QCD Lagrangian

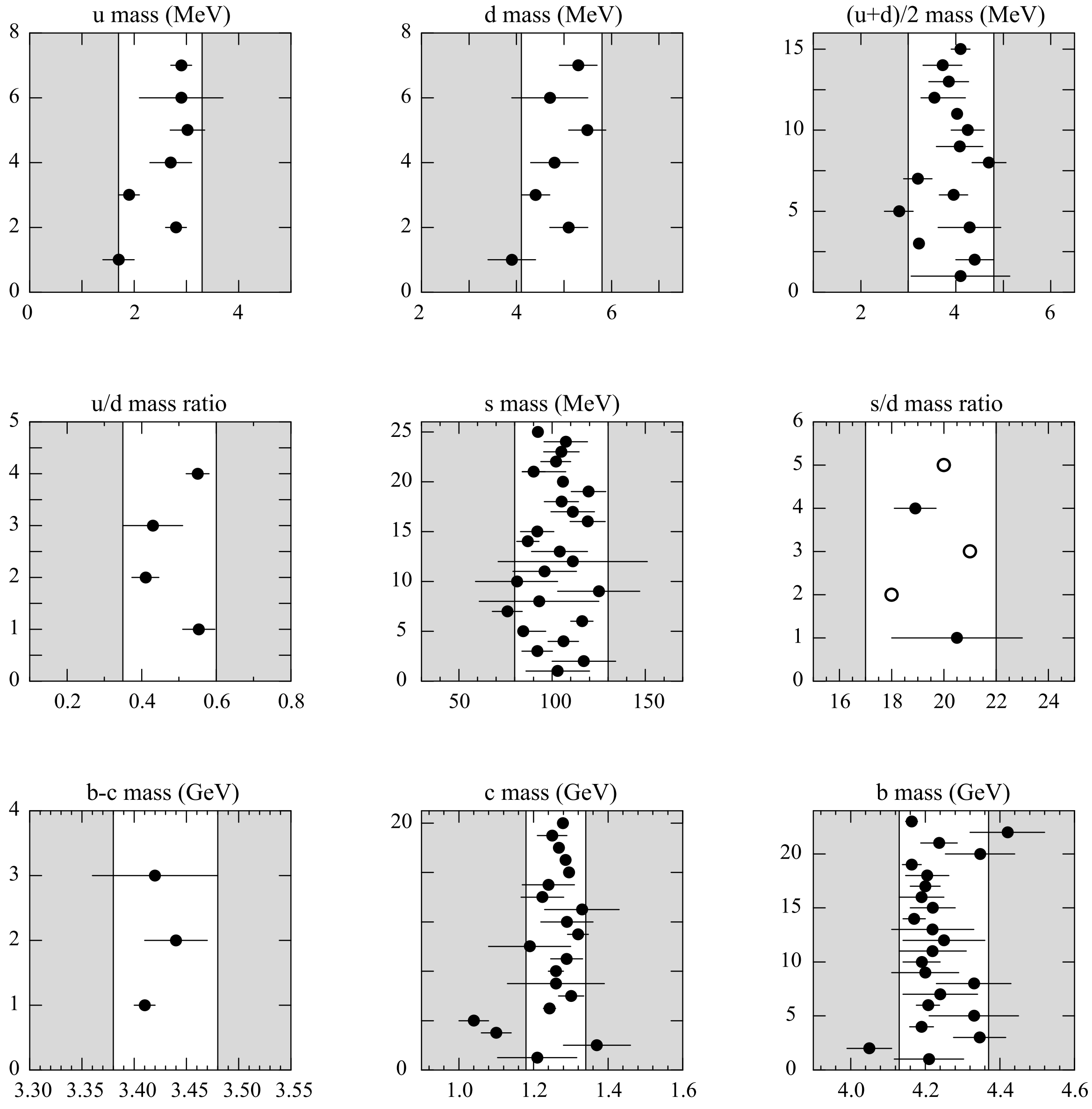
$$\mathcal{L}_{QCD} = \sum_q \left(\bar{\psi}_{qi} i\gamma^\mu \left[\delta_{ij}\partial_\mu + ig\left(G_\mu^\alpha t_\alpha\right)_{ij} \right]\psi_{qj} - m_q \bar{\psi}_{qi}\psi_{qi} \right) - \frac{1}{4} G_{\mu\nu}^\alpha G_\alpha^{\mu\nu}$$

$$\boxed{\mathcal{L}_{QED} = \bar{\psi}_e i\gamma^\mu \left[\partial_\mu + ieA_\mu\right]\psi_e - m_e\bar{\psi}_e\psi_e - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}}$$

