Quantum Theory 3

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Contents

Part I

Scattering Theory

Lecture 1 Introduction to Scattering

1.1 Two types of scattering

- elastic - initial particles remain and no new particles emerge in the collision
- inelastic - in the final state, there is more than just the initial particles

Will be using non-relativistic Quantum Mechanics for this part of the course, therefore will only be studying elastic non-relativistic scattering, e.g. Rutherford experiment, $\alpha + \mathrm{Au} \to \alpha + \mathrm{Au}$.

- Consider elastic $e^- e^+$ scattering
$$e^+ + e^- \to e^+ + e^- \tag{1.1}$$
Feynman diagrams of collision - both s-channel (particles meet) and t-channel (particles interact through virtual photon).
- Consider inelastic scattering
$$e^+ + e^- \to \mu^+ + \mu^- \tag{1.2}$$
Feynman diagram of collision and decay into muon and anti-muon - electron and positron collide and annihilate, their energy then carried by photon which decays into muon and anti-muon.

1.2 Scattering cross-sections

The scattering cross-section, σ, is a probabilistic quantity that characterises the 'strength' of the scattering (interaction between the particles). σ has dimension of area (m^2).

$$\frac{d\sigma}{d\Omega} = \frac{1}{F}\frac{dR}{d\Omega}, \tag{1.3}$$

where:

- F is the flux - number of incident particles per unit area per unit time ($s^{-1}m^{-2}$)
- dR is the rate - number of scattered particles (N) into $d\Omega$ per unit time (s^{-1})
- $d\Omega$ is the solid angle
- $\frac{d\sigma}{d\Omega}$ is the differential cross-section into the solid angle (m^2)

$$\frac{d\sigma}{d\Omega}(\theta,\phi) = \frac{1}{F}\frac{dR}{d\Omega} \tag{1.4}$$
$$\sigma_{tot} = \int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta \frac{d\sigma}{d\Omega}(\theta,\phi) \tag{1.5}$$
$$N = \sigma_{tot} \cdot \int F\, dt \tag{1.6}$$

- F is known for each experiment, part of the design
- σ_{tot} is measured in experiment
  - Measured in barns (b) - 1 barn = $10^{-24} cm^{-2}$
  - $\sigma_{\text{Thompson}} = 0.665\, b$
- LHC gluon fusion into Higgs:
  - $g + g \to H$
  - *Feynman diagram of gluon collision into Higgs boson*
  - $\sigma_{\text{Higgs}} \approx 10 pb$

Lecture 2 General Features of Potential Scattering in QM

2.1 The Schrodinger Equation

Time-dependent Schrodinger equation, and reduced mass:

$$i\hbar\frac{\partial\psi(r,t)}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V(r)\right)\psi(r,t) \tag{2.1}$$

$$m = \frac{m_A m_B}{m_A + m_B} \tag{2.2}$$

E = fixed and finite

$$E = \frac{p^2}{2m} \tag{2.3}$$

$$\psi(r,t) = e^{-iEt/\hbar}\psi(r) \tag{2.4}$$

Leads to time-independent Schrodinger equation

$$E\psi(r) = \left(-\frac{\hbar^2}{2m}\nabla^2 + V(r)\right)\psi(r) \tag{2.5}$$

$\frac{p}{\hbar} = k$, $U(r) = \frac{2m}{\hbar^2}V(r) \to$ Scattering equation:

$$\left(\nabla^2 + k^2 - U(r)\right)\psi(r) = 0 \tag{2.6}$$

For scattering:

- Looking for $\psi(r)$ s.t. as $r \to \pm\infty$,

$$\psi_{inc}(r) + \psi_{scat}(r) \equiv e^{ik\cdot r} + \frac{e^{i|k|\cdot|r|}}{r}\cdot f(k,\theta,\phi) \tag{2.7}$$

$$= e^{ikr\cos\theta} + \frac{e^{ikr}}{r}\cdot f(k,\theta,\phi) \tag{2.8}$$

- When scattering occurs, the incoming plane waves turn into spherical waves with $\frac{1}{r}$ amplitude from point of scattering
- $f(k,\theta,\phi)$ is the scattering amplitude - need to determine this in order to compute σ

$$\psi(r) \approx_{r\to\infty} 1\cdot e^{ik\cdot r} + f(k,\theta,\phi)\frac{e^{ik\cdot r}}{r} \tag{2.9}$$

Now consider the probability density for normalisation,

$$\rho_{inc}(r) \equiv |\psi_{inc}|^2 = 1 \tag{2.10}$$

What about flux?

$$F = \frac{\text{\# of incoming particles}}{\text{Area}\cdot\text{time}} \tag{2.11}$$

$$= v\cdot\rho = \frac{p}{m}\cdot\rho,\ \rho = 1 \tag{2.12}$$

$$= \frac{p}{m} \tag{2.13}$$

Recall $\frac{d\sigma}{d\Omega} = \frac{1}{F}\frac{dR}{d\Omega}$. So what is dR?

$$dR = j_r r^2 d\Omega \tag{2.14}$$

So j_r is the probability current density - the number of scattered particles crossing the unit area per unit time.

$$j_r \equiv \frac{\hbar}{2mi}\left(\psi^*_{scat}(r)\nabla\psi_{scat}(r) - (\nabla\psi_{scat}(r))^*\,\psi(r)\right) \tag{2.15}$$

$$= \frac{\hbar}{m}\mathrm{Im}\left(\psi^*_{scat}\nabla\psi_{scat}\right) = \frac{\hbar k}{m}\frac{|f|^2}{r^2} \tag{2.16}$$

$$= \frac{p}{m}|f|^2\frac{1}{r^2} = F\frac{|f(k,\theta,\phi)|^2}{r^2} \tag{2.17}$$

$$\implies \frac{d\sigma}{d\Omega} = \frac{1}{F}\frac{dR}{d\Omega} = |f(k,\theta,\phi)|^2 \tag{2.18}$$