Nuclear and Particle Physics

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Contents

Part I

Lecture 1

Use these because they are made by God himself: https://dmaitre.phyip3.dur.ac.uk/notes/NPP/

Use these notes only with link above, these will just be additional annotations

1.1 Units

Example: Do Broglie Wavelength

$$\lambda = \frac{\hbar}{p} \tag{1.1}$$
$$p = 20\text{GeV/c} \tag{1.2}$$
$$l = 0.05\hbar c\text{GeV}^{-1} \tag{1.3}$$
$$\hbar c = 0.19733\,\text{fm GeV} \equiv 1 \tag{1.4}$$
$$\lambda = 0.0987\,\text{fm} \tag{1.5}$$

1.2 Kinematics

High energies mean speed close to c, so use special relativity, and get the Lorentz transforms and Tensors.

Example: 4-Momenta
In the rest frame of a particle of mass m,

$$p = (m, \vec{0}) \tag{1.6}$$

How is it in a different frame?

$$p = (E, \vec{p}) \tag{1.7}$$
$$p \cdot p \equiv p^2 = \begin{cases} \text{Rest Frame} & m^2 - \vec{0}^2 - m^2 \\ \text{Other Frames} & E^2 \vec{p} \cdot \vec{p} = E^2 - |\vec{p}|^2 \end{cases} \tag{1.8}$$
$$m^2 = E^2 - |\vec{p}|^2 \tag{1.9}$$
$$E^2 = m^c + |\vec{p}|^2 \tag{1.10}$$
$$E = \sqrt{m^2 + |\vec{p}|^2} \tag{1.11}$$

Note: there will be factors of c in this, but set to 1, in natural units.

Part II

Lecture 1

1.1 Scattering

- de Broglie Wavelength

$$\bar{\lambda} = \frac{\hbar}{p} \tag{1.1}$$

  - higher resolution comes from larger momenta
- Elastic scattering - Number and particle type are conserved
- Inelastic scattering - Number and particle type are not conserved
- Total cross section,

$$\sigma_{tot} = \sigma_{el} + \sigma_{inel} = \frac{\dot{N}}{\phi_a N_b} \tag{1.2}$$

  - $\dot{N}$ - rate of collisions
  - ϕ_a - number density in the beam
  - N_b - number of targets
-

$$\dot{N} = \mathcal{L} \cdot \sigma_{tot}, \mathcal{L} = \phi_a N_b \tag{1.3}$$

  $\mathcal{L}$ is a purely experimental input, σ_{tot} purely theoretical.

$$\sigma_{tot} = \int_\Omega d\Omega \int_{E_{min}}^{E_{max}} dE \, \frac{d\sigma}{d\Omega \, dE} \tag{1.4}$$

- Fermi's Golden Rule:

$$\sigma = \frac{2\pi}{V_a} \left| M_{fi} \right|^2 g(E') V \tag{1.5}$$

$$M_{fi} = \langle \Psi_f | \mathcal{H}_{int} | \Psi_i \rangle \tag{1.6}$$

$$\Psi_i = \frac{1}{\sqrt{V}} e^{i \underline{p} \cdot \underline{x}} \tag{1.7}$$

$$\Psi_f = \frac{1}{\sqrt{V}} e^{i \underline{p}' \cdot \underline{x}} \tag{1.8}$$

$$\mathcal{H}_{int} = \frac{z \cdot Ze^2}{\underline{x} - \underline{x}'|} \exp -M |\underline{x} - \underline{x}'| \tag{1.9}$$

$$M_{fi} = \langle \Psi_f | \mathcal{H}_{int} | \Psi_i \rangle \tag{1.10}$$

$$= \int d^3 x \Psi_f^*(\underline{x}) \mathcal{H}_{int} \Psi_i(\underline{x}) \tag{1.11}$$

$$= \frac{z \cdot Ze^2}{V} \int d^3 x e^{i \underline{q} \underline{x}} \frac{e^{-M|\underline{x} - \underline{x}_0|}}{|\underline{x} - \underline{x}_0|} \tag{1.12}$$

$$= \frac{4\pi e^2 zZ}{V} e^{iqx_0} \frac{1}{|q|^2 + M^2} \rightarrow_{M \to 0} \frac{4\pi e^2 zZ}{V} e^{iqx_0} \frac{1}{|q|^2} \tag{1.13}$$

$$d\sigma = \frac{2\pi}{V_a} |M_{fi}|^2 dg(E') V = \frac{2\pi}{V_a} \left| \frac{4\pi e^2 zZ}{V} e^{ix_0 \cdot q} \frac{1}{|q|^2} \right|^2 V \frac{V}{(2\pi)^3} |p'|^2 d\Omega \tag{1.14}$$

$$\frac{d\sigma}{d\Omega} = \frac{4e^4 z^2 Z^2 E'^2}{|q|^4} = \frac{e^4 z^2 Z^2}{4E^2 \sin^4\left(\frac{\theta}{2}\right)} \tag{1.15}$$

1.2 Mott Scattering

The Rutherford scattering formula neglects spin. Spin is a purely relativistic property that distinguishes fermions ($s = \frac{1}{2}$) from bosons ($s = 0, 1, \ldots$). So the Mott cross-section can be written as

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott, no recoil}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Rutherford}} \left(1 - \beta^2 \sin^2\left(\frac{\theta}{2}\right)\right), \beta = \frac{v}{c} \tag{1.16}$$

$$= \left(\frac{d\sigma}{d\Omega}\right)_{\text{Rutherford}} \cos^2\left(\frac{\theta}{2}\right), \lim_{v \to c} \tag{1.17}$$

- Spin projection on $\underline{p}$ is called helicity:

$$h = \frac{\underline{s} \cdot \underline{p}}{|\underline{s}||\underline{p}|} \tag{1.18}$$

- Exception to this when target carries spin as well
- Key points:
    - For relativistic projectiles, spin has to be taken into account.
    - Helicity conservation suppresses backwards scattering.

Lecture 2

2.1 Nuclear Form Factors

- Pointlike charge distribution

$$g(x) = Ze\delta^2(x - x_0) \tag{2.1}$$

- Extended charge distribution

$$g(x) = Zef(x) \tag{2.2}$$

-

$$g(x) = \int g(y)\delta(x-y)dy - Ze\int f(y)\delta(x-y)dy \tag{2.3}$$

$$\phi(x) = Ze\int f(y)\frac{1}{|x-y|}dy \tag{2.4}$$

$$\Delta\phi(x) = -g(x) \tag{2.5}$$

$$\mathcal{H}_{int} = ze\cdot\phi(x) \tag{2.6}$$

$$M_{fi} = \langle\psi_f|\mathcal{H}_{int}|\psi_i\rangle \tag{2.7}$$

$$= \frac{e}{V}\int d^3x\; e^{iq\cdot x}\phi(x), q = p - p' \tag{2.8}$$

$$= \frac{e}{V}\int e^{iq\cdot x}d^3x\; Ze\int f(y)\frac{1}{|x-y|}d^3y \tag{2.9}$$

$$= \frac{Ze^2}{V}\int d^3y\; f(y)\int d^3x\;\frac{1}{|q|^2}e^{iq\cdot x}\underbrace{\triangle\frac{1}{|x-y|}}_{...\delta(x-y)} \tag{2.10}$$

$$= \frac{Ze^2}{V}\int d^3y\; f(y)\frac{4\pi}{|q|^2}e^{iq\cdot y} \tag{2.11}$$

$$= \frac{4\pi Ze^2}{V|q|^2}\int d^3y\; f(y)e^{iq\cdot y} \equiv \frac{4\pi Ze^2}{V|q|^2}F(q) \tag{2.12}$$

$$\Longrightarrow \left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott, point, no recoil}} |F(q)|^2 \tag{2.13}$$

