial}{\partial \theta}(\frac{\rho_0 sen^2\theta \cancel{r}}{r^{\cancel{3}}}) \right]$$

$$= \frac{1}{r^2 sen\theta}\left[\frac{-2\rho_0 cos\theta sen\theta}{r^2} + \frac{2\rho_0 sen\theta cos\theta}{r^2} \right] = \frac{1}{r^2 sen\theta}[0] = 0$$

6. **Demuestra que el Laplaciano en coordenadas cilíndricas y esféricas es el que se presenta, para ello tendrás que calcular los respectivos factores de escala.**

   de la ecuación (12) se sigue que:

$$\nabla^2 f = \frac{1}{h}\sum_{i=1}^{3} \frac{\partial}{\partial u_i}(\frac{h}{h_i^2}\frac{\partial f}{\partial u_i})$$

   con coordenadas esféricas, del ejercicio 1 y 2 se tiene:

$$h_r = 1; h_\theta = r; h_\psi = r \operatorname{sen}\theta$$

$$\therefore \nabla^2 f = \frac{1}{r^2 \operatorname{sen}\theta}\left[\frac{\partial}{\partial r}(r^2 \operatorname{sen}\theta \frac{\partial f}{\partial r}) + \frac{\partial}{\partial \theta}(\frac{\cancel{r^2} \operatorname{sen}\theta}{\cancel{r^2}}\frac{\partial f}{\partial \theta}) + \frac{\partial}{\partial \psi}(\frac{\cancel{r^2 \operatorname{sen}\theta}}{\cancel{r^2} \operatorname{sen}\theta^2}\frac{\partial f}{\partial \psi}) \right]$$

$$= \frac{1}{r^2 \operatorname{sen}\theta}\left[\operatorname{sen}\theta \frac{\partial}{\partial r}(r^2 \frac{\partial f}{\partial r}) + \frac{\partial}{\partial \theta}(\operatorname{sen}\theta \frac{\partial f}{\partial \theta}) + \frac{1}{\operatorname{sen}\theta}\frac{\partial^2 f}{\partial \psi^2} \right]$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \frac{\partial f}{\partial r}) + \frac{1}{r^2 \operatorname{sen}\theta}\frac{\partial}{\partial \theta}(\operatorname{sen}\theta \frac{\partial f}{\partial \theta}) + \frac{1}{r^2 \operatorname{sen}\theta}\frac{\partial^2 f}{\partial \psi^2}$$

en coordenadas cilíndricas, dado que:

$$\mathbf{r} = \hat{\imath}\rho\cos\psi + \hat{\jmath}\rho\operatorname{sen}\psi + \hat{k}z$$

entonces:

$$\frac{\partial\mathbf{r}}{\partial\rho} = \hat{\imath}\cos\psi + \hat{\jmath}\operatorname{sen}\psi$$

$$\frac{\partial\mathbf{r}}{\partial\psi} = -\hat{\imath}\rho\operatorname{sen}\psi + \hat{\jmath}\rho\cos\psi$$

$$\frac{\partial\mathbf{r}}{\partial z} = \hat{k}$$

así:

$$h_\rho = \|\frac{\partial\mathbf{r}}{\partial\rho}\| = \sqrt{\cos^2\psi + \operatorname{sen}^2\psi} = 1$$

$$h_\psi = \|\frac{\partial\mathbf{r}}{\partial\psi}\| = \sqrt{\rho^2\operatorname{sen}^2\psi + \rho^2\cos^2\psi} = \rho\sqrt{\operatorname{sen}^2\psi + \cos^2\psi} = \rho$$

$$h_z = \|\frac{\partial\mathbf{r}}{\partial z}\| = \sqrt{1} = 1$$

$$\therefore \nabla^2 f = \frac{1}{\rho}\left[\frac{\partial}{\partial\rho}(\rho\frac{\partial f}{\partial\rho}) + \frac{\partial}{\partial\psi}(\frac{\cancel{\rho}}{\rho^{\cancel{2}}\frac{\partial f}{\partial\psi}) + \frac{\partial}{\partial z}(\rho\frac{\partial f}{\partial z})}\right]$$

$$= \frac{1}{\rho}\left[\frac{\partial}{\partial\rho}(\rho\frac{\partial f}{\partial\rho}) + \frac{1}{\rho}\frac{\partial^2 f}{\partial\psi^2} + \rho\frac{\partial^2 f}{\partial z^2}\right]$$

$$= \frac{1}{q}\frac{\partial}{\partial\rho}(\rho\frac{\partial f}{\partial\rho}) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial\psi^2} + \frac{\partial^2 f}{\partial z^2}$$

