

## 参考答案

### 一、选择题

题号	1	2	3	4	5
答案	D	C	B	A	C

### 二、填空题

1.  $5\ \mu\text{F}$ ;  $20\ \mu\text{F}$

2.  $11/6$ ;  $11/6$

3.  $\ln 3/\ln 2$ ;  $\ln 2/\ln 3$

4. 减小;  $\varepsilon_r$

5.  $\frac{Q^2}{16\pi\varepsilon_0 R}$

### 三、计算题

1. 设 A、B 两极板分别带电  $Q$  和  $-Q$ , 利用电介质中的高斯定理  $\oint \vec{D} \cdot d\vec{S} = \sum q$  得球壳内的电位移矢量  $\vec{D}$  为

$$\vec{D} = \frac{Q}{4\pi r^3} \vec{r} \quad (R_A < r < R_B)$$

由此得

$$\vec{E}_1 = \frac{Q}{4\pi\varepsilon_0\varepsilon_1 r^3} \vec{r} \quad (R_A < r < R_C)$$

$$\vec{E}_2 = \frac{Q}{4\pi\varepsilon_0\varepsilon_2 r^3} \vec{r} \quad (R_C < r < R_B)$$

此时 A、B 两极板间的电势差  $U_{AB}$  为

$$\begin{aligned} U_{AB} &= \int_{R_A}^{R_C} \vec{E}_1 \cdot d\vec{r} + \int_{R_C}^{R_B} \vec{E}_2 \cdot d\vec{r} = \int_{R_A}^{R_C} \frac{Q}{4\pi\varepsilon_0\varepsilon_1 r^2} dr + \int_{R_C}^{R_B} \frac{Q}{4\pi\varepsilon_0\varepsilon_2 r^2} dr \\ &= \frac{Q}{4\pi\varepsilon_0\varepsilon_1} \left( \frac{1}{R_A} - \frac{1}{R_C} \right) + \frac{Q}{4\pi\varepsilon_0\varepsilon_2} \left( \frac{1}{R_C} - \frac{1}{R_B} \right) = \frac{Q}{24\pi\varepsilon_0 R_A} \left( \frac{2}{\varepsilon_1} + \frac{1}{\varepsilon_2} \right) \end{aligned}$$

故,

$$C = \frac{Q}{U_{AB}} = \frac{24\pi\varepsilon_0 R_A}{\left( \frac{2}{\varepsilon_1} + \frac{1}{\varepsilon_2} \right)} = \frac{24\pi\varepsilon_0\varepsilon_1\varepsilon_2 R_A}{2\varepsilon_2 + \varepsilon_1}$$

2. 先利用利用电介质中的高斯定理 $\oint \vec{D} \cdot d\vec{S} = \sum q$ 得球壳内的电位移矢量 $\vec{D}$

$$\vec{D} = \frac{q}{4\pi r^2} \vec{r}_0 \quad (R_A < r < R_B)$$

$$\vec{E}_1 = \frac{Q}{4\pi\epsilon_0\epsilon_1 r^2} \vec{r}_0 \quad (R_A < r < R_C)$$

$$\vec{E}_2 = \frac{Q}{4\pi\epsilon_0 r^2} \vec{r}_0 \quad (R_C < r < R_B)$$

$$dW_e = \frac{1}{2} E dV = \frac{1}{2} E 4\pi r^2 dr$$

$$W_e = \int_V dW = \int_{R_A}^{R_C} \frac{1}{2} \frac{q}{4\pi r^2} \frac{q}{4\pi\epsilon_0\epsilon_1 r^2} 4\pi r^2 dr + \int_{R_C}^{R_B} \frac{1}{2} \frac{q}{4\pi r^2} \frac{q}{4\pi\epsilon_0 r^2} 4\pi r^2 dr$$

$$= \frac{q^2}{8\pi\epsilon_0\epsilon_1} \left( \frac{1}{R_A} + \frac{\epsilon_1 - 1}{R_C} - \frac{\epsilon_1}{R_B} \right)$$