Nuclear and Particle Physics

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Part I

Lecture 1

Use these because they are made by God himself: https://dmaitre.phyip3.dur.ac.uk/notes/NPP/

Use these notes only with link above, these will just be additional annotations

1.1 Units

Example: Do Broglie Wavelength

$$\lambda = \frac{\hbar}{p} \tag{1.1}$$
$$p = 20\text{GeV/c} \tag{1.2}$$
$$l = 0.05\hbar c\text{GeV}^{-1} \tag{1.3}$$
$$\hbar c = 0.19733\,\text{fm GeV} \equiv 1 \tag{1.4}$$
$$\lambda = 0.0987\,\text{fm} \tag{1.5}$$

1.2 Kinematics

High energies mean speed close to c, so use special relativity, and get the Lorentz transforms and Tensors.

Example: 4-Momenta

In the rest frame of a particle of mass m,

$$p = (m, \underline{0}) \tag{1.6}$$

How is it in a different frame?

$$p = (E, \underline{p}) \tag{1.7}$$
$$p \cdot p \equiv p^2 = \begin{cases} \text{Rest Frame} & m^2 - \underline{0}^2 - m^2 \\ \text{Other Frames} & E^2 \underline{p} \cdot \underline{p} = E^2 - |\underline{p}|^2 \end{cases} \tag{1.8}$$
$$m^2 = E^2 - |\underline{p}|^2 \tag{1.9}$$
$$E^2 = m^c + |\underline{p}|^2 \tag{1.10}$$
$$E = \sqrt{m^2 + |\underline{p}|^2} \tag{1.11}$$

Note: there will be factors of c in this, but set to 1, in natural units.

Part II

Lecture 1

1.1 Scattering

- de Broglie Wavelength
$$\bar{\lambda} = \frac{\hbar}{p} \tag{1.1}$$
  - higher resolution comes from larger momenta
- Elastic scattering - Number and particle type are conserved
- Inelastic scattering - Number and particle type are not conserved
- Total cross section,
$$\sigma_{tot} = \sigma_{el} + \sigma_{inel} = \frac{\dot{N}}{\phi_a N_b} \tag{1.2}$$
  - $\dot{N}$ - rate of collisions
  - ϕ_a - number density in the beam
  - N_b - number of targets
- $$\dot{N} = \mathcal{L} \cdot \sigma_{tot}, \mathcal{L} = \phi_a N_b \tag{1.3}$$
  $\mathcal{L}$ is a purely experimental input, σ_{tot} purely theoretical.
$$\sigma_{tot} = \int_\Omega d\Omega \int_{E_{min}}^{E_{max}} dE \, \frac{d\sigma}{d\Omega \, dE} \tag{1.4}$$
- Fermi's Golden Rule:
$$\sigma = \frac{2\pi}{V_a} \left| M_{fi} \right|^2 g(E')V \tag{1.5}$$
$$M_{fi} = \langle \Psi_f | \mathcal{H}_{int} | \Psi_i \rangle \tag{1.6}$$
$$\Psi_i = \frac{1}{\sqrt{V}} e^{i\underline{p}\cdot\underline{x}} \tag{1.7}$$
$$\Psi_f = \frac{1}{\sqrt{V}} e^{i\underline{p}'\cdot\underline{x}} \tag{1.8}$$
$$\mathcal{H}_{int} = \frac{z \cdot Ze^2}{\underline{x} - \underline{x}'|} \exp -M|\underline{x} - \underline{x}'| \tag{1.9}$$
$$M_{fi} = \langle \Psi_f | \mathcal{H}_{int} | \Psi_i \rangle \tag{1.10}$$
$$= \int d^3x \Psi_f^*(\underline{x}) \mathcal{H}_{int} \Psi_i(\underline{x}) \tag{1.11}$$
$$= \frac{z \cdot Ze^2}{V} \int d^3x e^{i\underline{q}\underline{x}} \frac{e^{-M|\underline{x}-\underline{x}_0|}}{|\underline{x} - \underline{x}_0|} \tag{1.12}$$
$$= \frac{4\pi e^2 zZ}{V} e^{iqx_0} \frac{1}{|q|^2 + M^2} \to_{M\to 0} \frac{4\pi e^2 zZ}{V} e^{iqx_0} \frac{1}{|q|^2} \tag{1.13}$$
$$d\sigma = \frac{2\pi}{V_a} |M_{fi}|^2 dg(E')V = \frac{2\pi}{V_a} \left| \frac{4\pi e^2 zZ}{V} e^{ix_0 \cdot q} \frac{1}{|q|^2} \right|^2 V \frac{V}{(2\pi)^3} |p'|^2 d\Omega \tag{1.14}$$
$$\frac{d\sigma}{d\Omega} = \frac{4e^4 z^2 Z^2 E'^2}{|q|^4} = \frac{e^4 z^2 Z^2}{4E^2 \sin^4\left(\frac{\theta}{2}\right)} \tag{1.15}$$