Presentación 3

1. **Para el sistema de coordenadas esferoidales prolatas (ξ, η, ϕ), cuyas reglas de transformación son:**

$$x = a\operatorname{senh}\xi\operatorname{sen}\eta\cos\phi$$
$$y = a\operatorname{senh}\xi\operatorname{sen}\eta\operatorname{sen}\phi$$
$$z = a\cosh\xi\cos\eta$$

a) **Describe las superficies coordenadas del sistema**

Siguiendo el procedimiento de las notas, elevemos al cuadrado x, y, z:

$$x^2 = a^2 \operatorname{senh}^2 \xi \operatorname{sen}^2 \eta \cos^2 \phi$$

$$y^2 = a^2 \operatorname{senh}^2 \xi \operatorname{sen}^2 \eta \operatorname{sen}^2 \phi$$

$$z^2 = a^2 \cosh^2 \xi \cos^2 \eta$$

Ahora desarrollando:

$$\frac{x^2+y^2}{a^2 \operatorname{senh}^2 \xi} + \frac{z^2}{a^2 \cosh^2 \xi} = \frac{\cancel{a^2 \operatorname{senh}^2 \xi} \operatorname{sen}^2 \eta \cancel{(\cos^2 \phi + \operatorname{sen}^2 \phi)}}{\cancel{a^2 \operatorname{senh}^2 \xi}} + \frac{\cancel{a^2 \cosh^2 \xi} \cos^2 \eta}{\cancel{a^2 \cosh^2 \eta}}$$

$$= \cancel{\operatorname{sen}^2 \eta + \cos^2 \eta} = 1$$

$$\frac{x^2+y^2}{a^2 \operatorname{senh}^2 \eta} - \frac{z^2}{a^2 \cosh^2 \eta} = \frac{\cancel{a^2 \operatorname{sen}^2 \eta} \operatorname{senh}^2 \xi \cancel{(\cos^2 \phi + \operatorname{sen}^2 \phi)}}{\cancel{a^2 \operatorname{sen}^2 \eta}} - \frac{\cancel{a^2 \cos^2 \eta} \cosh^2 \xi}{\cancel{a^2 \cos^2 \eta}}$$

$$= \operatorname{senh}^2 \xi - \cosh^2 \xi = -\cancel{(\cosh^2 \xi - \operatorname{senh}^2 \xi)} = 1$$

y así,

$$\frac{x^2+y^2}{a^2 \operatorname{senh}^2 \xi} + \frac{z^2}{a^2 \cosh^2 \xi} = 1 \tag{1}$$

$$\frac{z^2}{a^2 \cos^2 \eta} - \frac{x^2+y^2}{a^2 \operatorname{sen}^2 \eta} = 1 \tag{2}$$

Si $\xi = cte$, la ec. (1) tiene la forma de una elipsoide degenerada (esferoide) $(\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{b^2} = 1)$ con ejes, $a \operatorname{senh} \xi$ y $a \cosh \xi$

Si $\eta = cte$, la ec. (2) tiene la forma de un hiperboloide(hiperboloide de revolución) $(-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1)$

Si

$$\phi = cte \Rightarrow r^2 = x^2 + y^2 + (z \pm a)^2 = x^2 + y^2 + z^2 \pm 2az + a^2$$

$$= a^2 \operatorname{senh}^2 \xi \operatorname{sen}^2 \eta \cos^2 \phi + a^2 \operatorname{senh}^2 \xi \operatorname{sen}^2 \eta \operatorname{sen}^2 \phi + a^2 \cosh^2 \xi \cos^2 \eta \pm 2a^2 \cos \xi \cos \eta + a^2$$

$$= a^2[\operatorname{senh}^2 \xi \operatorname{sen}^2 \eta \cancel{(\operatorname{sen}^2 \phi + \cos^2 \phi)} + \cosh^2 \xi \cos^2 \eta \pm 2 \cos \xi \cos \eta + 1]$$

