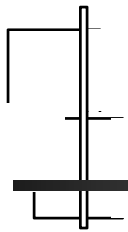


Introduction to Automata Theory



Reading: Chapter 1



What is Automata Theory?

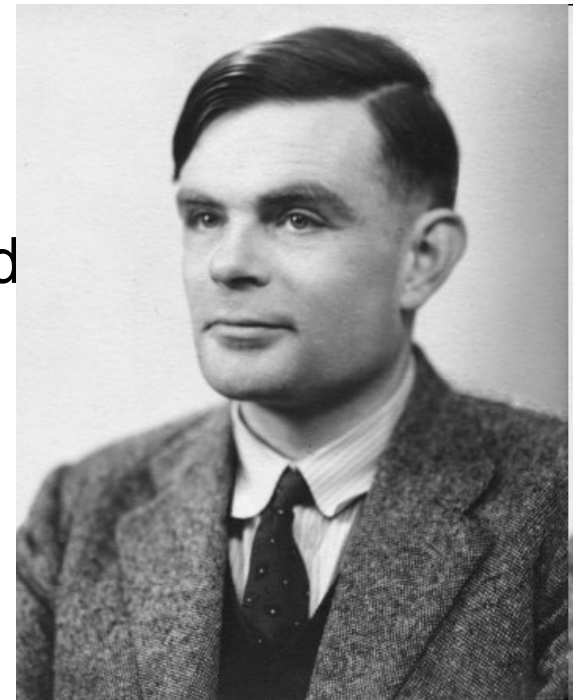
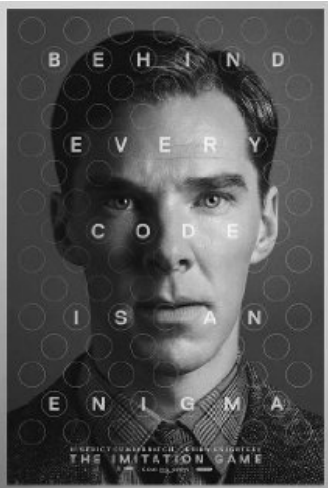
- *Study of abstract computing devices, or “machines”*
- Automaton = an abstract computing device
 - Note: A “device” need not even be a physical hardware!
- A fundamental question in computer science:
 - Find out what different models of machines can do and cannot do
- Computability vs. Complexity

(A pioneer of automata theory)

Alan Turing (1912-1954)

- Father of Modern Computer Science
- English mathematician
- Studied abstract machines called ***Turing machines*** even before computers existed

Heard of the Turing test?



Theory of Computation: A Historical Perspective

1930s	<ul style="list-style-type: none">• Alan Turing studies Turing machines• Decidability• Halting problem
1940-1950s	<ul style="list-style-type: none">• “Finite automata” machines studied• Noam Chomsky proposes the “Chomsky Hierarchy” for formal languages
1969	Cook introduces “intractable” problems or “NP-Hard” problems
1970-	Modern computer science: compilers, computational & complexity theory evolve

Languages & Grammars

An alphabet is a set of symbols:

$\{0,1\}$

Or “**words**”

↓ Sentences are strings of symbols:

0,1,00,01,10,1,...

A language is a set of sentences:

$L = \{000,0100,0010,.. \}$

A grammar is a finite list of rules defining a language.

$S \longrightarrow 0A$

$B \longrightarrow 1B$

$A \longrightarrow 1A$

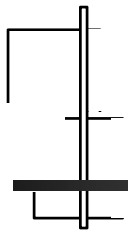
$B \longrightarrow 0F$

$A \longrightarrow 0B$

$F \longrightarrow \epsilon$

- Languages: “A language is a collection of sentences of finite length all constructed from a finite alphabet of symbols”
- Grammars: “A grammar can be regarded as a device that enumerates the sentences of a language” - nothing more, nothing less
- *N. Chomsky, Information and Control, Vol 2, 1959*

The Central Concepts of Automata Theory





Alphabet

An alphabet is a finite, non-empty set of symbols

- We use the symbol Σ (sigma) to denote an alphabet
- Examples:
 - Binary: $\Sigma = \{0,1\}$
 - All lower case letters: $\Sigma = \{a,b,c,\dots z\}$
 - Alphanumeric: $\Sigma = \{a-z, A-Z, 0-9\}$
 - DNA molecule letters: $\Sigma = \{a,c,g,t\}$
 - ...

