Observational Techniques in Astronomy

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Chapter 1

Introduction

1.1 Why study astronomy in a physics degree?

Ever wondered where the Sun and Earth came from? What processes produced the baryons you are made from? Why stars shine? Why galaxies have a range of shapes, colours and sizes? Why we know black holes exist and what quasars are? How we know about the existence of dark matter, or that the expansion of the Universe is accelerating (dark enegy)? Of course you have - that's why you're studying for a physics degree, and astronomy forms an important part of basic physics. In particular, observational astronomy allows us to probe physical regimes that we will never be able to recreate in a lab. For example: Does gravity behave like $1/r^2$ also on **very** large scales? What is the content of the Universe: are there elementary particles in the Universe that we have not seen in particle accelerators (dark matter makes up $\sim 70\%$ of the mass in the Universe, but only manifests itself on the largest scales and has not been detected on Earth). How do the laws of physics behave at extremely high energies? What happens to mass and energy in and around the accretion disk and event horzion of a black hole?

Since this is primarily an "observational" course, we start this brief exploration of the Universe in the solar system.

1.1.1 The solar system

The Sun is a star of mass $M_\odot \approx 2 \times 10^{30}$kg, radius $R_\odot \approx 7 \times 10^8$m, luminosity $L_\odot = 3.9 \times 10^{26}$ J s^{-1}, at a distance of $R \approx 1.5 \times 10^{11}$m. Given these numbers, the Sun extends an angle of approximately half a degree on the sky.

The orbit of the Earth around the Sun is in a plane: recall that this is because of conservation of orbital angular momentum[1]:

$$\begin{aligned} \frac{d}{dt}\mathbf{r} \times \mathbf{v} &= \mathbf{v} \times \mathbf{v} + \mathbf{r} \times \frac{d\mathbf{v}}{dt} \\ &= \mathbf{0}\,. \end{aligned} \tag{1.1}$$

[1]There is of course some disturbance due to the other planets, but this is very small.

The first term is identical to zero since $\mathbf{a} \times \mathbf{a} = \mathbf{0}$ for any vector $\mathbf{a}$, and so is the second since the gravitational acceleration $d\mathbf{v}/dt$ is parallel to $\mathbf{r}$. The plane of the orbit, perpendicular to the constant angular momentum vector $\mathbf{l} = \mathbf{r} \times \mathbf{v}$, is called the *ecliptic*. Within the ecliptic, the Sun's (apparent) orbit is an ellipse, with small eccentricity, (i.e. the Earths orbit is nearly circular).

The moon orbits the Earth in a tidally locked orbit. This means that the spin period of the moon is equal to its orbital period (or a lunar day equals a lunar month), which is why on Earth we always see the same side of the moon. The far side of the moon was not seen by humans until the early 1960's during the first Apollo missions. The angular extent of the moon is also about 0.5 degrees and so the moon can just cover the Sun during a full solar eclipse. Lunar phases are caused by the changing fraction of the moon illuminated by the Sun, as seen from Earth. Passage of the moon through the Earth's shadow causes a lunar eclipse.

The Earth's spin axis, $\mathbf{s}$, (with rotation period 1 day) is not exactly parallel to $\mathbf{l}$, but instead the angle between them is around 23 degrees, and this causes the Earth's seasons. The pole star happens to lie very close to where $\mathbf{s}$ punctures the firmament. The Sun and moon also exert tidal forces on the Earth, making $\mathbf{s}$ vary in time by a small amount. These tidal forces also cause the tides in the oceans.

The Sun used to have 9 planets in orbital planes close to the ecliptic. However astronomers kept on discovering more objects with masses similar (or smaller than) Pluto in the plane close to the ecliptic. Most of these are small (and called minor planets), but the discovery in 2005 of a new object outside Pluto's orbit, with a size at least as great as Pluto's, forced a debate whether this should be considered a new planet. The International Astronomical Union (IAU) debated this issue in 2006 and decided on a new definition of planet, which caused Pluto to lose its planet status. So then there were eight. Orbits of *comets* are not restricted to a plane close to the ecliptic.

Recent improvements in measurement techniques have lead to the detection of more than 200 extra-solar planets, with several stars now having more than one confirmed planet. This is probably one of the most exciting areas of research in astronomy: how many stars have planets in "habitable zones" (planets sufficiently close to stars that water is not frozen, but not so close that it should be boiled off the planet). In the next 20 years, significant effort will be made to take spectroscopic measurements of such planets, searching for water, methane and carbon dioxide in their atmospheres – some of the signatures of life.

1.1.2 The Milky Way

On a dark night you can see around several 1000 stars with the naked eye, a tiny fraction of the 10^{11} or so stars in the Milky Way galaxy. Most of these stars appear bright simply because they are (relatively) nearby. The Milky Way galaxy is a spiral galaxy, with a bulge (which is dominated by old stars, and contains about 70% of the total number of stars in our galaxy) and a disk. The disk is a relatively thin plane of stars, and because the Sun lies in this plane, we see the Milky Way as a faint band of light on the night sky.

Distances to nearby stars are extremely large compared to stellar sizes. Even with powerful telescopes, we can only measure the sizes directly for a handful of very nearby and massive stars (using special techniques which we will discuss in Lecture 6).

At the center of the Milky-Way galaxy, there is a massive black-hole. Direct evidence for the existence of the black hole comes from very high resolution observations of the stars at the center of our galaxy which have been show to be moving on elliptical orbits around an invisible, yet very massive object. By following their orbits over the last ~ 30 years, the black hole mass has been determined to by $M_{\rm BH} \sim 3\times 10^6\,{\rm M}_\odot$, and our distance from the center of the galaxy has been determined to be $R = 7.6 \pm 0.3\,{\rm kpc}$. We will discuss how these measurements are made in Lecture 8.

1.1.3 Star clusters and other nebulae

The Milky Way contains objects that appear extended on the sky. Examples include groups of gravitationally bound stars called "clusters", Planetary Nebulae (or PNe, final stages in the evolution of intermediate mass stars, when the mantle of the star is flung into space and fluoresces in many beautiful colours), supernova explosions (a much more dramatic version of a PNe), and star-forming regions where gas (mainly Hydogren) is lit-up by the re-radiation of UV light from massive stars.

1.1.4 Other galaxies

The Milky Way (MW) is orbited by about a dozen small galaxies, many of which can be seen with the naked eye. For example the Large (and Small) Magellanic clouds[2] are easy to see in the southern night sky. The Andromeda galaxy (M31) is the nearest galaxy of similar mass to the Milky Way. Andomeda and the Milky Way are two of the most massive galaxies in the "local group". Due to gravitational attraction, the Milky Way and Andromeda are on a collision course. Andromeda is approaching us at $\approx 170\,{\rm km\,s^{-1}}$ and the estimated time to impact ~ 5 Gyr (which is also about the remaining lifetime of the Sun).

The Milky Way is just one of the approximately 10^9 galaxies of similar size or greater in the visible Universe. About half of these galaxies are elliptical galaxies, which are very different from the MW[3]. The distribution of galaxies in the Universe has a characteristic structure, becoming more homogeneous on larger and larger scales. Some galaxies contain an active nucleus, with matter accreting onto a super-massive black hole with mass as large as $10^9 M_\odot$, out-shining the host galaxy by factors of several 1000s. These are the quasars (QSOs).

1.1.5 Cosmology

Cosmology refers to the study of the structure and evolution of the Universe on the largest scales. Amazingly, Einstein's theory of general relativity, developed before the study of galaxies began in earnest and even before Hubble's discovery of the expansion of the Universe, nevertheless describes the Universe well. The theory also provides the framework for interpreting redshifts and distances on very large scales.

[2] Guess which Western explorer mentioned those first.

[3] You will see more of this in the Galaxies part of the course next term.

In the 1930's, Edwin Hubble found a relation between distance, r, to a galaxy, and the redshift, z, of lines in its spectrum

$$\lambda = \lambda_0 \, (1 + z) \, , \tag{1.2}$$

where λ_0 is the laboratory wavelength of a line, and λ the wavelength measured for a distant galaxy. For relatively nearby galaxies $z = H_0 r/c$ defines Hubble's relation, where H_0 is called Hubble's constant. Because of the units of the Hubble constant (km s^{-1} Mpc^{-1}), the inverse of the Hubble constant represents the age of the Universe (about 13.7 Gyr). Interpretation of the redshift, z, in terms of a Doppler shift, $z = v/c$, while reasonable for nearby galaxies, breaks down for distant galaxies. The current redshift record for a galaxy has $z \approx 7$, which would imply velocities much larger than the speed of light, in apparent conflict with special relativity. However, the general theory accounts for these observations.

Looking further back in space, earlier generations of galaxies and their stars appear quite different from those we find locally. One of the goals of modern astrophysics is to understand how these galaxies form, what physical processes cause a big variety in their sizes, masses and shapes.

Just after the Big Bang (around 13.7 Gyr ago), the Universe was smooth, but quantum processes result in small pertubations in the primordial density field. Under the effects of gravity, these initially small perturbations grow in time, and ultimately give rise to the formation of the first stars and galaxies, and ultimately clusters of galaxies and the large scale structure of the Universe.

Most of the hydrogen and helium in the Universe were formed in the Big Bang, about 13 Gyr ago. As the Universe expands, it cools adiabatically, just like any gas in the lab would. Vice versa, going back in time, the Universe was hotter, and eventually, early enough in the evolution of the Universe, the hydrogen was fully ionised. This was the case at redshifts $z \geq 1000$. When ionised, photons and electrons couple and gas and photons get into equilibrium. The famous equilibrium distributions are Maxwell-Boltzmann for the gas, and a Planck distribution for the photons. When the gas recombined below $z = 1000$, photons stopped interacting with electrons (who were bound inside neutral hydrogen), and these photons have been redshifting ever since. They are observed as the microwave background (CMB) now. Observations of the CMB have allowed stringent constraints on the cosmological parameters that describe our Universe.

One of the may exciting recent discoveries in cosmology is the firm evidence that there is much more matter in the Universe than just in baryons (baryons are anything made up out of protons and neutrons, such as for example the Sun, the Earth, you, and the exam paper!). The existence of this so called 'dark matter' was first postulated by Fritz Zwicky in the 1930s, to explain the motion of galaxies in groups of galaxies. Gravitational lensing and CMB observations have provided strong support for this, yet we still do not know what type of particle makes up the dark matter.

However, even more puzzling is the recent discovery that even dark matter is not the main constituent of the Universe. Figure 1.1 shows that most of the Universe is in some even more elusive component called 'dark energy'. This dark energy was inferred from observations with some of the worlds largest telescope, measuring the redshifts and distances of supernovae. No one yet knows what this dark energy is, yet is appears to account for ∼74% of the energy density of the Universe. Hence, we don't know what dark matter is, yet it makes up most of the baryonic mass in the Universe, and have even

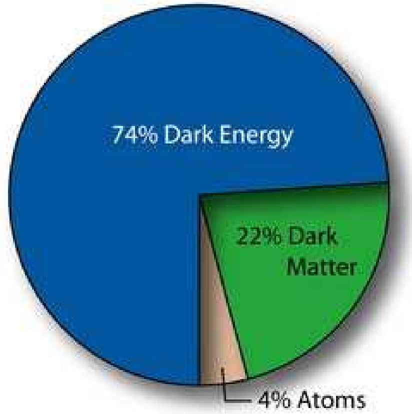

Figure 1.1: Pie chart showing the relative contributions of atmos (4%), dark matter (22%) and dark energy (74%) to the total mass and energy content of the Universe.

less idea what dark energy is, which dominates the energy density of the Universe. If you thought all of physics was nearly completely understood, you may want to reconsider!

Exercises

1. Demonstrate that the orbit in a two-body problem is in a plane.

2. Consider the motion of the moon, and determine what causes the lunar phases.

3. *Olbers paradox:* Demonstrate that in an infinite Universe with a uniform distribution of stars, the night sky is expected to be very bright. This is clearly in contradiction with observations. Discuss possible resolutions to this paradox.

1.2 Coordinate systems

You could describe the position of a star on the night sky by its altitude, a, above the horizon, and the angle, A, its projection makes on the horizon with the direction due north[4]. These angles (A, a) are called *topocentric* coordinates, and shown in the left panel of Fig. 1.2. Note that we chose two direction on the sky to characterise (A, a): the zenith (perpendicular to the horizon), and the direction due north.

As the sky rotates overhead, the coordinates (A, a) of a given star will change. These coordinates also depend where on Earth you are, and so not particularly useful for describing the location of a star or galaxy in the night sky. Instead, we need a set of coordinates that do not depend on time or position of the observer, and we use the location and tilt of the earth to define "sidereal coordinates",

[4]Each point in a 3D space can be characterised by its spherical coordinates (r, θ, ϕ). To characterise a direction, just the angles (θ, ϕ) suffice. In spherical coordinates these are defined as the angle with the z-axis (θ), and the angle of the projection in the z-plane with the direction of the x-axis.

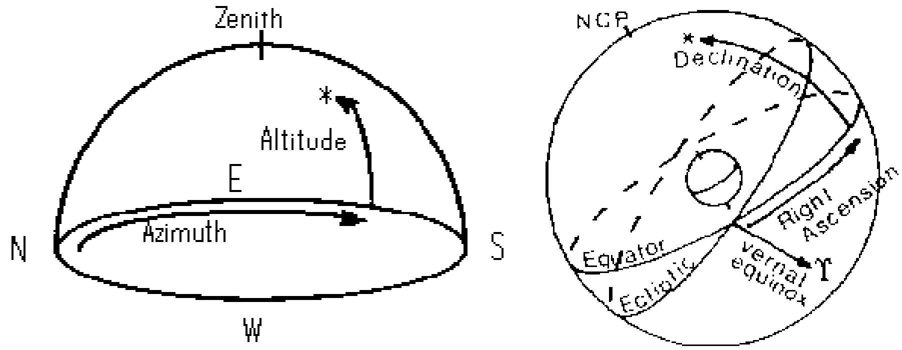

Figure 1.2: Coordinate systems: *Left:* Topocentric coordinates, in which the position of a celestial object depends on both the location of the observer and the time of day of observation. *Right:* Equatorial coordinates in which the co-ordinate system is invariant to the obserer location and time of day, but instead is defined by the location and tilt of the Earth.

which are two axes that are independent of time.

The Earth's spin axis, **s**, is (nearly) constant in time, hence so is the equatorial plane (the plane perpendicular to it). The angle of a star above the equatorial plane is called its declination δ, with $-\pi/2 \leq \delta \leq \pi/2$. To define the second angle we need to chose a direction in the equatorial plane. Since the Earth's orbital angular momentum vector, **l**, is constant, the ecliptic (the plane perpendicular to **l**) is also a constant. Recall that **s** and **l** are not parallel (the angle between them is around 23 degrees). Therefore the equatorial plane and the ecliptic are not parallel, but intersect in a line. This line intersects the firmament in two points, called vernal and autumnal equinox.[5] The angle between the projection of the star onto the equator with the vernal equinox is called *right ascension*, α, and is traditionally expressed in hours, $0 \leq \alpha \leq 24$ hours. Figure 1.2 (right hand panel) illustrates these *equatorial coordinates* α and δ

Although sidereal time is also measured in hours, minutes and seconds, one sidereal day is not equal to the time it takes the Earth to complete a full rotation (24 hours). This is because as the Earth rotates, it also moves around the Sun at a rate of about 1° per day. As such, the time takes for a star to reappear at exactly the same point in the sky on two consequtive nights is not exactly 24 hours, but instead is a little shorter: 23 hours, 56 minutes and 4.1 seconds. After a year, one additional sidereal "day" will have elapsed compared to the number of solar days that have gone by.

Tides from moon and Sun cause **s** to wobble in time (technically called precession and nutation), therefore **s** is not constant, and neither are (α, δ). Therefore when you look-up the equatorial coordinates of a given object, they will be given for a given date (e.q. 1 Jan 1950). Each observatory will have a computer programme to convert these into (α, δ) appropriate for the right date, and another programme to convert these into topocentric coordinates, which are useful to determine whether the star is actually visible!

[5] I suggest you play with a light source (representing the Sun), and a ball (the spinning Earth), to convince yourself that when the Sun appears to be in the vernal equinox, it is spring (and vice versa for the autumnal equinox.

The changes in **s** are not very rapid (the precession period is $\sim$ 26000 years, the nutation period $\sim$ 18 years) and its amplitude not very large, but enough that the pole star was not actually at the pole during the reign of the Egyptians say, and enough to cause some astrologer's star signs to be off by several. Tides due to the other planets also cause small changes in **l**, which can be taken into account if very accurate positions are required.

Exercises

1. What is the Right Ascension (RA) and Declination (Dec) co-ordinate system? What units are used in each direction? Why is one second of RA a different size to one second of Dec (and by how much)?

2. Approximately how many degrees per day does the Moon move relative to the "fixed" stars? On average how much later does the Moon rise each day?

3. Why do astronomical charts usually have the North-East direction pointing top+left rather than in the top+right? Precisely how long in hours, minutes, seconds does the Earth take to rotate relative to the "fixed" stars?

4. What is the ecliptic? What is the approximate RA and Dec of the Sun on 21st March, 21st June, 21st September and 21st December?

5. Why is the sidereal day different to a typical Earth day?

6. What is the angular separation between a star at right ascension (RA) 6 hr and declination (Dec) 60° and another star at RA 0 hrs and 60°?

7. If Andromeda is overhead at midnight, what is the right ascension of the Sun? (Andromeda is at RA = 1 hr, Dec = +40).

1.3 Magnitudes

1.3.1 Apparent Magnitude

A star emits a total luminosity, L, of electromagnetic (EM) energy per unit time (over all wavelengths). This is called its bolometric luminosity. For example the Sun has $L = L_\odot = 3.9 \times 10^{26}\,\mathrm{J\,s^{-1}}$. However, most instruments can only measure a small part of the EM spectrum and use filters to observe a specific range. For example, your eye uses chemical filters to respond to blue ($\sim$450nm), green ($\sim$550nm) and red ($\sim$650nm) light. However, this wavelength range represents a tiny fraction of the electromagetic spectrum. Just concentrating on the optical and infrared part of the spectrum, the wavelength ranges of a "standard" set of broad-band filters are given in Figure 1.3.

How much light an observed sees (the intensity, I) depends on the total luminosity of a source, (L), its distace, (r) by

$$I = \frac{L}{4\pi r^2}\,, \tag{1.3}$$

or when using a filter (e.g. the B-band filter)

Filter Name	U	B	V	R	I	J	K	L	M	N
Central λ (nm)	350	440	550	700	900	1250	2200	3400	4900	10200
Bandwidth (nm)	70	100	90	220	240	380	480	700	300	5000

Figure 1.3: Standard broadband filter wavelengths. The bands (U–N) span the optical–infrared part of the electromagnetic spectrum. The central wavelengths and bandwidths are given in nano-meters (nm). For example, the B-band is centered at $\lambda = 440\,\mathrm{nm}$ and a B-band filter will typically transmit light between 390–490 nm.

$$I_B = \frac{L_B}{4\pi r^2}\,, \tag{1.4}$$

The unit of intensity, I, is $\mathrm{J\,s^{-1}\,m^{-2}}$, or $\mathrm{J\,s^{-1}\,m^{-2}\,Hz^{-1}}$ if measured in a given band since we must account for the width of the filter (in Hertz). However, unfortunately(!) astronomers use *magnitudes* to express intensities. This is not just to annoy under-graduates, but instead spans back to the ancient greeks who decided that they would designate the star Vega as 0^{th} magnitude, and given the log-scaling in sensitivity of the human eye, they classed the faintest star they could see as 5^{th} magnitude, which is about $100\times$ fainter.

In astronomy, we therefore use the apparent magnitude (m) which defines a relative scale defined as

$$m_1 - m_2 = -2.5\log(\frac{I_1}{I_2})\,. \tag{1.5}$$

This magnitude system provides a measure of how much brighter source 1 appears with respect to source 2. For example, if $I_1 = 100\,I_2$, then $m_1 - m_2 = -5$. Note the minus sign: *brighter* stars have a *smaller* magnitudes.

Magnitudes are also used to characterise the *colour* of a source by measuring the intensities through two different filters. For example the $(B - V)$ colour is given by

$$m_B - m_V = -2.5\log(\frac{I_B}{I_V})\,. \tag{1.6}$$

As you will know from looking at stars in night sky (or if you haven't, you will find out later in this course!), some stars are blue and some are red, depending on their temperature. The star Rigel, in the constellation of Orion is a massive B-type star with a temperature of 12,000 K and a blue colour $(B - V) = -0.03$. Betelgeouse, another star in Orion is a red giant, with a much cooler temperature of 3,500 K and a colour of $(B - V) = 1.85$.

1.3.2 Absolute Magnitude

The magnitude system is very useful for describing the relative brightness of astronomical objects, but does not tell us how bright an object really is. For example, a very luminous star will appear dim if it is far enough away, whereas a low-luminosity star will appear bright if it close enough. We therefore relate the luminosity of a star relates to its absolute magnitude, which is the magnitude the star would be if placed at a "standard" distance of 10 pc from the Sun (1 pc = 3.09×10^{16} m). By convention, absolute magnitude is denoted by M (compared to apparent magnitude, m).

The inverse square law links the flux, f of a star at a distance d to the luminosity, L it would have if it were at distance $D = 10$ pc:

$$L\,/\,f = (d\,/\,D)^2 = (d\,/\,10)^2 \tag{1.7}$$

If M corresponds to L and m corresponds to f then equation 1.7 becomes

$$m - M = 2.5\,log(L\,/\,f) = 5\,log(d\,/\,10) \tag{1.8}$$

or as it is often expressed,

$$m - M = 5\,log(d) - 5 \tag{1.9}$$

Here, d is in parsecs (pc). The quantity m - M is called the distance modulus, since it is directly related to the stars distance.

Exercises

1. A variable star changes in brightness by a factor of 4. What is the change in magnitude?
2. What is the combined apparent magnitude of a binary system comprising two stars each of magnitude +3.0 and +4.0?
3. What is the distance (in pc) of a star whose absolute magnitude is +6.0 and apparent magnitude +16.0?
4. The Sun has an apparent visual magnitude of −26.75.

   (i) Calculate its absolute visual magnitude;

   (ii) Calculate its magnitude at the distance of Alpha Centauri (1.3 pc);

   (iii) The Palomar Sky Survey is complete to magnitudes as faint as $V = 19$ (i.e. it detects **all** stars to this limit). How far away, in parsecs, would a star identical to the Sun have to be in order to be just detected on this data?
5. A globular cluster has a total of 10^4 stars. 100 of them have a V-band magnitude of $M_v = 0$ and the rest have $M_{\rm V} = +5.0$. What is the integrated absolute magntiude of the cluster?

Chapter 2

Telescopes

Aim

After studying this chapter you should be able to answer questions about:

- Why lenses focus light (recap from Level 1)
- Draw ray diagrams for converging/diverging lenses (recap from Level 1)
- Focal length, real and virtual images, magnification, compound lenses (recap from Level 1)
- Why do most telescopes use mirrors and not lenses
- Why we build telescopes with large mirrors
- Describe the two main telescope mounts
- Which are the main telescope focus
- What is a Schmidt telescope
- What is angular resolution, and plate scale
- Where are the main observatories located and why

A good overview of telescope properties is in
http://www.telescopes-astronomy.com.au/telescopes003.htm, or
http://www.astro.ufl.edu/ oliver/ast3722/lectures/Scope%20Optics/scopeoptics.htm.

2.1 Geometric Optics

You should have seen the basics of geometric optics in Level 1 (*Young & Freedman* Chapter 34 p. 1114–1140), but since telescopes are based on lenses and mirrors, we'll briefly recap the basics.

2.1.1 Focusing light

A focusing lens (better called a converging lens) is thicker at the center. Light rays passing through the center take longer to travel through the lens than light rays on other parts. Fermats Principle states that *"the path taken between two points by a ray of light is the path that can be traversed in the least time"* and so all ray paths have the same optical path length. Although the path along the *optical axis* has to travel the shortest geometric distance, it also has to travel through the greatest thickness of glass. The simplest lenses of this type have two convex surfaces, although convex-planar, planar-convex lenses would have (approximately) the same effect if the glass thickness varied in the same way.

A focusing mirror has a concave curved surface. Again, the geometric path lengths are the same for all rays between the source and object.

2.1.2 Real and Virtual Images

Real images are formed when light rays from an object actually cross. If a screen in placed at this point, the image will be formed on it. A *virtual* image is formed when the light rays appear to come from a point, although they never actually come from a point and so they never actually cross (and so the image cannot be formed on a screen).

2.1.3 Ray Diagram Solution of the Lens and Mirror Systems

The focal point of a lens (or mirror) is defined as the point of convergence of light rays that are initially traveling parallel to the optical axis. The focal length of a lens is the distance of this point from the lens. For diverging lenses [mirrors] the focal length is negative, representing the apparent point from which the light rays seem to come.

Rules for construction of a ray tracing solution for a converging [diverging] lens:

1. The ray passing along the optical axis is undeviated.
2. The ray passing through the center of the lens is undeviated.
3. Any ray initially traveling parallel to the optical axis is deviated so that it passes through [appears to have passed through] the focal point.
4. Any ray that passes through the focal point [is traveling towards the focal point] is deviated to travel parallel to the optical axis.
5. The image of a source perpendicular to the optical axis is perpendicular to the optical axis.

2.1.4 The Lens Makers Formulae

An alternative method to relate the focal length to the object and image distance is to use the lens makers formulae:

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \tag{2.1}$$

where u is the object distance, v is the image distance, and f is the focal length. The same equation can be used for both lenses and mirrors. For lenses, positive v means the object is beyond the lens; for mirrors positive v means the object is in front of the mirror. f is the focal length of the lens (or mirror).

There is a sign convention that allows real and virtual images (and objects) to be incorporated without having to produce special cases of the formulae.

Quantity	+ve	-ve
u	real object	virtual object
v	real image	virtual image
f	converging lens/mirror	diverging lens/mirror

This makes sense if you compare with a converging lens/mirror producing a real image (from a real object). Note you should always sketch a ray diagram of the system as a double check!

For lenses, the focal length is related to the refractive index and radius of curvature by:

$$\frac{1}{f} = \frac{n-1}{R_1} + \frac{n-1}{R_2} \tag{2.2}$$

where n is the refractive index and R is the radius of curvature. R is positive for a convex surface.

For mirrors, the focal length is given by:

$$\frac{1}{f} = \frac{-2}{R} \tag{2.3}$$

Again, R is positive for a convex surface.

2.1.5 Magnification

It is also useful to talk about the relative size of the object and image. This can be described by the *linear magnification*. This is given by:

$$M_l = -v\,/\,u \tag{2.4}$$

If the magnification is negative, the image is inverted relative to the object.

