NPP Repetition Package

1. The differential cross section for the scattering of an electron from an extended charge distribution is

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{exp}} = \left(\frac{d\sigma}{d\Omega}\right)^*_{\text{Mott}} |F(\vec{q}^{\,2})|^2 \,,$$

where $\left(\frac{d\sigma}{d\omega}\right)^*_{\text{Mott}}$ is the Mott scattering cross section and the form factor

$$F(\vec{q}^{\,2}) = \int \exp(i\vec{q}\cdot\vec{x}) f(\vec{x})\, d^3x$$

where $\vec{q} = \vec{k} - \vec{k}'$ is the change in the momentum of the electron, $\vec{k}$ is the momentum of the incoming electron, $\vec{k}'$ is the momentum of the scattered electron and $f(\vec{x})$ is the charge distribution.

a) Show that for a spherically symmetric charge distribution the form factor can be written as

$$F(\vec{q}^{2}) = \frac{4\pi}{|\vec{q}|} \int f(r) \sin(|\vec{q}|r)\, dr$$

b) For a homogeneous charge distribution with radius R,

$$f(r) = \begin{cases} \frac{3}{4\pi R^3} & r \leq R, \\ 0 & r > R. \end{cases}$$

show that the form factor is given by

$$F(\vec{q}^{\,2}) = \frac{3}{(|\vec{q}|R)^3} \left[\sin\left(|\vec{q}|R\right) - |\vec{q}|R\cos\left(|\vec{q}|R\right)\right] .$$

2. Imagine that you have performed an experiment and measured the cross sections for the processes

$$p + N \to \mu^+\mu^- + \text{anything}\,,$$

$$\pi^+ + N \to \mu^+\mu^- + \text{anything}\,,$$

$$\pi^- + N \to \mu^+\mu^- + \text{anything}\,,$$

where $p = (uud)$ is a proton, $\pi^+ = (u\bar{d}), \pi^- = (\bar{u}d)$ are charged pions and N is a target nucleus with equal numbers of protons and neutrons.

a) Assume that there are only valence quarks in the hadrons involved in the proton and pion scattering off the nucleus N that share the momentum equally (no x-dependence) and show that

$$\frac{d\sigma_{pN}(s,m)}{dm} : \frac{d\sigma_{\pi^+N}(s,m)}{dm} : \frac{d\sigma_{\pi^-N}(s,m)}{dm} = 0 : 1 : 4.$$

b) Explain how the fact that the measurement to leading order agrees with the result of part a) supports the theory of the quark substructure of the pions!

c) The scattering cross sections can be used to determine the sea quark content of the proton and the neutron. One can do this by replacing $q \to (1-\epsilon)q + \epsilon\bar{q}$ in the proton and neutron and repeat the calculation of part b). Use this ansatz to show that for $\epsilon = 0.01$

$$\frac{d\sigma_{pN}(s,m)}{dm} : \frac{d\sigma_{\pi^+N}(s,m)}{dm} : \frac{d\sigma_{\pi^-N}(s,m)}{dm} = 0.17 : 1 : 3.85.$$

3. Consider the charged current deep inelastic scattering process $\nu_\mu(k) + N(P) \to \mu^-(k') + X(P')$, where N is a nucleon, X denotes a system of hadrons and the four momenta of the particles are given in brackets. The nucleon is initially at rest, the energy of the neutrino is E, the muon E' and mass of the muon may be neglected. The kinematic variables

$$q = k - k', \qquad Q^2 = -q^2, \quad \text{and} \quad x = \frac{Q^2}{2P\cdot q},$$

are used to describe the scattering process.

The cross sections for the scattering of a neutrino from a down quark (antineutrino from a down antiquark) and a neutrino from an up antiquark (antineutrino from an up quark) are:

$$\sigma(\nu_\mu d) = \sigma(\bar{\nu}_\mu \bar{d}) = \frac{G_F^2 xs}{\pi}; \qquad \sigma(\nu_\mu \bar{u}) = \sigma(\bar{\nu}_\mu u) = \frac{G_F^2 xs}{3\pi}.$$

$$\sigma(\nu_\mu u) = \sigma(\bar{\nu}_\mu \bar{u}) = \sigma(\nu_\mu \bar{d}) = \sigma(\bar{\nu}_\mu d) = 0$$

Assume that the nucleons only contain up and down quarks, and their corresponding antiquarks.

b) Show that the parton model predicts that the average cross sections for neutrino-nucleon and antineutrino-nucleon scattering are

$$\sigma(\nu_\mu N) = \frac{1}{2}\left(\sigma(\nu_\mu p) + \sigma(\nu_\mu n)\right) = \frac{G_F^2 s}{2\pi}\left[f_q + \frac{1}{3}f_{\bar{q}}\right],$$

$$\sigma(\bar{\nu}_\mu N) = \frac{1}{2}\left(\sigma(\bar{\nu}_\mu p) + \sigma(\bar{\nu}_\mu n)\right) = \frac{G_F^2 s}{2\pi}\left[\frac{1}{3}f_q + f_{\bar{q}}\right],$$

where G_F is the Fermi constant and $f_q = f_u + f_d$ and $f_{\bar{q}} = f_{\bar{u}} + f_{\bar{d}}$ are the average momentum fractions carried by the up and down quarks and antiquarks, respectively.

c) Experimentally, for $E \gg M$, $\frac{\sigma(\nu_\mu N)}{E_\nu} = 0.43\frac{G_F^2 M}{\pi}$ and $\frac{\sigma(\nu_{\bar{\mu}} N)}{E_{\bar{\nu}}} = 0.21\frac{G_F^2 M}{\pi}$. Calculate f_q and $f_{\bar{q}}$ and comment on the physical significance of your result.

