

The Strong Interaction and LHC phenomenology

Juan Rojo
STFC Rutherford Fellow
University of Oxford

Theoretical Physics Graduate School course

Lecture 2:

The QCD Lagrangian, Symmetries and Feynman Rules

The QED Lagrangian - recap

From the **QFT-II course** you know the form of the QED Lagrangian for a single fermion with mass m :

$$\mathcal{L}_{\text{QED}} = \bar{\psi} (i\gamma^\mu D_\mu - m) \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

where the **field-strength tensor** (classical electrodynamics) is defined from the four-vector potential

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

and the **covariant derivative** is given by

$$D_\mu = \partial_\mu + ieA_\mu$$

The form of the QED Lagrangian is the one required by **gauge invariance with respect to the abelian U(1) group gauge transformations**, where fermions and photons transform as

$$\psi(x) \rightarrow \psi'(x) = e^{i\phi(x)} \psi(x)$$

$$A^\mu \rightarrow A'^\mu = A^\mu + \partial^\mu \chi(x)$$

Exercise: show that the fermion sector is invariant under U(1) if:

$$\chi(x) = -\frac{\phi(x)}{e}$$

The field-strength tensor is also gauge invariant, so the full Lagrangian also

Exercise: show that the covariant derivative transforms as the fermion field (hence the name *covariant*)

Feynman rules in QED

Incoming electron

$$\begin{array}{c} \longrightarrow \\ p \rightarrow \end{array} \bullet = u_{\alpha}(p, s)$$

Outgoing electron

$$\bullet \longrightarrow \begin{array}{c} p \rightarrow \end{array} = \bar{u}_{\alpha}(p, s),$$

Incoming positron

$$\begin{array}{c} \longleftarrow \\ p \rightarrow \end{array} \bullet = \bar{v}_{\alpha}(p, s),$$

Outgoing positron

$$\bullet \longleftarrow \begin{array}{c} p \rightarrow \end{array} = v_{\alpha}(p, s).$$

Incoming photon

$$\begin{array}{c} \sim \\ k \rightarrow \end{array} \bullet = e_{\mu}(k, \lambda)$$

Outgoing photon

$$\bullet \begin{array}{c} \sim \\ k \rightarrow \end{array} = e_{\mu}^{*}(k, \lambda)$$

$\lambda \rightarrow$ photon polarization

Fermion
propagator

$$\bar{\Psi}_{\alpha} \bullet \begin{array}{c} \longleftarrow \\ q \end{array} \bullet \Psi_{\beta} = \left[\frac{i}{\not{q} - m + i0} \right]_{\alpha\beta}$$

Photon propagator
(Feynman gauge)

$$A^{\mu} \bullet \begin{array}{c} \sim \\ q \rightarrow \end{array} \bullet A^{\nu} = \frac{-ig^{\mu\nu}}{q^2 + i0}.$$

Electron-photon vertex

$$\begin{array}{c} \mu \sim \end{array} \bullet \begin{array}{l} \nearrow \beta \\ \searrow \alpha \end{array} = (+ie\gamma^{\mu})_{\beta\alpha}.$$

In QED, as will be the case in QCD, gauge-fixing is required, but results independent of choice of gauge

From QED to QCD

The QCD Lagrangian has the **same structure** as the QED Lagrangian but the invariance is now with respect to the **non-abelian group SU(3)**

Formally they are very similar, in practice the two theories are **extremely different**

Main difference is **non-abelian nature of the gauge group**, which implies that **gluons have self-interactions**, as opposed to QED, where photons couple only to fermions

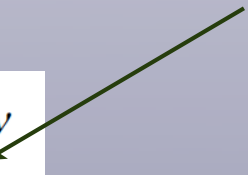
In QED the coupling constant grows with the energy, while in **QCD it decreases** (asymptotic freedom, as found in deep-inelastic scattering and electron-positron data)

QED is perturbative for all relevant energies, while in QCD the coupling becomes strong, and hence the **theory becomes non-perturbative, for scales < 1 GeV**

We now present the **QCD Lagrangian** and compare it in detail with the QED one

$$\mathcal{L}_{\text{QED}} = \bar{\psi} (i\gamma^\mu D_\mu - m) \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