Strings

A string or word is a finite sequence of symbols chosen from Σ

- ***Empty string is ε (or “epsilon”)***

- Length of a string w , denoted by “ $|w|$ ”, is equal to the *number of (non- ε) characters in the string*

- *E.g., $x = 010100$*

$|x| = 6$

- $x = 01 \varepsilon 0 \varepsilon 1 \varepsilon 00 \varepsilon$

$|x| = ?$

- xy = *concatenation* of two strings x and y



Powers of an alphabet

Let Σ be an alphabet.

- Σ^k = the set of all strings of length k
- $\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \dots$
- $\Sigma^+ = \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \cup \dots$

Languages

L is said to be a language over alphabet Σ , only if $L \subseteq \Sigma^$*

→ this is because Σ^* is the set of all strings (of all possible length including 0) over the given alphabet Σ

Examples:

1. Let L be *the* language of all strings consisting of n 0's followed by n 1's:

$$L = \{\epsilon, 01, 0011, 000111, \dots\}$$

2. Let L be *the* language of all strings of with equal number of 0's and 1's:

$$L = \{\epsilon, 01, 10, 0011, 1100, 0101, 1010, 1001, \dots\}$$

→
Canonical ordering of strings in the language

Definition: \emptyset denotes the Empty language

- Let $L = \{\epsilon\}$; Is $L = \emptyset$?

NO

The Membership Problem

Given a string $w \in \Sigma^$ and a language L over Σ , decide whether or not $w \in L$.*

Example:

Let $w = 100011$

Q) Is $w \in$ the language of strings with equal number of 0s and 1s?