1.2 Mott Scattering

The Rutherford scattering formula neglects spin. Spin is a purely relativistic property that distinguishes fermions ($s = \frac{1}{2}$) from bosons ($s = 0, 1, \ldots$). So the Mott cross-section can be written as

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott, no recoil}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Rutherford}} \left(1 - \beta^2 \sin^2\left(\frac{\theta}{2}\right)\right), \beta = \frac{v}{c} \tag{1.16}$$

$$= \left(\frac{d\sigma}{d\Omega}\right)_{\text{Rutherford}} \cos^2\left(\frac{\theta}{2}\right), \lim_{v \to c} \tag{1.17}$$

- Spin projection on $\underline{p}$ is called helicity:

$$h = \frac{\underline{s} \cdot \underline{p}}{|\underline{s}||\underline{p}|} \tag{1.18}$$

- Exception to this when target carries spin as well
- Key points:
  - For relativistic projectiles, spin has to be taken into account.
  - Helicity conservation suppresses backwards scattering.

Lecture 2

2.1 Nuclear Form Factors

- Pointlike charge distribution

$$g(x) = Ze\delta^2(x - x_0) \tag{2.1}$$

- Extended charge distribution

$$g(x) = Zef(x) \tag{2.2}$$

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$$g(x) = \int g(y)\delta(x-y)dy - Ze\int f(y)\delta(x-y)dy \tag{2.3}$$

$$\phi(x) = Ze\int f(y)\frac{1}{|x-y|}dy \tag{2.4}$$

$$\Delta\phi(x) = -g(x) \tag{2.5}$$

$$\mathcal{H}_{int} = ze\cdot\phi(x) \tag{2.6}$$

$$M_{fi} = \langle\psi_f|\mathcal{H}_{int}|\psi_i\rangle \tag{2.7}$$

$$= \frac{e}{V}\int d^3x\ e^{iq\cdot x}\phi(x), q = p - p' \tag{2.8}$$

$$= \frac{e}{V}\int e^{iq\cdot x}d^3x\ Ze\int f(y)\frac{1}{|x-y|}d^3y \tag{2.9}$$

$$= \frac{Ze^2}{V}\int d^3y\ f(y)\int d^3x\ \frac{1}{|q|^2}e^{iq\cdot x}\underbrace{\triangle\frac{1}{|x-y|}}_{...\delta(x-y)} \tag{2.10}$$

$$= \frac{Ze^2}{V}\int d^3y\ f(y)\frac{4\pi}{|q|^2}e^{iq\cdot y} \tag{2.11}$$

$$= \frac{4\pi Ze^2}{V|q|^2}\int d^3y\ f(y)e^{iq\cdot y} \equiv \frac{4\pi Ze^2}{V|q|^2}F(q) \tag{2.12}$$

$$\implies \left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott, point, no recoil}}|F(q)|^2 \tag{2.13}$$

- For point-like target, $F(q) = 1$.
- The shape of $F(q)$ yields information about the charge distribution.
  - $F(q)$ is the Fourier transform of $f(y)$.
  - e.g. spherically symmetric target, $f(x) = f(|x|)$

$$F(q) = \int e^{iq\cdot x}f(x)d^3x = \int f(v)e^{iqv\cos\theta}2\pi v^2 dv d\cos\theta \tag{2.14}$$

$$= \int_v \left(\frac{1}{iqv}e^{iqv\cos\theta}\right)_{-1}^{1} f(v)2\pi v^2 dv \tag{2.15}$$