- $G_\alpha^{\mu\nu} = \partial^\mu G_\alpha^\nu - \partial^\nu G_\alpha^\mu - gf^{\alpha\beta\gamma}G_\beta^\mu G_\gamma^\nu$ color fields tensor
- G_α^μ four potential of the gluon fields (α=1,..8)
- t_α 3x3 Gell-Mann matrices; generators of the SU(3) color group
- $f^{\alpha\beta\gamma}$ structure constants of the SU(3) color group
- ψ_i Dirac spinor of the quark field (*i* represents color)
- $g = \sqrt{4\pi\alpha_s}\ (\hbar = c = 1)$ color charge (strong coupling constant)

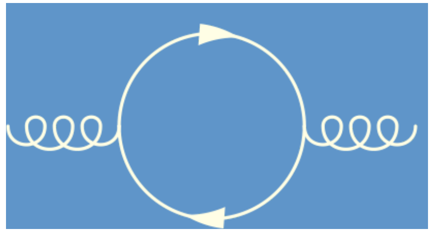
- The quarks have three basic color-charge states, which can be labeled as *i*=red, green, and blue. Three color states form a basis in a 3-dimensional vector space. A general color state of a quark is then a vector in this space. The color state can be rotated by 3 × 3 unitary matrices. All such unitary transformations with unit determinant form a Lie group SU(3).
- A crucial difference between the QED and QCD is that the gluon field tensors contain the additional term representing interaction between color-charged gluons.
- While sources of the electromagnetic field depend on currents that involve a small parameter, gluons are sources of the color field without any small parameter. Gluons are not only color-charged, but they also produce very strong color fields.

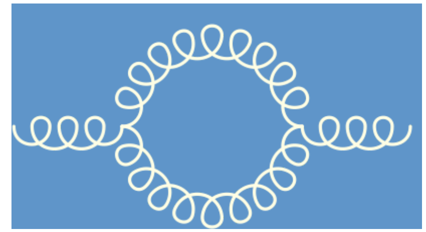
Quark masses



PDG, 2010

Vacuum polarization in QCD





The left diagram is shared by QED and QCD which renders the interaction stronger at shorter distance (screening). The second diagram arising from the nonlinear interaction between gluons in QCD has the antiscreening effect, which makes the coupling weaker at short distance.

- Color is anti-screened
- Color builds up away from a source
- Interaction becomes strong at large distances (low momenta)
- Confinement of quarks; quarks are not observed as isolated particles

Strong coupling constant $\alpha_s = \frac{g^2}{4\pi}$

In quantum field theory, the coupling constant is an effective constant, which depends on four-momentum Q^2 transferred. For strong interactions, the Q^2 dependence is very strong (gluons - as the field quanta - carry color and they can couple to other gluons). A first-order perturbative QCD calculation (valid at very large Q^2) gives:

running coupling constant!

$$\alpha_s(Q^2) = \frac{12\pi}{(22 - 2n_f) \cdot \ln(Q^2 / \Lambda^2_{\mathrm{QCD}})}$$