Quark and gluon fields have new degree of freedom: color



$$\mathcal{L}_{\text{QCD}} = \sum_f \bar{\psi}_i^{(f)} \left(i\gamma_\mu D_{ij}^\mu - m_f \delta_{ij} \right) \psi_j^{(f)} - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu}$$

Though the formal structure is the same as in QED, the new color degrees of freedom lead to a **strikingly different theory**

The SU(3) group

SU(N) is the special unitary groups of degree n: group of $n \times n$ unitary matrices with **determinant = 1**

$$U^\dagger U = 1, \quad \det U = 1.$$

The **Lie algebra** of these groups is defined by the commutation relations of the generators of the group

$$[t^A, t^B] = if^{ABC} t^C$$

An important representation of the group is the **fundamental representation**, with the so-called **Gell-Mann matrices**, which are traceless and Hermitian. In the case of SU(3)

$$\begin{aligned} \lambda_1 &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \lambda_2 &= \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \lambda_3 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\ \lambda_4 &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} & \lambda_5 &= \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} & \lambda_6 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \\ \lambda_7 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} & \lambda_8 &= \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}. \end{aligned}$$

$$t^A \equiv \frac{1}{2} \lambda^A$$

The QCD Lagrangian

Let us examine in detail the QCD Lagrangian:

$$\mathcal{L}_{\text{QCD}} = \sum_f \bar{\psi}_i^{(f)} \left(i\gamma_\mu D_{ij}^\mu - m_f \delta_{ij} \right) \psi_j^{(f)} - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu}$$

$i, j = 1, 3$ -> Color indices in the **fundamental representation** of SU(3)

$a = 1, 8$ -> Color index in the **adjoint representation** of SU(3)

The covariant derivative looks similar to the QED one, now acting in **color space** as well

$$D_{ij}^\mu = \partial_\mu \delta_{ij} + i g_s t_{ij}^a A_a^\mu$$

Generators of SU(3)

The field-strength tensor for QCD has a crucial difference wrt QED: the **self-interaction of gauge bosons**

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g_s f_{abc} A_\mu^b A_\nu^c$$

- ☑ All the interactions in QCD are proportional to the **strong coupling constant**, g_s
- ☑ QCD interactions do not distinguish between quark flavors
- ☑ The structure of the QCD Lagrangian is fixed by the requirement of the **invariance under SU(3)**

Gauge invariance

The structure of QCD is fully defined by the requirement of **SU(3) local gauge transformations**: the physical content of the theory is unchanged if quark and gluon fields transform under SU(3), and this transformation can be **different for each space-time point**

Quarks transform in the fundamental representation of SU(3):

$$\psi_i^{(f)} \rightarrow \psi_i^{(f)'} = U_{ij}(x) \psi_j^{(f)}$$

$$U_{ij}(x) = \exp(i\theta^a(x)t_{ij}^a)$$

Since fermion sector of the Lagrangian must be independently gauge invariant (because in principle there is an arbitrary number of fermions n_f) then the **covariant derivative** must transform like quark fields

$$D_{ij}^\mu \psi_j \rightarrow (D_{ij}^\mu \psi_j)' = U_{ik}(x) D_{kj}^\mu \psi_j$$

This condition determines the transformation property of the **gluon field under SU(3)**

$$t^a A_a^\mu \rightarrow t^a A_a^{\prime\mu} = U(x) t^a A_a^\mu U^{-1}(x) + \frac{1}{g_s} (\partial^\mu U(x)) U^{-1}(x)$$

Exercise: check that with this transformation the fermion sector of the QCD Lagrangian is gauge invariant
Special care must be taken with the fact that SU(3) matrices **do not commute**, as opposed to the QED case

Gauge invariance

Therefore, the gauge invariance of the fermion sector of the QCD Lagrangian requires **gluons to transform**

$$t^a A_a^\mu \rightarrow t^a A_a'^\mu = U(x) t^a A_a^\mu U^{-1}(x) + \frac{1}{g_s} (\partial^\mu U(x)) U^{-1}(x)$$