4. a) Draw the Feynman diagram for quark-antiquark pair production and a muon-antimuon pair in an electron-positron collider. Describe the behaviour of the cross section ratio

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

for energies up to below the Z boson mass. Explain the values taken by the ratio

b) The J/ψ is a $L = 0$ $c\bar{c}$ bound state with mass 3097 MeV and width 92.2 keV. It can decay into a muon-antimuon pair. What are the possible spin and orbital angular momentum of the muon pair? Calculate the momentum of the muon after decay in the rest frame of the J/ψ.

c) The J/ψ can also decay into three pions. Calculate the angle between the first and second pions as a function of the magnitudes of their momenta.

d) Given the partial widths

$$\Gamma(J/\psi \to \mu^+\mu^-) = 5.5\,\text{keV}\ , \quad \Gamma(J/\psi \to ggg) = 69.4\,\text{keV}\ ,$$

and the cross section formula for the cross section for $J/\psi \to f$ as a function of the centre of mass energy E

$$\sigma_{e^+e^-\to f}(E) = \frac{3\pi\lambda^2}{16\pi}\frac{\Gamma_{J/\psi\to e^+e^-}\Gamma_{J/\psi\to f}}{(E - M_{J/\psi})^2 + \frac{\Gamma_{tot}^2}{4}},$$

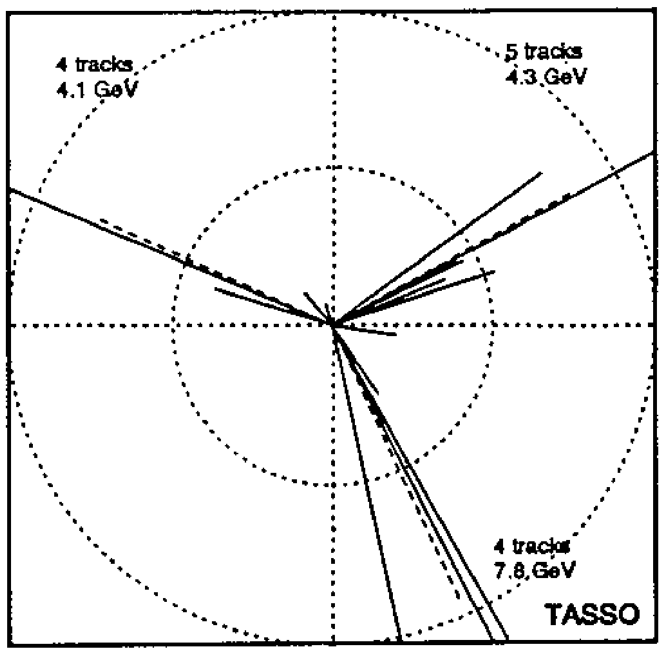
sketch on the same plot the energy dependence of the cross sections for $f = ggg$ and $f = \mu^+\mu^-$ around the J/ψ mass.

e) Derive an estimate of the number of quark colours from the partial widths above and the following approximations:

- The three-gluon and electromagnetic decay through a virtual photon are the only decay channels.
- Up to the factor due to the electric charge, the cross sections for the decay through a virtual photon for all allowed fermions are the same.

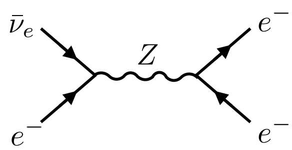
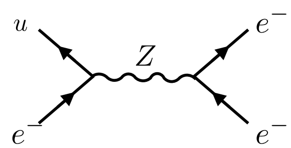
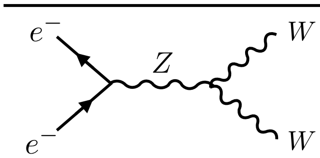
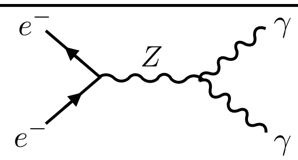
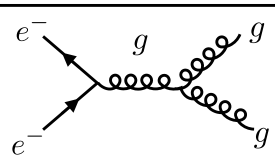
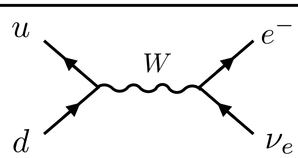
[Hint: $m_e = 0.51\,\text{MeV/c}^2$, $m_\mu = 105.7\,\text{MeV/c}^2$, $m_\tau = 1777\,\text{MeV/c}^2$]

5. The figure below shows an event display of a three-jet final state observed at an electron-positron collider. Draw the Feynman diagram that leads to this three-jet final state and explain why this picture is considered evidence for the existence of gluons.



6. Is it possible for an energetic photon γ to decay into an electron and a positron $\gamma \to e^+e^-$? Proof your answer using relativistic energy and momentum conservation. How does your answer change if the photon is virtual $\gamma^* \to e^+e^-$?

7. Some of the Feynman diagrams in the table below are not allowed by the conservation laws we discussed in the lecture. Write down which diagrams are allowed and which are not and identify the reason for why they are not allowed. Remember that the arrows on the fermion lines indicate fermions and antifermions and *not* the direction of travel. In all diagrams time flows from the bottom to the top and all (external) momenta follow this direction.

8. For the hadronic decays listed below determine which are possible, which force the decay occurs via and draw quark-line diagrams for the allowed decays:

i) $\rho^0 \to \pi^+\pi^-$; $\rho^0 \to \pi^0\pi^0$;

ii) $K^0 \to \pi^+\pi^-$; $\bar{K}^0 \to \pi^+\pi^-$;

iii) $K^0 \to \pi^0\pi^0$; $\bar{K}^0 \to \pi^0\pi^0$;

iv) $b_1^+ \to \omega\pi^+$; $b_1^+ \to \pi^+\pi^0$;

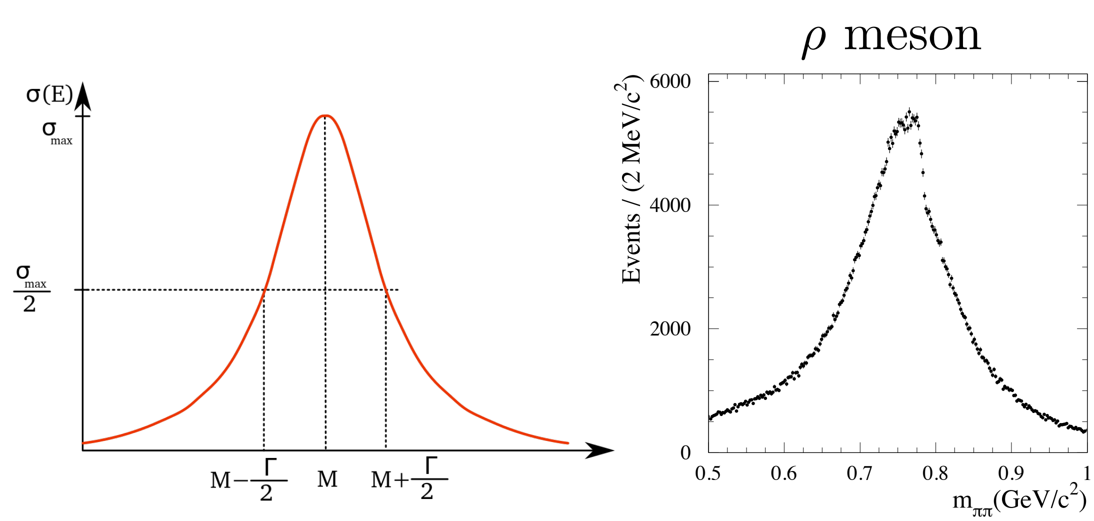
v) $B^0 \to D^-\pi^+$; $B^0 \to \pi^+\pi^-$; $B^0 \to J/\psi K^0$.