- For point-like target, $F(q) = 1$.
- The shape of $F(q)$ yields information about the charge distribution.
  - $F(q)$ is the Fourier transform of $f(y)$.
  - e.g. spherically symmetric target, $f(x) = f(|x|)$

$$F(q) = \int e^{iq\cdot x}f(x)\,d^3x = \int f(v)e^{iqv\cos\theta}2\pi v^2 dv d\cos\theta \tag{2.14}$$

$$= \int_v \left(\frac{1}{iqv}e^{iqv\cos\theta}\right)_{-1}^{1} f(v)2\pi v^2 dv \tag{2.15}$$

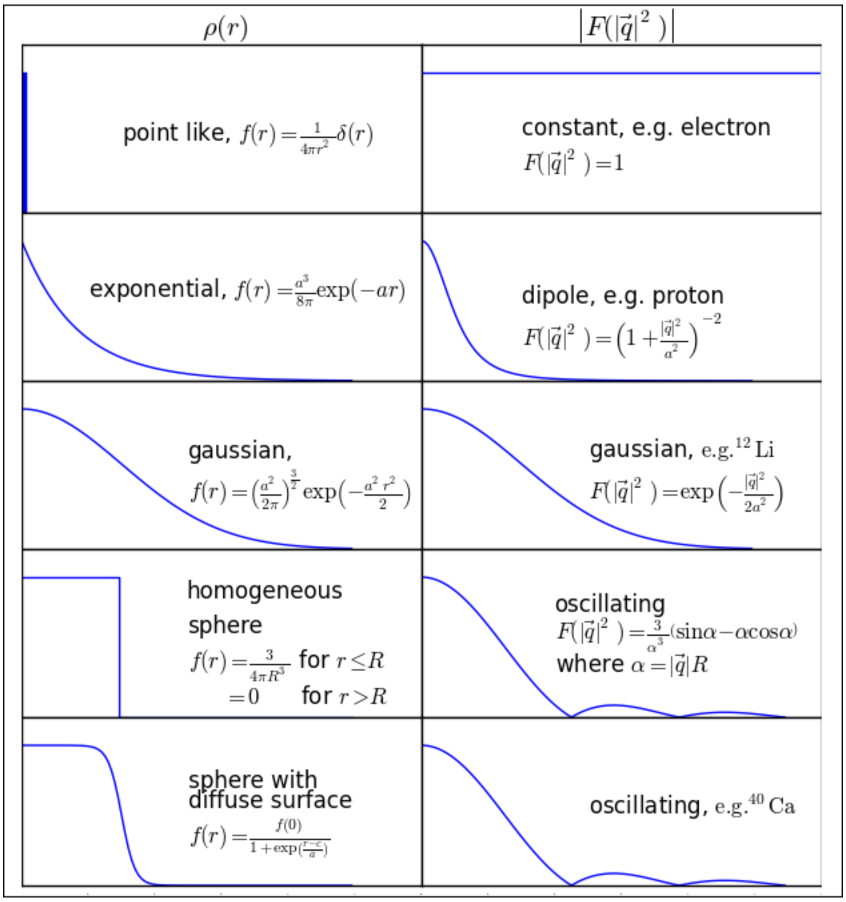
$$= \int_v \frac{1}{iqv}\left(e^{iqv} - e^{-iqv}\right)f(v)v^2 dv \tag{2.16}$$

$$= 2\pi\int 2\frac{\sin(qv)}{qv}f(v)v^2 dv \tag{2.17}$$

$$= 4\pi\int_0^1 \frac{\sin(qv)}{qv}\left(\frac{3}{4\pi R^3}\right)v^2 dv \tag{2.18}$$

$$= \frac{3}{R^3q^3}\left(\sin(qR) - qR\cos(qR)\right) \tag{2.19}$$

- Something about graphs leading to $R \approx \frac{4.5}{q_0}$.



Key points:

- Scattering off an extended charge distribution intoeuces a form factor $F(q)$. $F(q)$ is the Fourier transform of the charge distribution.
- Measure the ration between $d\sigma/d\Omega$ in experiment and Mott allows to determine the shape of $F(q)$.

2.2 Scattering Off Nucleons

- Require a resolution of $1\,fm = 10^{-15}m$ to perceive nucleons
- Requires $q = \frac{\hbar c}{\lambda} = 200\,MeV$
- $m_p = 938\,MeV$
- For such large momenta, the recoil has to be taken into account: $E \neq E'$

$$g(E) = \frac{dn}{dE} = \frac{dn}{dE'}\frac{dE'}{dE} \approx \frac{dn}{dE'} \cdot \frac{E'}{E} = g(E') \tag{2.20}$$

- With this modification, now we have recoil

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott, recoil}} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott, no recoil}} \cdot \frac{E'}{E} \tag{2.21}$$

- The target carries spin, and therefore a magnetic moment.
  - Recall,

$$\mu = I \cdot A = \frac{q}{2m}L \tag{2.22}$$

- For a spinning charge,

$$\mu = g\frac{e}{M} \cdot S,\ S = \frac{1}{2} \tag{2.23}$$

Lecture 3

Scattering Processes and their cross sections:

- pointlike charge (no spin) on pointlike charge (no recoil, no spin)

$$\left(\frac{d\sigma}{d\Omega}\right)_{Ruth} = \frac{e^4 z^2 Z^2}{4E^2 \sin^4(\theta/2)} \tag{3.1}$$

- pointlike charge (spin) on pointlike charge (no recoil, no spin)

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott, no recoil}} = \left(\frac{d\sigma}{d\Omega}\right)_R \cos^2(\theta/2) \tag{3.2}$$

- pointlike charge (spin) on extended charge (no recoil, no spin)

$$\left(\frac{d\sigma}{d\Omega}\right)_\rho = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott, no recoil}} |F(q^2)|^2 \tag{3.3}$$

- pointlike charge (spin) on pointlike charge (recoil, no spin)

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott, no recoil}} \frac{E'}{E} \tag{3.4}$$

- pointlike charge (spin) on pointlike charge (recoil, spin)

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \cdot \left(1 + 2\tau \tan^2(\theta/2)\right) \tag{3.5}$$