Let us now check that the purely gauge sector is also gauge invariant under this transformation

$$\mathcal{L}_{\text{QCD}} = \sum_f \bar{\psi}_i^{(f)} \left(i \gamma_\mu D_{ij}^\mu - m_f \delta_{ij} \right) \psi_j^{(f)} - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu}$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g_s f_{abc} A_\mu^b A_\nu^c$$

An useful relation (**exercise**: derive this expression) is provided by the commutator of two covariant derivs

$$[D_\mu, D_\nu] = i g_s t^a F_{\mu\nu}^a$$

(Recall that covariant derivatives are **operators** acting on fermion fields)

and since we just showed that under SU(3) transformations the **covariant derivative transforms as the quark field**, we have that

$$t^a F_{\mu\nu}^a \rightarrow t^a F_{\mu\nu}'^a = U(x) t^a F_{\mu\nu}^a U^{-1}(x)$$

Therefore, the gluonic sector is always gauge invariant since we can write, using

$$\text{Tr} [t^a t^b] = \frac{1}{2} \delta^{ab}$$

$$-\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} = -\frac{1}{2} F_{\mu\nu}^a F^{\mu\nu,b} \text{Tr} [t^a t^b] = -\frac{1}{2} \text{Tr} [F_{\mu\nu}^a t^a F^{\mu\nu,b} t^b]$$

trace is cyclic operator

Gauge invariance

Therefore, the gauge invariance of the fermion sector of the QCD Lagrangian requires **gluons to transform**

$$t^a A_a^\mu \rightarrow t^a A_a'^\mu = U(x) t^a A_a^\mu U^{-1}(x) + \frac{1}{g_s} (\partial^\mu U(x)) U^{-1}(x)$$

Let us now check that the purely gauge sector is also gauge invariant under this transformation

$$\mathcal{L}_{\text{QCD}} = \sum_f \bar{\psi}_i^{(f)} \left(i \gamma_\mu D_{ij}^\mu - m_f \delta_{ij} \right) \psi_j^{(f)} - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu}$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g_s f_{abc} A_\mu^b A_\nu^c$$

An useful relation (**exercise**: derive this expression) is provided by the commutator of two covariant derivs

$$[D_\mu, D_\nu] = i g_s t^a F_{\mu\nu}^a$$

and since we just showed that under SU(3) transformations the covariant derivative transforms as the quark field, we have that

$$t^a F_{\mu\nu}^a \rightarrow t^a F_{\mu\nu}'^a = U(x) t^a F_{\mu\nu}^a U^{-1}(x)$$