2.1.6 Compound Lens Systems I: The eyepiece

Systems comprising two or more lenses can be solved by dealing with each of the lenses in turn. The image formed by the first lens (the intermediate image) is then considered as the object of the second lens (etc). In some situations, this leads to the formation of a "virtual object" where the converging

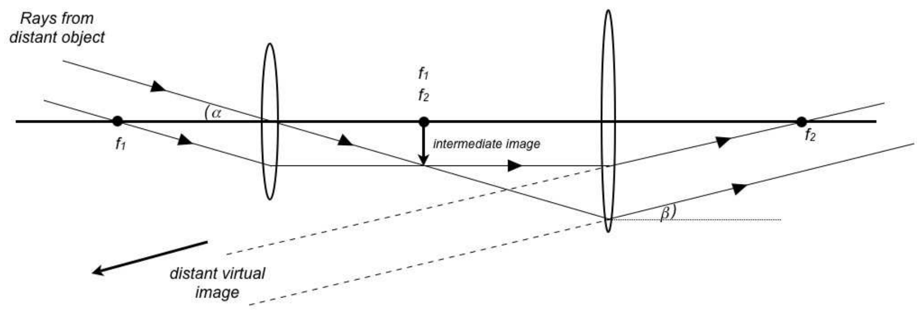

Figure 2.1: Ray diagram for an "eyepiece" compound lens. Incoming parallel rays from infinity are first focussed by the first lens to an intermediate image (between the two lenses), and then focussed to infitiny by lens 2. In this case, there is no linear magnification (since the object is at infinity). Instead we define angular magnification as $\beta\,/\,\alpha$ which is the ratio of outgoing-to-incoming angles.

light rays from the first lens are intercepted by the second lens before they actually cross. The object distance is then negative, and this situation requires no modification of the lens/mirror formulae.

To calculate the magnification of a compound lens system, you must separately calculate the overall magnification due to the first lens, and then due to the second lens. The overall magnification is then the produce of these.

Let's consider a simple compound lens system formed from two converging lenses (Figure 2.1). It is usually designed so that the light rays from a distant object are focused to form an intermediate real image at the focal point of the second lens. At first sight, nothing seems to have been achieved, since the final image is virtual and at infinity. However, the system achieves two important effects.

Firstly, a glass screen is places at the intermediate focus. This may be a simple cross hair, or it can be marked with an angular scale. The first lens plays the role of a "transducer", converting the angle (α) between the rays from the distant object and the optical axis into a distant (y_i) in the focal plane of the first lens. Since the angles are small, $\alpha = y_i/f_1$. The second lens plays the opposite role, converting the displacement at the intermediate focus into the angle between the rays and the optical axis, (β).

By using lenses of different focal length, the angular size of the distant object can be increased. For such systems (i.e. both object and final image at infinity), it is very useful to define the *angular magnification*:

$$M_\theta = \frac{\beta}{\alpha} = \frac{f_1}{f_2} \tag{2.5}$$

2.1.7 Compound Lens Systems II: The Zoom Lens

The zoom lens is a compound lens system which consists of both a converging and diverging lens (Figure 2.2). The first lens converges the light to form an intermediate image. Before reaching this image, however, the light is intercepted by the second lens so than an image is still formed,

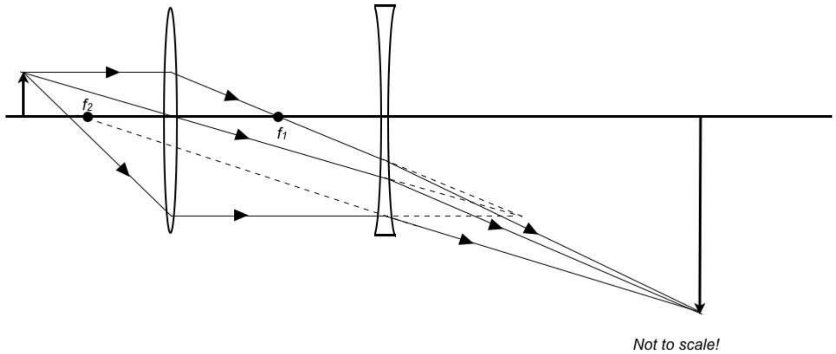

Figure 2.2: Ray diagram for a “zoom lens”. The image of the object is converged by the first converging lens, and then (slightly) diverged by the second (diverging) lens. This has the result of making the final image much larger than would be possible than with a single lens (without having a very long focal length).

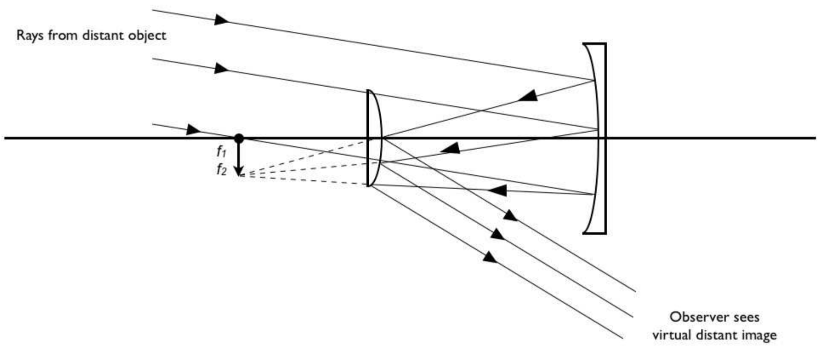

Figure 2.3: Simple ray diagram for a telescope. Incoming (parallel) rays are first reflected by a parablic mirror on to a secondary mirror before passing through an eyepiece.

but at much greater distance from the primary lens. The effect is to produce a system that has a long effective focal length (giving high magnification), but is physically compact. Furthermore, by adjusting the distance between the two lenses, the effective focal length of the system can be continuously varied.

2.1.8 Compound Lens Systems III: The Telescope

The telescope is similar to the zoom lens, but is designed to view distant objects (e.g. the stars) and to produce an image that is also at infinity (Figure 2.3). The effect, however is to produce a system that can have very high angular magnification (note that magnification is not variable since the image is at infinity).

Modern telescopes use mirrors, rather than lenses. This has the advantage that: *(i)* the design is very compact; *(ii)* mirrors do not suffer chromatic aberration due to the wavelength dependence of refractive index; *(iii)* it is easier (and cheaper) to produce large, accurate polished mirrors; *(iv)* The brightness of the final image formed increases as the square of the diameter of the primary mirror.

Exercises

1. A beam of parallel rays spread out after passing through a thin diverging lens, as if all the rays came from a point 20 cm from the center of the lens. You want to use this lens to form a virtual image that is $\frac{1}{3}$ of the height of the object. Where should the object be placed? Where will the image be?

2. A human eye has a power ($P = 1 / f$) of $P = 58.6\,m^{-1}$.

   (i) If a healthy eye can focus light from a star (i.e. at infinity), how far is the retina from the lens?

   (ii) An old person (like your lecturer) needs glasses to see the stars. These have a power of $-1.2\,m^{-1}$. In the absence of the glasses, how far behind the retina is the image being formed?

3. The zoom lens is made from two lenses. The first has a focal length of $f_1 = 20\,cm$. The second, diverging lens has a focal length of $f_2 = -30\,cm$. The separation of the lenses (D) can be altered to increase the magnification of the subject. The zoom is being used to photograph a rare orchid that is 1 m from the first lens.

   (i) What is the magnification due to the first lens?

   (ii) How does the magnification of the second lens depend on D? Determine the lens separation that is needed to give a combined magnification of -0.5.

   (iii) Show that as the lenses are brought together, (i.e. $D \to 0$), the combined magnification becomes that of a single lens with focal length f where $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$.

2.2 Telescopes

Distances to even nearby stars are large, with the closest stars at around 1pc[1]. This makes astronomy into a mostly observational (as opposed to experimental) science, with most observations constrained to detecting light from other stars. The exception to this rule is the detection of neutrinos (from Super Nova 1987A in the Large Magellanic Cloud), and the future promises the detection of gravitational waves. However, here we will concentrate on optical and infrared telescopes, although new technology and techniques have often led to new discoveries, for example:

1610	Refracting telescope	Discovery of moons of Jupiter (Galileo)
1820	Spectroscope	Helium (Fraunhoffer)
1920	Large refractors	Expansion of Universe (Hubble)
1930	Radio telescopes	Quasars
1965	CCDs	Gravitational lenses
1970	X-ray satellites	Black holes
1994	Hubble Space Telescope	Age of the Universe
2002	Adaptive optics	Direct Imaging of Exoplanets
2024	ESO Extremely Large Telescope	Spectroscopy of Exoplanets?

[1] A parsec is the distance at which an AU extends 1 arc-second on the sky, where 1AU is the distance to the Sun, $1\,AU = 1.494 \times 10^{11}$m

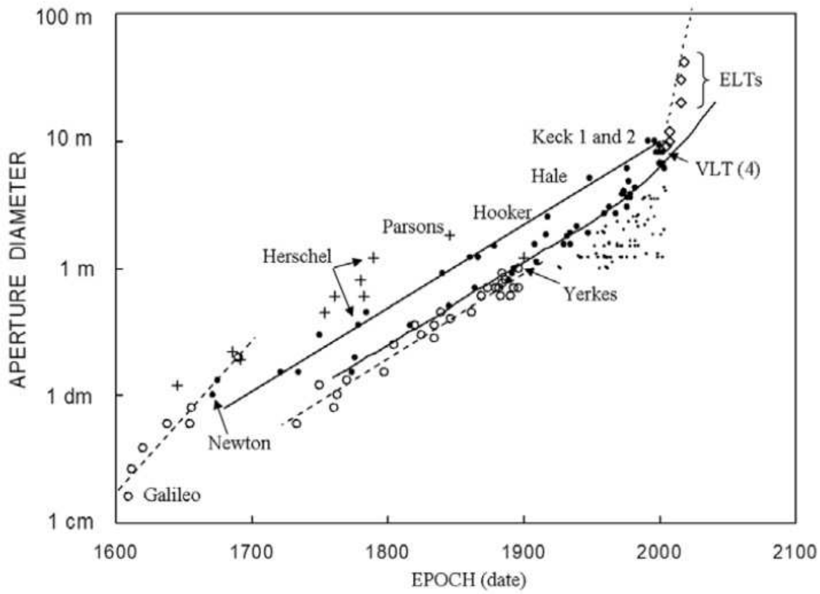

Figure 2.4: The growth of aperture size with time from the invention of the telescope to the present day. Note the log-scale on the y-axis.

2.3 Lenses versus mirrors

The aim of a telescope is to focus as much light as possible, with as little distortion as possible onto the detector, while costing as little as possible.

Early telescopes were from lenses and a telescope that uses a lens is called a *refractor*. However, large lenses are difficult to make, and hence expensive. Since the refractive index depends on wavelength, the focal point depends on wavelength (chromatic aberation), leading to blurred images.

A mirror also focuses light, and telescopes using mirrors are called *reflectors*. Telescope mirrors are made out of glass, which is polished to great accuracy to obtain the correct parabolic shape, which is then covered with a reflective coating (such as aluminium which has excellent reflectivity). However, there are problems with large mirrors. For example, a large mirror may weigh several tons and so will bend under its own weight, distorting the images. Different parts of the mirrors may expand or contract at different rates when heated or cooled during an observing night. To avoid these problems, mirror and support structures are made of a combination of materials that limit thermal distortions and are very stiff. In practice this still limits the mirror size to around 4 meters.

A plane wave (appropriate for observing an astronomically distant object) reflected by a *parabolic* mirror is focused to a point called *prime focus*. The distance from prime focus to mirror is called the *focal length*. The diameter of the mirror is called *aperture*. The ratio of the ratio focal length over aperture is called the focal ratio.

$$\text{focal ratio} = \frac{\text{focal length}}{\text{aperture}}\,. \tag{2.6}$$

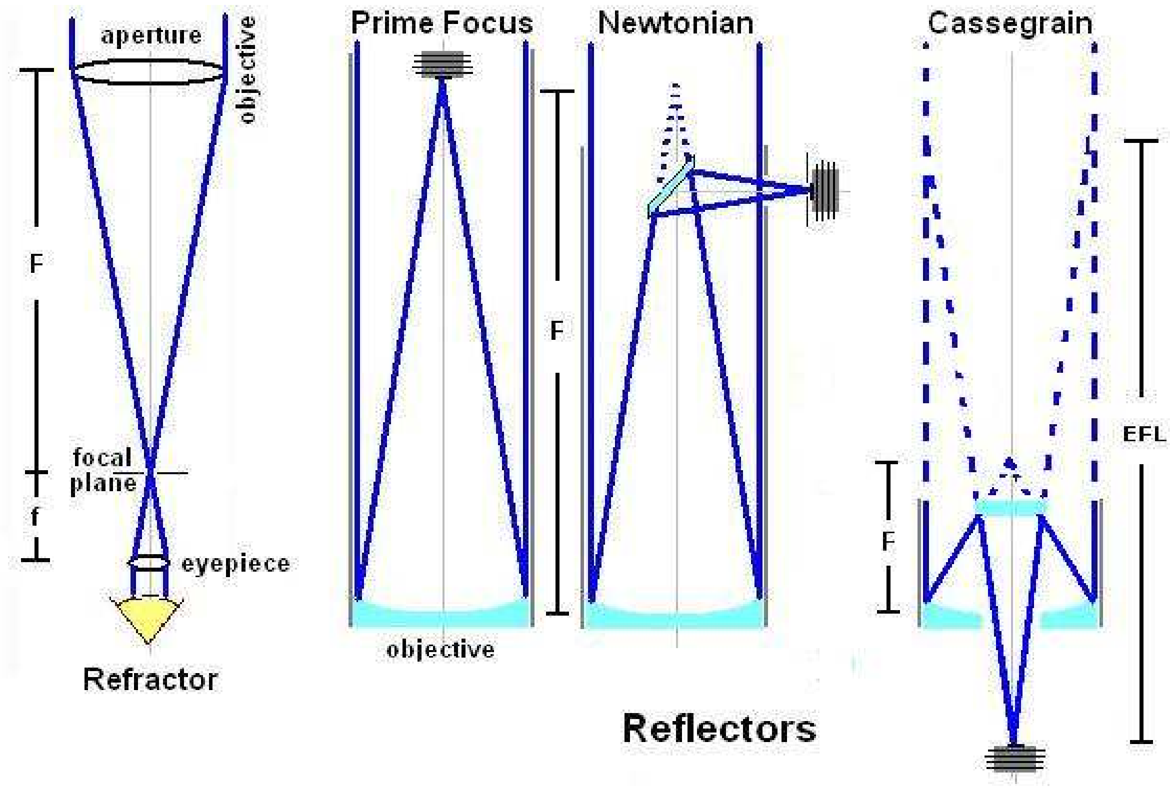

Figure 2.5: Example telescope designs: *Left:* refractor (using lenses); *Right:* Refractors (using mirrors) with different foci. Each of these designs has their own advantages/disadvantages.

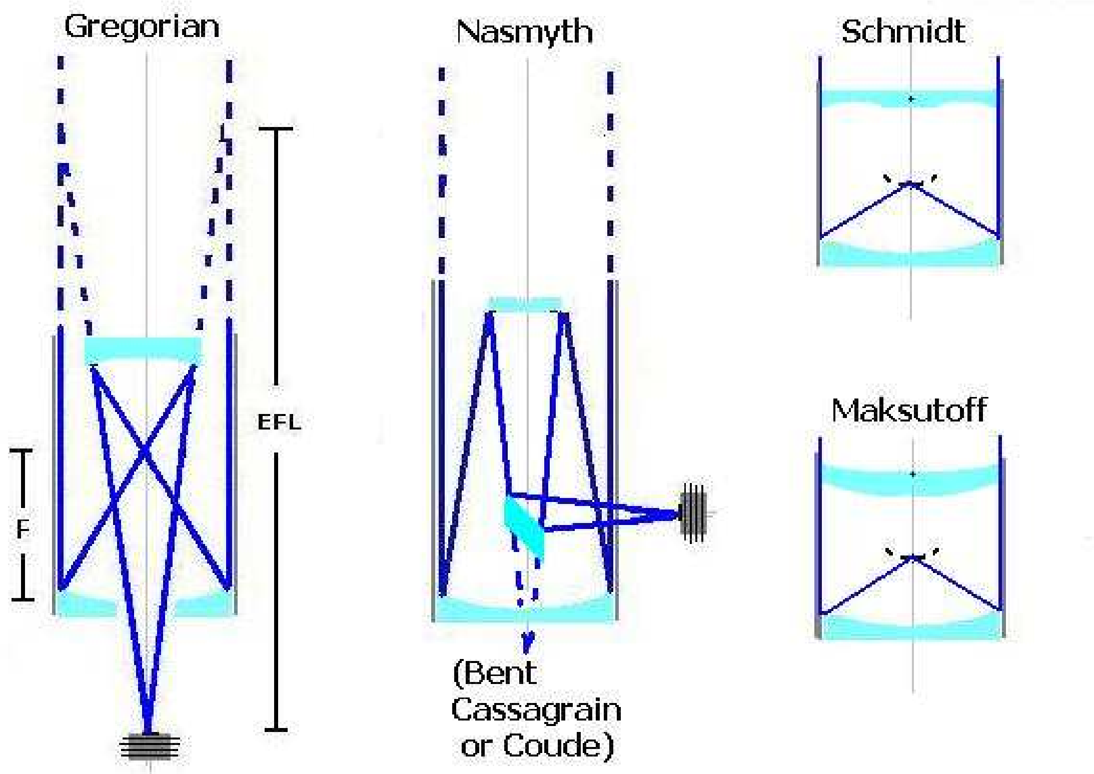

Figure 2.6: More commonly used foci of refracting telescopes. Most large telescopes use a Naysmith design since this allows the light to be passed out of the telescope on to a (gravity-stable) platform and in to heavy/bulky instruments. Most instruments on todays large telescopes weigh several tonnes.

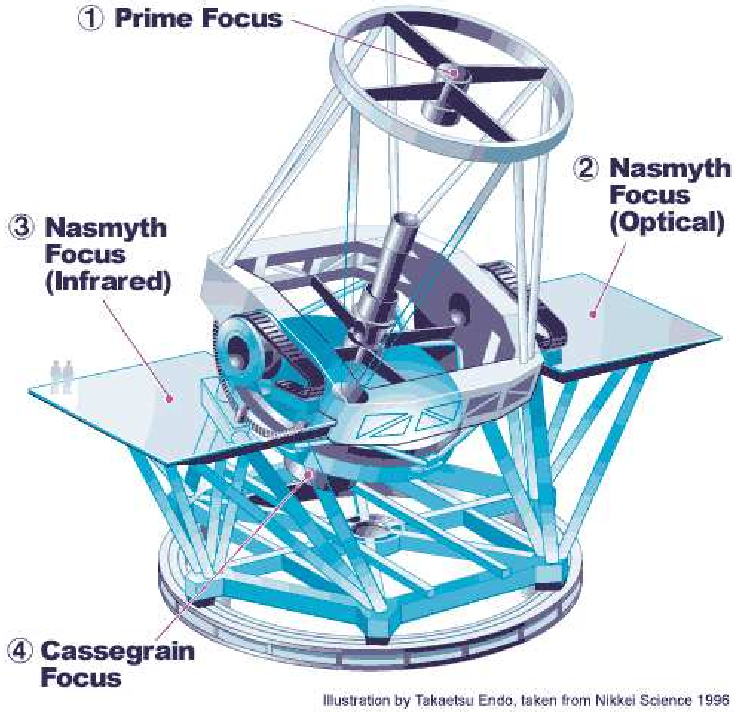

Figure 2.7: Telescope foci of the Subaru telescope. The prime focus offers the largest field of view, and so is often used for wide-field imagers. The Naysmith platform is used for heavy/bulky instruments, or instruments that need good (gravity) stability. The Cassegrain focus is used for light weight instruments.

Figures 2.5 and 2.6 illustrate some common telescope designs.

2.3.1 Telescope Focii

Prime Focus: Putting the detector at the focal point (*prime focus camera*) is the simplest design, and has the advantage that the instrument will have a large field of view, and the fewest mirrors. This is good because each extra mirror leads to some loss of light. However since the prime focus lies directly above the mirror, a large, heavy instrument will block light and will have to move with the telescope. Furthermore, only stars perfectly aligned with the telescope axis will be imaged in the prime focus (a prime focus camera requires a corrector lens to focus for this telescope aberration and so allow one to get a large field of view in focus).

Schmidt Focus: The *Schmidt* design is an extension of the prime focus. This uses a spherical (as opposed to parabolic) mirror, which focuses the light after it has passed through a glass lens (called a corrector), with a camera in the prime focus. This design allows for a very large field of view. However, this design is only used for relatively small telescopes.

Cassegrain Focus: Most small telescopes use a Cassegrain focus. This involves a parabolic primary

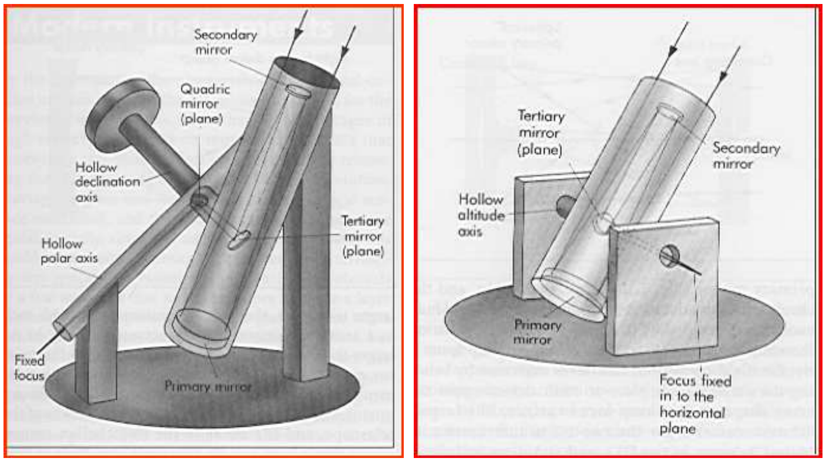

Figure 2.8: Examples of telescope mounts. *Left:* Equatorial mount; *Right:* Alt-Azimuth mount.

and convex, hyperbolic secondary mirror. The design is compact, but retains a long focal length. The eyepiece of a telescope is usuall at the cassegrain focus. However, the field of view in limited to 10–20 arcminutes.

Naysmith Focus: The *Nasmyth* foci is used for large, bulky instruments (which may weigh several tons and be the size of a mini-bus on large telescopes). This focii sits on a platform that rotates with the telescope and so is (relatively) stable.

Coude Focus: If very high stability is required, then instruments can use the Coude focus, effectively folding the light down to a fixed (stationary) position in the observatory. The major disadvantage is that the field of view is small (due to the long focal length) and the light needs to be folded through several mirrors, resulting in light losses.

2.4 Telescope mounts

A telescope mount is simply the structure that holds the mirror and the instruments. To be able to point to any object in the visible sky, a telescope should be able to rotate along two axes. As the telescope is pointing at a star, the star seems to move on the sky due to the Earth's rotation. To follow the star, the telescope needs to *track*.

Equitorial Mount: Chose one of the axes to be parallel to the Earth's spin axis, $\mathbf{s}$, and the other axis perpendicular to $\mathbf{s}$. With this choice, the pole star would need no tracking of the telescope at all[2]. To observe any other star needs an off-set for one of the axes to point to that star, and tracking now only requires rotating along the other axis, at constant speed (Fig. 2.8).

Alt-Az mount: The alt-azimuth mount shown in Fig. 2.8 has one of the axis vertical (towards the zenith), the other one perpendicular to it. Tracking a star now requires rotation around *both* axes,

[2]Assuming of course that the telescope is in the Northern hemisphere!

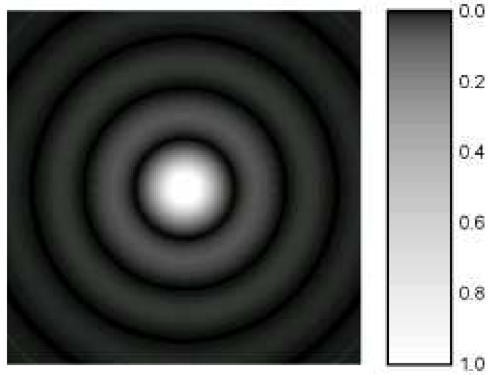

Figure 2.9: An example of an Airy pattern: the image of a point source as seen through a telescope. This is very similar to the diffraction pattern seen by waves as they pass through a slit, but now seen in two-dimensions.

at variable speeds. Although this sounds more complicated, the much simpler design of the mount means most, if not all, large telescope use it.

Advantages	**Disadvantages**
Equatorial Mounts	
Only one axis must be controlled (RA)	Large, bulky & Expensive
Tracking rate is constant (360° per sidereal day)	Gravity vector hard to predict
Star field does not rotate with time	
Alt-Azimuth Mounts	
Advantages	Disadvantages
Simple and more compact to construct	Non uniform tracking speed
Naysmith platform available	Requires two axis to be controlled
	Requires image derotator

2.5 Angular resolution

Angular resolution refers to the minimum angular separation between objects on the sky, that can still be seen separately.[3] Even without taking seeing into account (effects of the atmosphere are discussed in the next lecture), a telescope's angular resolution is limited due to *diffraction*.

Plane parallel ocean waves will diffract when entering a narrow harbor entrance: inside the harbor, it is as if circular waves emanate from the harbor's entrance. Similarly sound waves may diffract through doors in a concert hall. You may also be familiar with the characteristic diffraction pattern of peaks and troughs that occurs when an incident beam of a light encounters an obstructing wall with parallel slits. When the path-lengths between waves emanating from the two slits differ by a multiple, n, of the wavelength, λ, they interfere constructively, resulting in a maximum. When the

[3]As your eyes age, you will no longer be able to read the small print on packages for example: your eyes have lost resolution!

path-lengths differ by $(n + 0.5)\lambda$ they interfere destructively and cause a minimum.

The connection with telescopes is that the telescope mirror acts as a circular aperture (as opposed to the parallel slits above), and hence causes diffraction of the oncoming parallel light beam. The result is that the light is not focused in a single point, but in a central disc, called the 'Airy disc' which surrounded by rings (see Fig. 2.9, the rings correspond to the minima and maxima in the plane parallel slits case). The width θ_A of the Airy disc depends on the telescope aperture, D, and the wavelength of the light, λ: $\theta_A \propto \lambda/D$. Two stars in the field of view will both be imaged in their own Airy discs. Clearly the observer cannot distinguish the two stars if the angular distance between the stars is less than the size of their Airy discs. Hence the definition of diffraction limited resolution of a telescope as

$$\text{diffraction limited resolution} = 1.22\,\frac{\lambda}{D}\,. \tag{2.7}$$

(the factor 1.22 comes from the bessel function associated with the two-dimensional aperture, rather than a one-dimensional slit).