- pointlike charge (spin) on extended charge (recoil, spin)

$$\left(\frac{d\sigma}{d\Omega}\right)_{Rosenbluth} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left[\frac{G_E^2(q^2) + \tau G_M^2(q^2)}{1+\tau} + 2\tau G_M^2(q^2) \tan^2(\theta/2)\right] \tag{3.6}$$

- The electric form factor is the Fourier transform of the electric charge distribution.

$$G_E(q^2) = \int e^{iq\cdot y} f(y)\, d^3 y \tag{3.7}$$

- The magnetic form factor is the Fourier transform of the magnetic moment distribution.

$$G_M(q^2) = \int e^{iq\cdot y} \mu_z(y)\, d^3 y \tag{3.8}$$

- Expectation (for $Q^2 \to 0$) for proton and neutron:

$$G_E^p = 1 \qquad G_E^N = 0 \tag{3.9}$$
$$G_M^p = 1 \qquad G_M^N = 0 \tag{3.10}$$

- Measurement for proton and neutron:

$$G_E^p = 1 \qquad G_E^N = 0 \tag{3.11}$$
$$G_M^p = 2.79 \qquad G_M^N = -1.31 \tag{3.12}$$

- G_M^N most surprising as neutron should be neutral and not interact magnetically - suggests neutron composed of non-neutral particles
- Mind-blowing stuff

$$y = mx + c \tag{3.13}$$

- Dipole form factor

$$G_E^p, G_M^p, G_M^N = \frac{1}{1 + \frac{q^2}{a^2}} \tag{3.14}$$
$$G_E^N = 0 \tag{3.15}$$

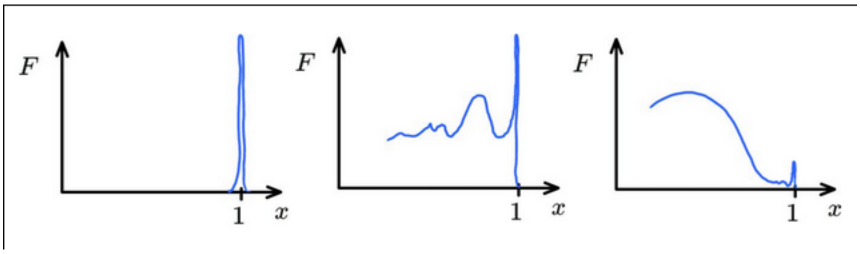
Lecture 4

Lecture 5

5.1 Repetition Inelastic Scattering

- $$w_p = P_\mu + p_\mu + p'_\mu \tag{5.1}$$
$$w^2 = (P+q)^2 = P^2 + 2P\cdot q - Q^2 \tag{5.2}$$
$$= M^2 + 2Mv - Q^2 \tag{5.3}$$
$$v = \frac{P\cdot q}{M} \tag{5.4}$$
- Further increasing the energy leads to inelastic scattering. Now the target can become excited and additional particles are produced. We need two parameters to describe the cross section.
- Deep inelastic scattering
$$\frac{d^\sigma}{d\Omega\, dE'} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott, no recoil}} \left[W_2(Q^2,v) + 2W_1(Q^2,v)\tan^2\frac{\theta}{2}\right] \tag{5.5}$$
W_i are known as structure functions - they are the "form factors" of inelastic scattering.
- Choose dimensionless structure functions. Define,
$$x = \frac{Q^2}{2Mv} \tag{5.6}$$
$$2Mv - Q^2 \geq 0 \tag{5.7}$$
$$0 < x \leq 1 \tag{5.8}$$
x is the Bjorken scale variable.
- F_1 is the magnetic ff, and F_2 is electric
$$F_1(x,Q^2) = MW_1(Q^2,v) \tag{5.9}$$
$$F_2(x,Q^2) = vW_2(Q^2,v) \tag{5.10}$$
- Expressed in dimensionless structure functions, the cross section reads
$$\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1 - y - \frac{M^2x^2y^2}{Q^2}\right)\frac{F_2(x,Q^2)}{x} + \frac{y^2}{2}\frac{2xF_1(x,Q^2)}{x}\right] \tag{5.11}$$
$$y = \frac{P\cdot q}{p\cdot q} \tag{5.12}$$
- In elastic scattering, both form factors of the proton are proportional to a dipole
$$\sigma \propto |G_{dipol}|^2 \propto \frac{1}{Q^3} \tag{5.13}$$
$$G_{dipol} = \frac{1}{\left(1 - \frac{|q|^2}{a^2}\right)^2} \tag{5.14}$$
- What do we observe for large momenta? The structure functions are apprxoimately flat in Q^2. This hints at a point-like substructure of the proton.
- What about the x dependence?
  - Use R as the radius of the proton
  - $Q^2R^2 << 1$ is the regime of elastic scattering, $x = 1$.
  - $Q^2R^2 \approx 1$, we start seeing structure and can excite proton to higher energy states.

- ➥ $Q^2R^2 >> 1$, can look deep into the structure of the proton.



- ➤ Key points
  - ➥ In a Deep Inelastic Scattering experiment, one collides large energy electrons with a proton.
  - ➥ There are 3 scattering regimes
  - ➥ Low energy $Q^2R^2 << 1$ is eleastic scattering
  - ➥ Moderate energy $Q^2R^2 \approx 1$ yields resonances
  - ➥ High energy $Q^2R^2 >> 1$ yields point-like substructure
- ➤ Parton model - there were hints of point-like substructure which were just called partons at first
- ➤ Assume that each parton carries a fraction of the Proton momentum, P. For massless ($0 \leq \eta \leq 1$) partons with momentum $k = \eta P$.

$$0 = k^2 = (\eta P + p - p')^2 = (\eta P + q)^2 \tag{5.15}$$

$$= \eta^2 M^2 + 2\eta P \cdot q - Q^2 \tag{5.16}$$

Assume $Q^2 >> M^2$:

$$\eta = \frac{Q^2}{2P \cdot q} = x \tag{5.17}$$

- ➤ This gives a physical interpretation of what x is - it is the fraction of the proton momentum carried by the individual parton that is struck in the scattering.
- ➤ Looking at the right-most figure above, the highest peak is at $x \approx 1/3$, suggesting that each parton has mass $m_p/3$, and therefore there must be three of them - assuming no one parton is any more special than the others.