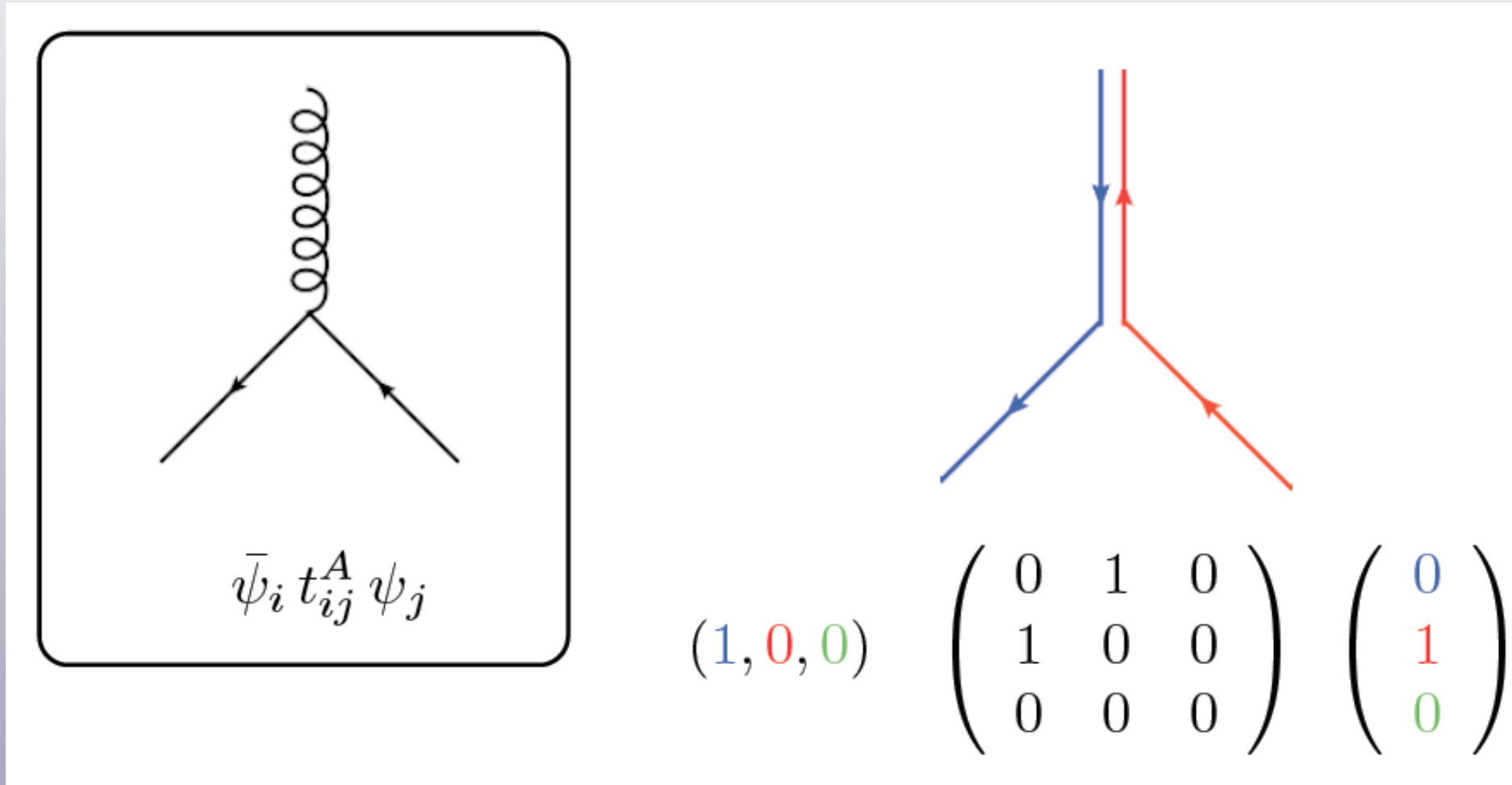
So also the **purely gauge sector of the QCD Lagrangian** is invariant under SU(3) transformations

Note that as opposed to QED, the field-strength tensor itself is **not** gauge invariant

As in QED, a **mass term for the gluon** is forbidden by gauge invariance (**exercise**: check explicitly)

Color Flow

The various contractions of color indices in the QCD Lagrangian can be interpreted as **color flows** between the different types of fields



In this specific example, an incoming quark with **red color** emits a gluon and is transformed into an outgoing quark with **blue color**

Gluons carry both **color** and **anti-color**: they change the color charge of quarks and of other gluons

Feynman rules in QCD

Now we turn to the **Feynman rules** derived from the QCD Lagrangian, in analogy with the QED ones

$$\mathcal{L}_{\text{QCD}} = \sum_f \bar{\Psi}_i^{(f)} \left(i\gamma_\mu D_{ij}^\mu - m_f \delta_{ij} \right) \Psi_j^{(f)} - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu}$$

The quark and gluon propagators are obtained from the **non-interacting Lagrangian**

Same recipe as in QED: **replace $i\partial$ with k** and take the inverse

$$\mathcal{L}_{\text{q,free}} = \sum_f \bar{\psi}_i^{(f)} (i\partial - m_f) \delta_{ij} \psi_j^{(f)}$$


$$\begin{array}{c} \alpha, i \\ \xrightarrow{k, m} \beta, j \end{array} = \left(\frac{i}{k - m} \right)_{\alpha\beta} \delta_{ij}$$

However this does not work for the gluon, since inverse does not exist

$$\mathcal{L}_{\text{g,free}} = \frac{1}{2} A^\mu (\Box g_{\mu\nu} - \partial_\mu \partial_\nu) A^\nu$$