2.6 Why Build Large Telescopes?

Large telescopes are constructed at remote mountain tops in order to *(i)* collect as much light as possible (mountain tops are lie above the cloud line, and above more of the Earths atmosphere) and *(ii)* resolve objects as close possible apart as possible (increasing D improves resolution since $\theta = 1.22\,\lambda\,/\,D$).

However, since the collecting area of a telescope is proprtional to D^2 and resolution is proportional to $\lambda\,/\,D$, the energy per unit area in an image is proportional to light-grasp / resolution$^2 \sim D^4$. Thus, increasing the size of a telescope by a factor of 2 can have a factor 16 increase in the energy per unit area collected by the detector. (although we will see in Chapter 3 & 4 that in real systems, atmospheric turbulence is the limiting factor).

2.7 Plate scale

The *plate scale* is the relation between an angular distance on the sky and a physical distance in the telescope's focal plane. To define plate scale, consider the distance, s, between an image of an object on the optical axis, and another image of an object a small angle u from the axis. The telescope diameter is D and its focal length f. For small angles,

$$\text{plate scale} = \frac{u}{s} = \frac{1}{f}\,. \tag{2.8}$$

In the above equation the angle u is expressed in radians so the plate scale is then in radians per meter, but usually this is converted to arcseconds per meter.

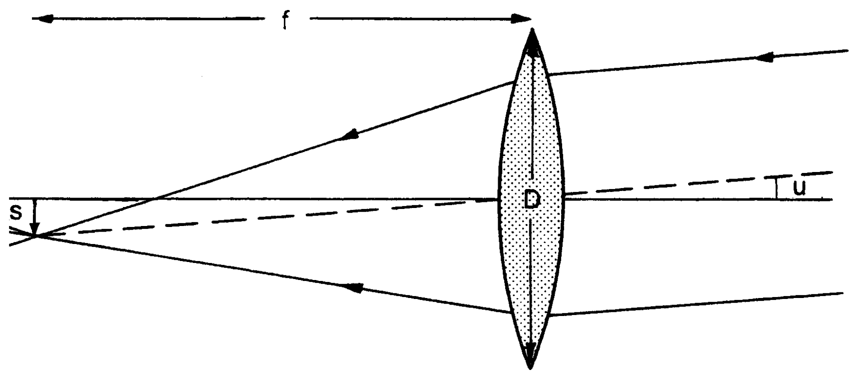

Figure 2.10: Relation between the distance between two points on the detector, s, versus their separation on the sky, u. The plate scale of a system is defined by plate-scale $= u\,/\,s$ (small angle approximation).

Exercises

1. Compute the size of a parsec (pc) in meters. Assuming the distance to the nearest star is 1 pc, how long would it take to get there by high-speed train ($300\,\mathrm{km\,h^{-1}}$), by a bullet (speed $300\,\mathrm{m\,s^{-1}}$) and by a rocket (speed $30\,\mathrm{km\,s^{-1}}$).

2. What is the angular extent of the solar disc as seen from this nearest star?

3. ESO's VLT telescope has a diameter $D = 8$m. What is the diffraction limit (in arcseconds) for (a) visible light ($\lambda = 5000$ Angstrom), (b) Infrared light ($\lambda = 1.2$ microns).

4. A telescope has focal length of 42 m, and a CCD camera of width 5 cm with 2048×2048 pixels. Compute the plate scale. Give your answer in arc seconds per pixel.

Chapter 3

The Turbulent Atmosphere

Aim

After studying this chapter you should be able to answer questions about:

- Which sources of absorption are there in the atmosphere?
- Which sources of emission and scattering are there in the atmosphere?
- What is the effect of refraction and dispersion?
- What is seeing?
- Which constraints are imposed on an observatory site?

3.1 Introduction

While the Earth's atmosphere provides the biosphere that we live in and protects life on the planet from harmful radiation from space, it is not so friendly to the pursuit of astronomy. The atmosphere absorbs and scatters incident electromagnetic radiation. It is the scattering of sunlight by air molecules that makes the sky seem blue; Lord Rayleigh (1842–1919) showed that the scattering is inversely proportional to wavelength to the fourth power (λ^{-4}) and so blue photons are scattered much more strongly and reach our eyes from all directions.

Scattered sunlight is also highly polarized (90°) from the Sun, a fact that is easily demonstrated with Polaroid sunglasses by tilting your head from side to side while looking at the blue sky to see that the intensity changes with the angle of your sunglasses. Under certain conditions the atmosphere also emits radiation. Of more concern is the fact that the atmosphere disturbs the incoming waves through turbulent air motion which in turn limits the ability of a telescope to achieve its ultimate angular resolution.

Thus, the atmosphere disturbs the view of stars due to absorption and dispersion, emission, and turbulence. Observatories are build at sites where these effects are minimised. This usually means

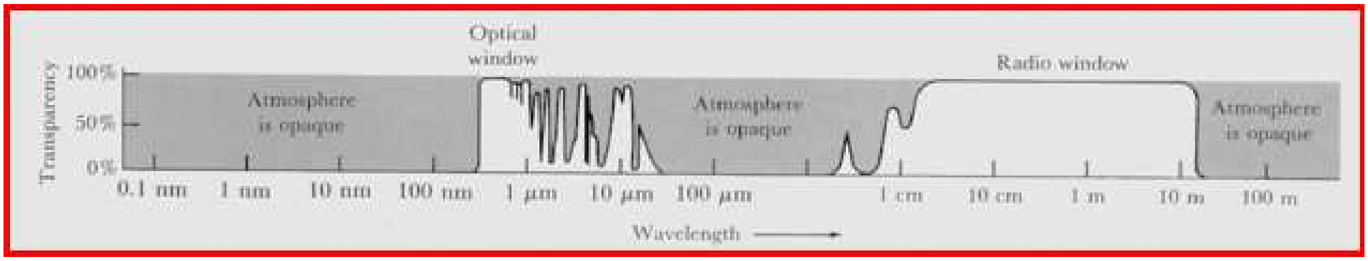

Figure 3.1: The transparency of the Earth's atmosphere as a function of wavelength. The atmosphere is opaque at wavelengths from the X-ray (0.1nm) up to the optical window ($\sim$400 nm). The atmosphere is again opaque in the infrared (from $\sim 20\,\mu$m to $\sim$1cm). In the radio regime, the atmosphere is transparent up to $\sim$10 m.

being on a high mountain top (little atmosphere overhead), good weather (no clouds; e.g. a desert), with little water vapour in the atmosphere, and dark site (e.g. away from cities).

The best sites on Earth are usually found on small volcanic islands (Hawaii, Canary islands) and where the Andes crosses the Atacama desert on the border between Chile and Argentina, in Namibia, and may be somewhat surprisingly on the South pole where the extreme cold means the water vapour content in the atmosphere is very low (most of the water in the atmosphere is frozen and so does not absorb incoming radiation as effectively as water vapour).

3.2 Transparency

Earth's atmosphere blocks some regions of the electromagnetic (EM) spectrum more than others, as shown in Fig. 3.1. There is an interesting variety in what causes that absorption. We'll start from long waves (radio signals) and work our way up to X-rays.

Low frequency radio-waves bounce off the ionosphere, the part of Earth's atmosphere ionised by the Sun at heights above $\sim$ 100km, and hence cannot be detected from outer space[1]. Wavelengths from about 10 m–1 cm do get through. At even shorter wavelengths (in the infrared, 10μm–1 cm), however, molecules in the atmosphere (mainly H_2O and CO_2), make the atmosphere opaque again. These molecules have so many transitions at these wavelengths that they block the light not just at specific frequencies, but at whole *molecular bands*. These molecules also block most of the Infrared region of the EM spectrum.

Moving to the near-infrared and visible (10μm–400 nm), the atmosphere is mostly transparent (in the absence of clouds), but moving to the UV, molecules again start to block most of the light (O_3 or ozone absorbs UV-light, as you know!). At even higher energies (X-rays and gamma-rays) it is not molecules, but the atoms in them (a process called photo-electric absorption) that block these very energetic rays.

[1]This phenomenon is used by radio-stations to broadcast signals around the world!

This leaves some windows in the radio, infrared, and the whole visible regions of the EM spectrum open for observations from the Earth's surface. Very long radio waves, most infrared, UV, X-ray and gamma-ray observations need to be performed from satellites.

3.3 Atmospheric turbulence: seeing

Even in the absence of clouds, visible light does not pass unaffected through the atmosphere. 'Seeing' refers to the blurring of the image of a point source, caused by the atmosphere. The atmosphere is not just layers of gas of different density and temperature. Within a given layer, pockets of gas of varying density and temperature move around. These temperature fluctuations in the atmosphere induce refractive index fluctuations. As an incident plane wave passes through the atmosphere, it becomes corrugated and distorted. The atmosphere can be considered as a super-position of many eddies of different scales. These range from the maximum, ~ 10–$100\,\mathrm{m}$ which break up in to progressively smaller eddies until they are dissipated by frictional forces at the inner scale, $l_0 \sim 1\,\mathrm{cm}$. These eddies all have different refractive indices. Thus, as the pockets move, the apparent position of the source seems to change: this is what causes the twinkling of stars[2]. The atmosphere can only be described statistically, and we do this using second order statistics (you will see this in level 4).

Integrating over a short time, (fraction of a second, say), the light path hardly changes, and the star is focused in a 'speckle'. Observed over longer times, the position of the speckle varies as it builds up the 'seeing disk'.

'Seeing 'thus leads to loss of resolution (just as diffraction did): two objects close together on the sky fail to be separated in the image because their seeing disks mostly overlap. The radius of the seeing disk is usually quoted in arc seconds, and is around 1 arc seconds for good sites, decreasing to 0.4 arc seconds for exceptionally good observing conditions. On the top of the physics department, we have four telescopes used mainly in the level 3 astrolab. Our best seeing (ever) is ∼1" and the average is about 3.5" seeing. Since seeing depends on atmospheric conditions, it varies through the night at a given observatory, and astronomers will constantly monitor the seeing by measuring the sizes of stellar images.

Interestingly, a rather large fraction of the seeing is generated by the atmosphere relatively close to the ground, and so can be affected by the shape of the mountain, and even the shape of the telescope dome (its housing). Before new observatories are build, astronomers may spend several years monitoring the seeing conditions on a range of potential sites.

3.4 Refraction, dispersion and emission

The atmosphere acts as a giant lens, which may change the apparent position of a star (*refraction*). Since this is wavelength dependent, it also causes chromatic abberation (the position of the star depends on the wavelength you chose the observe it), which effectively elongates the image (*dispersion*).

[2] A similar phenomenon happens when you look at the horizon above an asphalt road during a hot summer's day: pockets of hot air above the road deform objects on the horizon. This is called miraging.

To minimise this effect one should aim to observe a star when it is close to the zenith.

To calculate how big an effect atmospheric refraction has on the position of an object as viewed through the atmosphere, lets start by making the simplifying assumption that the Earth's surface is flat, which gives us a plane-parallel geometry (Fig. 3.2).

Assume that the atmosphere is a single homogeneous layer of refractive index n and that the observer is at position 'O'. The observed zenith distance of an object is $\sin z_0$ and $z_2 = z_0 + r$. Applying the sine law of refraction at the refracting upper surface gives

$$n \sin z_1 \;=\; \sin z_2 \tag{3.1}$$

but because all of the verticals are parallel in this model, z_0 and z_1 are equal, $n \sin z_0 = \sin z_2$.

Since the angle, r is small (usually less than about 0.5 degrees), we can assume $\sin r = r$ and $\cos r = 1$ and so

$$\sin z_2 \;=\; \sin z_0 \;+\; r \cos z_0 \tag{3.2}$$

Combining these equations and solving for r gives $r \cos z_0 = (n-1) \sin z_0$, or

$$r \;=\; (n \;-\; 1) \tan z_0 \tag{3.3}$$

This is the flat-Earth approximation for the refraction. The angle is proportional to the refractivity at the observer, and the tangent of the apparent (refracted) zenith distance.

But the refractive index, n, also has a wavelength dependence, so the atmospheric refraction also turns in to an atmospheric dispersion. Because of this variation of index of refraction with wavelength, every object actually appears as a little spectrum with the blue end towards the zenith. The amplitude of the spread in object position, A is given by

$$A \;=\; (n_{\mathrm{blue}} \;-\; n_{red}) \tan z_0 \tag{3.4}$$

Finally, the atmosphere also emits light. Sources of this usually unwanted light:

- fluorescent emission called *air glow* (emission from excited molecules – mainly OH – in the Earths atmosphere as they de-excite)
- scattered light, for example from the moon[3]
- light pollution (from ground (cities), satellites, air craft)
- zodiacal light (scattered from interplanetary dust in the solar system)

The telescope and its detectors itself radiate infrared light (thermal emission), clearly something to limit when observing in the infrared (since this wavelength corresponds to a temperature of $\sim$270 K in blackbody radiation).

[3]Telescope time is divided into dark, grey, and bright time, according to the phase of the moon

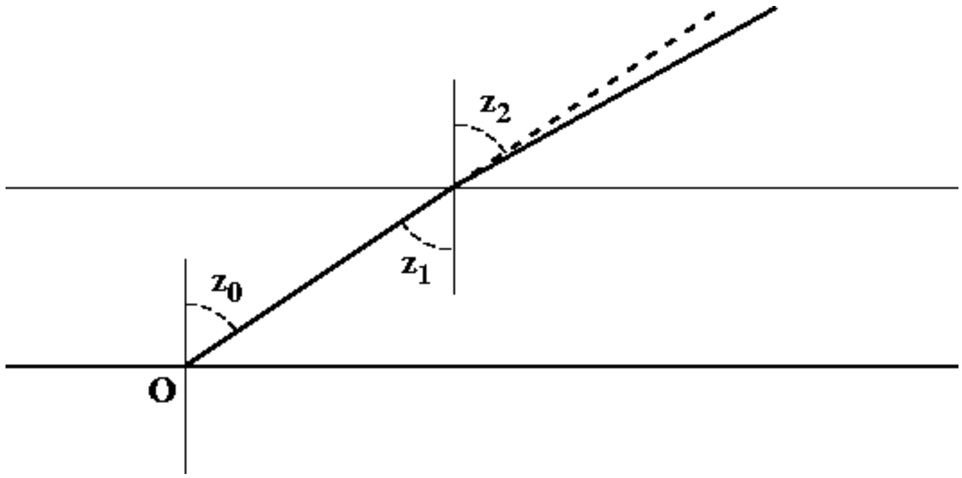

Figure 3.2: A plane-parallel geometry. This is the "flat-earth" model, with the observer at "O" and with the approximation that the atmosphere is a single slab of uniform density with a single refractive index. The angle z_0 is known as the zenith distance.

3.5 Telescope Sites

Given the effect of the atmosphere on observations, good observatory sites are selected to have:

- clear nights without clouds or dust
- good seeing conditions (minimal atmospheric turbulence)
- dark sky (away from sights causing light pollution)
- little water vapour (especially important for IR observations), so dry site

which usually means on top of a remote mountain in a dry place. Hence the popularity of volcanic islands, and the Atacama desert in Chile.

Chapter 4

Detectors

Aim

After studying this chapter you should be able to:

- Describe what a CCD is and how it works
- Describe the advantages and disadvantages of a CCD detector
- Explain what is quantum efficiency, read-noise, bias, dark current, gain, ADU & blooming

4.1 CCD detectors

The detector is the equipment that records the amount of light collected by the telescope through its instruments. Photographic plates are no longer used, the light collector is a *Charge-Coupled Device* (CCD) similar to that in your digital camera and probably in youur mobile phone.

A CCD consists of a two-dimensional array of light sensitive *pixels* that convert light into photo-electrons through the photo-electric effect. The size of the CCD is expressed in its number of pixels, for example 2048×2048. The physical size of the whole CCD is usually small, of the order of a few centimeters. You can think of a pixel as a potential well in which electrons can be stored.

A pixel hit by a photon will result in an electron being stored in that pixel. The efficiency of this process depends on wavelength. At its peak, the quantum efficiency in astronomical CCDs is very high, $\sim 90\%$

$$QE = \frac{\text{number of electrons generated}}{\text{number of incoming photons}}, \tag{4.1}$$

Once the shutter is closed at the end of the exposure, the CCD is read-out by an amplifier. Electrons will be shifted from one pixel to the next along rows, until the charge moves of the detector, is amplified by the electronics, and stored as a digital number.

4.2 Noise in detectors

Noise refers to any unwanted contamination of a measurement by signals not from the source, for example noise of the air-conditioning in a lecture theater versus the signal: the lecturer talking.

Even the best CCDs, combined with their amplifiers are not perfect, and the analysis of the data needs to take these limitations into account. In particular there are several sources of noise. Assume N_γ photons hit the pixel during the integration time t, producing N_e electrons, which, when read-out by the amplifier, result in N_c counts. In the simplest case

$$\begin{aligned} N_\gamma &= \dot{N}_\gamma\, t \\ N_e &= N_\gamma \\ N_c &= g\, N_e\,. \end{aligned} \tag{4.2}$$

The telescope registers $\dot{N}_\gamma$ photons from the source (for example a star) per unit time, and so N_γ after an integration time t. This translates into exactly $N_e = N_\gamma$ electrons, which in turn the amplifier registers as $N_c = g\, N_e$ counts. The gain, g, is the analogue-to-digital conversion, *i.e.* a measure of how many counts the amplifier registers per electron. In the simplest case $g = 1$.

Note that the telescope's optics generally lead to a loss of light from the source. For example every reflection (on a mirror) leads to some light losses. Each instrument will in addition usually have some lenses: these also lead to light losses. Depending on the complexity of the instrument, even a good telescope may lose as much as 60 per cent of the source's light.

Complications occur when $N_\gamma \neq \dot{N}_\gamma\, t$ (because not all pixels are equally sensitive, or because not all photons are form the source), $N_e \neq N_\gamma$ (not all electrons are due to photons), g varies (between pixels say), and even counts are registered without there being any electrons. The actual count value is encoded in an integer number with a finite number of bits. If the recorded value is too high, this will result in an overflow (with possibly negative values for N_c).

A more realistic description of the counts in a CCD must therefore include the noise, the main sources of which are:

- *Sky background:* the image of the star may be polluted with photons from another source, either coming from the night sky, or from light erroneously reflected in the telescope (e.g. scattered light from the moon, or from light pollution from cities).
- *Cosmic rays:* Not all electrons may be due to incoming photons: other particles may hit the detector and release electrons. These particles may be from outer space, called cosmic rays, or may simply result from radioactive decay of the concrete in the telescope building for example. These external sources are referred to as cosmic rays ($N_{\rm ext}$). These events are usually easy to spot, because they cause the pixel to saturate.
- *Sensitivity:* The sensitivity, a, will vary from pixel to pixel, in addition to being wavelength dependent.

- *Bias:* The amplifier that boosts the signal also adds an off-set, called bias[1], N_0.
- *Dark current:* even without light falling onto the detector, pixels will build-up an electric charge generated by thermal motions, which will then be amplified and contribute to the counts: this is the dark current $g\,N_D\,t$.
- *Read-out noise:* the electronics add noise (R) when reading out the CCD (the read-out noise). This term will fluctuate between successive read-outs of the CCD, and we will assume it has mean R and variance σ_R.
- *Bad pixels:* Some pixels may be broken and have $a = 0$.
- *Blooming:* charge from a saturated pixel may leak to neighboring pixels during read-out.

Taking these terms in to account, equation **??** therefore becomes:

$$\begin{aligned}
N_\gamma &= (\dot{N}_\gamma(\text{source}) + \dot{N}_\gamma(\text{background}))\,t && (4.3)\\
N_e &= aN_\gamma + N_{\text{ext}} + \dot{N}_d\,t \\
N_c &= N_0 + f(g\,N_e) + g\,R && (4.4)
\end{aligned}$$

i.e. some of the photons we detect may not be from the source, but are from the night sky (first equation). Some electrons are not due to detected photons (but are from an external source, N_{ext} or dark current, N_d; second equation). The registered counts may have an offset (N_0), be non-linear in the number of electrons (function f), and contain noise (R) (third equation). If this is not enough: a pixel may saturate when too many photons fall on top of it (you may have seen this when happen with your digital camera, if the exposure was too long and/or a source too bright).

4.3 Detector limits

Apart from noise, CCDs have other limitations:

1. *Depth of potential well:* maximum number of electrons on pixel before it saturates (up to 10^5 for some CCDs)
2. *Resolution:* The number of counts is stored by the amplifier as a binary number with a given number of bits. This translates to a maximum number of counts, for example
   - 8 bit $= 2^8 = 256$ levels
   - 16 bit $= 2^{16} = 64536$ levels
3. *Band-width:* the range of wavelengths the CCD is sensitive to.

[1]The reason for doing this is to ensure that the detector operates as much as possible in the linear region, with $f \equiv 1$.

Exercises

1) Given identical integration times, how much fainter can the Keck 10-meter telescope (equipped with a CCD with quantum efficiency of 70%) detect an object compared to the Mount Palomar 5-meter telescope with a photographic plate (quantum efficiency 1%)?

2) A CCD camera has a well depth of 200,000 photons.

- Describe what is meant by the gain of a CCD camera.
- What should the gain be so that the wells are full for a saturated 16-bit analogue-to-digital converter?
- The camera records an average output of 5,000 adu's per pixel across the whole field. What is the error on this, per pixel, assuming that photon noise is the only source of error?
- The camera records an average output of 5,000 adu's per pixel across the whole field. What is the error on this, per pixel, assuming that photon noise is the only source of error?

Chapter 5

Photometry

Photometry is the process of obtaining quantitative (numerical) values to characterise the brightness of celestial objects.

Aim

After studying this chapter you should be able to:

- Define what is photometry, and explain the magnitude system
- Explain what is noise, and how you take it into account
- Define and calculate noise using Poisson statistics
- Explain how a raw CCD image is processed to account for bias, read noise, dark current and flat field.
- Perform signal-to-noise calculations for given CCD and exposure parameters
- Explain how to account for absorption in the atmosphere when performing photometric observations

5.1 Measuring the magnitude of a star.

The image of a star as recorded on a CCD will be approximately circular, and extend over several pixels (due to seeing and / or diffraction). This section discusses how to take the CCD's properties into account to infer the photon arrival rate (and its error) from our object of interest ($\dot{N}_\gamma$) from the CCD counts (from N_c).

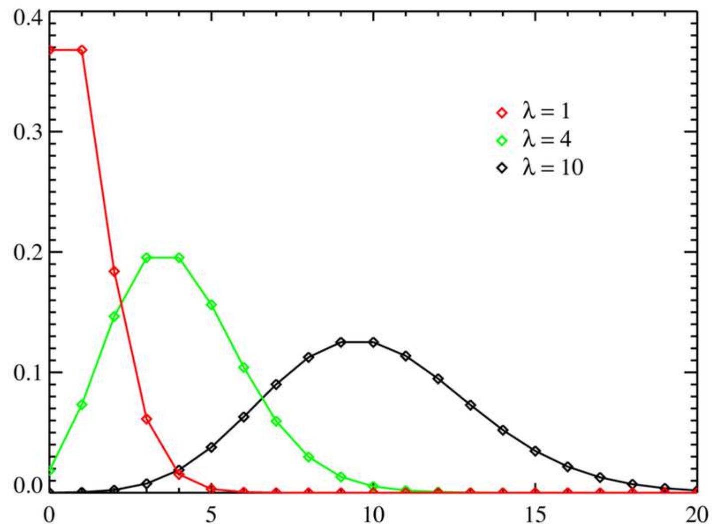

Figure 5.1: Poisson distributions as a function of the mean number, N. At large N, the distribution is closely approximated by a Gaussian profile.

5.2 Photon-statistics

Suppose a source emits $\dot{N}_\gamma$ photons per unit time *on average*. Then during time t, we expect to register $N_\gamma = \dot{N}_\gamma\, t$, on average, but with some dispersion. This dispersion is determined by Poisson statistics:

$$P(n, N) = \frac{\exp(-N)\,\mathrm{N}^n}{n!}\,. \tag{5.1}$$

This probability distribution is the probability of measuring n events for a Poisson process with mean N. The Poisson distribution is function (PDF) is plotted in Fig. 5.1 for $N = \lambda = 1$, 4 and 10. Note that a probability distribution characterises all the statistical properties of a given process, for example:

$$\begin{aligned}
\sum_n P(n, N) &= 1 \\
\sum_n nP(n, N) &= N \\
\sum_n n^2 P(n, N) - N^2 &= N
\end{aligned} \tag{5.2}$$

The first equation simply states that the probability of any value occurring is 1 (normalisation). The second equation tells us that the mean is N, and the third equation tells us that the dispersion about the mean is $\sigma_N = \sqrt{N}$.

Lets take a simple example. Suppose we detect $\mathcal{N} = 10^6$ photons during an integration time of $t = 10^6$ s. The mean photon rate is therefore $\dot{N} = \mathcal{N}/t = 1\,\mathrm{s}^{-1}$. Now perform another experiment and observe the source several times, but over a much shorter time interval, $t = 1\,\mathrm{s}$. The mean number of photons you expect $= N = \dot{N}\,t = 1$. As the photons arrive from the source, during an exposure sometimes we may not detect any photons during our 1 s exposure, sometimes we measure 1 photon, sometimes 2 photons, etc. The PDF Eq. (5.1), with $N = 1$, gives us the probability of the outcome of detecting n photons.

Clearly, different exposure times of the source will give you different estimates of the source emissivity, $\dot{N}$. A single measurement will give you an estimate of $\dot{N} = n/t$, with some error. That error is given by the dispersion, and so (as with any experiment), our measurement should be reported as:

$$\dot{N} = \frac{n}{t} \pm \sqrt{\frac{n}{t}}\,. \tag{5.3}$$

5.3 Data reduction process

Now that we know that our goal is to determine the photon arrival rate (and its error) from a source, we can start to account for some of the external sources of noise discussed in the previous chapter.

The steps required to obtain the signal, $\dot{N}_\gamma$, from the measurement, N_c, can be understood by looking again at the relations:

$$N_\gamma = \dot{N}_\gamma\, t \tag{5.4}$$

$$N_e = aN_\gamma + N_{\rm ext} + \dot{N}_d t \tag{5.5}$$

$$N_c = N_0 + g\,N_e + g\,R\,. \tag{5.6}$$

For simplicity, we will assume the gain $g = 1$, and leave it as an exercise to find the relations when $g > 1$. If a cosmic ray hits a pixel, $N_{\rm ext}$, no amount of data reduction can help: we will need to discard this pixel. Therefore assume $N_{\rm ext} = 0$.

Characterisiting a CCD measurement

There characteristics of a CCD we must account for are the bias, dark current and flat-field (illumination).

Bias and Readout noise:

The bias level of a CCD frame is an artificially induced electronic offset which ensures that the Analogue-to-Digital Converter (ADC) always receives a positive signal. All CCD data has such an offset which must be removed if the data values are to be truly representative of the counts recorded per pixel.