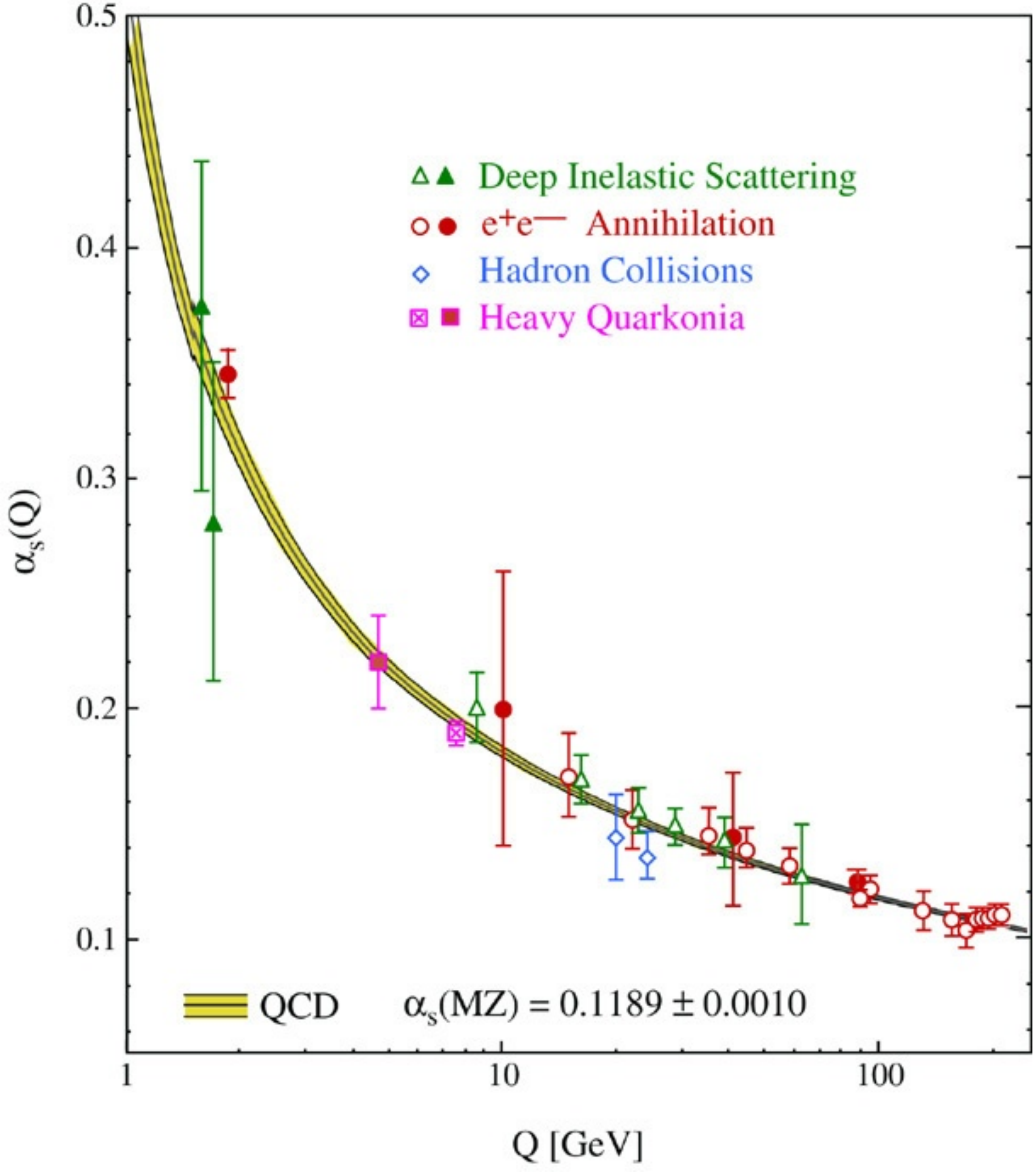
n_f =6 – number of quark flavors
Λ_{QCD} – a parameter in QCD (~0.22 GeV), an infrared cutoff

The spatial separation between quarks goes as

$$\lambda\!\!\!^{-} = \frac{\hbar}{\sqrt{Q^2}}$$

Therefore, for very small distances and high values of Q^2, the inter-quark coupling decreases, vanishing asymptotically. In the limit of very large Q^2, quarks can be considered to be "free" (asymptotic freedom). On the other hand, at large distances, the inter-quark coupling increases so it is impossible to detach individual quarks from hadrons (confinement).

Asymptotic freedom was described in 1973 by Gross, Wilczek, and Politzer (Nobel Prize 2004).



It is customary to quote α_s at the 91 GeV energy scale (the mass of the Z boson)

Chiral symmetry

For massless quarks, QCD Lagrangian preserves helicity. Indeed, since a massless quark travels at the speed of light, the handedness or chirality of the quark is independent of any Lorentz frame from which the observation is made.

$$\mathcal{L}_{QCD} = \mathcal{L}_{QCD}(\psi_L) + \mathcal{L}_{QCD}(\psi_R)$$

the QCD interaction does not couple the left and right-handed quarks

The mass term explicitly breaks the chiral symmetry as:

$$m_q \bar{\psi}_q \psi_q = m_q \bar{\psi}_{qL} \psi_{qR} + m_q \bar{\psi}_{qR} \psi_{qL}$$

The main origin of the chiral symmetry breaking, however, may be described in terms of the fermion condensate (vacuum condensate of bilinear expressions involving the quarks in the QCD vacuum) formed through nonperturbative action of QCD gluons.

