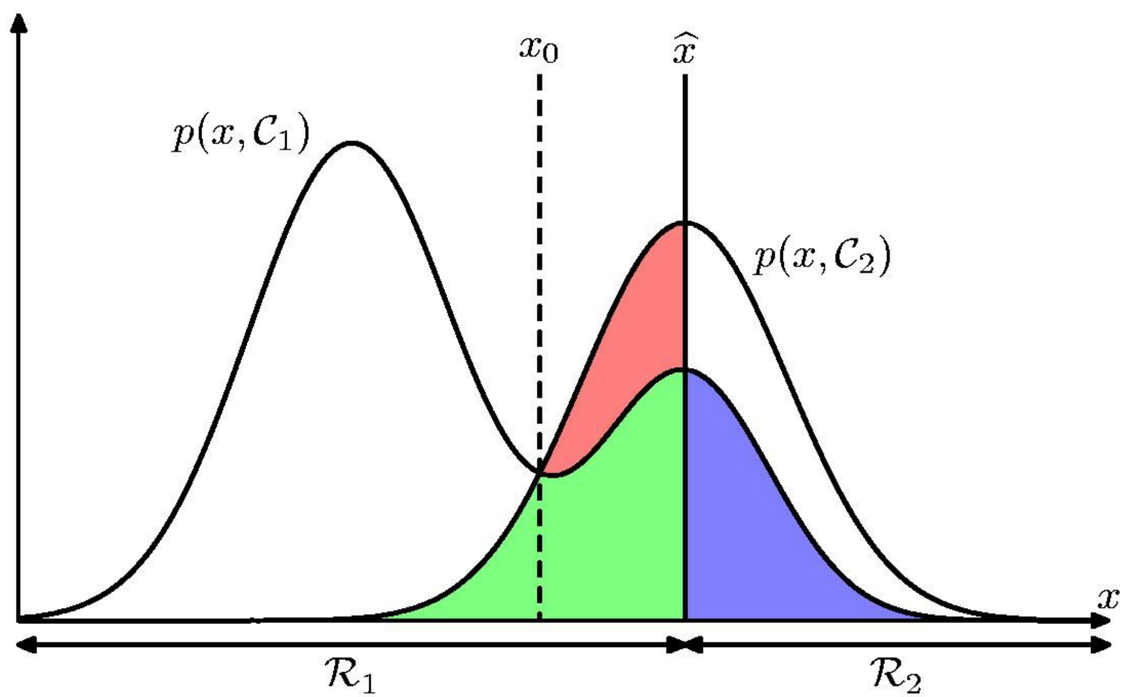
Reject Option

Minimizing the Misclassification Rate



$$p(error) = p(x \in R_1, C_2) + p(x \in R_2, C_1) =$$

$$= \int_{R_1} p(x, C_2)\, dx + \int_{R_2} p(x, C_2)\, dx$$

Minimum Risk Classification Rule (MRC rule)

- What can we do if there is a cost associated with a decision?

	cancer	**normal**	**rejected**
cancer	0	1000	1
normal	10	0	1

Cost Matrix $\{C_{ij}\}$

- From an operative point of view, the MRC rule, for a given sample *x*, chooses the class *k* such that:

$$k = argmin_j \sum_{i=1}^{N} C_{ij} P(i|x)$$

Cost Matrix Assumptions

- Cost Matrix → Indices Assumptions

Predicted Classes (j)

	Class1	**Class2**	**rejected**
Class1	(1,1)	(1,2)	(1,0)
Class2	(2,1)	(2,2)	(2,0)

$\{C_{ij}\}$

True Classes (i)

i=2 // j=0

A numeric example (1/3)

- If we suppose to work with a binary classifier and that:
  - $p(cancer|x)=0.2$
  - $p(normal|x)=0.8$
- By considering the following cost matrix:

	cancer	**normal**	**Rejected**
cancer	0	1000	1
normal	10	0	1

$$[0.2 \quad 0.8] * \begin{bmatrix} 0 & 1000 & 1 \\ 10 & 0 & 1 \end{bmatrix} = [8_{cancer} \quad 200_{normal} \quad \boxed{1_{Rejected}}]$$

A numeric example (2/3)

- If we suppose to work with a binary classifier and that:
  - $p(cancer|x)=0.0001$
  - $p(normal|x)=0.9999$
- By considering the following cost matrix:

	cancer	normal	Rejected
cancer	0	1000	1
normal	10	0	1

$$[0.0001 \quad 0.9999] * \begin{bmatrix} 0 & 1000 & 1 \\ 10 & 0 & 1 \end{bmatrix} = [9.999_{cancer} \quad \boxed{0.1_{normal}} \quad 1_{Rejected}]$$

Chow's rule

- The MRC rule particularizes to the well-known Chow's Rule when the costs do not depend on the class.
- Definition
  - E=Error cost $E = C_{ij}, i \neq j$
  - R=Reject cost $R = C_{i0}$
  - C=aCcuracy gain $C = C_{ii}$
- Reject
  - if $p(Class|x) \leqslant \frac{(E-R)}{(E-C)}$

A numeric example (3/3)

- If we suppose to work with a binary classifier and if we suppose that:
  - $p(cancer|x)=0.08$
  - $p(normal|x)=0.92$
- If we suppose to have the following cost matrix:

	cancer	normal	Rejected
cancer	0	10	1
normal	10	0	1

$$E=C_{ij}=10$$
$$R=C_{i0}=1$$
$$C=C_{ii}=0$$
$$(E-R)/(E-C)=0.9$$

$$[0.08 \quad 0.92]*\begin{bmatrix}0 & 10 & 1\\ 10 & 0 & 1\end{bmatrix}=[9.2_{cancer} \quad \boxed{0.8_{normal}} \quad 1_{Rejected}]$$

A possible Reject Architecture