The b_1^+ ($|u\bar{d}\rangle$) has $J^P = 1^+$. You can argue using parity and angular momentum conservation. Keep in mind that for weak decays parity must <u>not</u> be conserved!

9. Particles produced in experiments manifest themselves as resonances in the decay products. The lifetime τ of a particle is related to the decay width Γ at $\approx \sigma_{\max}/2$ (where $\sigma_{\max}$ is the peak of the cross-section at the resonance) via the uncertainty principle

$$\Gamma = \frac{\hbar}{\tau}\,.$$

This is illustrated on the left in the figure below. The right panel of the figure shows the number of events per bin for $\pi^+\pi^-$ pairs produced around the ρ resonance. Calculate the lifetime of the ρ meson (Use that $\hbar = 6.582 \times 10^{-16}\,\mathrm{eV\,s}$).
Is the result compatible with the diagram you found in part **8.**i)? What would the plot for the same experiment look like if it had searched for the decay $B^0 \to \pi^+\pi^-$ instead? Use that the mass of the B^0 is given by $m_B = 5280$ MeV and its lifetime $\tau_B = 1.52 \times 10^{-12}$ s. Why are the ρ and B lifetimes so different?



10. a) What are the possible decays of the W^- boson?

b) Using the values for the CKM matrix,

$$|V_{\mathrm{CKM}}| = \begin{pmatrix} |V_{ud}| & |V_{cd}| & |V_{td}| \\ |V_{us}| & |V_{cs}| & |V_{ts}| \\ |V_{ub}| & |V_{cb}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 1 & 0.22 & 0.003 \\ 0.22 & 1 & 0.04 \\ 0.003 & 0.04 & 1 \end{pmatrix}$$

calculate the relative size for all the different W^- branching ratios:

$$\mathrm{Br}(W^- \to \mu^- \bar{\nu}_\mu) : \mathrm{Br}(W^- \to e^- \bar{\nu}_e) : \mathrm{Br}(W^- \to s\bar{u}) : \ldots$$

where $\mathrm{Br}(W^- \to \mu^- \bar{\nu}_\mu) = \Gamma(W^- \to \mu^- \bar{\nu}_\mu)/\Gamma_{\mathrm{total}}$ and you can neglect effects from the fermion masses.

c) What is the most and least probable decay mode for the W^- boson?

d) The partial W^- boson decay width into electron and electron-neutrino is given by

$$\Gamma(W^- \to e^- \bar{\nu}_e) = \frac{g^2 M_W}{48\pi}\,.$$

Use $M_W = 80.4\,\mathrm{GeV}$ and $g = 0.65$, and calculate the total decay width of the W^- boson (you should find $\Gamma_{\mathrm{total}} = 2.095$ GeV).

e) The lifetime of the W^- boson is related to the width by $\tau = \frac{\hbar}{\Gamma_{\mathrm{tot}}}$. Use $\hbar = 6.58 \times 10^{-25}$ GeV s to calculate the lifetime of the W^-.

NPP Repetition Solutions

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1. a) For a spherically symmetric charge distribution

$$
\begin{aligned}
F(\vec{q}^{\,2}) &= \int \exp\left(i\vec{q}\cdot\vec{x}\right) f(\vec{x})\mathrm{d}^3x \\
&= 2\pi \int \mathrm{d}r \int_{-1}^{1} \mathrm{d}\cos\theta \exp\left(i|\vec{q}|r\cos\theta\right) f(r) \\
&= 2\pi \int \mathrm{d}r f(r) \left[\frac{\exp\left(i|\vec{q}|r\cos\theta\right)}{i|\vec{q}|r}\right]_{-1}^{1} \\
&= \frac{4\pi}{|\vec{q}|} \int f(r) r \sin|\vec{q}|r \mathrm{d}r
\end{aligned}
$$

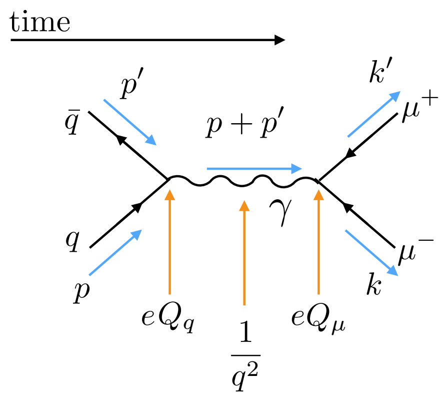
b) In this case

$$
F(\vec{q}^2) = \frac{4\pi}{|\vec{q}|}\frac{3}{4\pi R^3}\int_0^R r\sin|\vec{q}|r\mathrm{d}r.
$$

Integrating by parts gives

$$
\begin{aligned}
F(\vec{q}^2) &= \frac{3}{|\vec{q}|R^3}\left\{\left[-\frac{r}{\vec{q}}\cos|\vec{q}|r\right]_0^R + \frac{1}{|\vec{q}|}\int_0^R \cos|\vec{q}|r\mathrm{d}r\right\} \\
&= \frac{3}{|\vec{q}|R^3}\left[-\frac{r}{\vec{q}}\cos|\vec{q}|r + \frac{1}{|\vec{q}|^2}\sin|\vec{q}|r\right]_0^R \\
&= \frac{3}{|\vec{q}|^3R^3}\left[-R|\vec{q}|\cos|\vec{q}|R + \sin|\vec{q}|R\right].
\end{aligned}
$$