The readout-noise is the noise which is seen in the bias level. This is produced by the on-chip amplifier and other sources of noise in the data transmission before the signal is converted into a digital representation by the ADC. Typically this can be represented by one value which is an estimate of the standard deviation of the bias level values. In the best CCDs, this may be just a few electrons, but in an observation where we may only be detecting a few photons from a faint star or galaxy, we must still take this in to account.

To measure the bias and readout noise of a CCD, we can simply read the CCD many times (with a zero second exposure) and measure the average level (and dispersion) for each pixel. The counts from the bias + readout is:

$$N_c = N_0 + \langle R \rangle \,, \tag{5.7}$$

where N_0 is the bias level and ⟨R⟩ is the readout noise. Or, equivalently, the bias frame is:

$$\text{bias} = N_0 + \langle gR \rangle \,. \tag{5.8}$$

Dark current:

All CCDs exhibit the phenomenon of dark current. This is basically charge which accumulates in the CCD pixels due to thermal noise. The effect of dark current is to produce an additive quantity to the electron count in each pixel. The reduction of dark current is the main reason why all astronomical CCDs are cooled to liquid nitrogen temperatures. To measure the dark current, the CCD is exposed for the same integration time as the observations, but without opening the shutter. Our equation for the total number of counts we detect (bias + dark) now becomes:

$$N_c = N_0 + R + \dot{N}_d t \tag{5.9}$$

Flat field: uniform illumination

The sensitivity of a CCD to incident photon flux is not uniform across the whole of its surface (some pixels are more sensitive than others due to the manufacuring process). The variations in CCD response can be on the large scale (one end of the CCD to the other) and pixel-to-pixel.

Flatfield calibration frames are usually taken of a photometrically flat source using the same optical setup as that used to take the science observations. In the past, images of the interior of the telescope dome have been used for this purpose, however, it now generally thought that images of the twilight/dawn sky are more representative of a true flatfield, having the same global illumination as the data and having a good signal level (remember that calibration frames will be applied to the object data at some stage and hence will introduce a noise contribution to the final data values, it is therefore essential to get a good set of calibration frames with lots of signal if this process is to introduce the absolute minimum of noise).

Including these pixel-to-pixel variations (flat-field), our equation now becomes:

$$N_c = N_0 + R + \dot{N}_d t + aN_\gamma \tag{5.10}$$

If all pixels were equally sensitive, $a = 1$. If pixel 1 is only half as sensitive as all the others, then the value of the flat field for that pixel will be 0.5. By *dividing* counts by the flat field, we can correct for this variation in sensitivity.

Result: In an observation of a star, the total number of counts in any pixel is comprised from (i) the counts from the star, (ii) bias, (iii) dark, and (iv) with a correction required for pixel-to-pixel variations (flat-field). The final data frame is thus obtained from the raw data as:

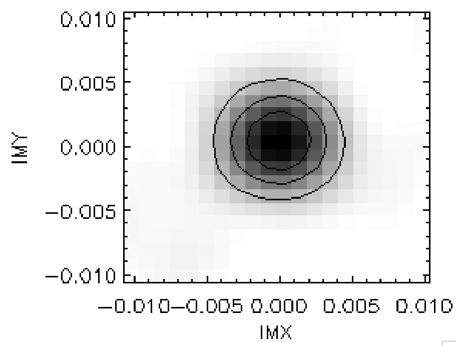

Figure 5.2: Seeing disk of a star image on a CCD detector. The image of the star should be a point dource (stars have angular extents that are <<1 pixel on a CCD, but the image is blurred due to atmospheric turbulence (or "seeing").

$$\text{result} = \frac{[N_c - (\text{bias} + \text{dark})])}{\text{flatfield}} \tag{5.11}$$

We now need to calibrate the data, and obtain an error estimate.

Calibration and detector noise

To calculate the signal (and its error), we now propograte the measurements and errors in equation 5.11. The sources of noise are:

1. readout noise from amplifier (R electrons per pixel, with variance σ_R). The measurement error in the read-out noise is Gaussian (not Poisson) since this measurement does not depend on a photon arrival rate.
2. Photon counting noise due to stars or galaxies. This is a Poission process (since it depends on the photon arrival rate) and so , variance $= \sqrt{(}\dot{N}_\gamma\, t)$.
3. Noise due to dark current. This is also a Poisson process since this depends on a photon arrival rate (even if those photons are generated from thermal processes). Hence, variance $= \sqrt{(}\dot{N}_d\, t)$
4. (Assume noise from bias = 0)
5. Sky background noise (which is also a Poisson process since this depends on the photon arrival rate from the night sky).

These sources of noise are independent, therefore their variances add in quadratures. For example for three sources of noise

$$\begin{aligned} s_1 &= x_1 \pm \sqrt{(}x_1) \\ s_2 &= x_2 \pm \sqrt{(}x_3) \\ s_3 &= x_3 \pm \sqrt{(}x_3) \end{aligned}$$

then

$$s = s_1 + s_2 + s_3 = x_1 + x_2 + x_3 \pm \sqrt{(}x_1 + x_2 + x_3)\,. \tag{5.12}$$

This equation can be applied to a single pixel, but the image of a star will be spread across many pixels (either due to diffraction or atmospheric seeing). For example, Figure 5.2 shows the image of a star as recorded by a CCD camera. Therefore the total intensity should be summed over all these pixels, but this is also true for the noise. Therefore summing counts over pixels, the signal is

$$S = \sum_i N_\gamma\,. \tag{5.13}$$

The noise (uncertainty) on this measurement is due to (a) photon noise, N_γ (Poisson), (b) dark current, N_d (Poisson), (c) readout noise, R, (Gaussian) and (d) sky background noise, N_{bg}, and hence:

$$N^2 = \sum_i N_\gamma \; + N_d \; + N_{bg} \; + R^2 \tag{5.14}$$

Therefore the signal-to-noise ratio is

$$\frac{S}{N} = \frac{\sum N_\gamma}{\left(\sum \left[N_\gamma + N_d + N_b + R^2\right]\right)^{1/2}} \tag{5.15}$$

Conversion to Magnitudes

We cannot yet relate our photon flux to the magnitude of the star, since we need to know how transparent the sky is. To finally determine the magnitude m of our object, we need to determine the net signal S_s of a standard star, with known magnitude, m_s. This is often called the "zero-point" magnitude, and is used to determine the magnitude of an object if the telescope + CCD detects 1 photon (since it is then easy to determine the magnitude if the telescope + CCD detects N-photons). The definition of magnitudes, Eq. (1.5), gives the final result

$$m = m_s - 2.5\log(\frac{S}{S_s})\,. \tag{5.16}$$

The last step cannot be done when the transparency varies rapidly in time (for example when there are clouds). To obtain good measurements of the magnitude of stars or galaxies requires observations when the atmosphere is stable with no clouds. Nights with good conditions for photometry are called 'photometric' nights.

Exercises

1. Use error propagation to find the error on the magnitude m in Eq. (5.16), given the error on S. Assume the error on S_s is negligible.

2. In a particular CCD exposure, the signal is 100 photo-electrons, the dark current is 30 photo-electrons, the read noise is 3 photo-electrons and the sky background gave 40 photo-electrons. Calculate the signal-to-noise ratio.

3. You are observing a faint star and would like 1% accuracy in your photometric measurement (i.e. a signal-to-noise ratio of 100). After 5 minutes of observation, you have a signal-to-noise of 10 (i.e. 10% accuracy). *(i)* What integration time is required to reach a signal-to-noise of 100? *(ii)* If you could increase the quantum efficiency of the detector by a factor of 10, what would the required exposure time be to reach a signal-to-noise of 100?

4. You are trying to detect a faint companion to a 15th magnitude star with Hubble Space telescope. A 1 minute integration results in 2^{16} count for the star. What is the limiting magnitude for the companion to be detected with S/N greater than 1? Assume the light from star and planet falls each onto a single pixel, with readout noise 2 electrons, dark current 0.1 electrons per second, and gain $g = 1$.

5. You are observing the image of a 25.5 magnitude star in the R-band, which is centered at a wavelength of 750 nm and has a 100 nm bandpass) on a 10 meter telescope with a camera that has a combined efficiency (CCD plus telescope plus atmosphere) of 13%. The plate scale of the CCD is $0.2''$ per pixel and the read noise is 8 electrons. *(i)* What is the photon detection rate per pixel assuming that the dark current is negligible? *(ii)* If the sky has a magnitude of $23\,\mathrm{mag}/\mathrm{arcsec}^2$, what integration time is required so that the observations are background limited? What integration time is required to give a signal-to-noise of 5? [useful information: a 0^{th} magnitude star produces $3.92 \times 10^{-8}\,\mathrm{W\,m^{-2}\,\mu m^{-1}}$ at the top of the Earths atmosphere. Planks constant is $h = 6.63 \times 10^{-34}\,\mathrm{J\,s}$ and $c = 2.997 \times 10^{8}\,\mathrm{m/s}$].

Chapter 6

Spectroscopy

Aim

After studying this chapter you should be able to answer questions about:

- Describe the key components of a spectrograph
- Explain what is spectroscopy and why it is used
- Explain the relation between wave-diffraction and a grating spectrograph
- Define spectral resolution

6.1 Introduction

Spectroscopy is the measurement of the intensity of a light source as function of wavelength, and is probably the most useful tool in astronomy. For example, spectroscopy allows us to measure:

- the contents (chemical make-up) of stars and galaxies by identifying spectral lines. Helium was discovered first in the spectrum of the Sun, hence its name.

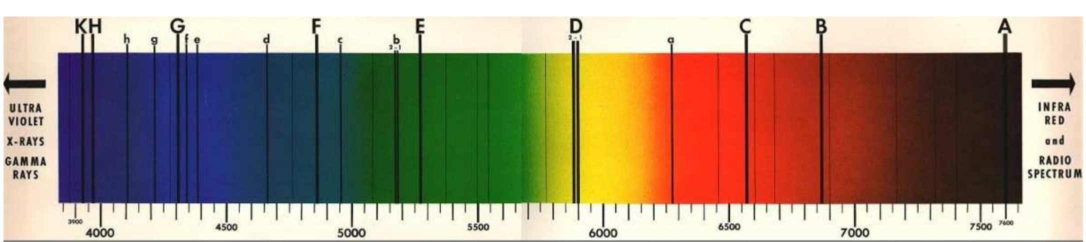

Figure 6.1: A spectrum of the Sun at visible wavelengths (between 400–750 nm). The dark bands at discrete wavelengths are absorption lines caused by elements such as hydrogren, helium oxygen, magnesium, calcium and iron in the solar atmosphere. These occur at wavelengths correcponding to the energy levels of the transitions.

- To infer kinematics, for example of stars in a galaxy.
- To infer distances (of very distant objects), using the redshift.

A spectrum of the Sun is shown in Fig. 6.2. The dark lines are absorption lines due to atoms and molecules in the stellar atmosphere due to elements such as Hydrogen, Oxygen, Sodium, Iron, Magnesium and Calcium. A selection of these lines in given in Table 6.1. However, a large fraction of the spectral lines in the Sun have not yet been identified! A galaxy spectrum is made up of the combined spectra of all its stars and gas. The galaxy spectrum can be analysed to determine its mix of stellar types (e.g. their ages and masses).

lines	Due To	λ (Å)
A - (band)	O2	7594 – 7621
B - (band)	O2	6867 – 6884
C	H	6563
a - (band)	O2	6276 – 6287
D - 1, 2	Na	5896 & 5890
E	Fe	5270
b - 1, 2	Mg	5184 5173
c	Fe	4958
F	H	4861
d	Fe	4668
e	Fe	4384
f	H	4340
G	Fe & Ca	4308
g	Ca	4227
h	H	4102
H	Ca	3968
K	Ca	3934

The strength of these absorption lines depending on the mass, temperature and chemical make up of the star. For example, A-stars (which have masses between 1.4–2.1 times that of the Sun and surface temperatures between 7,600 and 10,000 K) are dominated by Hydrogen Balmer absorption lines. In contrast, in the spectrum of the Sun shown in Fig. 6.2, there are many more absorption lines from heavier elements such as Oxygen, Sodium and Magnesium (any element heavier than Helium is usually referred to as a “metal’).

6.2 Spectrographs

To disperse the light from a star or galaxy and so create a spectrum requires a spectrometer. These come in three types:

- Refraction (i.e. using prisms)
- Diffraction (i.e. using a grating)

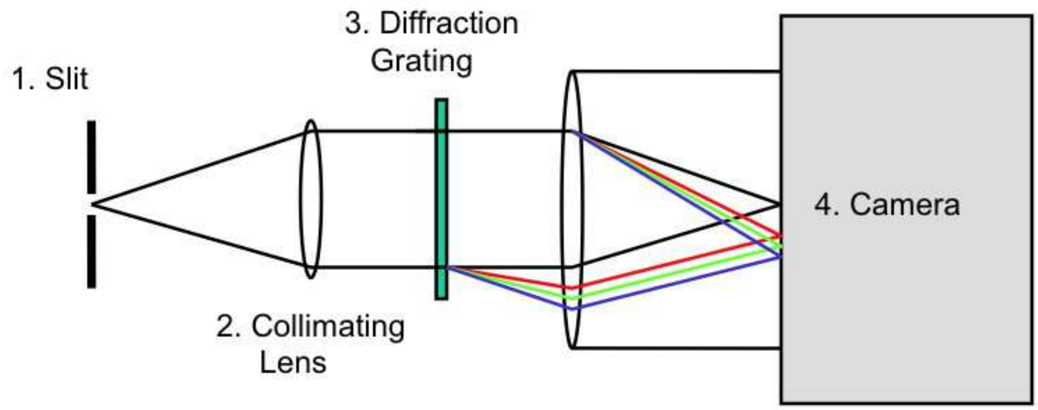

Figure 6.2: The key components of a spectrograph: the slit, the collimating lens, the diffraction grating and the camera.

- Interference (e.g. a Fabry-Perot interferometer)

Most spectrometers (or spectrographs) on telescopes use diffraction gratings, and so in this Chapter, this is what we will concentrate on. We will discuss interference interferometers in a later Chapter. There are four key components of a spectrograph:

- Slit
- Collimating lens
- Diffraction Grating
- Camera

6.3 Diffraction Through A Single Slit

Since we will concentrate on spectroscopy using diffraction gratings, we will briefly recap diffraction. You should have seen the basics of diffraction in Level 1 (*Young & Freedman* Chapter 36).

Diffraction occurs when a coherent wave passes through a small aperture to illuminate a screen. A simple example is Youngs double slit experiment. A coherent light source is created by illuminating a small hole (alternatively a laser can be used). This acts as the light source for a pair of holes (slits) in a second screen. The waves from this source overlap on a screen. Their interference produces a diffraction pattern.

[0]For interference to be observable, the relative phases of the waves must remain constant over long periods of time. Sources that obey this requirements are called coherent

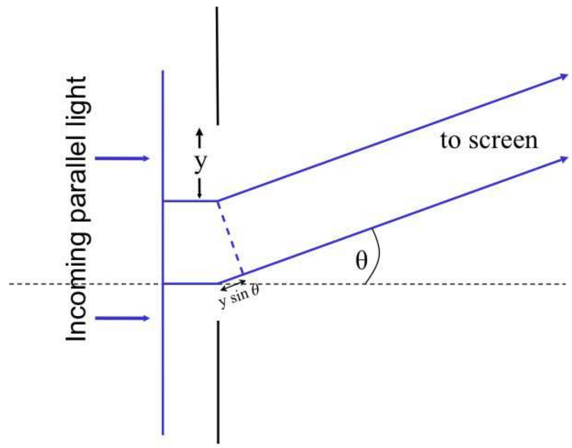

Figure 6.3: A parallel plane wave passing through a slit. The incoming parallel light is diffracted through an angle θ.

The *Frauenhoffer limit* refers to the diffraction pattern observed when the light source (the initial single hole) and the observing screen are placed at large distances from the diffraction mask (e.g. the two slits). In practice it is easier to use two lenses with the light source and screen placed at the focal points. The first lens ensures that the mask is illuminated by a parallel wavefront, and the second that light-rays diffracted through an angle θ are all focused to the same point on the screen.

To understand the diffraction pattern produced by a single slit, we need to think of a wavefront passing through the slit as a line of sources all of which have the same phase (Fig. 6.3). For light that passes through the mask undeflected, $(\theta = 0)$, the rays all arrive at the screen with the same phase where they interfere constructively. Light rays that travel to the screen at angle θ arrive at the screen slightly out of phase. To derive the intensity of the wave we need to combine the waves taking the phase shift into account their phase. The ray at distance y along the slit has a phase shift of $ky\sin\theta$ relative to the center of the slit $(k = 2\pi/\lambda)$. Since there are an infinite number of sources, the sum becomes as integral and the amplitude of the wave at the screen, $Y(\theta)$ is given by

$$Y(\theta) = \int_{-b/2}^{b/2} A_0\, cos(\omega t - k\,(R - y\sin\theta))\, dy \tag{6.1}$$

(R is the distance to the screen). To calculate this integral, it is easiest to use the complex number representation of the wave, and so we can write

$$Y(\theta)\, e^{i\omega t} = A_0\, e^{i\,\omega\, t - i\, k\, R} \int_{-b/2}^{b/2} e^{i\,k\,y\sin\theta}\, dy \tag{6.2}$$

If we make the change of variable $\beta = \frac{1}{2}\,k\,b\sin\theta$, then things look much simpler, and the result of the integral part of the equation is

$$-\,b\left(\frac{e^{i\,\beta} - e^{-i\,\beta}}{2\,i\,\beta}\right) = -b\,\frac{\sin\beta}{\beta} \tag{6.3}$$

The integral we have solved for is in fact the Fourier Transform of the slit (i.e. of a function that is 1 if $b/2 < y < b/2$ and 0 elsewhere).

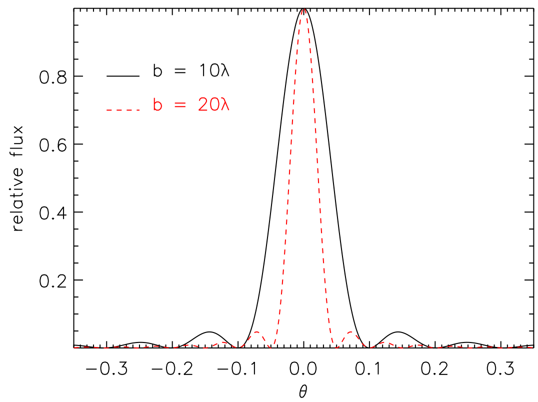

Figure 6.4: The diffraction pattern for a single slit of width (b) $10\,\lambda$ (solid line) and $20\,\lambda$ (dashed line). The narrower the slit, the sharper the intensity pattern.

We are not interested in the absolute intensity, but rather we want to know how the relative intensity changes as a function of θ (and thus position on the screen). The dependence of θ is expressed through β. Taking the square of the modulus of the β term, we can therefore write that

$$I(\theta) \propto \left(\frac{\sin\beta}{\beta}\right)^2 \tag{6.4}$$

This is the Frauenhoffer diffraction pattern due to a single aperture, and is shown in Fig. 6.4. Note that the width of the diffraction pattern (as a function of θ) depends inversely on the width of the slit. If the slit is wide, most of the light passes through without deviation. If the slit is narrow, light is spread out over a wide range of angles.

6.4 Diffraction Through a Double Slit

In the case of two slits, a plane wave incident on the slits will bright spots on the screen where the waves interfere constructively, and dark spots where the waves interfere destructively. If d is the distance between the slits then the path difference in direction θ is $d\sin(\theta)$. Constructive interference occurs when this is an integer n times the wavelength λ:

$$n\lambda = dsin(\theta)\,, \tag{6.5}$$

see Fig. 6.6.

To derive the intensity pattern as a function of distance angle from the slits to the screen, first consider a diffraction pattern due to two slits separated by distance d. Lets start by imagining the slits are very narrow. We can then deduce the diffraction pattern by considering just two light rays. In this approximation, the amplitude at the screen is given by:

$$Y(\theta) = A_0\, e^{i\omega t - ikR}\,.\,[\,1 + e^{ikd\sin\theta}\,] \tag{6.6}$$

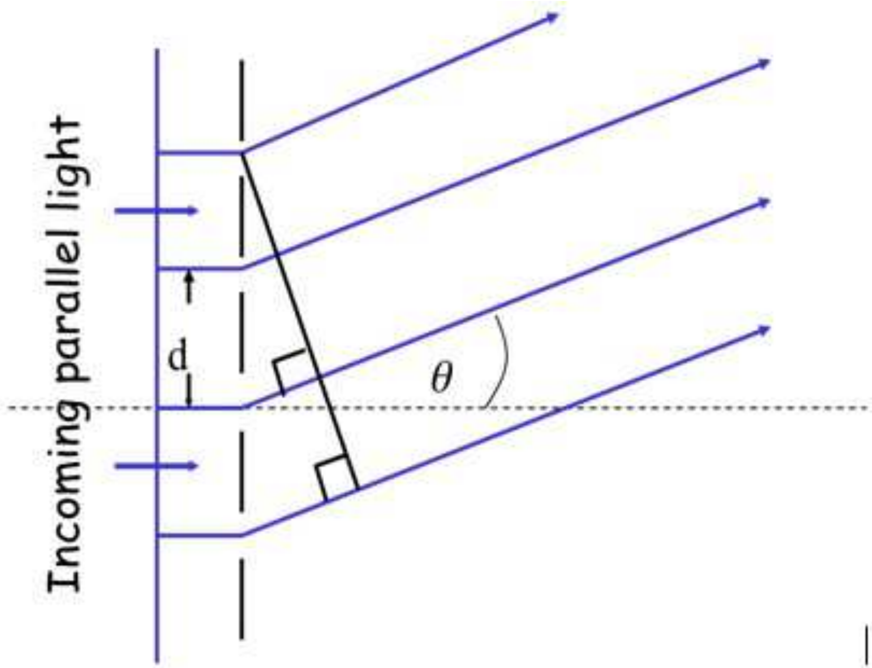

Figure 6.5: The diffraction grating. Incoming parallel light, incident on the grating is diffracted through the slits at an angle θ. The diffraction pattern that is formed om a screen depends on the wavelngth of the light, the slit width and the slit separation (see Fig. 6.6). Narrower and more closely packed slits produce higher resolution (sharper images).

using $\alpha = \frac{1}{2}\, k\, d \sin\theta$, the [ ]-bracketed term can be simplified to $e^{i\alpha}\,(e^{-i\alpha} + e^{i\alpha}) = 2\, i\, e^{i\alpha} \cos\alpha$, so that the intensity of the pattern is now

$$I(\theta) \;=\; \cos^2\alpha \tag{6.7}$$

for narrow slits.

If the slits have significant width, b, then the overall pattern becomes modulated by the diffraction pattern due to each slit, and we instead have

$$I(\theta) \;=\; \left(\frac{\sin\beta}{\beta}\right)^2 . \cos^2\alpha \tag{6.8}$$

6.5 Many Slits – The Diffraction Grating

With even more slits, the widths of the bright spots decrease, see Fig. 6.6. Such a dispersing element is called a diffraction grating. The diffraction grating is not a series of slits, but rather a series of rulings, or indentations on a glass plate. The position where the waves interfere constructively are called orders, and their position depends on wavelength. Hence when white light is diffracted by multiple slits, its constituent wavelengths are diffracted slightly differently producing a spectrum. However, the condition for constructive interference is the same as for a single (or double) slit, i.e. $d \sin\theta = n\lambda$

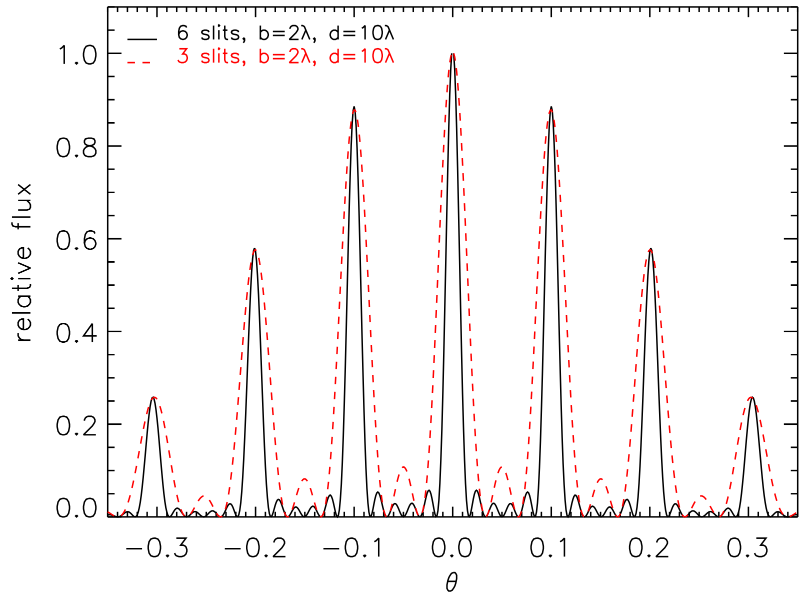

Figure 6.6: The diffraction pattern for a screen with 3 (dashed) and 6 (solid) slits. In both cases, the width of the slit is $2\,\lambda$ and they have spacing $10\,\lambda$.

To work out the intensity profile, we can generalise equation 6.6 further. To keep things simple, let assume the slits are narrow. If there are N slits, then we have

$$Y(\theta) \;=\; A_0\, e^{i\omega t - ikR} . [1 \,+\, e^{ikd\sin\theta} \,+\, e^{2ikd\sin\theta} ... \,+\, e^{(N-1)ikd\sin\theta}] \tag{6.9}$$

Using the previous definition of α, the term in the bracket can be rewritten as

$$1 \,+\, 2e^{i\alpha} \,+\, (2e^{i\alpha})^2 \,+\, ... + \,(2e^{i\alpha})^{(N-1)} \tag{6.10}$$

Using the result for the sum of a geometric series, $1 + x + x^2 + ... + x^{m-1} = (1 \text{-} x^m)\,/\,(1 \text{-} x)$, the bracketed term can be rewritten as

$$\frac{1 \,-\, e^{2iN\alpha}}{1 \,-\, e^{2i\alpha}} \;=\; \left(\frac{e^{iN\alpha}}{e^{i\alpha}}\right) \left(\frac{e^{-iN\alpha} \,-\, e^{iN\alpha}}{e^{-i\alpha} \,-\, e^{i\alpha}}\right) \tag{6.11}$$

The first term effects only the phase, so that the intensity is given by

$$I(\theta) \;=\; \left(\frac{\sin{(N\,\alpha)}}{\sin\alpha}\right)^2 \tag{6.12}$$

for narrow slits

For slits of general width, the pattern is again modulated.

$$I(\theta) \;\propto\; \left(\frac{\sin{(N\,\alpha)}}{\sin\alpha}\right)^2 . \left(\frac{\sin\beta}{\beta}\right) \tag{6.13}$$

6.5.1 Why add more slits?

As more slits are added to the diffraction pattern, the tallest maxima become sharper. These are called principle maxima and occur when $\sin\alpha = 0$ (i.e. $\alpha = n\pi$) [note that $\sin(N\alpha) = 0$ as well, so the intensity is not infinite]. At each principle maxima we have $d\sin\theta = n\lambda$, so that the angle at which the maximum is observed depends on wavelength. n is referred to as the *order* of the maximum. $n = 0$ corresponds to the undeviated light (not very interesting).