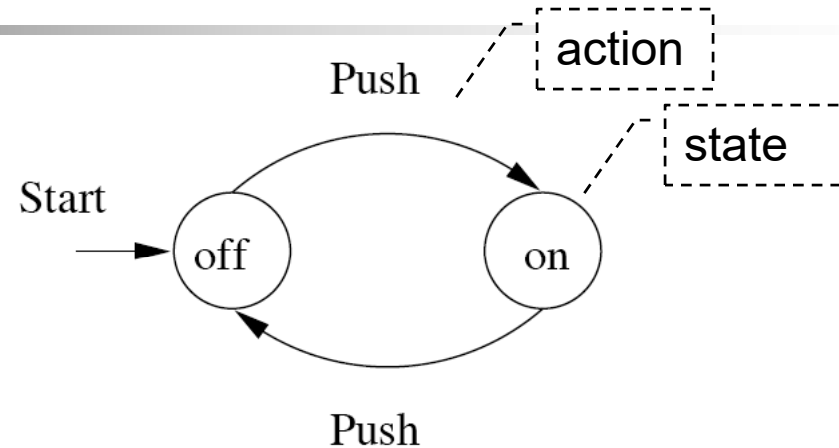


Finite Automata

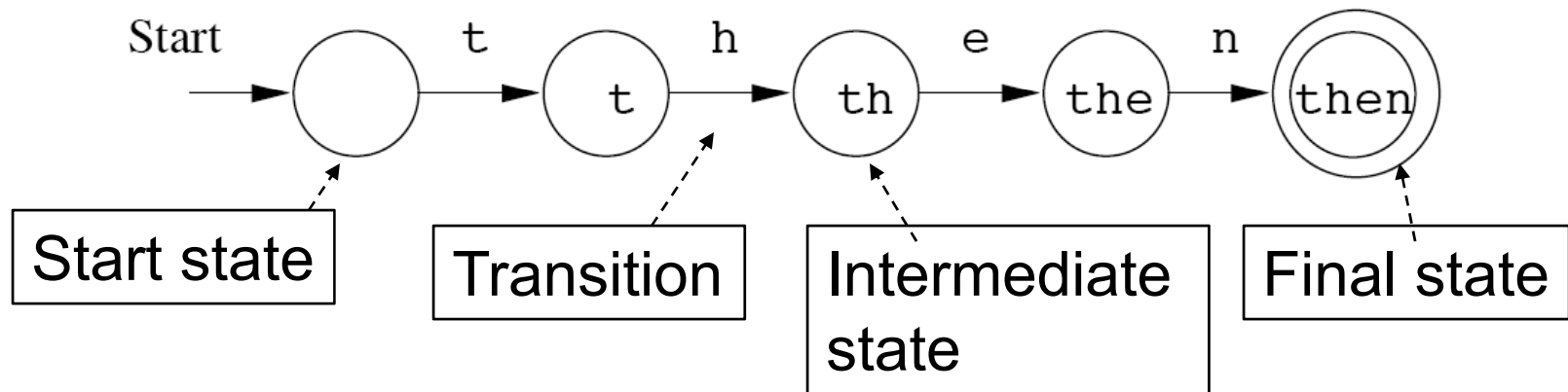
- Some Applications
 - Software for designing and checking the behavior of digital circuits
 - Lexical analyzer of a typical compiler
 - Software for scanning large bodies of text (e.g., web pages) for pattern finding
 - Software for verifying systems of all types that have a finite number of states (e.g., stock market transaction, communication/network protocol)

Finite Automata : Examples

■ On/Off switch

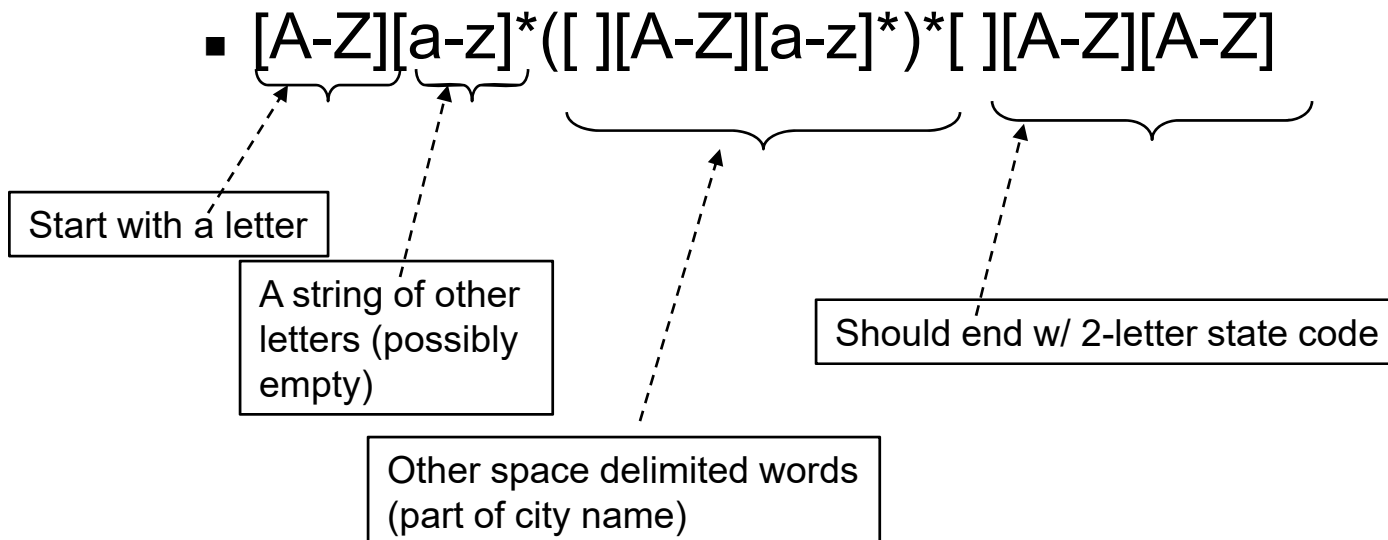


■ Modeling recognition of the word “*then*”



Structural expressions

- Grammars
- Regular expressions
 - E.g., unix style to capture city names such as “Palo Alto CA”:





Formal Proofs

Deductive Proofs

From the given statement(s) to a conclusion statement (what we want to prove)

- Logical progression by direct implications

Example for parsing a statement:

- “If $y \geq 4$, then $2^y \geq y^2$.”

given

conclusion

(there are other ways of writing this).