2. a) Using the Feynman rules discussed in the lecture, we can write for the matrix element and the cross sections



$$
\mathcal{M} \propto \frac{\alpha}{q^2}Q_qQ_\mu \quad \Rightarrow \quad \sigma(q\bar{q}) \propto \frac{\alpha^2}{q^2}Q_q^2Q_\mu^2
$$

where $\alpha = e^2/(4\pi)$ and the electric charge of the muon is $Q_\mu = -1$. Note that the cross section is proportional to the matrix element squared $\sigma \propto |\mathcal{M}|^2$, but the photon momentum only enters with $1/q^2$. The reason for this is that we are omitting factors of q from the full calculation. We know however that the scattering cross section has units of an area= length^2. In natural units this corresponds to inverse mass squared $1/q^2 = 1/s = 1/(p+p')^2 = 1/(k+k')^2 \equiv 1/m^2$. The scattering cross sections for $d - \bar{d}$ and $u - \bar{u}$ annihilation read

$$\begin{aligned}
\sigma(u\bar{u} \to \mu^+\mu^-) &\propto \frac{1}{m^2}\alpha^2 Q_u^2\,, \\
\sigma(d\bar{d} \to \mu^+\mu^-) &\propto \frac{1}{m^2}\alpha^2 Q_d^2\,.
\end{aligned}$$

with $Q_u = 2/3$ and $Q_d = -1/3$. All other cross sections vanish,

$$\begin{aligned}
&\sigma(\bar{u}d \to \mu^+\mu^-) = \sigma(ud \to \mu^+\mu^-) \\
&= \sigma(uu \to \mu^+\mu^-) = \sigma(dd \to \mu^+\mu^-) = 0\,,
\end{aligned}$$

because of charge conservation, and because we assume that there are only valence quarks in the target nucleus.

In the next step we need to calculate the probability of quarks and antiquarks meeting in the scattering process. In general this involves an integral over the momentum fraction of all target and projectile quarks, e.g.

$$\begin{aligned}
\sigma_{\pi^+N} = &\int_0^1 dx\, d_N(x)\left(\bar{d}_{\pi^+}(x)\sigma(d\bar{d} \to \mu^+\mu^-) + u_{\pi^+}(x)\sigma(du \to \mu^+\mu^-)\right) \\
&+ u_N(x)\left(\bar{d}_{\pi^+}(x)\sigma(u\bar{d} \to \mu^+\mu^-) + u_{\pi^+}(x)\sigma(uu \to \mu^+\mu^-)\right) \\
\propto &\int_0^1 dx\, d_N(x)\, \bar{d}_{\pi^+}(x)\sigma(d\bar{d} \to \mu^+\mu^-)\,,
\end{aligned}$$

where $d_N(x)$ and $u_N(x)$ are the particle distribution functions for the down and up quarks in the target, and $\bar{d}_{\pi^+}(x)$ and $u_{\pi^+}(x)$ are the particle distribution functions for the antidown and up quarks in the projectile and in the last line we kept only the term for which the cross section doesn't vanish. In general, these particle distribution functions are unknown and experimental input is needed to solve the x-integral. In this exercise, the particle distribution functions do not depend on x and the valence quark share the momentum equally, so that the x-integral becomes trivial

$$\begin{aligned}
\sigma_{\pi^+N} &= \int_0^1 dx\, d_N(x)\, \bar{d}_{\pi^+}(x)\sigma(d\bar{d} \to \mu^+\mu^-) \\
&= d_N\, \bar{d}_{\pi^+}\sigma(d\bar{d} \to \mu^+\mu^-) \int_0^1 dx \\
&= \frac{1}{2}\frac{1}{2}\sigma(d\bar{d} \to \mu^+\mu^-) \cdot 1 \\
&= \frac{\alpha^2}{4}\frac{1}{m^2}\frac{1}{9} .
\end{aligned}$$

Since none of the particle distribution functions depend on x, we can ignore the $x-$integral and just keep the $q\bar{q}$ cross sections that contribute (we can ignore universal constant factors as well, because we are only interested in ratios of cross sections).

The quark structure of the target can be written as $N = (uud+ddu)$, whereas the proton is $p = (uud)$ and the charged pion quark structure is given by $\pi^+ = (u\bar{d})$ and $\pi^- = (\bar{u}d)$. In this approximation, we find for the cross sections:

$$\sigma_{pN} = \sigma((uud) + (uud + ddu)) = 0 ,$$

because there is no antiquark available. And for the pions

$$\begin{aligned}
\sigma_{\pi^+N} &= \sigma((u\bar{d}) + (uud + ddu)) = \sigma(\bar{d}d) \propto \frac{1}{m^2}\alpha^2\frac{1}{9} , \\
\sigma_{\pi^-N} &= \sigma((\bar{u}d) + (uud + ddu)) = \sigma(\bar{u}u) \propto \frac{1}{m^2}\alpha^2\frac{4}{9} ,
\end{aligned}$$

and therefore

$$\frac{d\sigma_{pN}(s,m)}{dm} : \frac{d\sigma_{\pi^+N}(s,m)}{dm} : \frac{d\sigma_{\pi^-N}(s,m)}{dm} \;=\; 0 : 1 : 4.$$

Note that it is straightforward to calculate the differential cross section

$$\frac{d\sigma(d\bar{d} \to \mu^+\mu^-)}{dm} \propto -\frac{2}{m^3}\alpha^2\frac{1}{9} ,$$

but since we take ratio this is not necessary.

b) If the pions were pointlike particles, one would expect the annihilation cross section to vanish, because the target consists of protons and neutrons. The fact that the cross sections don't vanish is therefore a sign of a common substructure of pions, protons and neutrons.

c) We can write for the proton and the target nucleus

$$\begin{aligned}
p &= (1-\epsilon)(uud) + \epsilon(\bar{u}\bar{u}\bar{d}) , \\
N &= (1-\epsilon)(uud) + \epsilon(\bar{u}\bar{u}\bar{d}) + (1-\epsilon)(ddu) + \epsilon(\bar{d}\bar{d}\bar{u}) .
\end{aligned}$$