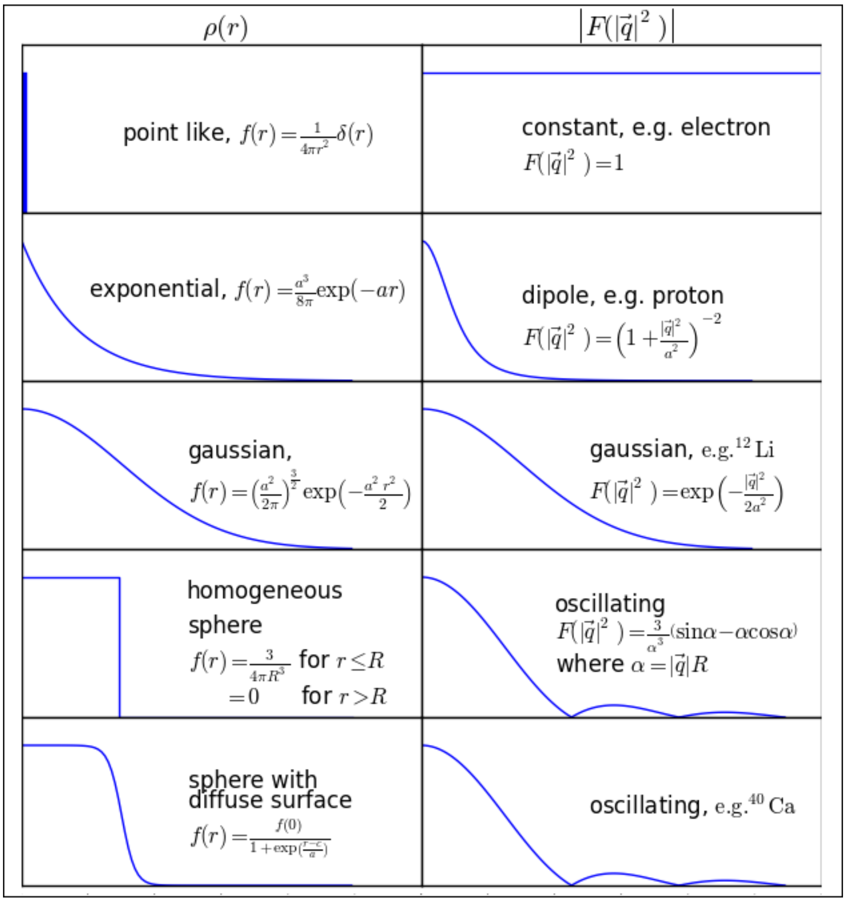
$$= \int_v \frac{1}{iqv}\left(e^{iqv} - e^{-iqv}\right)f(v)v^2 dv \tag{2.16}$$

$$= 2\pi\int 2\frac{\sin(qv)}{qv}f(v)v^2 dv \tag{2.17}$$

$$= 4\pi\int_0^1 \frac{\sin(qv)}{qv}\left(\frac{3}{4\pi R^3}\right)v^2 dv \tag{2.18}$$

$$= \frac{3}{R^3q^3}\left(\sin(qR) - qR\cos(qR)\right) \tag{2.19}$$

- Something about graphs leading to $R \approx \frac{4.5}{q_0}$.



Key points:

- Scattering off an extended charge distribution intoeuces a form factor $F(q)$. $F(q)$ is the Fourier transform of the charge distribution.
- Measure the ration between $d\sigma/d\Omega$ in experiment and Mott allows to determine the shape of $F(q)$.

2.2 Scattering Off Nucleons

- Require a resolution of $1\,fm = 10^{-15}m$ to perceive nucleons
- Requires $q = \frac{\hbar c}{\lambda} = 200\,MeV$
- $m_p = 938\,MeV$
- For such large momenta, the recoil has to be taken into account: $E \neq E'$

$$g(E) = \frac{dn}{dE} = \frac{dn}{dE'}\frac{dE'}{dE} \approx \frac{dn}{dE'} \cdot \frac{E'}{E} = g(E') \tag{2.20}$$

- With this modification, now we have recoil

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott, recoil}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott, no recoil}} \cdot \frac{E'}{E} \tag{2.21}$$

- The target carries spin, and therefore a magnetic moment.
  - Recall,

$$\mu = I \cdot A = \frac{q}{2m}L \tag{2.22}$$

- For a spinning charge,

$$\mu = g\frac{e}{M} \cdot S, \; S = \frac{1}{2} \tag{2.23}$$