The first minima, located either side of the principle maximum will be very important when we consider the spectral resolution of the diffraction grating. They are separated by $\Delta\alpha = \frac{\pi}{N}$. The larger the number of slits, the smaller this angle.

Between the principle maxima are a series of secondary maxima, and there are $N - 2$ of these. Their intensity also decreases as the number of slits increases.

In summary: increasing the number of slits (1) makes the principle maxima sharper; and (2) increases the contrast between the primary maxima and the secondary maxima.

6.5.2 The Resolving Power of a Diffraction Grating

Imagine that we are observing a light source that contains two spectral lines. Will we recognise that there are two lines, or will they appear as a single line? This depends on their separation in wavelength, and the *resolving power* of the spectrograph.

The peak of the first line occurs at $\theta_1 = n\lambda_1 / d$ (approximating $sin\theta \simeq \theta$) and the separation of the lines is $\Delta\theta = n\Delta\lambda / d$. According to *Rayleighs criterion*, the minimum separation that can be resolved corresponds to the separation of maxmima at the first minima, $\Delta\theta_{min} = \lambda / Nd$ (we should use the average λ in this formula, although the result does not change much if we use λ_1 or λ_2 instead).

Therefore, we will recognise the two lines if they differ in wavelength by more than

$$\Delta\lambda_{min} = \lambda / nN \tag{6.14}$$

otherwise they will not be resolved.

We therefore define the *Resolving Power*, R, as $R = \lambda / \Delta\lambda = nN$. The spectral resolving power can be found theoretically by differentiating equation 6.5 to derive

$$\frac{d\theta}{d\lambda} = \frac{n}{d\cos\theta} \tag{6.15}$$

where $\frac{d\theta}{d\lambda}$ is known as the *angular dispersion* and is usually measured in radians / nm. In terms of a spectrograph on a telescope, it is usually more convenient to calculate the *reciprocal linear dispersion*, $\frac{d\lambda}{dx}$ which measures the wavelength per unit distance, x, at the detector by multiplying the angular dispersion ($\frac{d\theta}{d\lambda}$) by the camera plate scale ($\frac{d\theta}{dx} = 1 / f_{cam}$) to derive:

$$\frac{d\lambda}{dx} = \frac{d\lambda}{d\theta}\frac{d\theta}{dx} = \frac{d}{f_{cam}\, n}\cos\theta \tag{6.16}$$

6.6 The Spectral Resolution Through a Finite Slit Width

In most textbooks, the *Resolving Power*, R, is defined by $R = \lambda / \Delta\lambda$. However, this assumes that the aperture is infinitely narrow, and that the incident light is coherent, neither of which are true when observing the light from a distant star. In practice, the resolution is limited by the width of the slit (or the image of the slit on the detector), and the fringe pattern on the detector is instead the incoherent sum of point sources that passed through the (finite) slit, each of which produces their own, slightly displaced, fringe pattern. The final fringe pattern is then the sum of all individual fringe patters.

In the case of a finite slit width, the spectral resolution is given by:

$$R = \frac{n\,\rho\,\lambda\,W}{\chi\,D_T} \tag{6.17}$$

where n is the diffraction order, ρ is the ruling density (lines / mm), λ is the wavelength, W is the grating size, χ is the angular size of the image of a star on the slit (which is usually the seeing) and D_T is the telescope size. At most wavelengths, this value of R is much smaller than that given by the theoretical maximum value of nN.

6.7 Reflection Gratings

Most spectrometers on telescopes use a reflection (rather than diffraction) grating. However, in most ways, these are similar to a diffraction grating, except the light is *reflected* from the grating surface, not transmitted[1]. In this case, the incoming light may be off-axis (Fig. 7.3), and so we must modify equation 6.5 to

$$n\lambda = d\,(sin\alpha + sin\beta) \tag{6.18}$$

This is often rewritten as

$$n\,\lambda\,\rho = sin\alpha + sin\beta \tag{6.19}$$

where $\rho = 1 / d$ is the ruling density (lines / mm) and n is the diffraction order.

Exercises

1. The sodium D-lines in the solar spectrum have wavelengths of 589.6 and 589.0 nm. Given a grating with 500 lines / mm.
   (a) what is the angular separation of the lines in the *first* order?
   (b) what is the angular separation of the lines in the *second* order?
   (c) in second order, what spectral resolution is required to resolve the two lines?

2. Explain what is meant by the *Resolving power* of a spectrograph? State the definition of spectral resolution in terms of wavelength.

[1] You can use a CD as a reflection grating: hold the CD at a shallow angle towards a light source to get the light to disperse. The incoming waves diffract on the dots that hold the information on the CD.

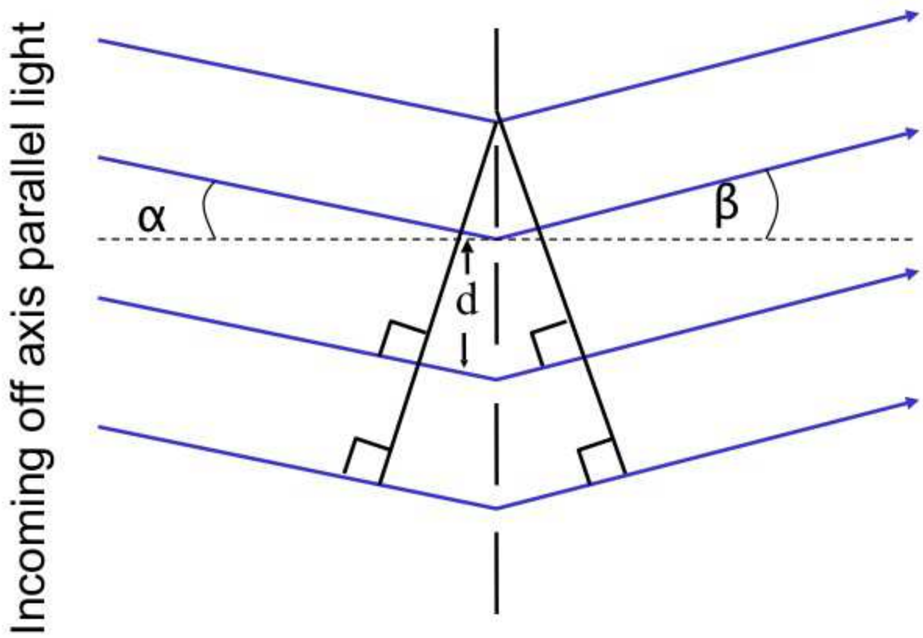

Figure 6.7: The diffraction grating when incoming light is not plane parallel to the grating surface, but instead makes an angle α to the grating normal. In this case, the standard diffraction grating equation is simply modified to $n\lambda = d\,(\sin\alpha + \sin\beta)$.

3. What is the resolving power of a spectrograph at 700 nm used on a 8-meter telescope in $0.5''$ seeing? The spectrograph uses a 20 cm grating with 1000 lines / mm and first order light.

4. A spectrograph mounted on the 10-meter Keck telescope uses a reflection grating with 600 lines / mm. Due to mechanical constraints, the mimimum angle between the camera and the collimator is $20°$ (i.e. the sum of the incident and reflected angles is $20°$). If a first order spectrum is observed, what is the maximum wavelength that can be observed? [Hint: use the reflection grating equation and the fact that $\frac{d\sin(A-x)}{dx} = -\cos(A-x)$ where A is a constant].

5. A spectrograph is being designed to study the motion of the stars using the doppler shift of a spectral line with rest wavelength 430 nm. It uses a diffraction grating that is 200 mm wide and is rules to 1000 lines per mm.

   (a) In order to detect Jupiter-like planets around these stars, the stars velocities must be measured to an accuracy of $1\,\mathrm{km\,s^{-1}}$. What resolving power is needed from the spectrograph? [Hint: the doppler shift formulae is $\Delta\lambda / \lambda = v / c$ (See Young & Freedman Chapter 44)].

   (b) In order to obtain this resolving power, what order spectrum should be observed? At what angular deviation will the rest wavelength of the line be observed?

   (c) Another astronomer suggests that it would be better to use a spectral line at 656 nm to perform this experiment. Are they correct?

   (d) You are trying to build a spectrograph for the 40-meter Extremely Large Telescope (ELT) which will be able to detect Earth-like planets around other stars (which requires sufficient resolution to detected $1\,\mathrm{m\,s^{-1}}$ velocity shifts). At which telescope focus would you put the instrument (and why)?

Chapter 7

Measuring Stars

7.1 Blackblody radiation

In previous chapters, we saw that individual atomic transitions produce absorption (or emission) lines. However, where does the thermal (continuous) spectrum come from? When an aggregate of atoms interact strongly with one another, they convert their thermal energy into electromagnetic energy. Since the interactions occur at a range of energies, any individual spectral features become washed out, and a thermal continuum results.

An object which is in thermal equilibrium with its surroundings is called a blackbody, with a spectrum which only depends on the absolute temperature. In particular, the gasses in the interior of a star are opaque (highly absorbent) to all radiation (otherwise we would see the stellar interior at some wavelength!), hence the radiation there is blackbody in character. When we look at a star, we see the radiation that is slowly leaking from the surface of the star, and to a first approximation, this continuum radiation is blackbody in nature.

The Planck blackbody spectrum radiation law is given by:

$$I(\nu)\Delta\nu \;=\; (2\,h\,\nu\,/\,c^2)\,[\,1\,/\,(e^{h\nu/kT}-1)\,]\,\Delta\nu \tag{7.1}$$

where $I(\nu)\Delta\nu$ is the intensity $(\mathrm{J}\,/\,\mathrm{m}^2\,/\,\mathrm{s}\,/\,\mathrm{sr})$ of radiation from a blackbody at temperature T in the frequency range between ν and $\nu+\Delta\nu$, h is Plancks constant, c is the speed of light and k is Boltzmaann's constant. Because the frequency, ν and wavelength λ are related by $c=\nu\lambda$, we can also express equation 7.1 in terms of intensity emitted per unit wavelength

$$I(\lambda)\Delta\lambda \;=\; (2\,h\,c^2\,/\,\lambda^5)\,[\,1\,/\,(e^{hc/\lambda kT}-1)\,]\,\Delta\lambda \tag{7.2}$$

Equation 7.2 follows because the intensity emitted per unit wavelength, $I(\lambda)\Delta\lambda$ equals the intensity per unit frequency $I(\nu)\Delta\nu$ and so $\Delta\nu=c\Delta\lambda\,/\,\lambda$.

Equation 7.2 is illustrated in Fig. 7.1 for several values of T. Note that both $I(\lambda)$ and $I(\nu)$ increase as the blackbody temperature increases – the blackbody becomes brighter. This effect is easily seen when interpreted in terms of equation 7.1, where $I(\nu)\Delta\nu$ is directly proportional to the number of

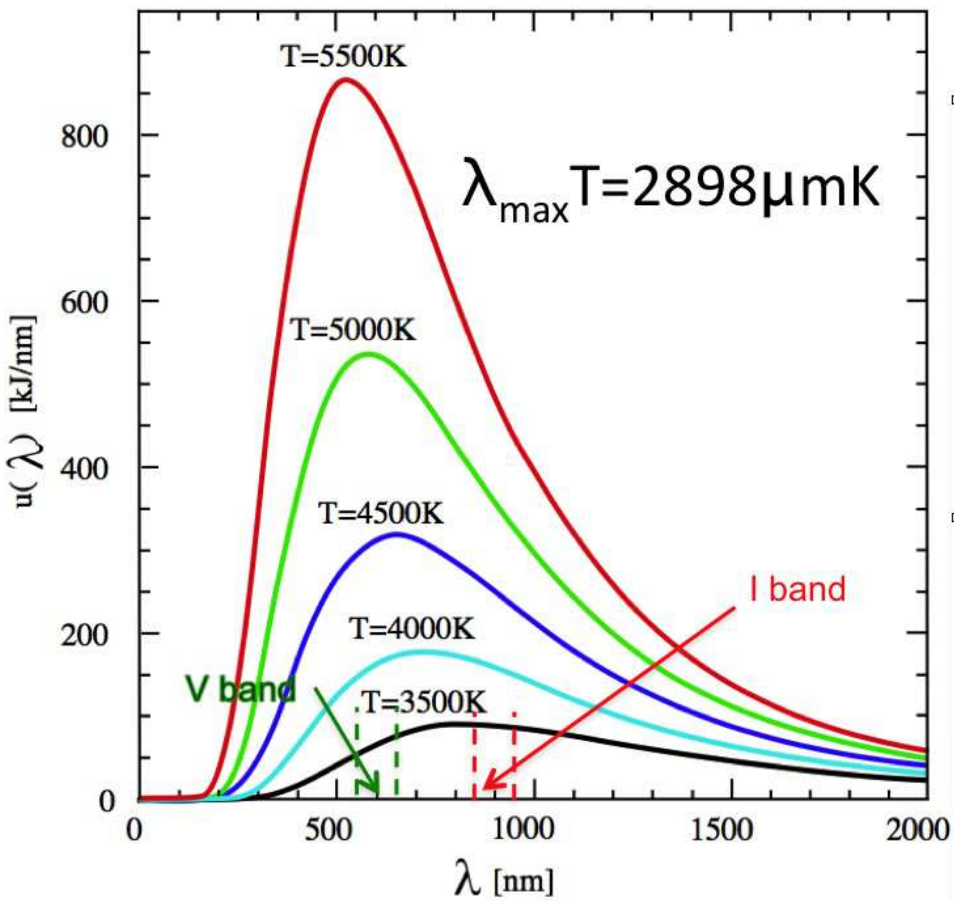

Figure 7.1: Schematic of the Planck function. The intensity as a function of wavelength for temperatures of T = 3500–5500 K (black to red). As the temperature is increased, the peak wavelength shifts to lower wavelengths (hotter objects appear bluer). The peak wavelength of the Planck function is related to the wavelength by $\lambda_{\rm max}\,{\rm T} = 2898\mu\,{\rm mK}$ (Weins law).

photons emitted per second near the energy $h\nu$. The Planck function is special enough that it is often given its own symbol, $B(\lambda)$, or $B(\nu)$.

Note that in the case of **low temperatures**, the exponential term becomes very large, and so the Planck function approximates to

$$B_\lambda(T) \;=\; I_\lambda(T) \;=\; (2\,h\,c^2\,/\,\lambda^5)\,e^{-hc/\lambda kT} \tag{7.3}$$

which is often called the *Wein distribution*. In the opposite case, when the temperature is high, we can use the power-law series expansion, $e^x - 1 \simeq x$ to derive the approximation

$$B_\lambda(T) \;=\; I_\lambda(T) \;=\; 2\,c\,k\,T\,/\,\lambda^4 \tag{7.4}$$

which is known as the *Rayleigh Jeans distribution*.

As can be seen from Fig. 7.1, a blackbody emits at a peak wavelength which shifts to shorter wavelengths as the temperature increases. If we set the derivative of equation 7.2 to zero, then we derive the Wein displacement law

$$\lambda_{max}\,T \;=\; 2.898\,\times\,10^{-3}\,m\,K \tag{7.5}$$

For example, the continuous spectrum from our Sun is approximately a blackbody, peaking at $\lambda_{max} = 500\,\mathrm{nm}$. Therefore the surface temperature is near 5800 K. The spectra of stars that are hotter than the Sun will peak at shorter wavelengths; i.e. they will appear bluer, such as Sirius – the Dog star – which has a surface temperature of $\sim 10{,}000\,\mathrm{K}$ and a colour of $(B - V) = 0$ whilst stars that are cooler than the Sun will have spectra that appear redder; e.g. Betelgeuse – a M-type star with a surface temperature 3600 K and a colour $(B - V) = 2$.

The area under the Planck curve (integrating the Planck function) represents the total energy flux, F ($\mathrm{W\,/\,m^2}$) emitted by a blackbody when summed over all wavelengths and solid angles:

$$F(T) \;=\; \sigma\,T^4 \tag{7.6}$$

where $\sigma = 5.669 \times 10^{-8}\,\mathrm{W\,m^2\,K^4}$. Note the strong temperature dependence – the brightness of a blackbody increases as the fourth power of the temperature. This expression is known as the Stefan-Boltzmann law. Approximating a star as a blackbody, the total energy output per unit time of the star (its power or luminosity in watts) is just

$$L \;=\; 4\,\pi\,R^2\,\sigma\,T^4 \tag{7.7}$$

since the surface area of a sphere of radius R is $4\,\pi\,R^2$.

Exercises

1. How much more energy is emitted by a star with effective surface temperature 20,000 K compared to the Sun ($T_\odot = 5{,}700\,\mathrm{K}$)?

2. What is the predominant colour of a star with effective surface temperature 20,000 K and 5,700 K? [use the Wein displacement law and express the answer in wavelengths]

7.2 Observables for stars

7.2.1 Stellar temperatures

If we are dealing with a true blackbody, then we can establish the temperature using *(i)* the shape of the Planck curve using two points on the curve and *(ii)* Wein's Law and the wavelength of the peak emission, or the Stefan-Boltzmann law if we know the luminosity and radius.

The colour of a star simply defines the ratio of flux in one band and another. For example, a $(B\text{-}V)$ colour refers to the flux density at ~ 450 nm to that at ~ 550 nm according to $-2.5\log(\frac{I_B}{I_V})$. However, if the star radiates as a blackbody, then the ratio of these fluxes (the colour) can be used to define the effective surface temperature of the star. Hotter stars are generally bluer, and also tend to be much more luminous than redder stars (see section 7.2.3 below).

7.2.2 Stellar spectral sequences

The properties seen in the spectra of stars also correlate with their temperature (and hence colour). The spectra can be (broadly) classified into seven spectral types, OBAFGKM. Stars near the beginning of the sequence (O-stars) are the bluest (and hence hottest stars) and are often called early-type stars whereas stars at the end of the sequence (M-stars) are red and (relatively) cool. Each spectral type is then divided further into early-type (0) and late-type (9). For example, ...F8 F9 G0 G1 G2... G9 K0... In this scheme, our Sun is a G2 star. In general, the spectral types are based on three criterion, *(i)* the presence or absence of absorption lines; *(ii)* the strength (equivalent width) of the Hydrogen Balmer absorption lines; *(iii)* the ratio of the strength of metal lines (such as CaII H&K) compared to the Balmer series. This spectral sequence corresponds closely to a surface temperature sequence (the spectral types does not provide a perfect correlation with temperature since in reality these are not perfect blackbodies – in some spectral types atomic and molecular lines blanket the continuum).

7.2.3 Stellar distances and luminosity

The most direct method to determine the distance to (nearby) stars is to use their parallax (the difference in the apparent position of an object viewed along two different lines of sight). As the Earth orbits the Sun, nearby stars appear to shift in position with respect to the more distant (apparently fixed) stars. If the observed parallax angle is p (in seconds of arc), then the star's distance is

$$d = (206265 \,/\, p'')\, AU \tag{7.8}$$

[Remember that 206265 is the number of arcseconds in 1 radian].

With an estimate of the distance to a nearby star, the total luminosity can then be derived using the observed flux and the $1\,/\,d^2$ law. For stars, we usually quote an absolute magnitude, M (see § 1.3.2). In 1911, Ejnar Hertzsprung-Russell plotted the absolute magnitude for stars with 5 pc of the Sun versus their spectral type. Independently, in 1913, Henry Norris Russell also made the same plot, and because the luminosity, temperature, colour and radius of stars are correlated, the

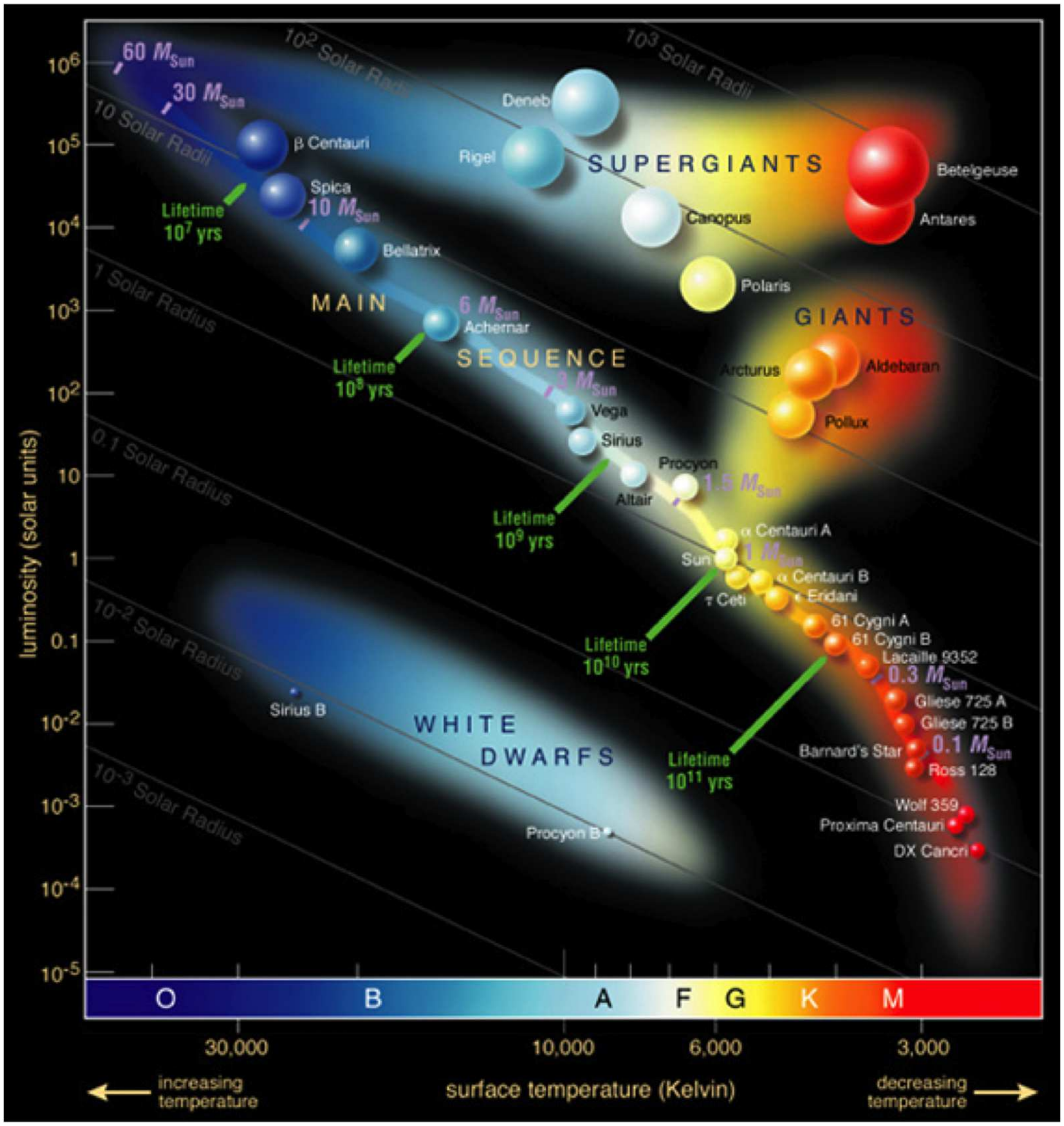

Figure 7.2: Hertzsprung-Russell Diagram with many of the best-known stars in the Milky-Way overplotted. Stars tend to fall into three certain regions of the diagram. The most prominent is the "main0sequence" which follows a diagonal from the lower-right (cool, low luminosity) to upper-left (hot, high luminosity). In the lower-left corner, we find white dwarfs, and above the main sequence are the subgiants, giants and supergiants.

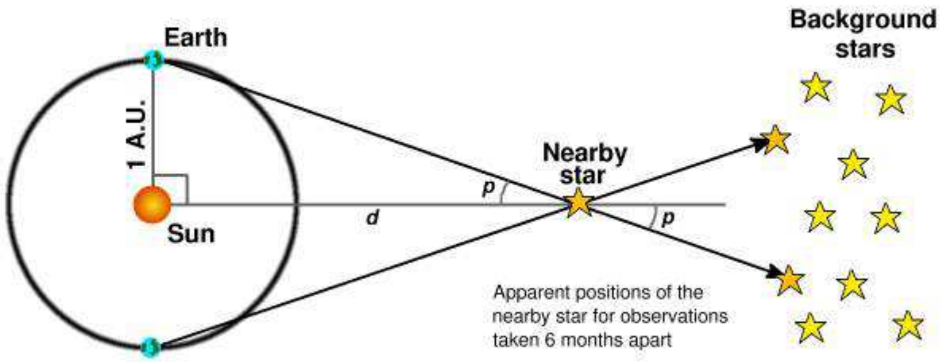

Figure 7.3: Schematic of parallax of stars. As the Earth orbits the Sun, the apparent positions of nearby stars is shifted with respect to much more distant stars. A star with a parallax of 1-arcsecond will have distance of 1 parsec.

relation between the stellar luminosity, temperature, magnitude and radius is universally called the Hertsprung-Russell (HR) diagram (Fig. 7.2).

The HR diagram represents one of the great observational synthesis in astrophysics. You will study the physics of the HR diagram in lots of detail in the "stars" part of this lecture course. Briefly, first, note the log-scale on both axis of Fig. 7.2. In particular, a typical M-star is $10^9\times$ less luminous than an O-star (i.e. a single O-star shines as brightly as 10^9 M-stars, although a massive O-star will only live for a few million years – compared to several billion years for an M-star). This plot also shows a "main" sequence of stars, from massive, blue, hot (30,000 K) stars (top left) which runs down to low-mass, cool (3,000 K) red stars (bottom right). The Sun, a main-sequence G2V star sits in the middle of this sequence.

Branching off from this sequence are the giant (and super-giant) stars (top right) which represent "main-sequence" stars coming to the end of their lives, and white-dwarf stars (lower-left) which represent the cooling remnants of massive stars.

7.3 Measuring Stellar Radii

7.3.1 Interference by Division of Amplitude

Interferometry work by combining light rays which have traveled through slightly different path lengths. The basic type of interferometer is the Michelson Interferometer. By using a beam splitter (e.g. a glass block or a half silvered mirror), a light ray can has two possible paths through the instrument. When the light rays are recombined (e.g. using a second beam splitter), the intensity of the output wave depends on the difference in optical path length of the two routes. If the waves differ in phase by an integer number of wavelengths, $2\,m\pi$, they interfere constructively. If they differ by a half number of wavelengths $(2m+1)\pi$, the interference is destructive and the output is a minimum.