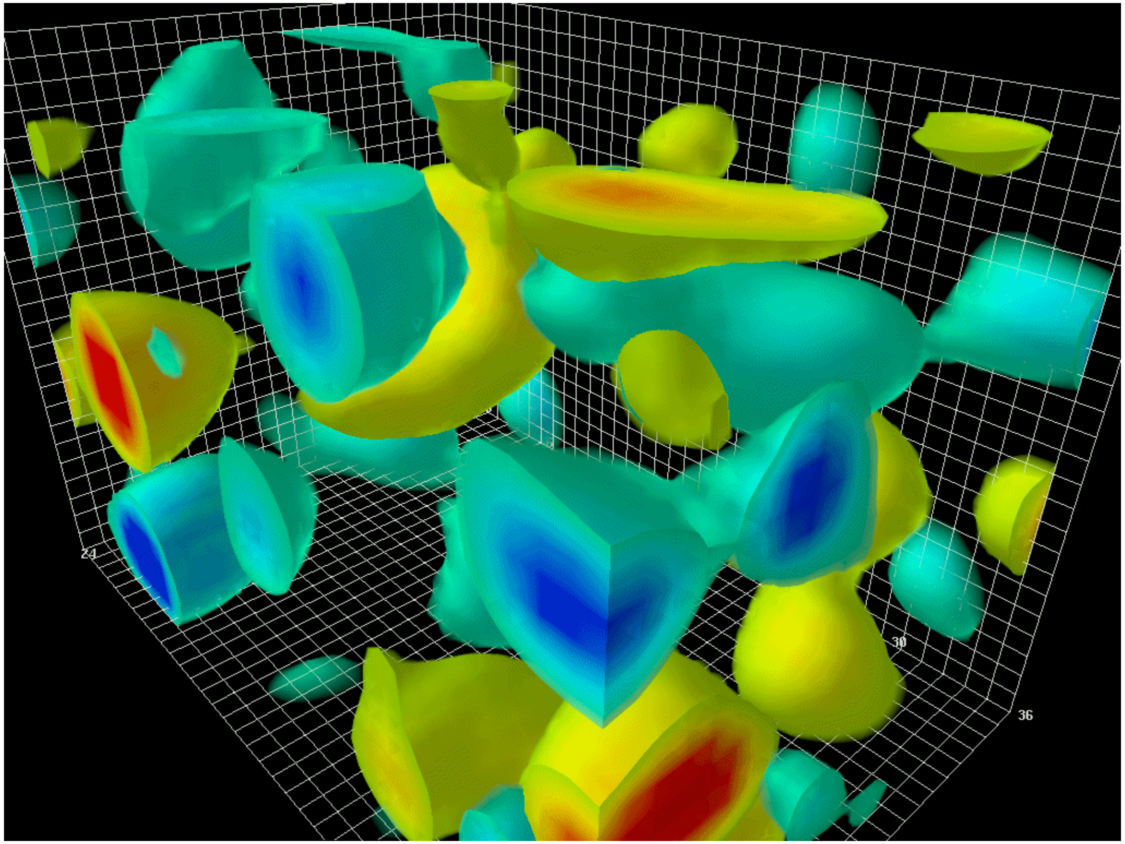
Spontaneous symmetry breaking due to the strong low-energy QCD dynamics, which rearranges the QCD vacuum:

$$\left\langle \bar{\psi}_{qL} \psi_{qR} \right\rangle \propto \Lambda_{QCD}^3 \neq 0$$

QCD vacuum

In QED vacuum polarization effects are extremely weak, because the electron has a small charge and a non-zero rest mass. On the other hand, the QCD gluons are massless, and their strong interaction is not damped by a small parameter. As a result, the QCD vacuum polarization effect is extremely strong, and the empty space is not empty at all - it must contain a soup of spontaneously appearing, interacting, and disappearing gluons. Moreover, in the soup there also must be pairs of virtual quark-antiquark pairs that are also color-charged, and emit and absorb more virtual gluons. It turns out that the QCD ground state of an "empty" space is extremely complicated. At present, we do not have any glimpse of a possibility to find the vacuum wave function analytically. Some ideas of what happens are provided by the QCD lattice calculations, in which the gluon and quark fields are discretized on a four-dimensional lattice of space-time points, and the differential field equations are transformed into finite-difference equations solvable on a computer.