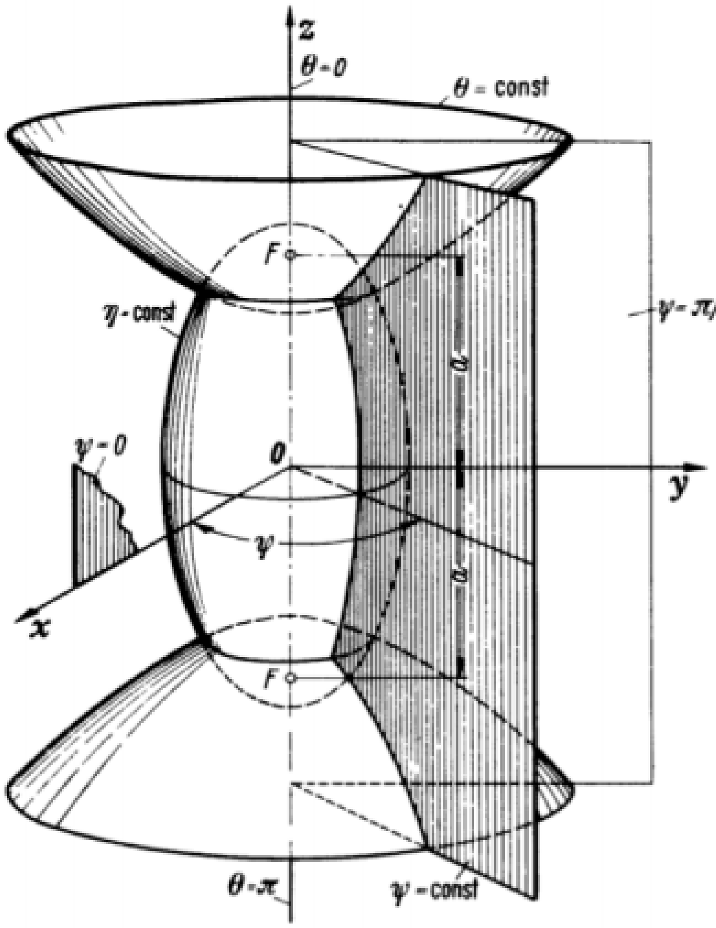
$$= a^2[(\cosh^2\xi - 1)\operatorname{sen}^2\eta + \cosh^2\xi\cos^2\eta \pm 2\cos\xi\cos\eta + 1]$$

$$= a^2[cosh^2\xi\cancel{(\operatorname{sen}^2\eta + cos^2\eta)} - \operatorname{sen}^2\eta \pm 2\cos\xi\cos\eta + 1]$$

$$= a^2[\cosh^2\xi + \cos^2\eta - \cancel{1} \pm 2\cos\xi\cos\eta + \cancel{1}]$$

$$= a^2(\cos\xi \pm \cos\eta)^2$$

Lo que describe un plano (o la mitad de uno)



b) **Calcula de manera explícita los factores de escala** (h_ξ, h_η, h_ϕ)
Como $\mathbf{r} = a\operatorname{senh}\xi\operatorname{sen}\eta\cos\phi\hat{\imath} + a\operatorname{senh}\xi\operatorname{sen}\eta\operatorname{sen}\phi\hat{\jmath} + a\cosh\xi\cos\eta\hat{k}$
así,

$$h_\xi = \|\frac{\partial\mathbf{r}}{\partial\xi}\| = \sqrt{a^2\operatorname{sen}^2\eta\cos^2\phi\cosh^2\phi + a^2\operatorname{sen}^2\eta + \operatorname{sen}^2\phi\cosh^2\xi + a^2\cos^2\eta\operatorname{senh}^2\xi}$$

$$= a\sqrt{\operatorname{sen}^2\eta\cosh^2\xi\cancel{(\cos^2\phi + \operatorname{sen}^2\phi)} + \cos^2\eta\operatorname{senh}^2\xi}$$

$$= a\sqrt{\operatorname{sen}^2\eta(\operatorname{senh}^2\xi + 1) + \cos^2\eta\operatorname{senh}^2\xi}$$