Lecture 6

- What is the parton spin? For spin $\frac{1}{2}$,

$$F_2(x, Q^2) = 2xF_1(x, Q^2) \tag{6.1}$$

  This is the Callan-Gross relation.
- The differential cross-section can be rewritten as

$$\frac{d\sigma}{dQ^2 dx} = \sum_{\text{constituents}} f_c(x) \frac{d\sigma}{d\Omega}(e^-, c; p, x, P, p') \tag{6.2}$$

- $f_c(x)$ is the parton distribution function. This is the number of partons of type c with momentum fraction in an infinitesimal interval around x.

$$dn_c(x) = f_c(x)\, dx \tag{6.3}$$

- The structure functions can now be expressed as

$$F_2(x, Q^2) = 2xF_1(x, Q^2) \tag{6.4}$$
$$= x \sum_{\text{constituents}} Q_c^2 f_c(x) \tag{6.5}$$

- Quarks

Name	Symbol	Charge
up	u	$+\frac{2}{3}$
down	d	$-\frac{1}{3}$
strange	s	$-\frac{1}{3}$
charm	c	$+\frac{2}{3}$
bottom	b	$-\frac{1}{3}$
top	t	$+\frac{2}{3}$

- Gluons are the force carriers of the strong force. In contrast to photons, they self-interact.
- Can draw similar field lines to that of electromagnetism dipoles, but field lines increase in density as you pull them apart, until you've provided enough energy by pulling them apart to generate new particles, which form new bound states with the old quarks, i.e. pions
- Nuclear decay arises due to short range of strong force but infinite range of electromagnetism - eventually over a large enough nucleus, the strength of the summed electromagnetic repulsion overcomes the strong force coupling.
- Partons can be produced for a short amount of time through Heisenberg uncertainty, $\delta E \delta t \geq \frac{\hbar}{2}$. These are known as sea quarks.
  - Quark-antiquark pairs can form as part of a nucleon for a short amount of time as they do not change the overall charge
  - The higher the mass of the quarks, the less likely they are to form $\implies$ higher mass requires more energy, so smaller lifetime
- We cann predict parton distribution functions from first principles, but some can be derived.
  - Using all constituents

$$\int_0^1 \sum_c x f_x(x)\, dx = 1 \tag{6.6}$$

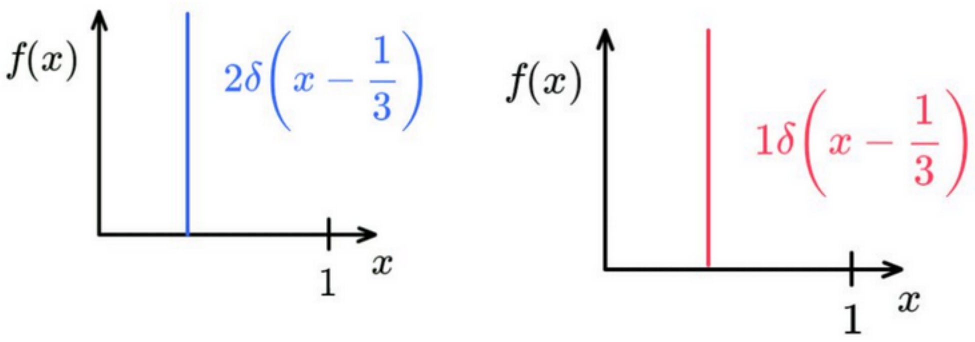
➥ Using charged constituents

$$\int_0^1 \sum_{\text{quarks}=q} Q_q f_q(x)\, dx = 1 \tag{6.7}$$

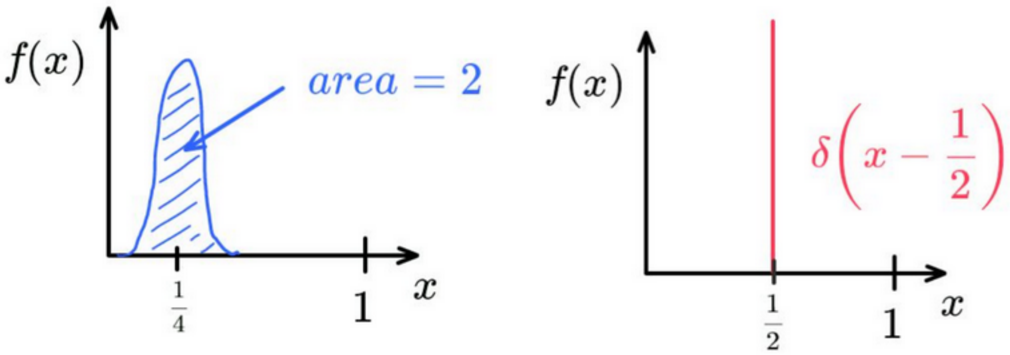
That 1 is the proton charge.

$$f_q(x) = f_q^v(x) + f_q^s \tag{6.8}$$

Assume no gluons and $f_q^s(x) = 0$.



➤ What if the 2 up-quarks share $\frac{1}{2}$ of the momentum and the down quark carries the other $\frac{1}{2}$?



➤ Educated guess

$$f(x) = x^a(1-x)^b,\ a, b > 0 \tag{6.9}$$

$$\lim_{x\to 0} f(x) = 0 \tag{6.10}$$

$$\lim_{x\to 1} f(x) = 0 \tag{6.11}$$

Tells us that they cannot be at rest and cannot carry all the momentum.

Lecture 7

- The parton model stipulates that the proton is made up from constituent particles called partons (quarks)
- Callan-Gross relation implies partons carry spin-$\frac{1}{2}$
- The constituents carry momentum fractions described the PDFs
- There are two types of quark in the nucleon
  - Valence quarks that determine the quantum numbers
  - Ion quarks that appear in $q-\bar{q}$ pairs

7.1 Quark Structure of the Nucleus

- Consider $e^{-}p$ scattering

$$F_2(x) = x\sum_f Q_f^2(q_f(x)+\bar{q}_f(x)) \tag{7.1}$$

$$\begin{aligned} F_2(x) = x\bigg[&\frac{1}{9}\left(d_v^p(x)+d_s^p(x)+\bar{d}_s^p(x)\right) \\ &+\frac{4}{9}\left(u_v^p(x)+u_s^p(x)+\bar{u}_s^p(x)\right) \\ &+\frac{1}{9}\left(s_s^p(x)+\bar{s}_s^p(x)\right)+\cdots\bigg] \end{aligned} \tag{7.2}$$

- Can use relations

$$u_v^p = d_v^n \qquad d_v^p = u_v^n \qquad s_s^p = s_s^n \tag{7.3}$$

- Now $e^{-}n$ scattering - similar with more down quarks than up quarks, use the above relations to give it the same variables

$$\begin{aligned} F_2^{en}(x) = x\bigg[&\frac{1}{9}\left(d_v^n+d_s^n+\bar{d}_s^n\right) \\ &+\frac{4}{9}(u_v^n+u_s^n+\bar{u}_s^n) \\ &+\frac{1}{9}(s_s^n+\bar{s}_s^n)\bigg] \end{aligned} \tag{7.4}$$

$$\begin{aligned} F_2^{en}(x) = x\bigg[&\frac{1}{9}\left(u_v^p+u_s^p+\bar{u}_s^p\right) \\ &+\frac{4}{9}\left(d_v^n+d_s^p+\bar{d}_s^p\right) \\ &+\frac{1}{9}\left(s_s^p+\bar{s}_s^p\right)\bigg] \end{aligned} \tag{7.5}$$

- Can take the average of these two

$$\begin{aligned} F_2^{eN} = \frac{F_2^{ep}+F_2^{en}}{2} = x\bigg[&\frac{5}{18}\left(d_v^p+d_s^p+\bar{d}_s^p\right) \\ &+\frac{5}{18}\left(u_v^p+u_s^p+\bar{u}_s^p\right) \\ &+\frac{1}{9}\left(s_s^p+\bar{s}_s^p\right)\bigg] \end{aligned} \tag{7.6}$$