This results in the cross sections (ignoring the common pre-factor)

$$
\begin{aligned}
\sigma_{pN} &= \sigma\Big\{\big[(1-\epsilon)(uud)+\epsilon(\bar{u}\bar{u}\bar{d})\big]+\big[(1-\epsilon)(uud)+\epsilon(\bar{u}\bar{u}\bar{d})\\
&\quad +(1-\epsilon)(ddu)+\epsilon(\bar{d}\bar{d}\bar{u})\big]\Big\}\\
&= (1-\epsilon)\epsilon(4+4+2+2)\sigma(u\bar{u})+(1-\epsilon)\epsilon(1+2+2+1)\sigma(d\bar{d})\\
&= (1-\epsilon)\epsilon\left(12\frac{4}{9}+6\frac{1}{9}\right)\\
&= 6(1-\epsilon)\epsilon\,,\\
\sigma_{\pi^+N} &= \sigma\Big\{(u\bar{d})+\big[(1-\epsilon)(uud)+\epsilon(\bar{u}\bar{u}\bar{d})+(1-\epsilon)(ddu)+\epsilon(\bar{d}\bar{d}\bar{u})\big]\Big\}\\
&= (2+1)\epsilon\sigma(u\bar{u})+(1+2)(1-\epsilon)\sigma(d\bar{d})\\
&= \epsilon\,3\,\frac{4}{9}+(1-\epsilon)\,3\,\frac{1}{9}=\frac{1}{3}(1+3\epsilon)\,,\\
\sigma_{\pi^-N} &= \sigma\Big\{(\bar{u}d)+\big[(1-\epsilon)(uud)+\epsilon(\bar{u}\bar{u}\bar{d})+(1-\epsilon)(ddu)+\epsilon(\bar{d}\bar{d}\bar{u})\big]\Big\}\\
&= (2+1)(1-\epsilon)\sigma(u\bar{u})+(1+2)\epsilon\sigma(d\bar{d})\\
&= (1-\epsilon)\,3\,\frac{4}{9}+\epsilon\,3\,\frac{1}{9}=\frac{1}{3}(4-3\epsilon)\,,
\end{aligned}
$$

As a result, the relative fractions read

$$
\frac{d\sigma_{pN}(s,m)}{dm}:\frac{d\sigma_{\pi^+N}(s,m)}{dm}:\frac{d\sigma_{\pi^-N}(s,m)}{dm}\quad=18(1-\epsilon)\epsilon:(1+3\epsilon):(4-3\epsilon)
$$

And for $\epsilon = 0.01$ this gives (setting $(1-\epsilon)\to 1$)

$$
18(1-\epsilon)\epsilon:(1+3\epsilon):(4-3\epsilon)\longrightarrow 0.178:1.03:3.97=0.17:1:3.85
$$

3. The cross section for neutrino scattering from the nucleon is

$$
\sigma(\nu_\mu)=\int \mathrm{d}x\,\big[d_N(x)\sigma(\nu_\mu d)+\bar{u}_N(x)\sigma(\nu_\mu\bar{u})\big]
$$

Therefore averaging over proton/neutron

$$
\sigma(\nu_\mu N)=\frac{1}{2}\int \mathrm{d}x\,\big[(d_p(x)+d_n(x))\sigma(\nu_\mu d)+(\bar{u}_p(x)+\bar{u}_n(x))\sigma(\nu_\mu\bar{u})\big]
$$

Now isospin symmetry implies $d_p(x)=u_n(x)$, $u_p(x)=d_n(x)$, $\bar{d}_p(x)=\bar{u}_n(x)$, $\bar{u}_p(x)=\bar{d}_n(x)$.

Therefore.

$$\begin{aligned}\sigma(\nu_\mu N) &= \frac{1}{2}\int \mathrm{d}x\,\left[(d_p(x)+u_p(x))\sigma(\nu_\mu d)+(\bar{u}_p(x)+\bar{d}_p(x))\sigma(\nu_\mu \bar{u})\right]\\ &= \frac{G_F^2 s}{2\pi}\int \mathrm{d}x\left[x(d_p(x)+u_p(x))+\frac{1}{3}x(\bar{u}_p(x)+\bar{d}_p(x))\right]=\frac{G_F^2 s}{2\pi}\left[f_q+\frac{1}{3}f_{\bar{q}}\right].\end{aligned}$$

Similarly for the antineutrino the calculation proceeds in the same way with quarks/antiquarks exchanged giving the stated result.

c) Using $s = 2EM$ gives:

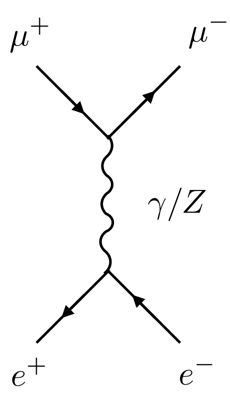
$$A_\nu = \frac{\pi\sigma(\nu_\mu N)}{G_F^2 E_\nu M} = f_q + \frac{1}{3}f_{\bar{q}};\qquad A_{\bar{\nu}} = \frac{\pi\sigma(\bar{\nu}_\mu N)}{G_F^2 E_{\bar{\nu}} M} = \frac{1}{3}f_q + f_{\bar{q}}.$$

Therefore

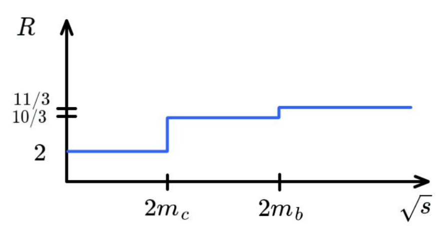
$$\begin{aligned} A_\nu - \frac{1}{3}A_{\bar{\nu}} &= \frac{8}{9}f_q &&\Rightarrow f_q = \frac{9}{8}\left[A_\nu - \frac{1}{3}A_{\bar{\nu}}\right] = 0.40\\ A_{\bar{\nu}} - \frac{1}{3}A_\nu &= \frac{8}{9}f_{\bar{q}} &&\Rightarrow f_q = \frac{9}{8}\left[A_{\bar{\nu}} - \frac{1}{3}A_\nu\right] = 0.08\end{aligned}$$

The physical significance is that the sum only amounts to $\sim 50\%$ of the total momentum, so the rest must be carried by the gluons.

4. a) The Feynman diagram and the R-ratio plot are given by



(same for $q\bar{q}$)



The values of the R-ratio correspond to $3 * \sum q_f^2$ where the sum runs over the quarks with $\sqrt{s} > 2m_f$.

b) The J/ψ has $J^P = 1^-$ for the final state the muon pair has intrinsic $P = -1$ so the angular momentum has to be even. For $L = 0, 2$ we can combine with the spin-one combination of the muon spins to get $J = 1$, but not with the $S = 0$ combination. In this frame both momenta are equal in length and opposite.