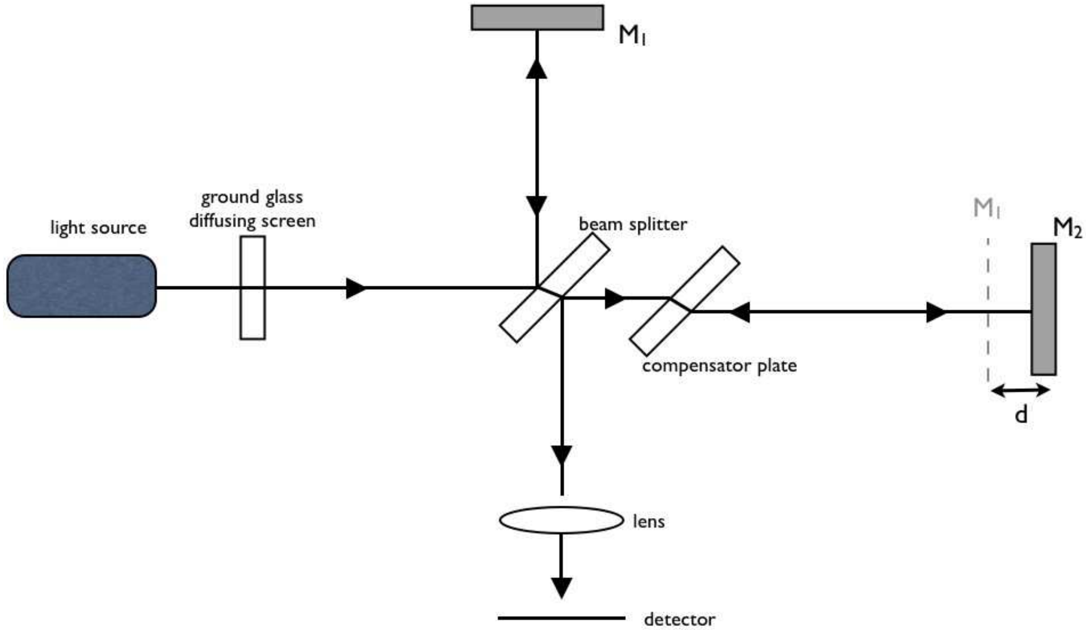

Figure 7.4: Schematic of the Michelson Interferometer. The Mirror M2 is shifted by a small distance, d to create a path difference with respect to M1. As the light is recombined with the light reflected back from M1, this creates and interference pattern.

7.3.2 The Michelson Interferometer

In the 1920s, Albert Michelson designed and used an interferometer to measure the radii of nearby stars. The Michelson Interferometer uses a beam splitter to send a single ray of light to two mirrors (M_1, M_2; Fig. 7.4). The mirrors reflect the light back to the beam splitter which serves to recombine them. The difference in path length is $2\,d$, where d is the extra distance to M_2. For the maximum signal, $2\,d = m\lambda$.

This type of interferometer can be used to control cutting or drawing machines with incredible accuracy (ever wondered how the lines were drawn on a diffraction grating?). Using blue light, the relative separation of the mirrors can be controlled to <500 nm. If the mirrors are moved, the change in distance can be measured by counting the number of bright *fringes* (how many times the signal passes through maximum).

The optical path length between the mirrors also depends on the refractive index along the path. Variations in this type of interferometer are used to study turbulence in wind tunnels (the refractive index of air depends on its density), or the physical state of plasmas in fusion reactors.

Such an interferometer was also used by Michelson & Morley to show that the speed of light does not depend on the reference frame in which it is measured. This led Einstein to formulate Special Relativity.

7.3.3 The Stellar Interferometer

The stellar interferometer is closely related to the Michelson Interferometer, although there is no actual beam splitter. A distant star (or galaxy) is observed with two telescopes, separated by a distance D (Fig. 7.5). By combining the light from the two telescopes, this device can be used to measure the diameters of stars even though this is much less than can be measured with a single telescope.

Imagine that you are using the stellar interferometer to determine the size of a distant star. The pattern you observe depends on the angle between the wavefronts from opposite limbs of the star; this intersection angle increases as the stellar angular size increases. If the star is very small (lets assume it's a point source), the two patterns coincide (for any telescope separation, D), and no fringes will be observed.

However, if the star has a finite size, then as the telescopes are moved apart, the two individual diffraction patterns will start to overlap until the minimum from one star overlaps with the maximum of the second. At this point, the fringes will disappear completely.

For the star Betelgeuse, Michelson found that the fringes vanished when the separation of the telescopes was 3.07 m. Since the resolution of an interferometer is given by

$$\phi = \lambda / D \tag{7.9}$$

with $\lambda = 570\,\text{nm}$, this gives
$\phi = 1.22\,(570\,\text{nm}) / 3.07\,\text{m} = 0.047$". If the distance ($d$) is known (from parallax measurement, for example), then the diameter of the star, D (using the small angle approximation) is simply $D = \phi\, d$. The distance to Betelgeuse is 200 pc, so the diameter is
$D = 0.047 \times (200\,\text{pc}) = 1.4 \times 10^9\,\text{km}$, which is approximately 1500 $\times$ larger than radius of the Sun (so Betelgeuse is a red giant).

Exercises

1. A Michelson Interferometer is used in the construction of a diffraction grating to rule a series of parallel lines 2×10^{-6} meters apart. The movable mirror controls a cutting head. How many fringes do you have to count to move the cutting head from one line to the next? [assume the device is illuminated with laser light at 800 nm]

2. One of the properties of an interferometer is that it is insensitive to objects with angular sizes much larger than the minimum resolvable angle of the shortest baseline (essentially because the different portions of the source destructively interfere with each other). The flux from a source of angular diameter, θ measured by an interferometer of baseline, L (in kilometers) is approximately $F_{measured} = F_{true}\exp[-0.4(L\theta/\lambda^2)]$ where λ is in centimeters. For a wavelength of 6 cm, what is the largest angular size of a source than can be observed with an interferometer of minimum baseline 1000 km if the lowest detected flux is 0.1 F_{true}? What would be the properties of source that you would *not* want to observe using an interferometer?

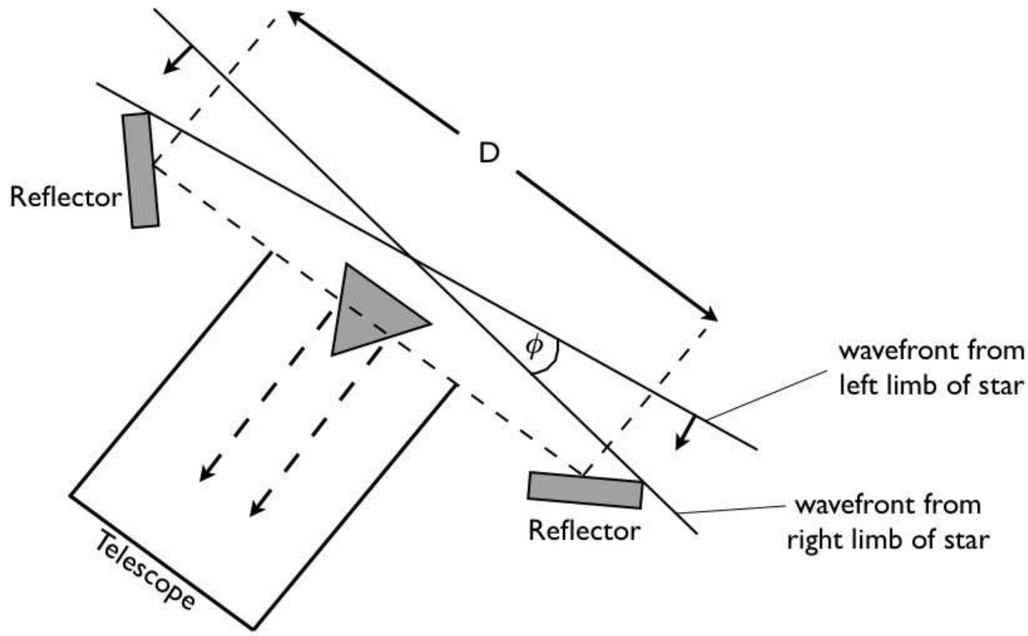

Figure 7.5: Schematic of the Stellar Interferometer. The widely spaced mirrors send starlight into the telescopes to produce the interference patterns. Light wave from opposite ends of the star arrive simultaneously, intersecting each other at an angle ϕ, which is the angular size of the star.

3. The star, Vega is observed with a stellar interferometer (operating at 700 nm), and it is found that the fringes disappear when the telescopes are 55 meters apart. *(a)* What is the stars angular size on the sky (in mill-arcseconds)? *(b)* The measured flux on Earth from Algol is $2.84 \times 10^{-8}\,\mathrm{W}\,/\,\mathrm{m}^2$ and it display a parallax of 0.12". *(c)* What is its distance (in parsecs)? *(d)* What are Vega's diameter and effective temperature?

4. *(a)* How much brighter is an A-type star (with effective surface temperature 10,000 K) compared to a K-type star (with effective surface temperature 4000 K) at a wavelength of 5000Å? *(b)* What is the ratio of the total luminosities of the A star compared to the K star? *(c)* What is the $(B-V)$ colour of a star with effective surface temperature 30,000 K and how does this compare to the Sun (effective temperature 5,700 K).

Chapter 8

Multi-Wavelength Techniques

8.1 Radio Astronomy

In 1929, Karl Jansky built an antenna to receive radio signals (wavelengths ranging from $\sim 5\,\mathrm{cm}$ to $10\,\mathrm{m}$). This antenna system rotated, scanning the whole sky in 20 minutes. Jansky noticed that the signal he recorded was comprised of three sources: *(i)* A weak signal caused by distant thunder storms; *(ii)* a more powerful burst due to local thunder storms; and *(iii)* a steady hiss.

Jansky originally believed the steady hiss to be radiation coming from the Sun, but discovered that the radio signal repeated every 23 hours and 56 minutes 4 seconds (remember the sidereal day from lecture #1?). The signal was in fact coming from the center of the Milky Way in the direction of the constellation of Sagittarius. This discovery led to the birth of modern radio astronomy.

Radio telescopes are similar in design to optical telescopes, only their surface are constructed from materials that exhibit very low absorption and reflection of radio waves. Since the accuracy of the surface scales with the wavelength we are observing, at radio wavelength, the telescope surface is lighter (it is not a giant piece of glass!) and the surface does not need to be as precise as in the optical (to ensure diffraction limited imaging, the surface must be smooth the $\lambda\,/\,20$). Hence, the telescopes can be be bigger. The biggest steerable radio telescope is the 100-meter Green Bank Telescope, whilst the Arecibo radio telescope (which is build into a mountain in Puerto Rico) is 305 meters across.

8.1.1 Single Dish Radio Astronomy

Why are radio telescopes big? The equation for the diffraction limit, $\theta = 1.22\,\lambda\,/\,D$ holds at any wavelength. The Jodrell Bank 76 meter telescope operating at 21 cm has a resolution of $\theta = 1.22 \times 0.21\,/\,76 \times 206265 =$ (about 11.6-arcminutes). In comparison, the Hubble Space Telescope (which has a 2 meter mirror) has a diffraction limit of about $0.1''$ at optical wavelengths. Hence, a single dish radio telescope with the same resolving power as Hubble, operating at 21 cm would have to be 500 km across!

Hence radio astronomers use interferometers to increase the resolving power, with baselines of several kilometers to reducing the diffraction limit.

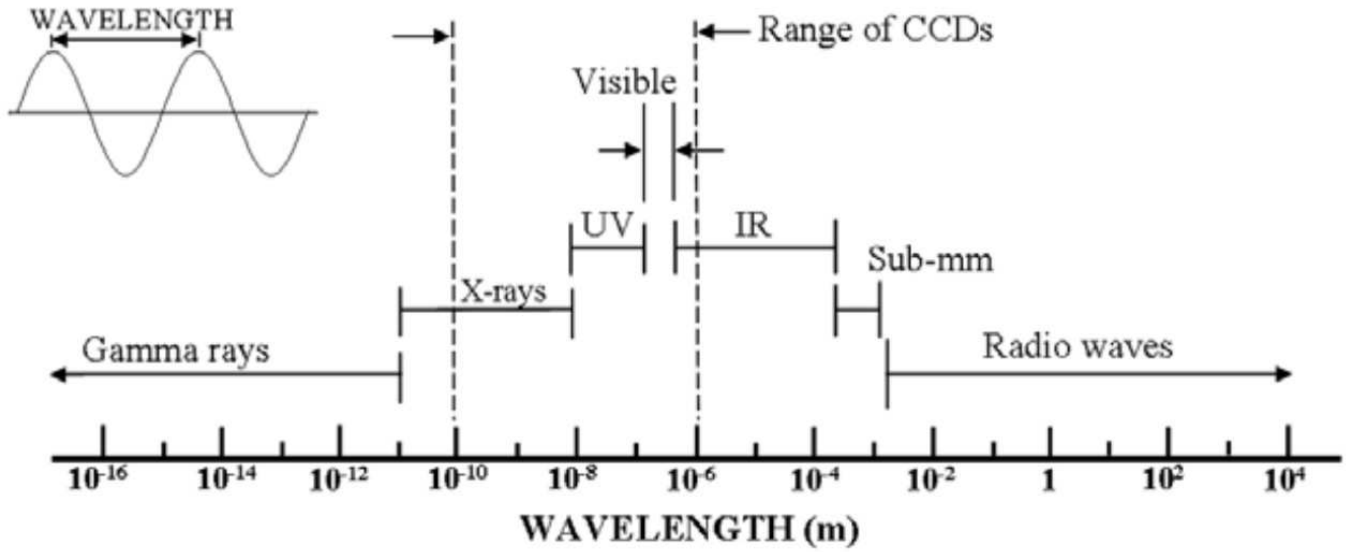

Figure 8.1: The electromagnetic spectrum from Gamma-rays, X-rays, UV, visible and infrared light, sub-mm and radio waves.

8.1.2 Radio Interferometry

As we have already seen, the essence of interferometry is to consider the phases of wavefronts reaching the two dishes of a simple interferometer. If a source is directly overhead, the waves arrive at both telescopes in phase. When the signals are combined (electronically in a mixer), they constructively interfere to reinforce a strong signal. Now assume that the source is somewhat west of overhead, so that the path lengths of the two antennas differ by exactly $\lambda / 2$. They arrive 180° out of phase and destructively interfere. As the Earth turns, the change in phase is recorded, and the way in which the phase changes reflects the size and structure of the object of interest. The resolution of an interferometer is set by the separation of the dishes. The equation $\theta = 1.22\,\lambda / D$ is still valid, only now D is the separation of the telescopes.

However, a two telescope interferometer only gives one part of the separation between points on the sky (in the same direction as the baseline). To map a source completely with uniform coverage (and hence build up an image) requires a trick: twisting the object with respect to the interferometers pattern; a technique known as **aperture synthesis**. This twisting can be accomplished naturally by observing the source as it moves across the sky – it's orientation will twist with respect to the interferometers baselines. To help this process, the baseline of the interferometer can also be twisted by arranging the telescopes in a "Y"-shaped pattern (e.g. look at the antennae configuration of the Jansky Very Large Array (JVLA).

8.1.3 Interstellar Radio Lines: the Neutral Hydrogen line at 21 cm

Where interstellar atomic gas is cold, hydrogen is neutral and in its ground state. This ground state has two energy levels, separated by a very small energy difference, and this energy difference corresponds to a wavelength of 21 cm. The reason for this phenomenum lies in the fact that both the proton and electron have intrinsic spin. A moving charge produces a magnetic field. Because

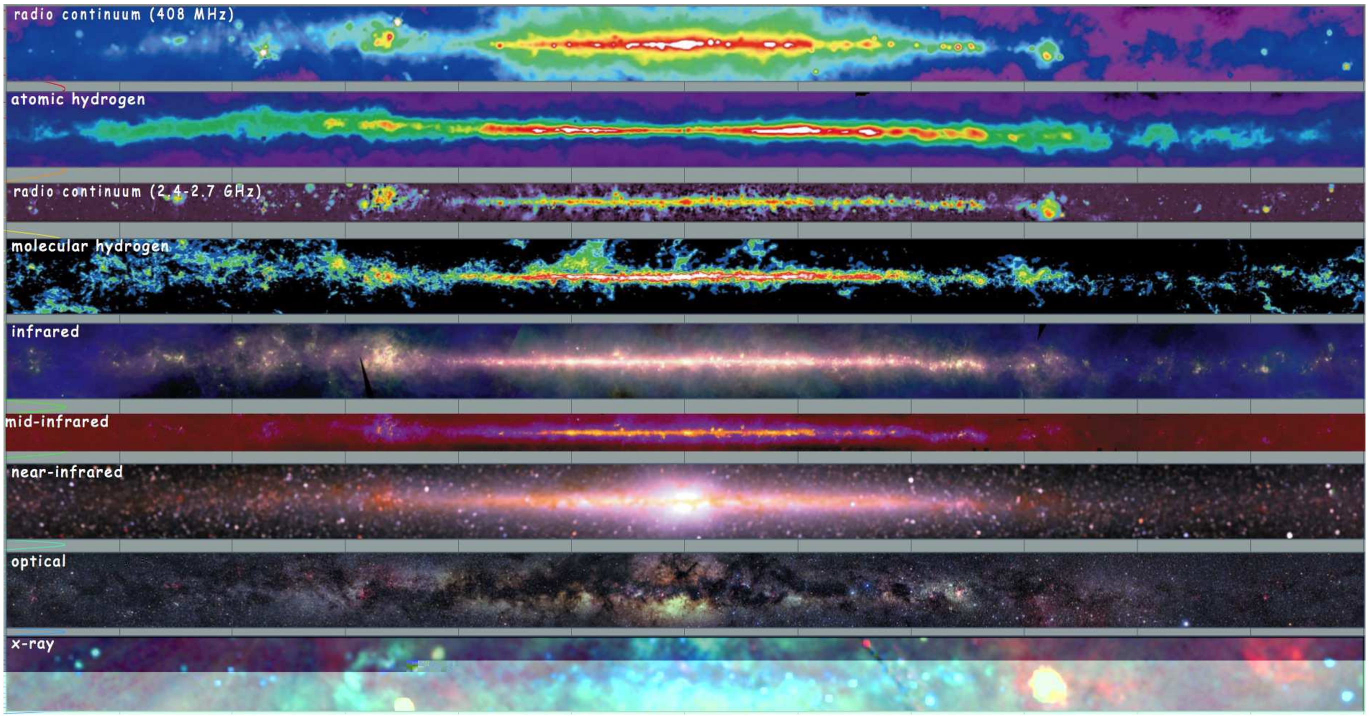

Figure 8.2: The Milky-Way as a function of wavelength. We view the Milky Way edge-on from our perspective near the plane of the disk 7 kpc from the Galactic center. These multi-wavelength maps are in Galactic coordinates with the direction of the Galactic center in the center of each. **Radio continuum (2.4–2.7 GHz):** Intensity of radio continuum emission from hot, ionized gas and high-energy electrons in the Milky Way, The majority of the bright emission seen is from hot, ionized regions, or produced by energetic electrons moving in magnetic fields. **Atomic hydrogen:** Column density of atomic hydrogen, derived from radio surveys of the 21-cm transition of hydrogen. **Molecular hydrogen:** The molecular gas is mainly H_2, but H_2 is difficult to detect directly at interstellar conditions and ^{12}CO, the second most abundant molecule, is observed as a surrogate. **Visible (0.4–0.6μm) light:** due to the strong obscuring effect of interstellar dust, the light is primarily from stars within 1–2 kpc of the Sun, nearby on the scale of the Milky Way. **mid- and far-infrared intensity (12–100 μm:** most of the emission is thermal, from interstellar dust warmed by absorbed starlight, including star-forming regions embedded in interstellar clouds. **Near-Infrared (1–3μm** Most of the emission at these wavelengths is from relatively cool giant K stars in the disk and bulge of the Milky Way. **Mid-infrared (6.8 - 10.8 μ m):** most of the diffuse emission arises in complex molecules of polycyclic aromatic hydrocarbons (PAH), which are commonly found both in coal and interstellar gas clouds. **X-ray image:** In the Milky Way, extended soft X-ray emission is detected from hot, shocked gas.

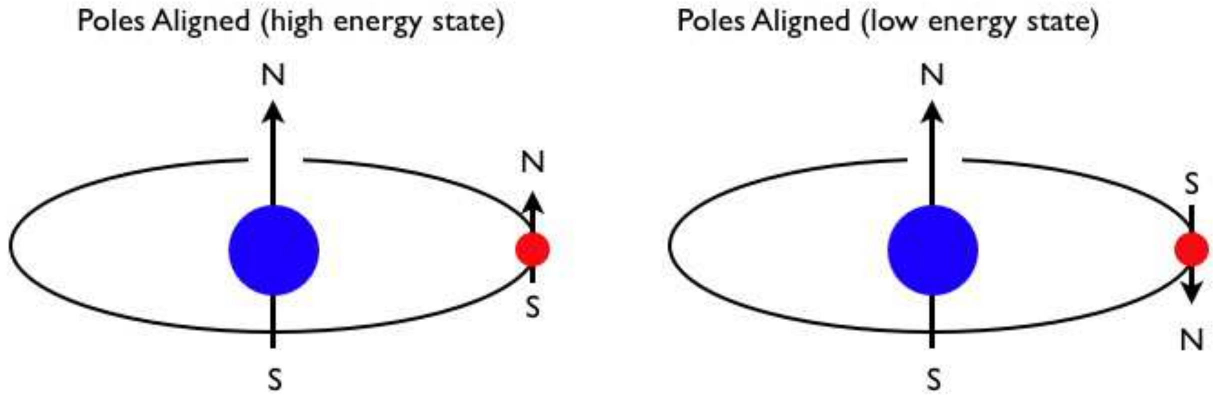

Figure 8.3: Spin alignment for hydrogen. The spin alignment of the proton and electron can either be aligned *(left)* or anti-parallel *(right)* which is the lower energy state.

both the proton and electron are charged particles, their spin motion generates a dipolar magnetic field (like the field of a tiny bar magnet) that we can characterise by the term magnetic moment. The magnetic moment of a spinning particle is represented by a vector, which is proportional to the vector angular momentum of the particle.

Two possible ground state configurations of the neutral Hydrogen atom exist (Fig. 8.3). In one configuration, the magnetic moment vectors of the proton and electron are parallel (or aligned). Because vectors add, this configuration has a high state of magnetic energy. If the magnetic moment vectors are anti-parallel (or opposed), we have the second configuration, which is characterised by less magnetic energy and a more tightly bound orbit. Since the aligned and opposite states are at slightly different energy levels, we refer to this effect as hyperfine splitting of the ground state of the Hydrogen atom. A spontaneous transition from the higher hyperfine state to the lower hyperfine state can occur (when the spin of the electron flips from a parallel to anti-parallel), accompanied by the emission of a low energy photon. The difference in energy between the two states corresponds to a photon at a wavelength of 21 cm, (i.e. in the radio wavelengths).

Since the transition from upper- to lower- hyperfine state is strongly forbidden, a Hydrogen atom in the upper hyperfine state will flip to the lower state once every few million years (on average). However, in the mean time, collisional excitation and de-excitation of the atom Hydrogen within the inter-stellar medium will also occur about once every 400 years. However, most of the baryonic matter in galaxies is in the form of Hydrogen, and so the total 21 cm emission from a galaxy is often bright enough to be detected from Earth, allowing measurements of galaxy structure (spiral arms) and galaxy rotation curves (dynamics).

8.2 Mid-infrared and Sub-mm Techniques

The sub-millimeter regime is at the interface between the radio and optical / infrared regions, and techniques from both camps are combined. For example, the use of mirrors, lenses, and filters are essentially the same as for the optical / infrared, whereas the application of antennas, feed horns, and

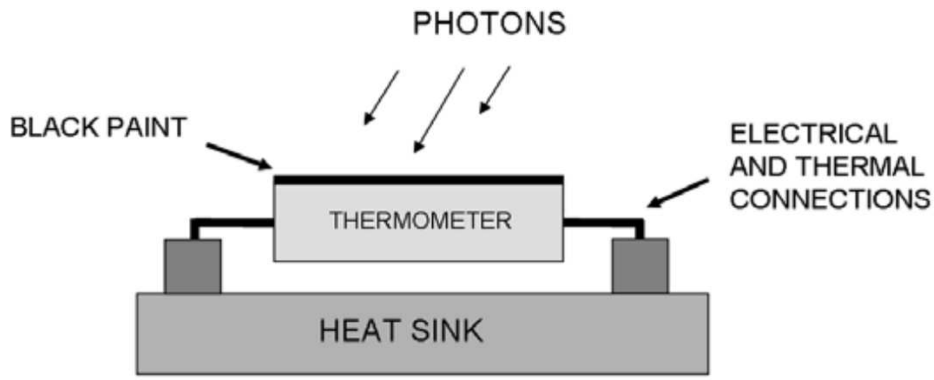

Figure 8.4: Schematic of a simple bolometer element. The bolometer is designed to detect electromagnetic radiation at millimeter wavelengths by detecting the (very small) increase in temperature as phtons are absorbed by the thermometer. The small change in temperature causes a change in resistivity, which can be measured with an electical current.

wave-guides is derived from radio technology.

Cold interstellar clouds with a temperature of 10 K will emit thermal radiation with a peak at about 200 μm, and this is the material from which stars form. Unlike many other parts of the electromagnetic spectrum, all-sky mapping has not occurred yet. One difficulty is that the atmosphere is almost totally opaque at these wavelengths except for some weak windows that partially open at high, dry observing sites like the summit of Mauna Kea in Hawaii or the Atacama Desert in Chile. In addition, detector development has been slower than optical / infrared or radio because there are no commercial or military applications! Moreover, extremely low detector temperatures are required, typically well below liquid helium (4 K) and into the milli-Kelvin range.

Detectors for the sub-millimeter fall into two main classes: bolometers or continuum detectors for broad-band imaging, and heterodyne systems for spectroscopy and narrow-band observations. Here we will focus on bolometers.

A bolometer measures the energy (power) from a radiation field by measuring the change in electrical conductivity of a device as it is heated by the radiation. Typically, a metal-coated dielectric material is used as the absorber and a semiconductor is used as the thermometer. Each pixel is illuminated by a feed horn, and a small focal plane array can be built up by stacking individual bolometers side by side. A schematic layout of a bolometer is shown in Fig. 8.4. The key features are a a heat sink held at a fixed temperature. The incoming photon causes a very small temperature change, and hence change in reistence and so a current will flow, and so we measure a voltage. Operating in the 350 μm and 450 μm atmospheric windows, these bolometers must be cooled to 300 mK (0.3 degrees above absolute zero) by a ^{3}He refrigerator.