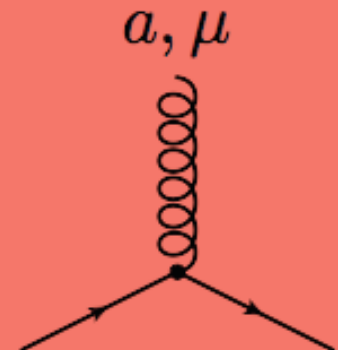
Exercise: check this is the form of the QCD Lagrangian and verify that it is non-invertible

Need to **fix a gauge** to obtain the gluon propagator. In the **covariant gauges** it reads

$$\frac{-i}{k^2} \left(g_{\mu\nu} - (1-\xi) \frac{k_\mu k_\nu}{k^2} \right) \delta^{ab} =$$
A Feynman diagram representing a gluon propagator. It consists of two vertices connected by a horizontal wavy line. The left vertex is labeled \$a, \mu\$ and the right vertex is labeled \$b, \nu\$. Below the wavy line is a right-pointing arrow labeled \$k\$, indicating the momentum flow.

All gauge dependence must explicitly cancel in the final calculations
Same as in QED

Feynman rules in QCD

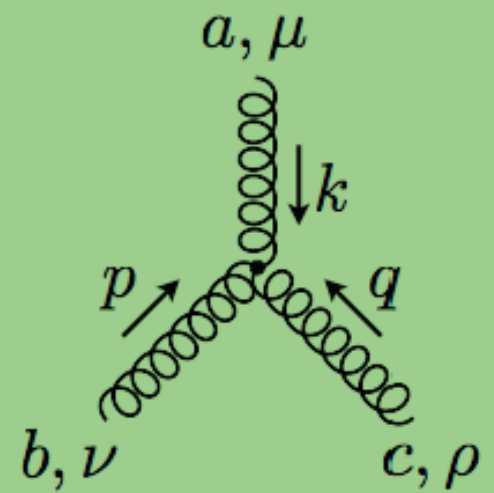


$$= i g_s \gamma^\mu t^a$$

The **quark-gluon vertex** in QCD is analogous to the fermion-photon vertex in QED

The gluon mixes the color charge of the incoming and outgoing quark

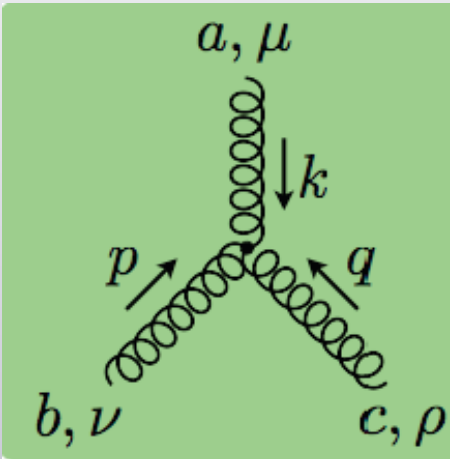
The **genuine non-abelian interactions** are the triple and quartic couplings between the gluons



$$= g_s f^{abc} \left[g^{\mu\nu} (k - p)^\rho + g^{\nu\rho} (p - q)^\mu + g^{\rho\mu} (q - k)^\nu \right]$$

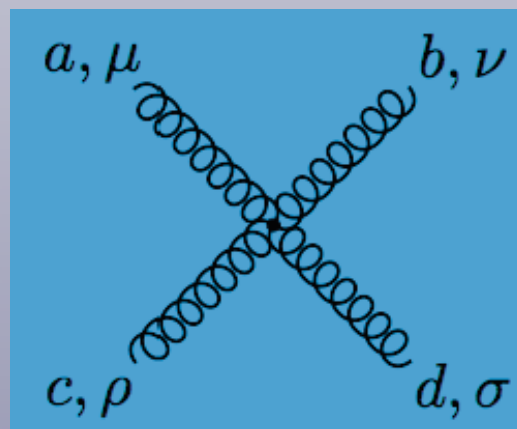
Feynman rules in QCD

The **genuine non-abelian interactions** are the triple and quartic couplings between the gluons



$$= g_s f^{abc} \left[g^{\mu\nu} (k - p)^\rho + g^{\nu\rho} (p - q)^\mu + g^{\rho\mu} (q - k)^\nu \right]$$