Through energy conservation we have

$$M_{J/\psi} = 2\sqrt{m_\mu^2 + p^2} \Rightarrow p = \frac{1}{2}\sqrt{M_{J/\psi}^2 - 4m_\mu^2} \; ,$$

c) The three momenta are

$$(p_1, 0, 0) \; , \quad (p_2 \cos\theta, p_2 \sin\theta, 0) \; , \quad (-p_1 - p2\cos\theta, -p_2 \sin\theta, 0) \; .$$

Equating the energy components we get

$$\begin{aligned} M_{J/\psi} &= \sqrt{p_1^2 + m^2} + \sqrt{p_2^2 + m^2} + \sqrt{(p_1 + p_2\cos\theta)^2 + (p_2 \sin\theta)^2 + m^2} \\ &= \sqrt{p_1^2 + m^2} + \sqrt{p_2^2 + m^2} + \sqrt{p_1^2 + 2p_1p_2\cos\theta + p_2^2 + m^2} \\ &\Rightarrow \cos\theta = \frac{1}{2p_1p_2}\left[\left(M - \sqrt{p_1^2 + m^2} - \sqrt{p_2^2 + m^2}\right)^2 - p_1^2 - p_2^2 - m^2\right] \end{aligned}$$

d) The two curves have a bell shape and the two width at half minimum are the same.

e) The mass of the J/ψ is to low for it to decay into tau leptons so the branching fraction to decay through a quark pair is given by:

$$B(J/\psi \to q\bar{q}) = 1 - B_{\text{strong}} - 2B(J/\psi \to \mu^+\mu^-) = 12.8\%$$

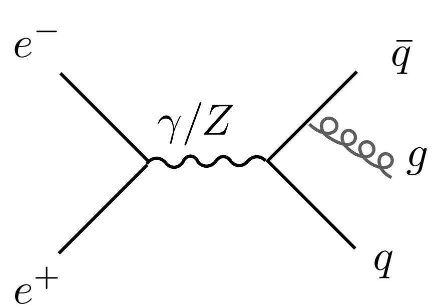
The quarks invoved can be either u, d or s but not c, b or t as they are too heavy. The ratio of the cross section for quarks to that of the muons will therefore be

$$R = \left(\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 = \frac{6}{9} = \frac{2}{3}$$

So the estimated number of colours is

$$N_C = \frac{B(q\bar{q})}{RB(\mu^+\mu^-)} = 3.2$$

5. The initial state is colour-neutral, therefore only quark-antiquark pairs can be produced in the final state. That would result in an even number of jets, unless a gluon is radiated off the final state quarks, see the Feynman diagram below



6. Energy conservation gives

$$E_\gamma = E_+ + E_-$$
$$p_\gamma c = \sqrt{p_e^2 c^2 + m_e^2 c^4} + \sqrt{p_p^2 c^2 + m_p^2 c^4}$$

We know that electron and positron have the same mass and also it follows that $p_e = p_p$, because the transversal components of the momenta (perpendicular to the original photon direction) need to be conserved: $0 = p_e \sin\theta - p_p \sin\theta$. Therefore

$$p_\gamma c = 2\sqrt{p_e^2 c^2 + m_e^2 c^4}$$

Momentum conservation in the direction of travel of the photon gives

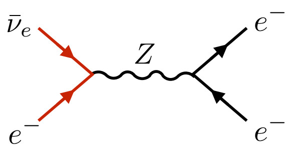
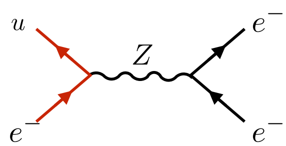
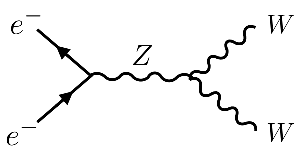
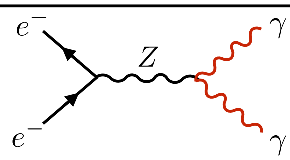
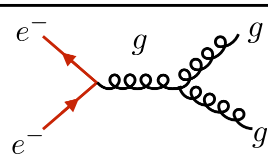
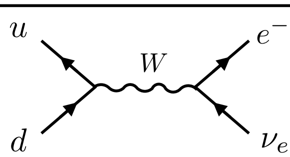
$$p_\gamma = p_e \cos\theta + p_p \cos\theta = 2 p_e \cos\theta$$

Equating these equations yields

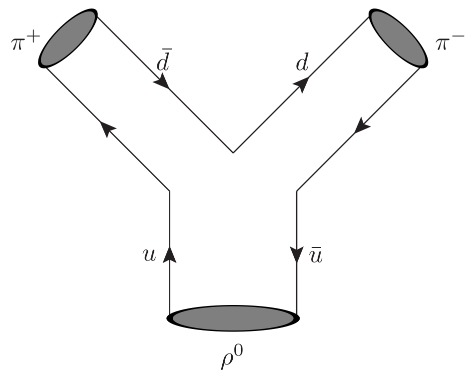
$$p_e \cos\theta = \sqrt{p_e^2 + m_e^2 c^2}$$

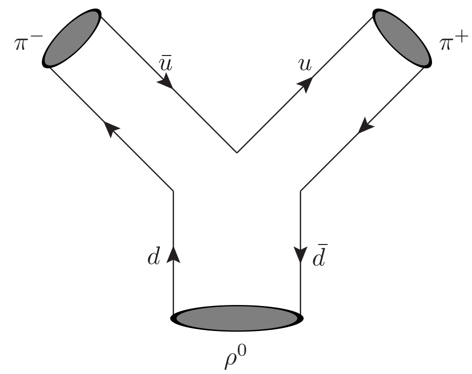
Since $\cos\theta \leq 1$ and $m_e c > 0$, momentum and energy cannot be conserved simultaneously. For a virtual photon this argument is not true, because $E_{\gamma^*} \neq p_{\gamma^*} c$. A virtual photon can "decay" into an electron-positron pair.

7. The table reads

	Not allowed because of lepton number conservation. On the left vertex: $L[e^-] - L[\bar{\nu}_e] = 1 - (-1) \neq 0$
	Not allowed because of lepton- and baryon number conservation, as well as electric charge conservation: $-Q[u] + Q[e] = -2/3 + (-1) \neq 0$
	allowed
	Not allowed,because the photon only couples to charged particles and $Q[Z] = 0$.
	Not allowed,because gluons couple only to colour charged particles and leptons are uncoloured.
	allowed

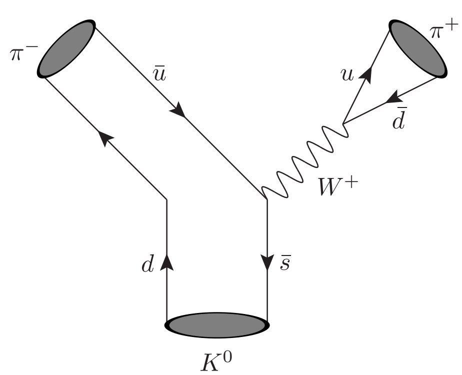
8. i) For $\rho^0 \to \pi^+\pi^-$ the initial state has $J^P = 1^-$ therefore the final state must have odd parity and $J = 1$. Due to angular momentum conservation the final state must have $\ell = 1$. Therefore it has $P = (-1)^\ell(-1)^2 = -1$. Therefore the decay is allowed.