$$= a\sqrt{\operatorname{senh}^2\xi(\operatorname{sen}^2\eta + \cos^2\eta) + \operatorname{sen}^2\eta} = a\sqrt{\operatorname{senh}^2\xi + \operatorname{sen}^2\eta}$$

$$h_\eta = \|\frac{\partial \mathbf{r}}{\partial \eta}\| = \sqrt{a^2\operatorname{senh}^2\xi\cos^2\eta\cos^2\phi + a^2\operatorname{senh}^2\xi\cos^2\eta\operatorname{sen}^2\phi + a^2\cosh^2\xi\operatorname{sen}^2\eta}$$

$$= a\sqrt{\operatorname{senh}^2\xi\cos^2\eta\cancel{(\cos^2 phi + \operatorname{sen}^2\phi)} + \cosh^2\xi\operatorname{sen}^2\eta}$$

$$= a\sqrt{\operatorname{senh}^2\xi(1-\operatorname{sen}^2\eta) + \cosh^2\xi\operatorname{sen}^2\eta}$$

$$= a\sqrt{\operatorname{senh}^2\xi + \operatorname{sen}^2\eta\cancel{(\cosh^2\xi - \operatorname{senh}^2\xi)}} = h_\xi$$

$$h_\phi = \|\frac{\partial \mathbf{r}}{\partial \phi}\| = \sqrt{a^2\operatorname{senh}^2\xi\operatorname{sen}^2\eta\operatorname{sen}^2\phi + a^2\operatorname{sen}^2\xi\operatorname{sen}^2\eta\cos^2\phi}$$

$$= a\sqrt{\operatorname{senh}^2\xi\operatorname{sen}^2\eta\cancel{(\operatorname{sen}^2\phi + \cos^2\phi)}} = a\operatorname{senh}\xi\operatorname{sen}\eta$$

2. **Ocupando el mismo sistema de coordenadas esferoidales prolatas (ξ, η, ψ) del ejercicio anterior:**

   a) **Calcula los operadores diferenciables $\nabla\phi$, $\nabla \cdot B$, $\nabla \times B$ y $\nabla^2\phi$**
   Usaremos los facores de escala calculados anteriormente:

$$\nabla\phi = \sum \frac{\hat{e_i}}{h_i}\frac{\partial\phi}{\partial u_i} = \frac{\hat{e_\xi}}{h_\xi}\frac{\partial\phi}{\partial\xi} + \frac{\hat{e_\eta}}{h_\eta}\frac{\partial\phi}{\partial\eta} + \frac{\hat{e_\psi}}{h_\psi}\frac{\partial\phi}{\partial\psi}$$

$$= \frac{\hat{e_\xi}}{a\sqrt{\operatorname{senh}^2\xi + \operatorname{sen}^2\eta}}\frac{\partial\phi}{\partial\xi} + \frac{\hat{e_\eta}}{a\sqrt{\operatorname{senh}^2\xi + \operatorname{sen}^2\eta}}\frac{\partial\phi}{\partial\eta} + \frac{\hat{e_\psi}}{a\operatorname{senh}\xi\operatorname{sen}\eta}\frac{\partial\phi}{\partial\psi}$$

$$= \frac{1}{a}\left[\frac{1}{\sqrt{\operatorname{senh}^2\xi + \operatorname{sen}^2\eta}}(\hat{e_\xi}\frac{\partial\phi}{\partial\xi} + \hat{e_\eta}\frac{\partial\phi}{\partial\eta}) + \frac{\hat{e_\psi}}{\operatorname{senh}\xi\operatorname{sen}\eta}\frac{\partial\phi}{\partial\psi}\right]$$

$$\nabla \cdot B = \frac{1}{h}\sum\frac{\partial}{\partial u_i}\left(\frac{B_i h}{h_i}\right) = \frac{1}{h}\left[\frac{\partial}{\partial\xi}(\frac{B_\xi \cancel{h}}{\cancel{h_\xi}}) + \frac{\partial}{\partial\eta}(\frac{B_\eta \cancel{h}}{\cancel{h_\eta}}) + \frac{\partial}{\partial\psi}(\frac{B_\psi \cancel{h}}{\cancel{h_\psi}}\right]$$