- Now we turn our eyes to the mystical creature known as the neutrino - cousin to the mighty unicorn

$$F_2^{\nu N} = x \sum_f (q_f(x) + \bar{q}_f(x)) \tag{7.7}$$

- We neglect the strange quark

$$\frac{F_2^{eN}}{F_2^{\nu N}} = \frac{5}{18} \tag{7.8}$$

- Now look at the momentum distribution - $q_f(x)$ (or $f_(x)$) is the probability of finding a parton with a momentum fraction $x \cdot \underline{P}$, where $\underline{P}$ is the momentum of the proton. We can estimate the momentum carried by the quarks f and anti-quarks $\bar{f}$.

$$p_f = \int_0^1 (q_f(x) + \bar{q}_f(x)) x \underline{P}\, dx \tag{7.9}$$

- The fraction of the total momentum carried by quarks is then

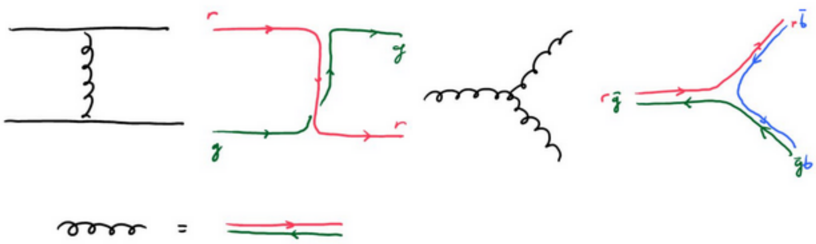
$$\frac{\sum_f p_f}{\underline{P}} = \sum_f \int_0^1 x(q_f(x) + \bar{q}_f(x))\, dx \tag{7.10}$$

$$= \int_0^1 F_2^{\nu N}(x)\, dx = \frac{18}{5} \int_0^1 F_2^{eN}(x)\, dx \tag{7.11}$$

  If there were only quarks, this would be $= 1$. The fact that it is ≈ 0.5 is indirect evidence of gluons.
- Gluons will decay into a quark and antiquark which can then recombine into a gluon again so depending on when you take your measurement, the part of the momentum not occupied by normal quarks could either be gluons or sea quarks.

7.2 Colour

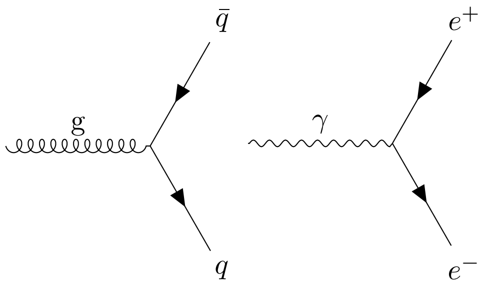
- Up, down, and strange quarks have been introduced to understand the properties of baryons (3 quarks).
  - 2 ups, 1 down - proton
  - 1 up, 2 downs - neutron
  - 2 ups, 1 strange - Σ^+ (+1 nuclear charge)
  - 3 ups - Δ^{++} (+2 nuclear charge)
- How to explain the Δ^{++} with all three spins aligned? What about Pauli?
- The strong charge is called colour.
- For the Δ^{++}, the three up quarks have the same spins, but one is red-, one is blue-, one is green-colour-charged
- Gluons carry colour-anticolour combinations, i.e. red-antigreen



- The experimental consequence is that only globally colour-neutral particles can be observed.

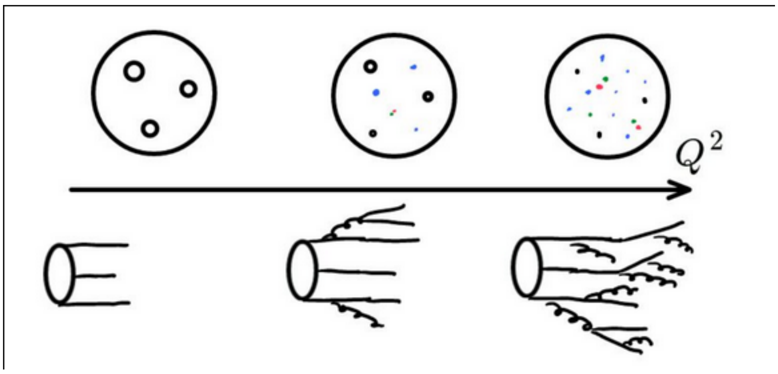
Lecture 8

- We can get information about the PDFs using DIS with different targets and projectiles
- The total function of momentum carried by quarks is ≈ 0.5. The other half is carried by gluons.
- The Δ^{++} resonance only agrees with the Pauli Principle, if we introduce **colour**.
- Both quarks and gluons carry colour.
- Gluons mediate the strong force.
- Only colour-neutral objects above ≈ 1 fm.



8.1 Scaling Violation

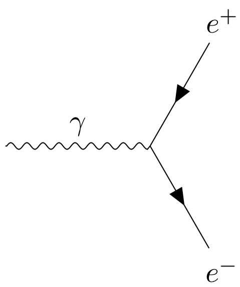
There is a mild Q^2-dependence in the PDFs, because the "resolution" of the photon in deep inelastic scattering is increasing with Q^2 revealing more sea quarks and gluons than at low Q^2.



8.2 Feynman Diagrams

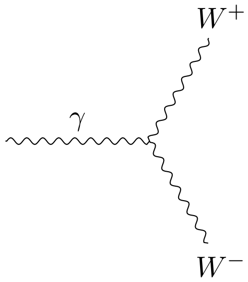
Feynman diagrams are a way of illustrating interactions in particle physics.

- Fermions are represented by plain lines. Arrows in direction of time represent fermions. Arrows against the direction of time represent anti-fermions.

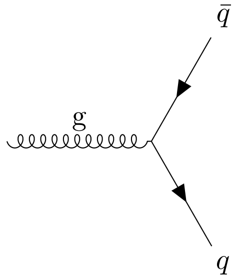


- Bosons

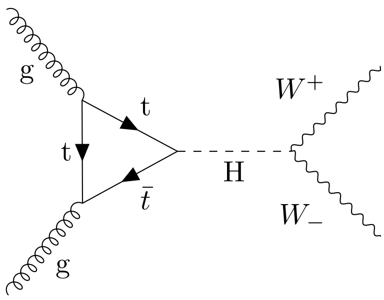
- ➥ Electroweak bosons are wiggly lines



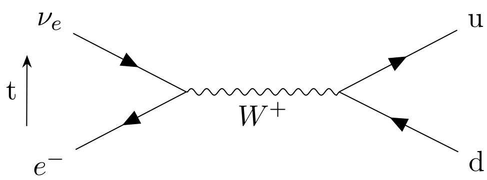
- ➥ Strong (gluons) is curly



- ➥ Higgs is dashed



- ➤ Vertices - interaction points. Each vertex conserves:
  - ➥ Electric charge
  - ➥ Lepton number
- ➤ Photons couple to all electrically charged particles. Each vertex carries $Q\sqrt{\alpha}$ with $\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$.
- ➤ W-Boson - All W-interactions have a different strength depending on the flavours involved.
- ➤ The Z-Boson couples to all weak charges.
- ➤ The gluon couples to all coloured particles with an interaction strength of $\sqrt{\alpha_s}$.
- ➤ Internal Particles:



Internal lines bring a factor

$$\frac{1}{p^2 - m^2 + im\Gamma}$$

- ➤ Standard model vertices - 4-vertices and Higgs interactions.