8.3 Infrared techniques

Because of the intrinsic band-gap of silicon, CCDs do not respond to radiation beyond 1.1 μm. To cover the huge infrared range out to at least 150 μm requires different materials and techniques.

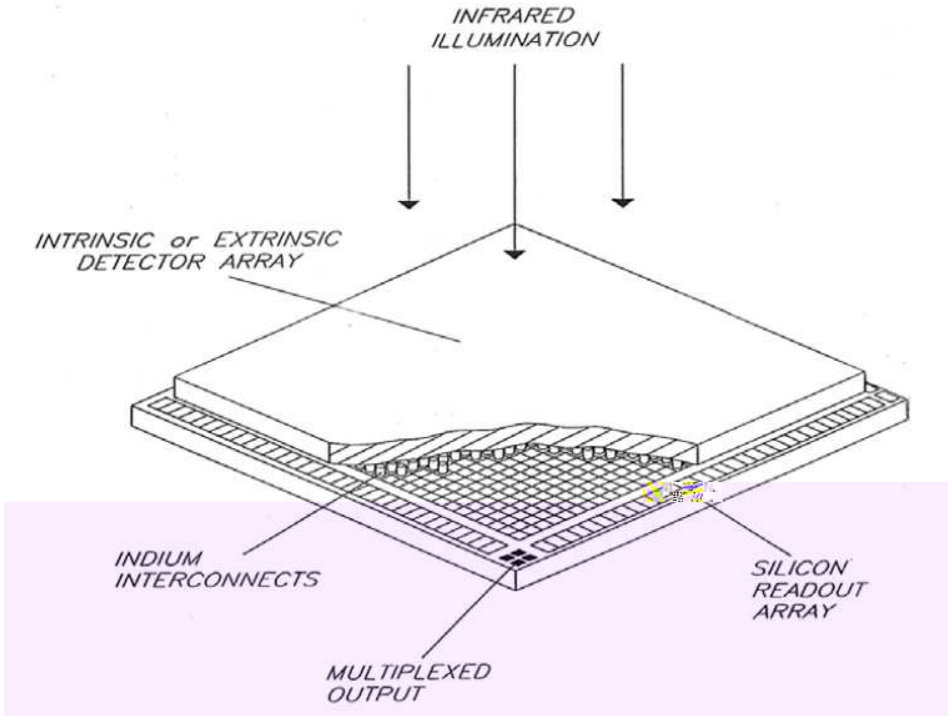

Figure 8.5: The “hybrid” structure of infrared array devices. The two slabs are separated by a grid of tiny indium bumps that remain soft at cryogenic temperatures.

Infrared observations are extremely important in astrophysics for many reasons. For example, because of the Hubble expansion of the Universe, the visible light from distant galaxies is stretched, effectively moving the observed spectrum into the infrared for the most distant objects. Equally important, infrared wavelengths are much more penetrating than visible light, and can therefore reveal the processes at work in star-forming regions which are typically enshrouded in clouds of gas and dust. Similarly, infrared observations can allow us to “see” all the way to the center of the Milky Way and reveal the nature of the central mass of the Galaxy. Cold interstellar material emits no visible light but it does emit in the far-infrared, which provides a means to study the dust itself. Finally, energy transitions in molecules that involve quantized rotation and vibration states result in the emission of low-energy infrared photons, and thus infrared spectroscopy is a powerful diagnostic tool to probe the chemistry of the interstellar medium and the coolest stars.

The near- and mid- infrared are generally considered to be the region just beyond the red limit of sensitivity of the human eye, at a wavelength of about 1μm (near-infrared) and $>3\mu$m (infrared) (up to about 200μm which we usually call the sub-mm). Development of detectors in the infrared we driven by military requirements for heat-seeking devices (which is why early detectors had the words “tank buster” written up the side – they were designed to find tank exhausts in fog!).

Infrared array detectors are not based on the charge-coupling principle of the silicon CCD. This is an important distinction with some practical implications. For example, infrared arrays do not “bleed” along columns when a pixel saturates, and bad pixels do not block off others in the same column. Also, “non-destructive” readout schemes are possible and very effective. On the other hand, on-chip charge binning and charge-shifting are not possible. Finally, infrared detectors must be kept cold ($\sim$100 K) since the thermal emission from the detector itself is close to the band-gap energy of the photons we are trying to detect.

8.4 UV techniques

The ultraviolet region spans roughly 300 nm to 10 nm, X-rays from 10 nm to 0.01 nm, and gamma rays the regime below that down to and below nuclear dimensions (one-millionth of 1 nm). Photons with wavelengths shorter than 300 nm (3,000Å) are not transmitted by the Earth's atmosphere. Consequently, observations in the ultraviolet (UV), X-ray, and gamma-ray regimes must be carried out from space. If normal optical telescopes with parabolic mirrors can be used then the diffraction-limited formula for angular resolution with a circular aperture ($\theta = 1.22\,\lambda\,/\,D$) suggests that ultraviolet observations should have better resolution than visible light images for a given telescope aperture because of the smaller wavelength. However, the surface quality of the mirror becomes very important as the wavelength is decreased. For example, to achieve the diffraction limit, the r.m.s. amplitude of the surface must be smooth to within $\sigma = \lambda\,/\,20$ (a $\lambda\,/\,20$ surface will scatter 33% of the light out of the diffraction spot). At 500 nm this corresponds to a 25 nm rms surface smoothness, but at 100 nm in the UV, it would imply a surface must be smooth to 5 nm. The Hubble Space Telescope possesses one of the smoothest primary mirrors ever polished with a surface roughness of $\sim$2–3 nm which allows it to perform into the UV. Clearly, for even smaller wavelengths, achieving diffraction-limited performance becomes challenging.

One of the major concerns for UV work is that the detector should be "solar-blind" (i.e., it should not be sensitive to visible light photons). The reason for this restriction is simply that many astronomical sources emit 10^4 to 10^8 visible photons for every UV photon in the 1,000Å to 2,000Å wavelength range, and if the detector actually has its maximum sensitivity at visible light wavelengths, then there will be an enormous background unless the visible light is heavily filtered out. Unfortunately, even blocking to reduce visible light by a factor of 10^5 still isn't sufficient in many cases and such filters also absorb UV photons. A CCD with a 20% quantum efficiency (QE) in the ultraviolet would end up with a detector quantum efficiency (DQE) of only 1–4% when it was made solar-blind; (the DQE takes into account all losses). The toughest wavelength for CCDs is around 2,500Å because the absorption depth is only about 3 nm (30Å), much less than the wavelength of the UV light, and so about 70% of the incident radiation is simply reflected!

Alternative UV CCD technologies include the use of very thin gate structures (known as "thin-poly" gates) to minimize absorption. Indeed, the considerable progress on thinning and coating of CCDs, and with readout noise around 1 e^- r.m.s, CCD detectors have become very important for UV and NUV astronomy, and several space missions include UV-sensitive CCDs.

8.5 X-ray techniques

One of the major successes of the space era for astronomy has been the ability to study X-ray emissions from cosmic sources. As a reminder, around a wavelength of 7–10 nm (or 70–100Å), the extreme ultraviolet merges into the X-ray region which extends to wavelengths as short as 0.01 nm

(0.1Å) beyond which the photons are called gamma rays. At such short wavelengths it becomes more convenient to describe the photon by its energy $\mathrm{E} = h\nu = hc\,/\,\lambda$ and convert this to electron-volts. A useful relation to remember is that the energy in keV is given by $\mathrm{E} = 1.24\,\mathrm{keV}\,/\,\lambda\,(\mathrm{nm})$. The range of energies is from 0.1 keV to 125 keV. Lower energies (<10 keV) are called "soft" X-rays and the high-energy end is the "hard" X-ray region.

Finding materials that reflect photons of these energies is difficult. Hence, X-ray telescopes are not usually built from mirrors, but rather use "grazing incidence" mirrors to gradually focus the light on to the detector. This means that X-ray telescopes tend to have very long focal lengths and a small field of view.

Exercises

1. Why must a radio telescope be much larger than optical telescopes to achieve moderate angular resolution on the sky? Calculate the angular resolution of a radio telescope of diameter 26 m (a single dish on the VLA) working at a wavelength of 6 cm.
2. Show that $h\nu/kT << 1$ for most astronomical sources [Hint: take the temperature of the stellar photo-sphere to be 5800 K]
3. What is the advantage of combining radio signals from two well separated radio telescopes observing the same source at the same time?
4. Explain the term "aperture synthesis"
5. Explain the term "atmospheric window" and "thermal background" as they apply to infrared astronomy.
6. Use the Plank function to calculate the monochromatic flux from a blackbody at a temperature of 300 K. Convert this to photons at a wavelength of 2.2 μm.
7. List three important differences between a CCD and infrared detector.

Chapter 9

Frontiers in Astronomy and Instrumentation

9.1 Beating the Atmosphere

Images of point-like astronomical sources formed on a CCD camera appear as a point spread function, and as we have seen, in the absence of other degrading effects, the spreading of the image is determined by the diffraction of light. In practice, for ground-based observatories, the light must pass through the atmosphere, which has a major impact on image quality because of turbulence.

As everyone who has looked through a telescope knows, star images are always much more blurred than the theoretical limit of the diffraction limit. Time-dependent turbulence blurs the tiny diffraction-limited image by rapid, random shifts of position resulting in a fuzzy "seeing" disk of light that can be 10 to 100 times larger in diameter depending on the site of the telescope and atmospheric conditions. Incoming waves are distorted by randomly moving cells of air with different densities (and hence refractive indices), which in turn arise from temperature variations.

Turbulence is characterized by the size of the typical atmospheric cell. It turns out that these cells, even at a very good site, are usually much less than 1 meter across (20 cm is typical), much less than the diameter of a modern large telescope, and it is thus this length that determines the size of the fuzzy image or seeing disk. Astronomers compare seeing-limited and diffraction-limited images using the Strehl ratio which is defined as the intensity at the peak of the actual seeing disk divided by the intensity at the peak of the true Airy diffraction pattern. Typically, the Strehl ratio is $\sim$0.01 in ground based observations. If this ratio could be increased to nearer unity, then most of the light would be in the central spike of the Airy diffraction pattern and the contrast against the sky background would be increased enormously. Smaller image sizes also mean that narrower slits can be used in spectrographs, which in turn implies that the whole spectrometer can be made more compact and achieve higher spectral resolution. To correct for this atmospheric blurring and achieve high resolution images is the ultimate goal of *adaptive optics*, a ground-based method of achieving space-based image quality.

What is the effect of the atmosphere on image quality? The Earth intercepts only a tiny fraction of the spherical wave emitted from a distant, point-like source such as a star. When these waves arrive at the Earth the wavefronts are essentially flat and parallel to each other. These plane wavefronts are distorted randomly by moving cells of air, each with a slightly different index of refraction. Variations in the refractive index are caused by variations in density, which in turn arise (mainly) from temperature variations in a fully developed turbulent atmosphere. In a turbulent atmosphere each patch of air with a slightly different index of refraction acts like a lens, bending the incoming rays this way or that by small amounts (as if the atmosphere is full of drifting and merging lenses). When the light finally reaches the telescope, the rays are no longer parallel and the wavefront is no longer flat (plane). Atmospheric turbulence is usually visible to the naked eye in the form of twinkling of starlight, also called scintillation.

Air turbulence occurs in several different locations:
• Heat sources within the telescope dome cause local turbulence.
• Wind patterns around the telescope dome can also cause turbulence. During night time the boundary layer can be a few hundred meters high.
• There is a strong temperature gradient up through the troposphere (the lowest portion of the atmosphere; tropos means turning or mixing) to the tropopause at $10-12$ km where the stratosphere begins, and therefore wind shear will cause mixing and index of refraction fluctuations.

Image quality is directly related to the statistics of the random perturbations on the incoming wavefront. To make any predictions about the turbulence requires a statistical model of the mixing, and this is called "Kolmogorov theory of turbulence". Briefly, the fluctuations are described statistically in terms of a structure function $D_n(r)$ which describes the variation in refractive index (n) between two points on the wavefront separated by a distance $r = (r_1 - r_2)$

$$D_n(r) = < |n(r_1) - n(r_2)|^2 > = C_n^2 r^{2/3} \tag{9.1}$$

where the symbol $<>$ indicate the average value of the square of the difference in the refractive indices between the two points, and it turns out that this average is proportional to the separation r to the two-thirds power. The factor C_n^2 is a measure of the strength of turbulence and its value can range from 10^{-14} to $10^{-15}\,\mathrm{m}^{-2/3}$ near the ground to about $10^{-18}\,\mathrm{m}^{-2/3}$ above 10 km. C_n^2 is not a constant, but varies with site and conditions, especially the local conditions right above the ground. It is usually written as $C_n^2(h)$ to show that it is a function of height, h.

The size of the turbulent cells are characterized by a length scale, known as the Fried parameter (r_0) which is the length over which the wavefront is not significantly perturbed. The Fried parameter is a critically important quantity, and is effectively the aperture which has the same "resolution" (as defined by Fried) as a diffraction-limited aperture in the absence of turbulence. The Fried parameter is given by

$$r(\lambda) = \left(\frac{\lambda}{\lambda_0}\right)^{6/5} r_0 \tag{9.2}$$

. Note the wavelength dependence. Typically, $r_0 \sim 20$ cm at $\lambda = 5000$Å (in the visible part of the spectrum) on Mauna Kea. Thus, without any form of correction, the Keck 10 m telescope will have a resolution resolution which be no better than a 20 cm (8-inch) telescope. This is a serious problem!

9.2 Adaptive Optics

Is there a more general way to undo the effects of r_0 and get the angular resolution in long exposures to approach $\lambda\,/\,\mathrm{D}$? The first step is to compensate for the random wandering of the center (or centroid) of the seeing disk by means of a simple tip-and-tilt motion of a mirror, which could even be the secondary mirror of the telescope, and in this way redirect the overall wavefront to the same (fixed) place in the image.

The second step is more complex. An optical arrangement is used to project a real, de-magnified image of the primary mirror of the telescope onto a much smaller mirror whose detailed "shape" can be changed or "deformed" by the forces applied from numerous small actuators behind it. To do this, a high-speed CCD camera acting as a wavefront sensor detects the changes in the slope of the coherent areas of turbulence, also referred to as sub-apertures. By analyzing the wavefront distortions very rapidly, it is possible to control the shape of the deformable mirror in such a way as to compensate for the wavefront distortions (i.e. by applying an equal, but opposite "shape" of the atmosphere). The basic layout is illustrated in Fig. 9.1 which shows the disturbed wavefront entering the telescope aperture and being re-imaged onto the deformable mirror.

The correction on the deformable mirror must be made quickly. The rate of change of the turbulence occurs on time-scale according to

$$\tau_0 \;=\; 0.314\,\frac{r_0}{v} \tag{9.3}$$

where v is the average velocity of the turbulence (which is a function of height, but averages to about $10\,\mathrm{m\,s^{-1}}$). τ_0 is known as the "coherence time". Typically τ_0 is only a few milliseconds. Thus, to achieve a "good" correction first requires a sufficiently bright star to be used to measure the "shape" of the atmosphere every few milli-seconds in order that the correct shape can be applied to the deformable mirror. Typical the limiting magnitude of natural guide stars is $R = 14$. Stars fainter than this provide poor correction since the time needed to obtain a high signal-to-noise measurement of the "shape" of the atmosphere becomes longer than the coherence time. Second, the effectiveness of the compensation decreases as the angular distance from the guide star increases, simply because the wavefront distortions are not the same. Perfect compensation is limited to an angular patch of radius a few arcseconds around the guide star. This region is known as the isoplanatic region. The isoplanatic angle is defined as the radius of a circle over which the wavefront disturbance is essentially identical and is given approximately by

$$\theta_0 \;=\; 0.314\,\frac{r_0}{H} \tag{9.4}$$

where H is the average distance to the seeing layer. A typical isoplanatic patch has a radius of $<25''$.

Assuming that the isoplanatic patch is about 20 arcseconds, then there are 10^9 patches in the sky, but only 150,000 suitable guide stars. In general, no natural guide star will be close enough to a particular target. Thus natural guide stars have poor sky coverage. However, recently, it has become possible to create a "fake" star using pulsed lasers.

A pulsed laser beam tuned to the wavelength of the orange-colored sodium D resonance line (589.0 nm and 589.6 nm) is projected through a telescope and focused on the so-called sodium (Na) layer

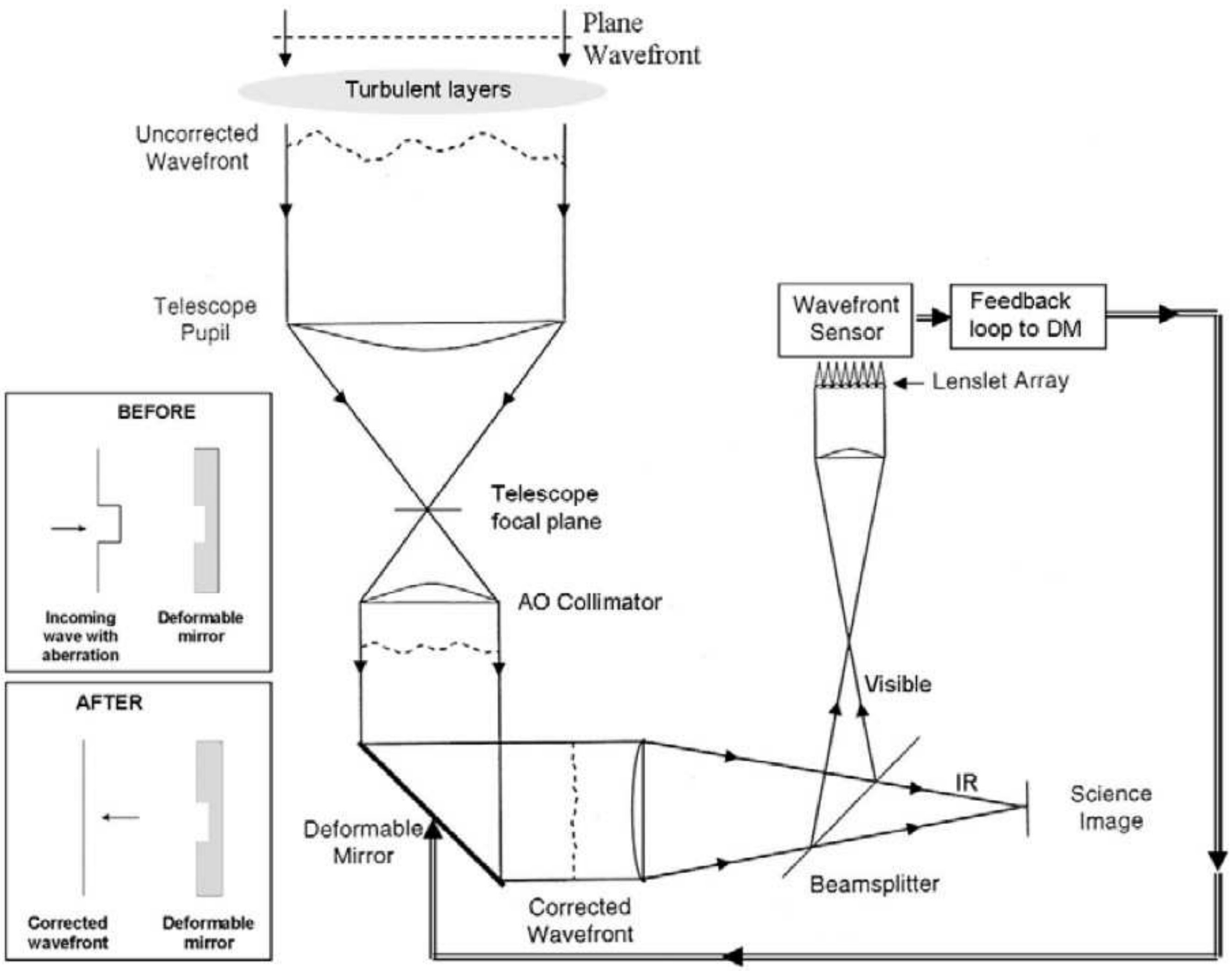

Figure 9.1: The basic layout of an AO system. Plane waves arriving at the Earth from a distant star become distorted by randomly moving cells of air with different indices of refraction. These distortions are sensed at high speed and corrections are sent to change the shape of a deformable mirror which corrects the wavefront at reflection.

in the upper atmosphere at an altitude of $\sim 90\,\text{km}$. Resonance fluorescence in this layer produces a glowing, artificial star. Outgoing laser pulses return from the Na layer in only 700 ms, retracing their path exactly, enabling the adaptive optics system to update the shape of the deformable mirror.

9.3 Multi-Object and Integral Field Spectroscopy

9.3.1 Multi-Object Spectroscopy

Spectroscopic measurements can be painfully slow, even with very sensitive CCD detectors. Typically, the signal at any point in the spectrum of an astronomical source will be at least several hundred times fainter than the signal in a direct image. Thus, the possibility of recording spectra from several objects at once is very attractive.

Slit-less spectroscopy and objective prism spectroscopy are two similar ways to observe the spectra of many objects at the same time. Removing the slit is okay for point sources, the spectral resolution then being determined by the seeing disk rather than the slit width, but this method only works at wavelengths where the sky background is very dark and the field not too dense to avoid overlapping spectra.

Another way to solve the problem is the multi-object spectrograph which employs an entrance slit composed of multiple sub-sections which can be positioned by computer to pick up many different objects in the field of view. Today, many observatories have a spectrograph with a "multi-slit" system. Slits are cut by lasers onto a mask (a think sheet of metal) at different angles and in different locations on the mask and therefore the spectra do not line up perfectly on the detector, increasing the complexity of the data reduction. Slit-mask technology has also been extended to the near-infrared part of the spectrum where the challenge is increased because the mask must be cold ($<150\,\text{K}$) to prevent the infrared detectors from seeing its thermal glow. Several different technologies are possible including exchangeable slit-masks that are loaded through a vacuum-cryogenic airlock system, or movable opposing slit bars that quantize the y axis but allow any slit location in the x-direction, or micro-shutters.

Of even greater impact on multi-object spectroscopy has been the technology of transparent optical fibers. Developed mainly for the telecommunications industry, these slim, flexible glass conduits, which resemble normal electrical cables on the outside, can be used as "light-pipes" to transmit light over very long distances with only slight losses or attenuation. The basic idea is to position one end of each of several optical fibers in the focal plane of the telescope at points corresponding to interesting objects, such as a cluster of distant galaxies, and to stack up the other ends along the entrance slit of the spectrograph; the spectrum of each source is therefore recorded simultaneously.

9.3.2 Integral Field Spectroscopy

Even the multi-object spectrograph suffers from the fact that in order to record the dispersed spectrum of any object in the field of view of the telescope the remainder of the field, which is most of

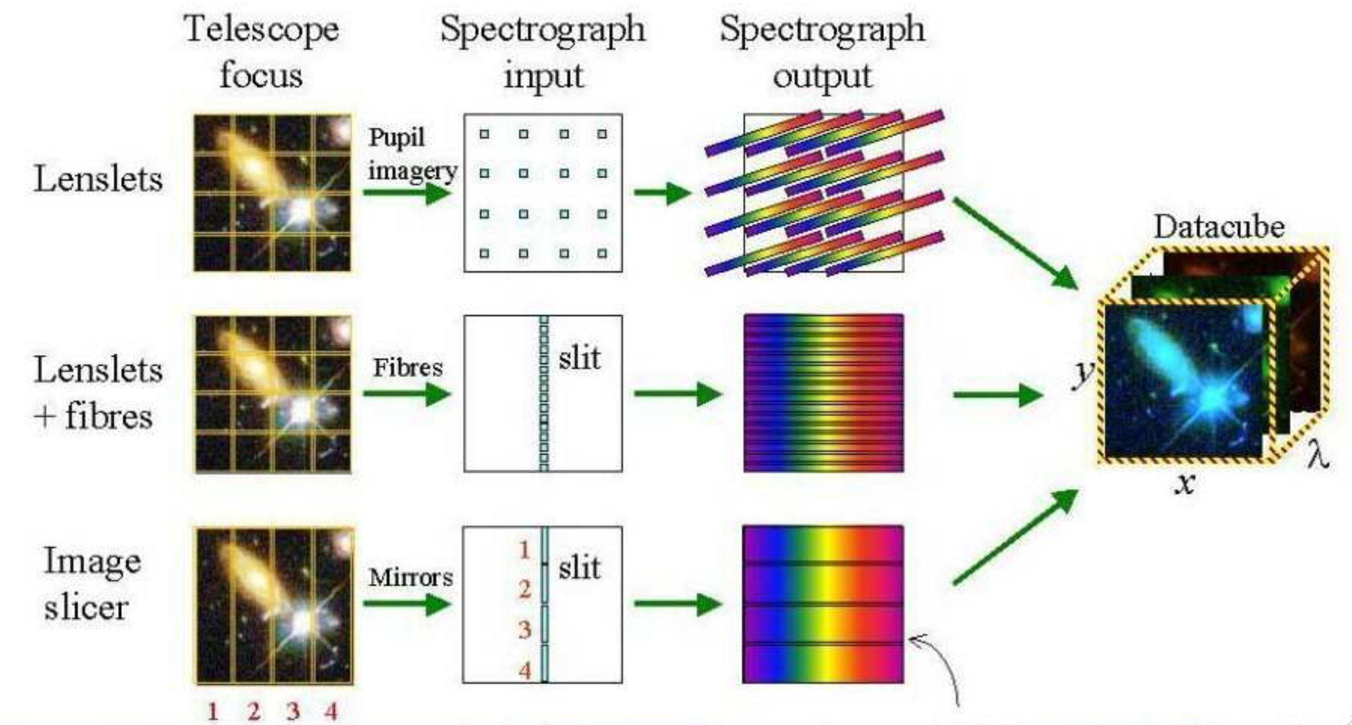

Figure 9.2: Three examples of methods used to construct three-dimensional datacubes. Lenslet array: The input image is split up by a microlens array. Light from each element of your observed object is then concentrated into a small dot and dispersed by the spectrograph. Fibres (with or without lenslets): this is currently the most common technique in use. The input image is formed at the entrance to a two-dimensional bundle of optical fibres which transfer the light to the slit of the spectrograph. The flexibility of the fibres allows the round/rectangular field-of-view to be reformatted into one (or more) "slits", from where the light is directed to the spectrograph, and the spectra are obtained without wavelength shifts between them. Image-slicer: The input image is formed on a mirror that is segmented in thin horizontal sections, sending each 'slice' in slightly different directions. A second segmented mirror is arranged to reformat the slices so that, instead of being above each other they are now laid out end to end to form the slit of the spectrograph.

it, must be hidden from the spectrometer by the slit-mask.