$$= \frac{1}{h_\xi h_\eta h_\psi}\left[\frac{\partial}{\partial\xi}(B_\xi h_\eta h_\psi) + \frac{\partial}{\partial\eta}(B_\eta h_\xi h_\psi) + \frac{\partial}{\partial\psi}(B_\psi h_\xi h_\psi)\right]$$

$$= \frac{\cancel{a^2}}{\cancel{a^3}(\operatorname{senh}^2\xi + \operatorname{sen}^2\eta)\operatorname{senh}\xi\operatorname{sen}\eta}[\frac{\partial}{\partial\xi}(B_\xi\sqrt{\operatorname{senh}^2\xi + \operatorname{sen}^2\eta}\operatorname{senh}\xi\operatorname{sen}\eta) +$$

$$\frac{\partial}{\partial\eta}(B_\eta\sqrt{\operatorname{senh}^2\xi + \operatorname{sen}^2\eta}\operatorname{senh}\xi) + \frac{\partial}{\partial\psi}(B_\psi(\operatorname{sen}^2\xi + \operatorname{sen}^2\xi)]$$

$$\nabla \times B == \frac{1}{h}\begin{vmatrix} h_\xi\hat{e}_\xi & h_\eta\hat{e}_\eta & h_\psi\hat{e}_\psi \\ \frac{\partial}{\partial\xi} & \frac{\partial}{\partial\eta} & \frac{\partial}{\partial u_\psi} \\ B_\xi h_\xi & B_\eta h_\eta & B_\psi h_\psi \end{vmatrix}$$

$$= \frac{1}{h}[h_\xi(\frac{\partial}{\partial\eta}(B_\psi h_\psi) - \frac{\partial}{\partial\psi}(B_\eta h_\eta))\hat{e}_\xi + h_\eta(\frac{\partial}{\partial\psi}(B_\xi h_\xi) - \frac{\partial}{\partial\xi}(B_\psi h_\psi))\hat{e}_\eta +$$

$$h_\psi(\frac{\partial}{\partial\xi}(B_\eta h_\eta) - \frac{\partial}{\partial\eta}(B_\xi h_\xi))\hat{e}_\psi]$$

$$= \frac{1}{h_\xi h_\psi}(\frac{\partial}{\partial\eta}(B_\psi h_\psi) - \frac{\partial}{\partial\psi}(B_\eta h_\eta))\hat{e}_\xi + \frac{1}{h_\xi h_\psi}(\frac{\partial}{\partial\psi}(B_\xi h_\xi) - \frac{\partial}{\partial\xi}(B_\psi h_\psi))\hat{e}_\xi +$$

$$\frac{1}{h_\xi h_\eta}(\frac{\partial}{\partial\xi}(B_\eta h_\eta) - \frac{\partial}{\partial\eta}(B_\xi h_\xi))\hat{e}_\psi$$

$$= \frac{1}{\cancel{a^2}\operatorname{senh}\xi\operatorname{sen}\eta\sqrt{\operatorname{senh}^2\xi + \operatorname{sen}^2\eta}}(\frac{\partial}{\partial\eta}(B_\psi\cancel{a}\operatorname{senh}\xi\operatorname{sen}\eta) - \frac{\partial}{\partial\psi}(B_\eta\cancel{a}\sqrt{\operatorname{senh}^2\xi + \operatorname{sen}^2\eta})\hat{e}_\xi$$

$$+ \frac{1}{\cancel{a^2}\operatorname{senh}\xi\cancel{\operatorname{sen}\eta}\sqrt{\operatorname{senh}^2\xi + \operatorname{sen}^2\eta}}(\frac{\partial}{\partial\psi}(B_\xi\cancel{a\operatorname{sen}\eta}\operatorname{sen}\xi) - \frac{\partial}{\partial\xi}(B_\psi\cancel{a\operatorname{sen}\eta}\operatorname{senh}\xi))\hat{e}_\eta$$