The **non-abelian nature of QCD** is shown by the presence of the SU(3) structure constants f_{ABC} in the Feynman rules: these structure constants arise from the **non-commutativity** of SU(3) matrices



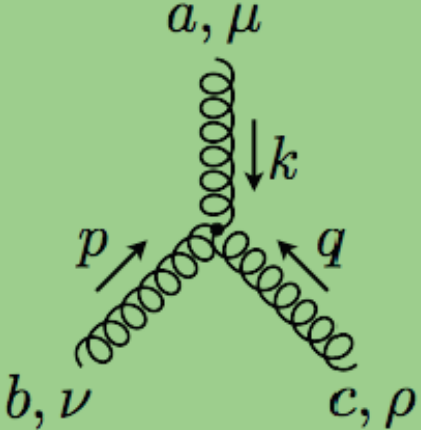
$$= -ig_s^2 \left[f^{abe} f^{cde} (g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}) + f^{ace} f^{bde} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\rho}) + f^{ade} f^{bce} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma}) \right]$$

The structure of these Feynman rules can be traced back directly to the **QCD field-strength tensor**

Feynman rules in QCD

The structure of these Feynman rules can be traced back directly to the **QCD field-strength tensor**

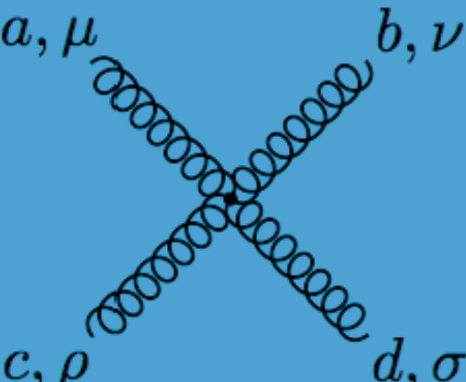
$$F_a^{\mu\nu} F_{\mu\nu}^a \rightarrow \dots + g_S^2 f_{abc} f_{ade} A^{\mu,b} A^{\nu,c} A_\mu^d A_\nu^e \\ - g_S f_{abc} A^{\mu,b} A^{\nu,c} [\partial_\mu A_\nu^a - \partial_\nu A_\mu^a] - g_S f_{abc} A_\mu^b A_\nu^c [\partial^\mu A^{\nu,a} - \partial^\nu A^{\mu,a}]$$



$$= g_s f^{abc} \left[g^{\mu\nu} (k - p)^\rho + g^{\nu\rho} (p - q)^\mu + g^{\rho\mu} (q - k)^\nu \right]$$

The **three-gluon vertex** is proportional to the **four-momentum** of one of the legs

The **four-gluon vertex** Feynman rule accounts for all possible permutations allowed by symmetry



$$= -ig_s^2 \left[f^{abe} f^{cde} (g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}) + f^{ace} f^{bde} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\rho}) + f^{ade} f^{bce} (g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma}) \right]$$

Feynman rules in QCD

The Feynman rules in QCD can also be interpreted as **governing the (conserved) color flow** between the incoming and outgoing legs of a diagram

$$\begin{array}{c} \text{gluon line} \\ \mu(j_1, i_1) \quad \nu(j_2, i_2) \end{array} = \frac{-ig^{\mu\nu}}{p^2} \delta_{j_1}^{i_2} \delta_{j_2}^{i_1} \quad \begin{array}{c} i_1 \text{---} j_2 \\ j_1 \text{---} i_2 \end{array}$$

$$\begin{array}{c} \text{photon line} \\ \mu(j_1, i_1) \quad \nu(j_2, i_2) \end{array} = -\frac{1}{N} \frac{-ig^{\mu\nu}}{p^2} \delta_{j_1}^{i_1} \delta_{j_2}^{i_2} \quad \begin{array}{c} i_1 \text{---} j_2 \\ j_1 \text{---} i_2 \end{array}$$