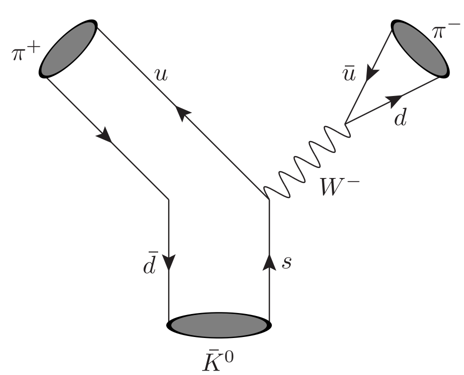




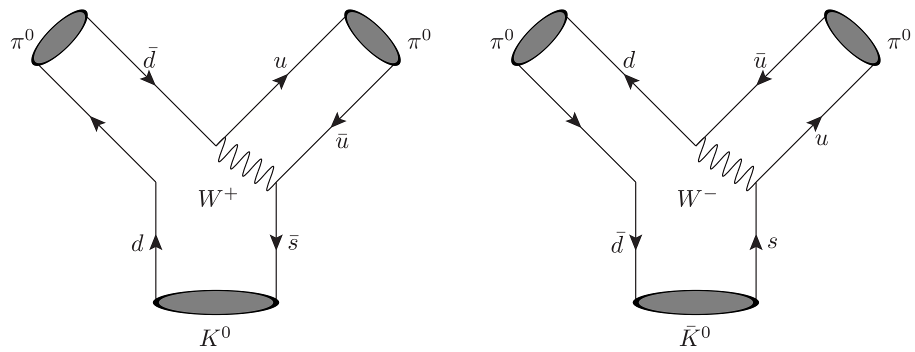
For $\rho^0 \to \pi^0\pi^0$ the same arguments apply however we now have a pair of identical bosons in the final state so that the final wavefunction must be symmetric under the exchange of the two neutral pions. However this requires that the orbital angular momentum is even, which is inconsistent with the other conservation laws so the decay is not allowed.

ii) The initial state has an anti-strange quark or strange whereas the final state does not, therefore any decay must be weak. As in a weak decay parity does not have to be conserved and angular momentum can be conserved with $\ell = 0$. All the decays are allowed.

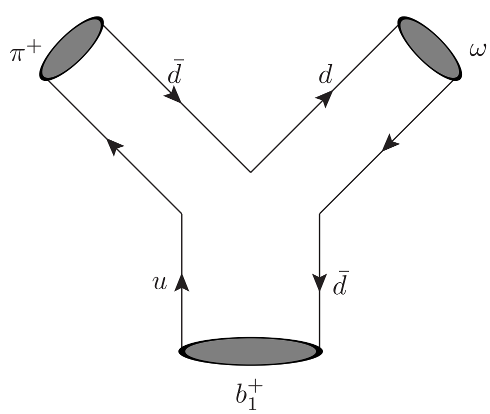




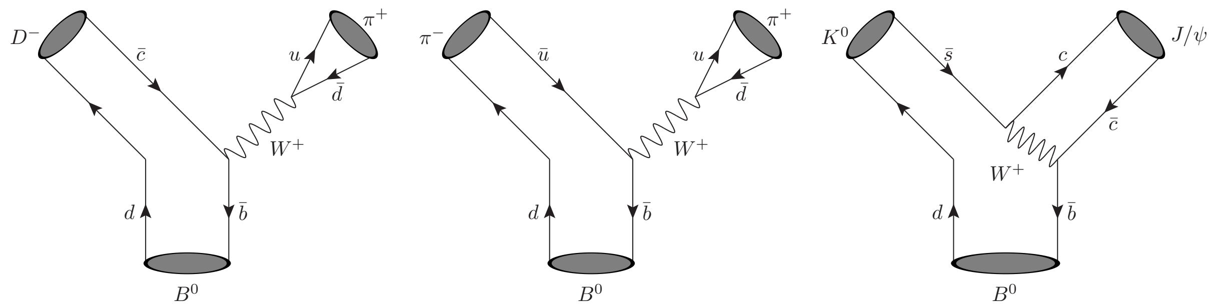
iii) The initial state has an anti-strange quark or strange whereas the final state does not, therefore any decay must be weak. As in a weak decay parity does not have to be conserved and angular momentum can be conserved with $\ell = 0$. Therefore the wavefunction is symmetric under the interchange of the identical pions as required. All the decays are allowed.



iv) The final-state system must have even parity parity $+1 = (-1)^\ell \times (-1)^2$ and therefore ℓ is even. In the first case $\ell = 0, 2$ then allows the conservation of angular momentum, i.e. $J = \ell + s = 0 + 1 = 1$ or $J = \ell + s = 2 + 1 = 1, 2, 3$, so the decay is allowed. While for the second decay as $s = 0$ and therefore $J = \ell$ so angular momentum and parity cannot be simultaneously conserved and the decay is not allowed by the strong force. In principle a weak decay is possible but would be very small as there are allowed strong decays.



v) In all the cases the initial state has an anti-bottom quark while the final state does not, therefore there must be a weak decay. In the first two cases angular momentum is conserved if the particles are in an $\ell = 0$ state while in the final case $\ell = 1$. All the decays are allowed.