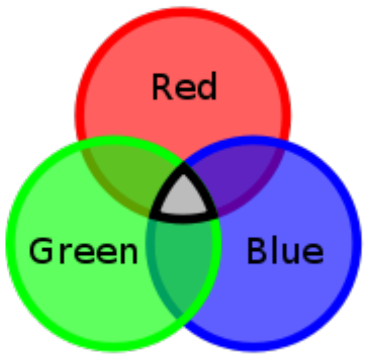
http://www.physics.adelaide.edu.au/theory/staff/leinweber/VisualQCD/Nobel/index.html

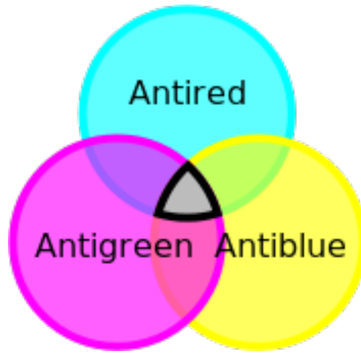


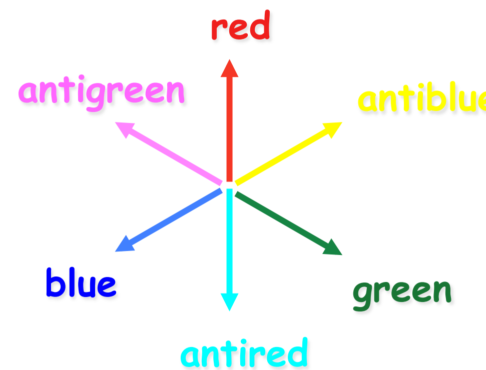
The typical four-dimensional structure of gluon-field configurations averaged over in describing the vacuum properties of QCD. The volume of the box is 2.4 by 2.4 by 3.6 fm, big enough to hold a couple of protons.

Color, Gluons

Gluons are the exchange particles which couple to the color charge . They carry simultaneously color and anticolor.







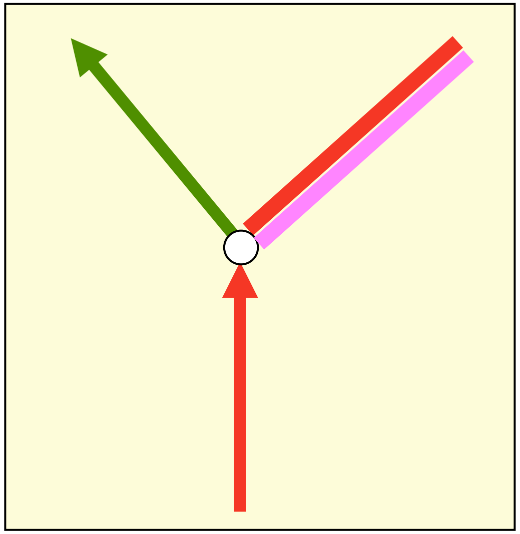
What is the total number of gluons? According to SU_3, 3x3 color combinations form a singlet and an octet. The octet states form a basis from which all other color states can be constructed. The way in which these eight states are constructed from colors and anticolors is a matter of convention. One possible choice is:

$$|R\bar{G}\rangle,\ |R\bar{B}\rangle,\ |G\bar{B}\rangle,\ |G\bar{R}\rangle,\ |B\bar{R}\rangle,\ |B\bar{G}\rangle,$$
$$\sqrt{1/2}\left(|R\bar{R}\rangle - |G\bar{G}\rangle\right),\ \sqrt{1/6}\left(|R\bar{R}\rangle + |G\bar{G}\rangle - 2|B\bar{B}\rangle\right)$$

The color singlet:

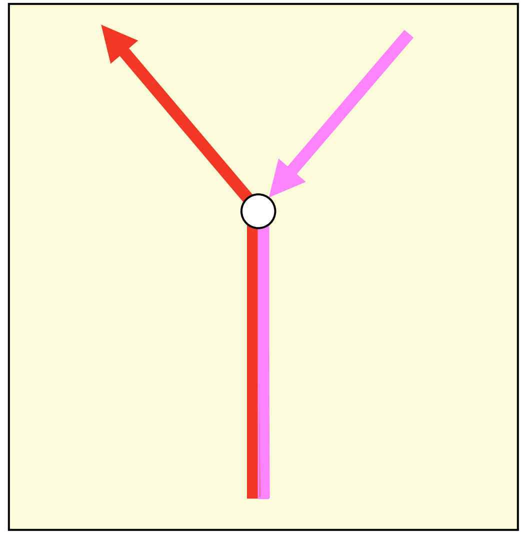
$$\sqrt{1/3}\left(|R\bar{R}\rangle + |G\bar{G}\rangle + |B\bar{B}\rangle\right)$$

is invariant with respect of a re-definition of the color names (rotation in color space). Therefore, it has no effect in color space and cannot be exchanged between color charges.



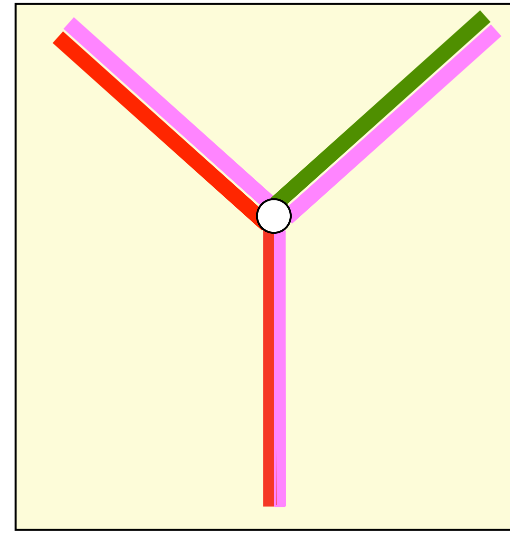
emission of a gluon
by a quark

$$q \rightarrow q + g$$



splitting of a gluon
into a quark-antiquark pair

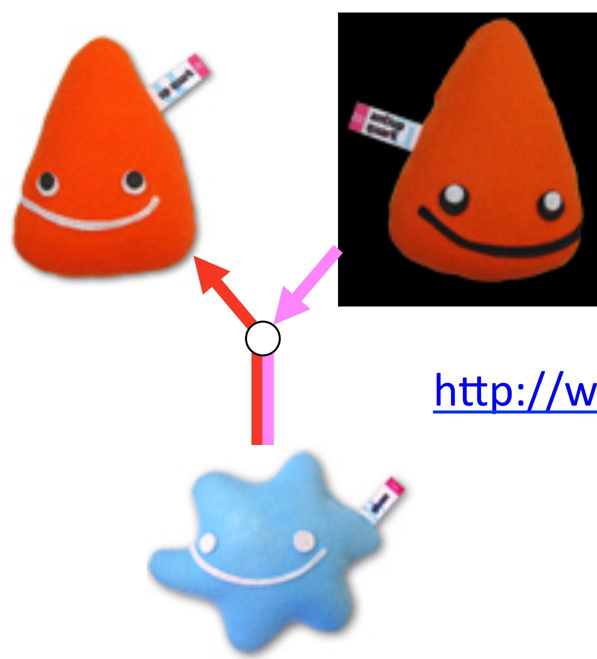
$$g \rightarrow q + \overline{q}$$

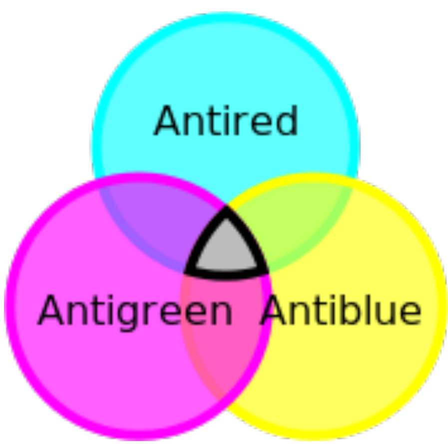


self-coupling of gluons

$$g \rightarrow g + g$$

$$g + g \rightarrow g + g$$





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