One way to implement an integral field mode (in which a continuous area of sky is mapped in x, y, λ) is to employ an image slicer, or bundle of fibers. The slicer technique uses a complex mirror with many tilted facets to subdivide the image in the focal plane of the telescope into narrow strips and then another similar mirror to "stack" these parts one beside the other along the length of a spectrograph slit (Fig. 9.2). Thus, every region of the image produces a spectrum. With new technology and large are CCDs, the field of view that can be covered can be up to 1×1-arcminute (e.g. the MUSE IFU on the ESO VLT).

9.4 Future Developments

There are several overarching topics that are typically repeated in decadal reviews that exploit these technologies. This list of key topics includes the following:

• the discovery of nearby Earth-like worlds, the statistics of planetary systems, and the evidence for biological activity elsewhere
• the detection and tracking of near-Earth asteroids
• a deeper understanding of the origin and formation of stars and planetary systems
• the Black Hole at the center of the Milky Way and tests of General Relativity
• the origin and evolution of the super-massive black holes in quasars
• the origin of cosmic gamma-ray bursts
• the nature and distribution of dark matter in the cosmos
• the nature and distribution of dark energy in the cosmos
• detection of the first starlight in the early Universe
• formation of the first galaxies.

Stars and Galaxies

Observational Techniques Homework Set 1

1) A star with an apparent magnitude of 10.0 undergoes an outburst and triples in luminosity for a short period of time. What is the apparent magnitude of the star during this outburst? [1 mark]

2) Name two commonly used telescope foci and state and advantage of each focus position. [2 marks]

3) Calculate the focal ratio of a 8 meter telescope which has a plate scale of 2.34 arcsec mm^{-1} [2 marks]

4) A 4-m reflecting telescope has f-ratio $f/3$ at prime focus. What is the maximum field-of-view in arcseconds that could be achieved by a detector that is 1000 pixels across and where each pixel is square of side 15μm? [2 marks]

5) *(i)* Calculate the resolution limit of a 8 meter telescope in the V-band (with a central wavelength of 550 nm) in the diffraction limited case. Give your answer in milli-arcseconds. [1 mark]

(ii) A V-band image of a star is recorded using this telescope fitting with a detector with pixels which are $20\,\mu\mathrm{m} \times 20\,\mu\mathrm{m}$ in size. The effective focal length of the telescope is 500 m. Calculate approximately how many pixels the image of the star will cover. [2 marks]

Stars and Galaxies

Observational Techniques Homework Set 2

1) Assume that the signal level in a CCD exposure is high enough such that the dark current and readout noise are negligable, so that the signal and background are the only sources of noise. The following table shows the output from a 5×5 pixel segment of the CCD, which has a gain of 1.

1030	970	986	994	992
983	952	1021	992	1030
941	1048	1100	992	986
961	994	1051	1005	1037
1061	1028	1020	922	973

(i) Calculate the probability that the reading from the central pixel is due to the presence of a star, and not just random occurence. [2 marks]

(ii) Imagine that the CCD had 1000 pixels with a mean of 999. How many pixels would you expect to have a value greater than 1100? [1 marks]

2) *(i)* Write down the equation relating the signal-to-noise (S / N) to the number of photons per second from the object and sky, exposure time, read-noise, dark current. [1 mark]

(ii) Describe what is meant if an observation is: **(a)** sky noise limited, **(b)** read-noise limited, **(c)** dark-noise limited. [1 mark]

(iii) You observe a 15^{th} magnitude star for 600 seconds with a 1-meter telescope on a night when the moon is 1/4 full, and hence there is some scattered light. The count rate from the star is 10 photons / second on a CCD with a gain of 1 photo-electron per ADU. Calculate the S / N in the sky-noise limited case if the sky background has a count rate of 20 photons / second [1 mark]

3) Becuase the stars and galaxies course is so exciting, you decide to persue a career in astronomical research. For your PhD project project, you want to search for extra-solar planets around nearby stars like the Sun, (a G-type star). You hope that one day, you will take a spectrum of one of these planets and detect the signs of life, which will win you the Nobel prize.

You decide to use the VIMOS camera on the ESO Very Large Telescope and continually monitor the magnitude of the star over a period of 8 hours. Your goal is to search for and detect the small change in the apparent brightness as a planet transits in front of the star.

However, you have a catalog of 100,000 G-type stars with a range of observed magnitudes to choose from, and need to narrow the catalog down. Assuming you need 1% accurary in the photometry to detect the planet, what is the limiting magnitude of the star that you can choose? [4 marks]

[Hint: You will need to use the VIMOS expousre time calculator to answer this question:
`https://www.eso.org/observing/etc/`
and select "VIMOS"). You will also need to decide on your observing conditions, and how often you need an new exposure for these observations].

Stars and Galaxies
Observational Techniques Homework Set 3

1) What are the four key components of a spectrograph? Draw a sketch to show how these are put together to make a spectrometer. [1 marks]

2) Write down the reflection grating equation of a spectrograph. Calculate the difference in angle for light reflected from a grating for two wavelengths of 550nm and 560nm. Assume the grating is used in the first order, the angle of incidence is 45°, and the ruling density is 1000 lines / mm. [2 marks]

3) The resolution of a spectrograph, R, is given by:

$$R = \frac{n\rho\lambda W}{\chi D_{\rm T}},$$

where n is the diffraction order, ρ is the ruling density of the grating, λ is the wavelength, W is the size of the grating, χ is the angular size of the target star and $D_{\rm T}$ is the size of the telescope.

Which three parameters can the designer change to improve this resolution for that particular telescope? [3 marks]

Assuming diffraction limited performance of both telescopes, how will the resolution of a particular spectrograph change if it is moved from a 4 m to 8 m telescope? [1 mark]

4) When attending a conference on *"The first galaxies that formed in the Universe"*, someone shows you a *Hubble Space Telescope* J-band image (which is centered at $1.2\,\mu$m) of what they believe to be the most distant galaxy ever discovered. They claim that this galaxy is likely to be at redshift, $z = 10$, which would mean it formed just 500 Myr after the Big Bang.

To confirm this discovery, you decide to take a spectrum of this object using the GNIRS spectrograph on the Gemini-North Telescope. You want to search for the redshifted Lyα emission (the emission from the Hydrogen atom as electrons cascade from the $n = 2$ to $n = 1$ energy level). You estimate that the galaxy must be forming stars at rate of about 1 solar mass per year, which means that the cooling gas within the inter-stellar medium would produce a total Lyα line luminosity of $\rm L_{Ly\alpha} = 7 \times 10^{35}$ W.

You aim for a signal-to-noise rate of SNR = 10 in the emission to be sure of a detection. How much time do you need to ask for on this 8-meter telescope? [5 marks]

Some information you might need:

The rest-frame wavelength of Lyα is $\lambda = 0.1215\,\mu$m.

The distance to a galaxy at $z = 10$ is 3.2×10^{27} m.

The line width (caused by random gravitational motions of the gas and stars within the inter-stellar medium) of a galaxy at $z = 10$ is expected to be approximately $250\,{\rm km\,s^{-1}}$.

Assume all of the emission from the galaxy in the broad-band image is from the emission line (i.e. assume there is no continuum emission).

The GNIRS exposure time calculator can be found at: `http://www.gemini.edu/node/10546`

Stars and Galaxies
Observational Techniques Homework Set 4

1) a) Explain why X-ray telescopes have very long focal lengths [1 mark]

b) One of the instruments onboard the NuStar X-ray Observatory has a detector with 32×32 pixels and a field of view of 400×400 arcseconds. If each pixel is $625 \times 625\,\mu$m in size, calculate the plate scale (in arcsec / mm) and hence the focal length of the telescope. [2 marks]

2) The star, Vega, is observed with a stellar interferometer (operating at 700 nm), and it is found that the fringes disappear when the telescopes are 55 meters apart. The measured flux on Earth from Vega is $3.92 \times 10^{-8}\,\mathrm{W}/\mathrm{m}^2$ and it displays a parallax of 130 mill-arcseconds.

a) What is the stars angular size on the sky (in milli-arcseconds)? [1 mark]

b) What is its distance (in parsecs)? [1 mark]

c) What are Vega's diameter, luminosity and effective temperature? Where does Vega lie on a Hertzsprung-Russel diagram, and hence what type of star is it? [2 marks]

The stephan-boltman constant is $\sigma = 5.669 \times 10^{-8}\,\mathrm{W\,m^2\,K^4}$.

3) Researchers are often asked to peer review proposals. Below are three (shortened) science cases that were submitted to a peer review panel for observing time on an 8-meter telescope. You have two tasks:

a) Rank them scientifically. If you could only give one of these observing time, which one would you suggest is carried out? [state your reasons] [1 mark]

b) Each of these proposals request one full night of observing time. You have three nights to give out, but the weather forcast is such that:

– night one will be excellent conditions (no cloud, very stable atmosphere);

– night two will be average conditions (some thin cloud at times, some low-level wind and so less stable atmosphere)

– night three will be patchy cloud and poor seeing (lots of turbulent atmosphere).

Which proposal should be given which night to maximise scientific return? [3 marks]

Proposal 1: How do massive stars form?

Massive stars ($M > 10\,M_\odot$) dominate the cycle of star-formation in their host galaxies, from the large amounts of ionizing radiation they emit, to the kinetic energy they pump back into the interstellar medium through their winds and supernova explosions. However, it is still unknown how such stars form. Massive stars are intrinsically rare, and much of the formation process happens on very short timescales ($<10^5$ years) while the proto-star is still heavily embedded in its dust cloud and hence difficult to observe. In addition, classical theory suggests that radiation pressure from the protostar inhibits further accretion once the stellar mass reaches $10\,M_\odot$.

One option for their formation is that massive stars may form through a mechanism similar to that of lower mass stars, whereby matter is accreted from a circumstellar disk, with the outward radiative force preferentially escaping via the poles. Some numerical simulations have succeeded in creating stars with masses $>10\,M_\odot$ this way (e.g. Yorke & Sonnhalter 2002, ApJ 569, 846; Krumholz et al. 2009, Science, 323, 754). The observational evidence to support this scenario however is limited. Large-scale bi-polar outflows have been observed to originate from heavily embedded objects (Beuther et al. 2002, A&A 383, 892); and anecdotal studies have found evidence of accretion disks in individual massive protostars (Patel et al. 2005, Nature 437, 109; Beltran et al. 2006, Nature 443, 427).

To measure the morphology of a younng, massive stars in formation we propose adaptive optics assisted observations with the NIFS integral field spectrograph on the 8-meter Gemini Telescope. These observations aim to spatially resolve the proto-star and its immediate environment on small ($<0.05''$) scales. These observations are designed to search for a rotationally-flattened 'torus', search for a bipolar winds and test whether they are aligned perpendicular to the disks. We have selected 10 stars that have luminosities ranging from (1-6) $\times 10^4\,L_\odot$, and hence cover a predicted mass range of ~ 10–$20\,M_\odot$. This program will be the first step creating a firm statistical footing for the accretion-disk-plus-bipolar-jet formation scenario for massive stars over a range of stellar masses.

Proposal 2: The formation of Lenticular Galaxies

Lenticular, or S0, galaxies make up $\sim 25\%$ of the galaxy population in the local Universe (e.g. Dressler, 1980, ApJ, 236, 351), so understanding how they form must constitute a significant element of any explanation of galaxy evolution. Their location at the crossroads between ellipticals and spirals in Hubble's tuning-fork diagram underlines their importance in attempts to develop a unified understanding of galaxy evolution, but also means that it is not even clear to which of these classes of galaxy they are more closely related. One option is that S0's represent a transitional phase between spiral- and elliptical- galaxies If the transformation simply involves a spiral galaxy losing its gas content and fading into an S0, then clearly S0s and spirals are closely related. However, it is also possible that mergers can cause such a transformation: while equal-mass mergers between spirals create elliptical galaxies, more minor mergers can heat the original disk of a spiral and trigger a brief burst of star formation, using up the residual gas and leaving an S0. In such a merger scenario, the mechanism for creating an S0 is much more closely related to that for the formation of ellipticals.

Clues to which mechanism is responsible can be found in the "archaeological record" that can be extracted from spectral observations of nearby S0 galaxies. For example, the present-day stellar dynamics should reflect the system's origins, with the gentle gas stripping of a spiral resulting in stellar dynamics very similar to the progenitor spiral, while the merger process will heat the stars, resulting in kinematics more dominated by random motions, akin to an elliptical. In addition, the absorption line strengths can be interpreted through stellar population synthesis to learn about the metallicity and star formation histories of these systems. These dynamical and stellar properties can be compared to see if a consistent picture can be constructed for the formation of each system.

Here, we propose to measure the spectral properties of 20 S0 galaxies in the Virgo cluster. We will definitively tie down how closely S0 galaxies are linked evolutionarily to spirals, and to ascertain the degree to which this relationship depends on mass.

Proposal 3: The proper motion of a magnetar

Magnetars are young neutron stars, with extremely strong magnetic fields ($B>10^{14}$ Gauss), whose extotic properties offer a window in to the extreme physics underying the final stages of stellar evolution. They are identified by periods of prolific bursting activity in the X-rays, with bursts lasting 10–100 milliseconds. Timing studies of their X-ray persistent emission have revealed coherent periodic modulations, interpreted as being due to the neutron star rotation. These periods range between 2–12 seconds, much slower than in the majority of pulsars with comparable ages.

Here we propose observations to constrain the properties of SGR 0501+4516 – one of the nearest magnetars by direct measurement of its proper motion. The location of SGR 0501+4516 is close to a supernovae remnant, and it's age of ∼8000–20 000 years, and offset from the centre of this remnant (∼80 arcmin) suggests a proper motion in the range of 0.2–0.4″ per year if the two are related (i.e. was the magnetar the progenitor star that was ejected when the supernovae occured?). We already have two epochs of imaging, obtained ~ 7 months apart, which provide evidence of proper motion at the 2σ level, with a direction consistent with the centre of the supernovae remnant. Here, we propose a further epoch of imaging which will have a baseline of 18 months to cleanly confirm or reject these results, and provide the first proper motion measurement for a magnetar.

Finally, a proper motion measure would also constrain the neutron star tangential kick velocity, which in itself depends crucially on the processes underlying core collapse. The measured offset from this star already suggests that this may be very high (>1100 km/s).

Stars and Galaxies

Observational Techniques Workshop 1

1) What is the sidereal day? Why is one sidereal day not equal to 24 hours?

2) Astronomers often use a rule of thumb that a change in brightness of 1% correspomds to a change of 0.01 magnitudes. Justify this.

3) The most distant galaxy that has ever been detected is approximately 30^{th} magnitude. How much fainter (in linear scale) is this than you can see with the naked eye (assume you are on a dark site and your eyes are well adapted to the dark).

4) The V-band magnitude of two stars are both observed to be 7.5, but their blue magnitudes are $B_1 = 7.2$ and $B_2 = 8.7$.
(i) What is the colour index, $(B - V)$, of each star?
(iii) If these two stars are in a binary system at a distance of 100 pc, what are their absolute magnitudes?

5) A certain globular cluster is comprised of 10^4 stars. When observed at very high resolution, it is found that 100 of the stars have apparent magnitude, $m_v = +1.0$. The rest have $m_v = +6.0$. However, when observed from Durham, due to bad seeing, this star cluster appears unresolved. What will the total apparent magnitude of the cluster be when observed from Durham?

6) List the four main atmospheric factors that dominate the choice of a telescope site.

7) A telescope with a 0.5 m primary mirror has a focal length of 15 meters and a CCD camera which is 3 cm across and comprises 1000×1000 pixels.
(a) Calculate the plate scale in arcseconds / pixel and the field of view of the instrument.
(b) If this telescope is observing at a wavelength of 600 nm, calculate approximately how many pixels will the image of the star cover if:
(i) if the telescope is placed in space (i.e. observing at the diffraction limit)
(ii) if the telescope is observing from the ground, where atmospheric turbulence results in seeing of $1''$

8) What size will the Moon appear, in mm, on a detector placed in the focal plane of a telescope with an aperture of 1.2 m and a focal ratio of $f/2.5$? Assume that the Moon has an angular diameter of 0.5 degrees.

9) (a) State three advantages of modern CCD detector technology over photographic films.

(b) A 4 m telescope has been equipped with a CCD camera which is read out using a 16-bit controller with a gain of 1 and a bias of 200 ADU. The CCD is used to perform a set of observations designed to link the positions of bright ($V = 8$–10) stars to those of much fainter stars ($V > 15$). To achieve this the CCD must detect the faintest stars without saturating the brightest stars.

(i) What is the maximum number of photo-electrons that can be registered before the CCD is saturated?

(ii) If the zero point of the system is 16.5 magnitudes, what is the longest exposure that can be made where a $V = 8$ magnitude star will be unsaturated? You may assume that all the counts fall on one pixel.

Stars and Galaxies

Observational Techniques Workshop 2

Warm-up Question 1

1) A star with V-band magnitude of $V = 23$ mag is measured with a CCD mounted on a 4-m telescope to give S / N = 10 in a certain exposure time in 1 arcsec FWHM seeing. Assume throughout that all observations are sky noise limited.

(a) What would the S/N have been for a $V = 25.5$ mag star under the same conditions?

(b) If the sky brightness had doubled due to moonlight, what S / N would result?

(c) If the seeing had been 0.5 arcsec FWHM what S / N would have been measured?

(d) What S / N for the star would have been measured by the same CCD mounted on an 8-m telescope in the same conditions and with the same exposure time?

(e) If the 4-m and 8-m telescopes were observing from above the atmosphere, by what factor would the 4-m telescope's exposure have to be increased relative to the 8-m exposure to ensure the same S / N?

Long Question #1 from June 2010

2) Determine the diffraction limit performance in arcseconds for a 1 meter diameter telescope operating at 20μm. [3 marks]

What is the diffraction limit for a 10 m telescope operating at the same wavelength? [1 mark]

The two telescopes are equipped with imaging cameras at 20μm with a square field of field of 20 arcminutes across.

For the two telescopes, determine the exposure time required to detect a source at a signal to noise ratio of 5 that has a magnitude of 24.0 at 20μm, assuming that the sky background is 20.0 mag per square arcsecond, the seeing is 0.5 arcseconds, the zeropoint for the 1m telescope is 15.5 mag and for the 10 m is 20.5 mag. [Hint: ensure that you take the diffraction limit of the images into account when determining the background contribution. Also the dark current and read noise contributions to the background are negligible.] [7 marks]

Long Question #2 from June 2012

Write down the major sources of statistical error in a CCD measurement of a star magnitude. [4 marks]

Which errors are usually governed by Poisson statistics and which are governed by Gaussian statistics? [2 marks]

For Poisson noise, if a process produces an average of N counts what will be the standard deviation? [1 mark]

Write down the general equation for the signal-to-noise (S/N) of a star's flux, f, measured using a CCD detector in a set exposure time, t. Include all the above four sources of noise. [2 marks]

ATLAS is a new CCD imaging survey aiming to reach the same depth as the previous SDSS survey. Both survey telescopes and instruments are identical except that the ATLAS CCD pixels are two times smaller in each of their linear dimensions. Relative to SDSS, what ATLAS exposure time will be required to reach the same S/N as SDSS in a passband where the observations are purely read-out noise limited? Assume that the star flux is completely contained in just one pixel for SDSS and 4 pixels for ATLAS. [4 marks]

In another passband, the sky noise is more significant relative to the readout noise. The readout noise is ± 2.5 ADU for both SDSS and ATLAS CCDs and the sky background is 50 ADU in an SDSS pixel in a set exposure time. If the star flux is now assumed small compared to the sky background, how much fainter a star magnitude can be observed to the same S/N limit in the same exposure time? Assume the same observing conditions for ATLAS and SDSS, a gain of 2 electrons/ADU and zero dark current for both ATLAS and SDSS CCDs and that the star flux is completely contained in one SDSS pixel and 4 ATLAS pixels. [6 marks]

Stars and Galaxies

Observational Techniques Workshop 3

1) What are the four main components of a spectrograph?

2) State the resolving power, R, in terms of wavelength. An astronomical slit spectrograph has a resolving power of $R = 7000$. What is the smallest wavelength difference that can be discerned at a wavelength of 700 nm?

3) The spectrum of a nearby star peaks at a wavelength of 3000 Å. It has a parallax of 379 milli-arcseconds and a V-band magnitude of $V = -1.5$. Sketch a HR diagram and place the star on it. What type of star is it? [The Sun has a V-band absolute magnitude of 4.8]

4) Write down the reflection grating equation of a spectrograph, and define the symbols. The sodium D line is a doublet of wavelengths 589.0 nm and 589.6 nm. A grating produces a first order spectrum where these two lines are diffracted at angles of 42.69° and 42.78° (with respect to the grating normal). Calculate the ruling density of the grating (in units of lines mm^{-1}).

5) The two Keck telescopes are each 10 m across and separated by 85 meters. What is the maximum distance from Earth that an extra-solar planet will be resolved if observed with *(i)* a single telescope; *(ii)* both telescopes when used as an interferometer? Assume the extra-solar planet is located 1 AU from the star ($1\,\text{AU} = 1.50 \times 10^{11}$ m).

6) (i) Write down two properties of a star or galaxy that can be measured with a spectrograph.

(ii) The resolving power, R, of a slit spectrograph that employs a diffraction grating is given by $R = m\rho\lambda W/(\chi D_T)$. Define all the terms in this expression for R.

(iii) For a spectrograph equipped with a reflection grating of length 10.4 cm with input angle $\alpha = 20°$ and output angle $\beta = 15°$, find the spectrograph's resolving power if used on an 8-m telescope with a slit width of 0.5 arcsec.

(iv) Calculate the spectral resolution at a wavelength of 500 nm if the telescope is put in space.

7) [from 2013 June Exam]

An astronomical slit spectrograph has a resolving power of $R = 7000$. What is the smallest wavelength difference that can be discerned at a wavelength of 700 nm? [3 marks]

The same spectrograph is used to observe an active galaxy nuclear emission line with flux 1×10^{-20} W m^{-2}. What is the total power that reaches the spectrograph detector when it is used on a telescope with a 6-m diameter mirror? You may assume that the telescope and spectrograph combination have 100% quantum efficiency. [4 marks]

If the emission line is just contained within a single pixel on the spectrograph CCD detector, what is the signal-noise ratio (S/N) of the measurement of the emission line strength in a $t = 1000$ s exposure if the sky flux is 10 times higher than the total emission line flux? You may assume that the dark noise is negligible and that the read-out noise is ± 5 electrons per pixel. [6 marks]

Is the S/N of the observation approximately sky-noise or read-out noise limited? [2 marks]

If the slit width doubles what is the new resolving power, R, of the spectrograph? [2 marks]

What is the new S/N of the measured emission line strength? [3 marks]

Stars and Galaxies
Observational Techniques Workshop 4

1) Can the Human eye detect a single photon?

2) The Sun has an apparent visual magnitude of -26.75. *(i)* Calculate its absolute visual magnitude; *(ii)* Calculate its magnitude at the distance of Alpha Centauri (1.3 pc); *(iii)* The Palomar Sky Survey is complete to magnitudes as faint as $V = 19$. How far away, in parsecs, would a star identical to the Sun have to be in order to be just detected on this data?

3) A nearby star is 5^{th} magnitude, but undergoes an outburst where it (instantly) increases in brightness by a factor of 20. What is its peak magnitude? It then fades by 0.3 magnitudes per day. How bright it is after 2 days?

4) Give two reasons why most modern telescopes are reflectors rather than refractors.

5) Name three types of telescope focus and give an advantage of each.

6) You are trying to detect the doppler shift of a star caused by an Earth-like planet. This requires a very stable spectrohraph and extremely high spectral resolution. At which telescope focus would you recommend putting the instrument, and why?

7) Write down four reasons why optical and infrared astronomical observatories are often located at remote, high altitude sites.

8) The Cassegrain focus of the 4.2m William Hershel Telescope on La Palma has a focal ratio of f/11. What is the field of view of the telescope if the detector size is 5×5 cm? What is the plate scale in arcsecs/pixel if the detector has 4096×4096 pixels?

9) Calculate the focal ratio of a 8.00 m telescope which has a plate scale of 2.34 arcsec mm^{-1}.

10) For a 2.5 m telescope with an f-ratio of f/10, what is the effective focal length? Determine the plate scale in arcseconds per millimetre for this telescope?

11) What type of observations is a Schmidt telescope used for? How are its optics different from a conventional telescope?

12) Show that the aparant magnitude of an object, m_{corr} is related to the observed magnitude, m_{obs} via $m_{\text{corr}} = m_{\text{obs}} - A_\lambda(z = 0)\sec(z)$ where z is the zenith distance, $\mathrm{A}_\lambda(z = 0)$ is the absorption coefficient at wavelength λ and at a zenith distance $z = 0$ the airmass is unity.

13) Give four ways that noise in an astronomical CCD image can be minimised.

14) Explain how the dark current, sky background and variations in the sensitivity of a CCD from pixel to pixel (flat fielding) are measured and corrected for in an astronomical image.

15) A 10^{th} magnitude calibration star gives a count rate of 100 photons/second (after corrections for the various noise sources). What is the zero-point magnitude?

16) Using the zero point magnitude and count rate from Q.14, find the ratio of the signal-to-noise ratios for these two stars for the following cases: (a) photon noise limited and (b) sky background limited.

17) Briefly describe the functions of the entrance slit and collimator in a simple transmission grating spectrograph.

18) The condition for constructive interference is $n\lambda = d\sin\theta$. Define the terms in this expression. Derive an expression for the reciprocal linear dispersion, $d\lambda\,/\,dx$.

19) A spectrograph mounted on a 8-meter telescope uses a reflection grating with 600 lines / mm. Due to mechanical constraints, the mimimum angle between the camera and the collimator is 30° (i.e. the sum of the incident and reflected angles is 30°). If a first order spectrum is observed, what is the maximum wavelength that can be observed?

20) What is the absolute magnitude of a star measured to have apparent magnitude, $m = 9.5$, and an annual parallax of $p = 0.5$ arcsec?

21) Why must a radio telescope be much larger than optical telescopes to achieve moderate angular resolution on the sky? Calculate the angular resolution of a radio telescope of diameter 26 m (a single dish on the VLA) working at a wavelength of 6 cm.

22) What is the advantage of combining radio signals from two well separated radio telescopes observing the same source at the same time?

23) Explain the term "aperture synthesis"

24) Why do X-ray telescopes have very long focal lengths? Why do the images of galaxies appear different in X-rays compared to those at UV, optical, infrared and radio wavelengths?

25) In a telescope system, how is radiation detected in the sub-mm regime? For blackbody radiation, what temperature are observations at 500μm sensitive to?

26) Define the terms coherence length, coherent time and isoplanatic patch as they relate to the atmosphere.

27) Discuss two ways that observations can be made to be diffraction limited.

28) The European Extremely Large Telescope (E-ELT) will have a primary mirror that is 39 m across. Show that the effective power of a telescope operating at the diffraction limit scales at D^4. Discuss one science goal that E-ELT will address that is not possible with current technology.