Lecture 9

9.1 How to draw a Feynman diagram

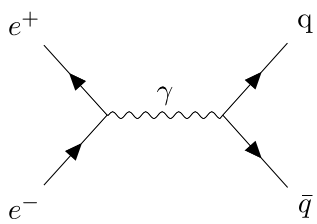
1. Start with fermion lines
2. Dress it with the right bosons

Key points:

- All vertices conserve:
  - electric charge
  - Baryon number
  - Lepton number

9.2 Quarkonia

Just like e^-p+ and e^+e^- can form bound states under electromagnetism, so can $q\bar{q}$ pairs under the strong force. Explore the spectrum of $(c\bar{c})$ and $(b\bar{b})$ states to learn about the $q-\bar{q}$ potential. Produced by:



Set energy of e^+e^- right $\to$ produce resonances. Lowest lying resonance (lightest) $J/\psi = (\bar{c}c)$, $m_{J/\psi} = 3.097\,\text{GeV}$ ($3\times$ proton mass) - $J/\psi = 1^-$. Heavier states decay into lighter states through radiating off a photon in analogy to e^+e^-, e^-p^+.

This "fine-structure" splitting between the two 1S states in e^+e^- originates from spin-spin interactions.

$$V_{ss}(e^+e^-) = \frac{8\pi}{3}\alpha\frac{\vec{S}_1\cdot\vec{S}_2}{m_e^2}\delta(\vec{x}) \tag{9.1}$$

$$V_{ss}(q\bar{q}) = \frac{32\pi}{9}\alpha_s\frac{\vec{S}_q\cdot\vec{S}_{\bar{q}}}{m_q m_{\bar{q}}}\delta(\vec{x}) \tag{9.2}$$

To calculate the energy or mass shift between the two 1S states, we need the value of $\langle\vec{S}_1\cdot\vec{S}_2\rangle$:

$$S(S+1) = \langle(S_1+S_2)^2\rangle \tag{9.3}$$
$$= \langle S_1^2\rangle + 2\langle S_1\cdot S_2\rangle + \langle S_2^2\rangle \tag{9.4}$$
$$= \frac{1}{2}(\frac{1}{2}+1) + 2\langle S_1\cdot S_2\rangle + \frac{1}{2}(\frac{1}{2}+1) \tag{9.5}$$
$$= \frac{3}{2} + 2\langle S_1\cdot S_2\rangle \tag{9.6}$$
$$\implies \langle S_1\cdot S_2\rangle = \frac{2S(S+1)-3}{4} = \begin{cases} -\frac{3}{4} & S=0 \\ \frac{1}{4} & S=1 \end{cases} \tag{9.7}$$

$$E_{ss} = \langle\psi|V_{ss}|\psi\rangle = \frac{8\pi\alpha_s}{gm_q m_{\bar{q}}}\langle\psi|\delta(\vec{x})|\psi\rangle\begin{cases} -3 \\ 1 \end{cases} \tag{9.8}$$

$$= \frac{8\pi}{gm_q^2}|\psi(0)|^2\begin{cases} -3 \\ 1 \end{cases} \tag{9.9}$$

The mass split,

$$\Delta E_{ss} = E_{ss}(S=1) - E_{ss}(S=0) \tag{9.10}$$
$$= \frac{32\pi}{g}\frac{\alpha_s}{m_q^2}|\psi(0)|^2 \tag{9.11}$$

9.3 Quark-Antiquark Potential

For small distances,

$$V(r) \propto \frac{1}{r} \tag{9.12}$$

For large distances, $V(r)$ has to grow.

$$V(r) = \underbrace{-\frac{4}{3}\frac{\alpha_s(r)}{r}}_{\text{short-range}} + \underbrace{kr}_{\text{long-range}} \tag{9.13}$$

Lecture 10

- Four ways for Quarkonia to decay:
  - EM decays
  - virtual/real photon emission and gluon emission
  - Decays through quarkonia production
  - Weak decays (change of quark)
- Parity is the spatial inversion at the origin

$$\psi(x,t) \to \psi'(x,t) = \hat{P}\psi(x,t) = \psi(-x,t) \tag{10.1}$$

$$\tag{10.2}$$

- Parity is conserved in QED (photons) and QCD (gluons), not in weak interactions (W,Z)
- All (anti-)fermions in the Standard Model have $P(\psi) = 1,\ P(\bar{\psi}) = -1$
- Vector bosons have $P(\gamma) = P(g) = P(W^{\pm}) = P(Z) = -1$
- $P(\vec{E}) = -1,\ P(\vec{B}) = 1$

10.1 EM Decays

For a state $n^{2S+1}L_J$, parity is given by

$$P(n^{2S+1}L_J) = \underbrace{P(q)P(\bar{q})}_{-1}(-1)^L = (-1)^{L+1} \tag{10.3}$$

Key points:

- Charm-anticharm bound states are charmonium
- Bottom-antibottom bound states are bottomium
- The spectrum of these states teaches us about the quarkonium potential
- At small distances, $V_{q\bar{q}} \propto \frac{1}{r}$
- At large distances, $V_{q\bar{q}} \propto r$
- Large $L = 0$ splittings are a result of spin-spin interactions
- There are four decay channels:
  - De-excitation (photon decay)
  - Virtual gluon/photon decay (several)
  - Hadronic (split into more quarkonia of lower mass)
  - Weak interaction (change of quark)

Lecture 11

11.1 Light Quark Mesons (u,d,s)

These are more complicated, because the motion of the light quarks in the bound state is relativistic. Additional tool: symmetry $u \leftrightarrow d \leftrightarrow s$.

11.1.1 Properties of Light Mesons

Consider $L = 0$ states (1S).