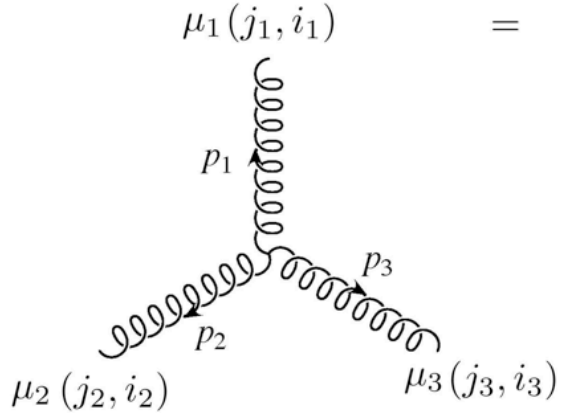
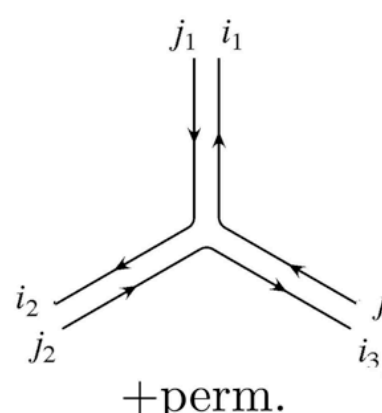
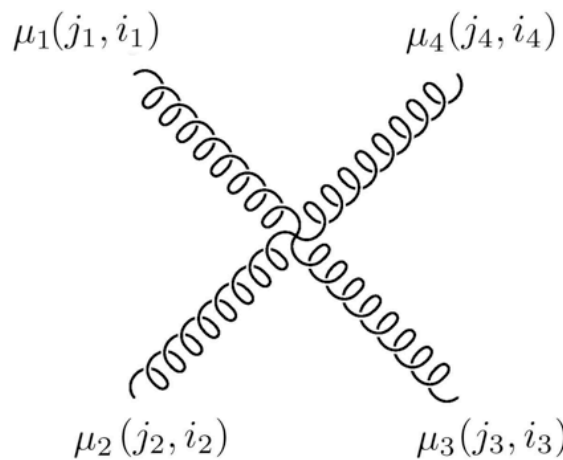
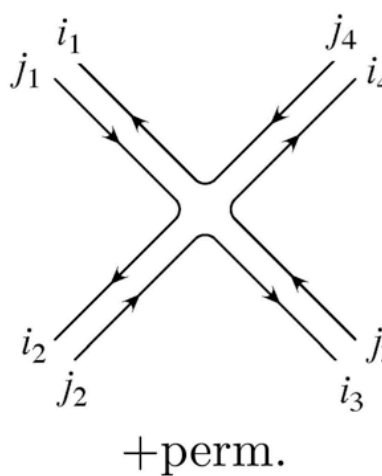
$$\begin{array}{c} \text{gluon vertex} \\ \mu(j_1, i_1) \\ i_q \text{---} j_q \end{array} = -i \frac{g}{\sqrt{2}} \gamma^\mu \delta_{j_1}^{i_q} \delta_{j_q}^{i_1} \quad \begin{array}{c} j_1 \text{---} i_1 \\ i_q \text{---} j_q \end{array}$$

Note that **color lines are never interrupted** (note the delta functions in color space)

For the propagation of a color-singlet object (like the photon), **color-anticolor annihilation** must take place

Feynman rules in QCD

The color-flow interpretation is also valid for the **gluon self-interactions**

	$= -i \frac{g}{\sqrt{2}} \left[(p_2 - p_1)_\mu g_{\mu_1 \mu_2} \right. \\ + (p_3 - p_2)_{\mu_1} g_{\mu_2 \mu_3} \\ + (p_1 - p_3)_{\mu_2} g_{\mu_3 \mu_1} \left. \right] \\ \times \delta_{j_3}^{i_1} \delta_{j_2}^{i_3} \delta_{j_1}^{i_2} \\ + \text{perm.}$	
	$= i \frac{g^2}{2} \left[2 g_{\mu_1 \mu_3} g_{\mu_2 \mu_4} \right. \\ - g_{\mu_1 \mu_2} g_{\mu_3 \mu_4} \\ - g_{\mu_1 \mu_4} g_{\mu_2 \mu_3} \left. \right] \\ \times \delta_{j_4}^{i_1} \delta_{j_3}^{i_4} \delta_{j_2}^{i_3} \delta_{j_1}^{i_2} \\ + \text{perm.}$	