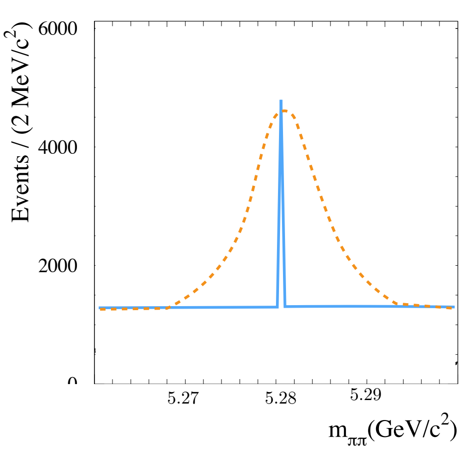
9. From the panel on the right of the figure one can estimate $\Gamma \approx 0.13$ GeV. Using the relation between width and lifetime one can estimate the lifetime as

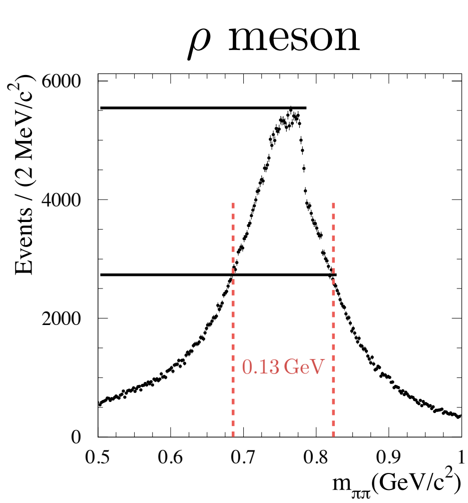
$$\tau_\rho = \frac{\hbar}{\Gamma_\rho} = \frac{6.582 \times 10^{-16}\,\text{eV s}}{0.13 \times 10^{9}\,\text{eV}} \approx 5 \times 10^{-24}\,\text{s}\,.$$

This tiny lifetime is a sign of a particle decaying through the strong interaction (compare it with the results in Table 1 in the lecture notes). This is in agreement with the diagram found in **8.**i).
For the B meson, the decay width is given by

$$\Gamma_B = \frac{\hbar}{\tau_B} = \frac{6.582 \times 10^{-16}\,\text{eV s}}{1.52 \times 10^{-12}\,\text{s}} \approx 4.3 \times 10^{-3}\,\text{eV}\,.$$

Naively one would expect the corresponding plot to look similar to the blue contour in the plot on the left below (the width is exaggerated). In reality it looks closer to the dashed orange contour. The experimental resolution is (by far) not good enough to measure such tiny widths and the particle width is then smeared out.





10. a) The W^- boson can decay into leptons

$$\Gamma(W^- \to e^-\bar{\nu}_e),\ \Gamma(W^- \to \mu^-\bar{\nu}_\mu),\ \Gamma(W^- \to \tau^-\bar{\nu}_\tau)\,,$$

but no terms mixing two different generations because of the separetely conserved lepton numbers L_e, L_μ and L_τ. It can also decay into the quarks

$$\begin{aligned}
&\Gamma(W^- \to d\bar{u}),\ \Gamma(W^- \to c\bar{s})\,,\\
&\Gamma(W^- \to d\bar{c}),\ \Gamma(W^- \to s\bar{u})\,,\ \Gamma(W^- \to b\bar{u})\,,\ \Gamma(W^- \to b\bar{c})\,,
\end{aligned}$$

where the decay width in the second line are CKM-suppressed. Decays into top quarks are not allowed, because the top is heavier than the W^-.

b) The partial widths of the W^- scale as $\Gamma(W^- \to d\bar{u}) \propto 3g^2|V_{ud}|^2 M_W$ for quarks and $\Gamma(W^- \to e^-\bar{\nu}_e) \propto g^2 M_W$, where the factor 3 comes from the 3 indistinguishable colours of the quark. The relative size of the partial decay width is then

$$\Gamma(W^- \to e^-\bar{\nu}_e) \propto g^2 M_W,\ \Gamma(W^- \to \mu^-\bar{\nu}_\mu) \propto g^2 M_W,\ \Gamma(W^- \to \tau^-\bar{\nu}_\tau) \propto g^2 M_W\,,$$

and

$$\begin{aligned}
\Gamma(W^- \to d\bar{u}) &\propto 3g^2|V_{ud}|^2 M_W \propto 3g^2 M_W,\\
\Gamma(W^- \to s\bar{c}) &\propto 3g^2|V_{cs}|^2 M_W \propto 3g^2 M_W\,,\\
\Gamma(W^- \to d\bar{c}) &\propto 3g^2|V_{cd}|^2 M_W \propto 3g^2(0.22)^2 M_W,\\
\Gamma(W^- \to s\bar{u}) &\propto 3g^2|V_{us}|^2 M_W \propto 3g^2(0.22)^2 M_W\,,\\
\Gamma(W^- \to b\bar{u}) &\propto 3g^2|V_{cd}|^2 M_W \propto 3g^2(0.003)^2 M_W\,,\\
\Gamma(W^- \to b\bar{c}) &\propto 3g^2|V_{cd}|^2 M_W \propto 3g^2(0.04)^2 M_W\,,
\end{aligned}$$

and therefore (the branching ratio for the decay $W^- \to xy$ is defined as $\mathrm{Br}(W^- \to xy) = \frac{\Gamma(W^- \to xy)}{\Gamma_{\text{tot}}}$, where Γ_{tot} is the total width of the W boson)

$$\begin{aligned}
&\mathrm{Br}(W^- \to \tau^-\bar{\nu}_\tau) : \mathrm{Br}(W^- \to \mu^-\bar{\nu}_\mu) : \mathrm{Br}(W^- \to e^-\bar{\nu}_e) :\\
&\mathrm{Br}(W^- \to d\bar{u}) : \mathrm{Br}(W^- \to s\bar{c}) :\\
&\mathrm{Br}(W^- \to d\bar{c}) : \mathrm{Br}(W^- \to s\bar{u}) : \mathrm{Br}(W^- \to b\bar{u}) : \mathrm{Br}(W^- \to b\bar{c})\\
&=\\
&\frac{1}{9.3} : \frac{1}{9.3} : \frac{1}{9.3} :\\
&\frac{3}{9.3} : \frac{3}{9.3} :\\
&\frac{0.146}{9.3} : \frac{0.146}{9.3} : \frac{8.1 \times 10^{-8}}{9.3} : \frac{0.048}{9.3}\,.
\end{aligned}$$

c) The most probable decay modes are $\mathrm{Br}(W^- \to s\bar{c}) = \mathrm{Br}(W^- \to d\bar{u}) = 32.26\%$. The least probable decay mode is $\mathrm{Br}(W^- \to b\bar{u}) = 8.7 \times 10^{-9}$.

d) We can use that $\Gamma_{\text{tot}} = 9.3 \times \Gamma(W^- \to e^- \bar{\nu}_e) = 2.095\,\text{GeV}$.

e) The total width can be used to calculate the W^- lifetime

$$\tau = \frac{\hbar}{\Gamma_{\text{tot}}} = \frac{6.58 \times 10^{-25}\,\text{GeV s}}{2.095\,\text{GeV}} = 3.14 \times 10^{-25}\,\text{s}\,.$$