$$+ \frac{1}{\cancel{a^2}(\operatorname{senh}^2\xi + \operatorname{sen}^2\eta)}(\frac{\partial}{\partial\xi}(B_\eta\cancel{a}\sqrt{\operatorname{senh}^2\xi + \operatorname{sen}^2\eta}) - \frac{\partial}{\partial\eta}(B_\xi\cancel{a}\sqrt{\operatorname{senh}^2\xi +^2\eta}))\hat{e}_\psi$$

$$= \frac{1}{a\sqrt{\operatorname{senh}^2\xi + \operatorname{sen}^2\eta}}(\frac{\frac{\partial}{\partial\eta}(B_\eta\operatorname{sen}\eta)}{\operatorname{sen}\eta} - \frac{\sqrt{\operatorname{senh}^2\xi + \operatorname{sen}^2\eta}\frac{\partial}{\partial\xi}B_\eta}{\operatorname{senh}\xi\operatorname{sen}\eta} + \frac{\partial}{\partial\psi}B_\xi - \frac{\frac{\partial}{\partial\xi}(B_\psi\operatorname{senh}\xi)}{\operatorname{senh}\xi}$$

$$+ \frac{\frac{\partial}{\partial\xi}(B_\eta\sqrt{\operatorname{senh}^2\xi + \operatorname{sen}^2\eta})}{\sqrt{\operatorname{senh}^2\xi + \operatorname{sen}^2\eta}} - \frac{\frac{\partial}{\partial\eta}(B_\xi\sqrt{\operatorname{sen}^2\xi + \operatorname{sen}^2\eta})}{\sqrt{\operatorname{senh}^2\xi + \operatorname{sen}^2\eta}})$$

$$\nabla^2\phi = \frac{1}{h}\sum \frac{\partial}{\partial u_i}\left(\frac{h}{h_i^2}\frac{\partial \phi}{\partial u_i}\right)$$

$$= \frac{1}{h}\left[\frac{\partial}{\partial \xi}(\frac{h}{h_\xi^2}\frac{\partial \phi}{\partial \xi}) + \frac{\partial}{\partial \eta}(\frac{h}{h_\eta^2}\frac{\partial \phi}{\partial \eta}) + \frac{\partial}{\partial \psi}(\frac{h}{h_\psi^2}\frac{\partial \phi}{\partial \psi})\right]$$

$$= \frac{1}{h}\left[\frac{\partial}{\partial \xi}(h_\psi\frac{\partial \phi}{\partial \xi}) + \frac{\partial}{\partial \eta}(h_\psi\frac{\partial \phi}{\partial \eta}) + \frac{\partial}{\partial \psi}(\frac{a(\operatorname{senh}^2\xi + \operatorname{sen}^2\eta)}{\operatorname{senh}\xi \operatorname{sen}\eta}\frac{\partial \phi}{\partial \psi})\right]$$

$$= \frac{1}{a^{\cancel{3}}(\operatorname{senh}^2\xi + \operatorname{sen}^2\eta)\operatorname{senh}\xi \operatorname{sen}\eta}[\frac{\partial}{\partial \xi}(\cancel{a}\operatorname{senh}\xi \operatorname{sen}\eta\frac{\partial \phi}{\partial \xi}) + \frac{\partial}{\partial \eta}(\cancel{a}\operatorname{senh}\xi \operatorname{sen}\eta\frac{\partial \phi}{\partial \eta}) +$$

$$\frac{\cancel{a}(\operatorname{sen}^2\xi + \operatorname{sen}^2\eta)}{\operatorname{senh}\xi \operatorname{sen}\eta}\frac{\partial^2 \phi}{\partial \psi^2}]$$

$$= \frac{1}{a^2(\operatorname{senh}^2\xi + \operatorname{sen}^2\eta)\operatorname{senh}\xi \operatorname{sen}\eta}[\operatorname{sen}\eta\frac{\partial}{\partial \xi}(\operatorname{senh}\xi\frac{\partial \phi}{\partial \xi}) + \operatorname{senh}\xi\frac{\partial}{\partial \eta}(\operatorname{sen}\eta\frac{\partial \phi}{\partial \eta}) +$$

$$\frac{\operatorname{senh}^2\xi + \operatorname{sen}^2\eta}{\operatorname{senh}\xi \operatorname{sen}\eta}\frac{\partial^2 \phi}{\partial \psi^2}]$$