- Parity

$$P = \underbrace{P(\bar{q})P(q)}_{-1}(-1)^L = -1 \tag{11.1}$$

- $L + S = J$, $J^P = O^-$ - pseudoscalar mesons
- $J^P = 1^-$ vector mesons
- We have 3 light quarks (u,d,s) and 3 light anti-quarks ($3 \times 3 = 9$)
- For massless quarks, there is a symmetry: $u \leftrightarrow d \implies$ Isospin

11.1.2 Isospin

The Isospin operator is the same as the spin operator, but it acts on flavour: $I_a = \frac{\sigma_a}{2}$.

$$I_1 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad I_2 = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad I_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{11.2}$$

$$I^2 = I_1^2 + I_2^2 + I_3^2,\ EV = I(I+1) \tag{11.3}$$

$$\begin{pmatrix} u \\ d \end{pmatrix} \implies I_3 = \begin{cases} +\frac{1}{2} \\ -\frac{1}{2} \end{cases},\ I = \frac{1}{2} \tag{11.4}$$

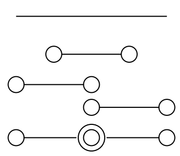
$$\begin{pmatrix} -\bar{d} \\ \bar{u} \end{pmatrix} \implies I_3 = \begin{cases} +\frac{1}{2} \\ -\frac{1}{2} \end{cases},\ I = \frac{1}{2} \tag{11.5}$$

For spin:

$$\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1 \tag{11.6}$$

Now:

$$2 \otimes \bar{2} = 1 \oplus 3 \tag{11.7}$$



11.2 Meson Multiplets

We can decompose the tensor-product of a triplet and an anti-triplet under SU(3).

$$3 \otimes 3 = 8 \oplus 1 \tag{11.8}$$

Graphical representation: what is going on?

11.3 Meson Masses

Vector mesons are heavier due to the spin-spin interactions.

$$E_{ss} = \frac{8}{9}\frac{\pi\alpha_s}{m_q m_{\bar{q}}}|\psi(0)|^2 \begin{cases} -3 & J=0 \\ 1 & J=1 \end{cases} \tag{11.9}$$

Assume $m_u = m_d$ and $|\psi(0)|^2$ is roughtly the same $\implies$

$$m_{u,d} \approx 310 MeV, \ m_s \approx 483 MeV \tag{11.10}$$

$$m_{\text{constituent}} = m_{\text{intrinsic}} + m_{\text{dynamic}} \tag{11.11}$$

Lighter quarks are mainly dynamic mass, heavier quarks mainly intrinsic mass.

Lecture 12

Lecture 13

13.1 Lepton Pair Production

- *put some Feynman diagrams in here*
- e^- are stable
- μ^- are unstable - rather long-lived on these scales, $t_\mu \approx 2\mu s$
- τ^- are unstable - short-lived, $t_\tau \approx 3 \times 10^{-13} s$

13.2 Hadron Production

- *some Feynman diagrams here*
- Leptons have $Q = -1$, Quarks have $Q = -\frac{1}{3}, \frac{2}{3}$
- Quarks carry colour quantum numbers: red, green, blue

$$\sigma(e^+e^- \to \text{ hadrons}) = \sum_f \sigma(e^+e^- \to q_f \bar{q}_f)\Theta(\sqrt{s} - 2m_f) \tag{13.1}$$

$$\sigma(e^+e^- \to q_f \bar{q}_f) = N_c Q_f^2 \sigma(e^+e^- \to \mu^+\mu^-) \tag{13.2}$$

- N_c is the number of colours (3)
- Can take the ratio

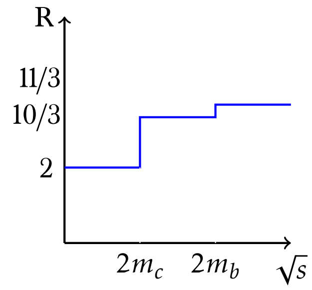
$$R = \frac{\sigma(e^+e^- \to \text{ hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \tag{13.3}$$

$$= 3 \sum_{2m_f \leq \sqrt{s}} Q_f^2 \tag{13.4}$$

$$\sqrt{s} > 2m_s \qquad R = 3\left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9}\right) = 2 \tag{13.5}$$

$$\sqrt{s} > 2m_c \qquad R = 3\left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9} + \frac{4}{9}\right) = \frac{10}{3} \tag{13.6}$$

$$\sqrt{s} > 2m_b \qquad R = 3\left(\frac{4}{9} + \frac{1}{9} + \frac{1}{9} + \frac{4}{9} + \frac{1}{9}\right) = \frac{11}{3} \tag{13.7}$$



The differential cross section

$$\frac{d\sigma(e^+e^- \to \mu^+\mu^-)}{d\Omega} = \frac{\alpha^2}{4s}(1 + \cos^2\theta) \tag{13.8}$$

$$\sigma = \frac{4\pi}{3}\alpha^2 \frac{1}{s} \tag{13.9}$$

Neglect the Z-Boson

$$\mu \propto \frac{1}{q^2 - M_\mu^2} + \frac{1}{q^2 - M_Z^2} \tag{13.10}$$

$$= \frac{1}{s} - \frac{1}{M_Z^2} \tag{13.11}$$

$$\sigma \propto S\left(\frac{1}{s} - \frac{1}{M_Z^2}\right)^2 = \frac{1}{S} - \frac{2}{M_Z^2} + \frac{s}{M_Z^4} = \frac{1}{S} \tag{13.12}$$

13.3 Resonances

Put Feynman diagram in here

➤ Virtual photons can only produce resonances of $q\bar{q}$ pairs with $Q_R = 0$ and $J_R = 1$.

Lecture 14

14.1 Key Point

- e^+e^- collisions are clean (initial state has a well-defined center-of-mass energy, no structure functions)
- Hadron production in e^+e^- collisions occur through the production of $q-\bar{q}$ pairs, for $\sqrt{s} \geq 2m_q$ ($\sqrt{s}$ the center-of-mass energy), compatible with γ^*, $J^P = 1^-$
- The R-ratio,
$$\frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}, \tag{14.1}$$
shows the quark thresholds and yields information on the number of colours, $N_c = 3$, and the quark charges
- At $91.2\,GeV$, the Z-boson is produced on-shell
- The heigh of the resonance peak is related to its width

$$\sigma = \frac{1}{(p^2+M_Z^2)+\Gamma^2 M_Z^2} \propto \left(\lim_{p^2 \approx M_Z^2}\right)\frac{1}{\Gamma^2} \tag{14.2}$$
$$\Gamma_{tot} = \Gamma(Z \to e^+e^-) + \Gamma(Z \to \mu^+\mu^-) + \Gamma(Z \to \tau^+\tau^-) + \Gamma(Z \to q\bar{q}) + \Gamma(Z \to \nu\bar{\nu}) \tag{14.3}$$

14.2 Weak Interaction

Name	Interaction	Mass
Photon, γ	E/M	0
Gluons, g	strong	0
$W^\pm, Z$	weak	$80.4\,GeV,\ 91.2\,GeV$

Why weak? Interactions are suppressed by the heavy mediator masses.
Which particles take part in the weak interaction?

14.2.1 Charged Leptons

Name	Mass	Charge
e^-	$0.511\,MeV$	-1
μ^-	$105.65\,MeV$	-1
τ^-	$1776.8\,MeV$	-1

Charged leptons share all quantum numbers apart from mass. If leptons were composite, you would expect $\mu \to e\gamma$, but this has never been observed. They decay through other channels:

$$\mu^- \to e^-\bar{\nu}_e\nu_\mu \tag{14.4}$$
$$\tau^- \to e^-\bar{\nu}_e\nu_\tau \tag{14.5}$$
$$\to \mu^-\bar{\nu}_\mu\nu_\tau \tag{14.6}$$
$$\to \pi^-\nu_\tau \tag{14.7}$$

draw some Feynman diagrams here for tau to pion and mu to electron.