Example: Deductive proof

Let Claim 1: If $y \geq 4$, then $2^y \geq y^2$.

Let x be any number which is obtained by adding the squares of 4 positive integers.

Claim 2:

Given x and assuming that Claim 1 is true, prove that $2^x \geq x^2$

■ Proof:

1) Given: $x = a^2 + b^2 + c^2 + d^2$

2) Given: $a \geq 1, b \geq 1, c \geq 1, d \geq 1$

3) $\rightarrow a^2 \geq 1, b^2 \geq 1, c^2 \geq 1, d^2 \geq 1$ (by 2)

4) $\rightarrow x \geq 4$ (by 1 & 3)

5) $\rightarrow 2^x \geq x^2$ (by 4 and Claim 1)

“implies” or “follows”



On Theorems, Lemmas and Corollaries

We typically refer to:

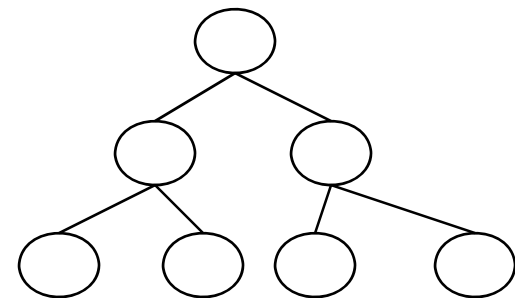
- A major result as a “**theorem**”
- An intermediate result that we show to prove a larger result as a “**lemma**”
- A result that follows from an already proven result as a “**corollary**”

An example:

Theorem: *The height of an n -node binary tree is at least $\text{floor}(\lg n)$*

Lemma: *Level i of a perfect binary tree has 2^i nodes.*

Corollary: *A perfect binary tree of height h has $2^{h+1}-1$ nodes.*



Quantifiers

“For all” or “For every”

- Universal proofs
- Notation= \forall

“There exists”

- Used in existential proofs
- Notation= \exists

Implication is denoted by \Rightarrow

- E.g., “IF A THEN B” can also be written as “ $A \Rightarrow B$ ”

Proving techniques

- By contradiction
 - Start with the statement contradictory to the given statement
 - E.g., To prove $(A \Rightarrow B)$, we start with:
 - $(A \text{ and } \sim B)$
 - ... and then show that could never happen

What if you want to prove that “ $(A \text{ and } B \Rightarrow C \text{ or } D)$ ”?

- By induction
 - (3 steps) Basis, inductive hypothesis, inductive step
- By contrapositive statement
 - If A then B \equiv If $\sim B$ then $\sim A$



Proving techniques...

- By counter-example
 - Show an example that disproves the claim
- Note: There is no such thing called a “proof by example”!
 - So when asked to prove a claim, an example that satisfied that claim is *not* a proof

Different ways of saying the same thing

- “*If H then C*”:
 - i. *H implies C*
 - ii. $H \Rightarrow C$
 - iii. *C if H*
 - iv. *H only if C*
 - v. *Whenever H holds, C follows*

“If-and-Only-If” statements

- “A if and only if B” ($A \iff B$)
 - (*if part*) if B then A (\implies)
 - (*only if part*) A only if B (\implies)
(same as “if A then B”)
- “If and only if” is abbreviated as “iff”
 - i.e., “A iff B”
- Example:
 - Theorem: *Let x be a real number. Then floor of x = ceiling of x if and only if x is an integer.*
- Proofs for iff have two parts
 - One for the “if part” & another for the “only if part”



Summary

- Automata theory & a historical perspective
- Chomsky hierarchy
- Finite automata
- Alphabets, strings/words/sentences, languages
- Membership problem
- Proofs:
 - Deductive, induction, contrapositive, contradiction, counterexample
 - If and only if
- Read chapter 1 for more examples and exercises