14.2.2 Neutrinos

- Almost massless, $m_\nu < 2\,eV$
- electrically neutral

- colour-less
- Introduced because of β-decay

$$n \to p + e^- + \bar{\nu}_e \tag{14.8}$$

- Anti-neutrinos are produced by reactions

$$\bar{\nu}_e + p \to n + e^+ \tag{14.9}$$
$$\nu_e + n \to p + e^- \tag{14.10}$$

  The first has been observed, while the second is not observed. Therefore, $\bar{\nu}_e \neq \nu_e$.
- Each charged lepton comes with a neutrino partner

$$\begin{pmatrix} e^- \\ \nu_e \end{pmatrix}, \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix}, \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix} \tag{14.11}$$

- Lepton number is conserved

$$L_l = N(l) - N(\bar{l}) + N(\nu_l) - N(\bar{\nu}_l) \tag{14.12}$$
$$L = L_e + L_\mu + L_\tau \tag{14.13}$$

Lecture 15

15.1 Coupling Strength of the Weak Interaction

$$\frac{g^2}{q^2 - M_W^2} = g^2\left(-\frac{1}{M_W^2} - \frac{g^2}{M_W^2} + \cdots\right) \tag{15.1}$$

- Contracting the W-propagator

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{M_W^2} \tag{15.2}$$

- G_F is the Ferm constant

15.2 Quarks and the CKM matrix

It should also be possible to extract the Fermi constant from semi-leptonic processes.

15.3 Key Points

- Weak interaction is mediated by $W^{\pm}$ and Z bosons
- It is responsible for β decay, and the decays of heavy quarks and leptons
- There are 3 charged leptons and 3 neutrinos
- Leptons can be organised in generations, like quarks, with a neutrino associated with each of the charged leptons
- Lepton number is conserved for each generation, and therefore total lepton number also
- Weak interactions can be charged ($W^{\pm}$), or neutral (Z)
- Z interactions never change flavour; charged interactions can change quark flavour, because the mass basis and the weak basis are not the same
- Flavour changing transitions are proportional to CKM elements

Lecture 16

16.1 Parity Violation

Parity is the symmetry associated with space inversion.

$$\vec{x} \to -\vec{x}, \text{ vector} \tag{16.1}$$
$$\vec{p} \to -\vec{p}, \text{ vector} \tag{16.2}$$
$$\vec{L} = \vec{x} \times \vec{p} \to (-\vec{x}) \times (-\vec{p}) = \vec{L}, \text{ axial vector} \tag{16.3}$$
$$E_{kin} = \frac{1}{2}m\vec{x}^2 \to \frac{1}{2}m(-\vec{x})^2 = E, \text{ scalar} \tag{16.4}$$
$$h = \frac{\vec{s} \cdot \vec{p}}{|\vec{s}||\vec{p}|} \to \frac{\vec{s} \cdot (-\vec{p})}{|\vec{s}||(-\vec{p})|} = -h, \text{pseudo-scalar} \tag{16.5}$$

For a massless fermion, helicity (spin along direction of momentum) is conserved, either as $h = +1$ for along momentum (right-handed), or $h = -1$ for anti-along the momentum (left-handed).
Remember Mott scattering

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \left(\frac{d\sigma}{d\Omega}\right)_{Ruth} \cos^2\theta \tag{16.6}$$

Helicity changes sign from scattering, so must have mass.
What happens to anti-fermions? *Draw Feynman diagrams in lecture 29 on notes.* The amplitude of the two diagrams are equivalent, so left-handed fermions and right-handed anti-fermions must be equivalent. Interactions can be classified like vectors (axial-vectors) depending on left- and right-handed fermions and anti-fermions.

- **Vector interactions** only couple between the same handedness (f_L and f_L, or f_L and $\bar{f}_R$, or $\bar{f}_R$ and $\bar{f}_R$, etc)
- **Axial-vector interactions** have couplings between opposite handedness (f_L and f_R, or f_L and $\bar{f}_L$, etc)

Each interaction can be split up into a vector part (c_V) and an axial-vector part (c_A). If $c_V = 0, c_A \neq 0$ or $c_A = 0, c_V \neq 0$, parity is conserved.
If $c_A = c_V$, the so-called $V + A$ interaction, only $f_R - \bar{f}_L$ couplings exist and $f_L - \bar{f}_R$ couplings are forbidden.
If $c_A = -c_V$, the so-called $V - A$, only $f_L - \bar{f}_R$ interact, and $f_R - \bar{f}_L$ is forbidden.
$V - A$ and $V + A$ interactions maximally violate parity. In the standard model:

Force	Carrier	Coupling
EM	γ	$c_A = 0$
Strong	g	$c_A = 0$
Weak	$W^{\pm}$	$c_A = -c_V$
	Z	$c_A \neq c_V \neq 0$

For massive particles,

$$f_R = c|h=1\rangle + c'|h=-1\rangle \qquad f_L = c|h=-1\rangle + c'|h=1\rangle \tag{16.7}$$
$$c = \frac{1}{2}\left(1 + \frac{|\vec{p}|}{E+m}\right) \qquad c' = \frac{1}{2}\left(1 - \frac{|\vec{p}|}{E+m}\right) \tag{16.8}$$

16.2 Parity Violations

Consider a negative pion, π^-, decaying into a leption and a lepton neutrino. Since the neutrino is massless, it has to be right-handed, and since the pion is at rest with spin zero, the lepton has negative neutrino momentum and therefore must be right-handed also.

$$(\bar{\nu}_l) = (\bar{\nu}_l)_R \qquad\qquad (l^-) = (l^-)_L \tag{16.9}$$

but $h(l_L^-) = -1$ is not possible, because of spin conservation. Therefore, the $h(l_L^-) = +1$ part takes part in the interaction.

$$e_L^- = c_e|h=-1\rangle + c_e'|h=+1\rangle \qquad\qquad \mu_L^- = c_\mu|h=-1\rangle + c_\mu'|h=+1\rangle \tag{16.10}$$

$$c_e' = \frac{1}{2}\left(1 - \frac{|\vec{p}|}{E+m_e}\right) \approx 0.00377 \qquad\qquad c_\mu' = \frac{1}{2}\left(1 - \frac{|\vec{p}|}{E+m_\mu}\right) \approx 0.44 \tag{16.11}$$

So the decay of a pion into an electron and its anti-neutrino is much more suppressed compared to the decay into a muon and its anti-neutrino. The branching fraction shows this:

$$B(\pi^- \to e^-\bar{\nu}_e) \approx 0.0123\% \tag{16.12}$$

$$B(\pi^- \to \mu^-\bar{\nu}_\mu) \approx 99.9\% \tag{16.13}$$

16.3 Muon Decay

Consider a muon decaying into a muon neutrino, an anti-electron neutron, and an electron. Neutrinos are massless so are always left-handed, and anti-neutrinos are always right-handed. The electron must be left-handed, $h(e_L^-) = \pm 1$, depending on which direction it goes, but $h = -1$ is much more dominant.