Note also that the **color structure** of the QCD amplitudes factorizes from the **space-time structure**

Symmetries of the Lagrangian

The form of the QCD Lagrangian is completely fixed by the requirement of **invariance under SU(3)**

$$\mathcal{L}_{\text{QCD}} = \sum_f \bar{\psi}_i^{(f)} \left(i\gamma_\mu D_{ij}^\mu - m_f \delta_{ij} \right) \psi_j^{(f)} - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu}$$

In addition, it was known from a long time that the strong interaction has other **approximate symmetries**. In particular, particles in the same **isospin** (isotopic spin) multiplet have similar mass and **scattering amplitudes** (Wigner-Eckart theorem). For example, protons and neutrons, or neutral and charged pions

How do these properties arise from **the QCD Lagrangian**?

In the quark model, **isospin** relates to the content in terms of up and down quarks

$$I_3 = \frac{1}{2} [(n_u - n_{\bar{u}}) - (n_d - n_{\bar{d}})]$$

So **proton** has $I=1/2$, and **neutron** has $-1/2 \rightarrow$ **protons and neutrons form a SU(2) isospin multiplet**

Also **pions** have $I=1$ (positively charged pion, quark content $u\bar{d}$), $I=0$, (neutral pion, quark content $u\bar{u} + d\bar{d}$) and $I=-1$, (negatively charged pion, quark content $\bar{u}d$)

Ultimately, isospin arises because QCD interactions are **flavor-blind**, and the (accidental) fact that **up and down quarks have very close masses**

Symmetries of the Lagrangian

Formally, an **isospin transformation** acts into the quark field as a **unitary matrix**

$$\psi_i^{(f)} \rightarrow \sum_{f'} U^{ff'} \psi_i^{(f')} \quad \longrightarrow \quad \text{SU(2) rotation in flavor space}$$

Seems similar to color transformations, but note that this is a **global transformation**, as opposed to gauge transformations that are **local** in space-time

Also, isospin acts only in the **up and down** quark flavors

Exercise: check under which conditions the (fermion sector) QCD Lagrangian is invariant under these transformations

$$\begin{aligned} \mathcal{L}_{\text{QCD}} = & \bar{\psi}_i^{(u)} \left(i\gamma_\mu D_{ij}^\mu - m_u \delta_{ij} \right) \psi_j^{(u)} + \bar{\psi}_i^{(d)} \left(i\gamma_\mu D_{ij}^\mu - m_d \delta_{ij} \right) \\ & + \sum_{f, f \neq u, d} \bar{\psi}_i^{(f)} \left(i\gamma_\mu D_{ij}^\mu - m_f \delta_{ij} \right) \psi_j^{(f)} - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} \end{aligned}$$

The transformed Lagrangian (restrict to up/down sector) is only invariant if i) **up and down quark masses vanish** or ii) **they are identical**

$$\sum_{f', f''} \sum_f (U_{f'f}^T U_{ff''}) \bar{\psi}_i^{(f')} \left(i\gamma_\mu D_{ij}^\mu - m_f \delta_{ij} \right) \psi_j^{(f'')}$$

Experimentally, we know that

$$m_{u,d} \ll \Lambda_S$$

Symmetries of the Lagrangian

In the limit of **vanishing quark masses**, the QCD Lagrangian has a larger set of **invariance properties**

In this limit, we can separate the fermion fields in left-handed and right-handed chiralities

$$\psi = \psi_R + \psi_L \quad \psi_L = \frac{1}{2}(1 - \gamma_5)\psi \quad \psi_R = \frac{1}{2}(1 + \gamma_5)\psi$$

and see that the right-handed and left-handed pieces of the Lagrangian are **separately invariant**

$$\sum_f \left(\bar{\psi}_R^{(f)} (i\gamma_\mu D^\mu) \psi_R^{(f)} + \bar{\psi}_L^{(f)} (i\gamma_\mu D^\mu) \psi_L^{(f)} \right)$$

This is a direct consequence that for **massless fermions chirality is conserved**

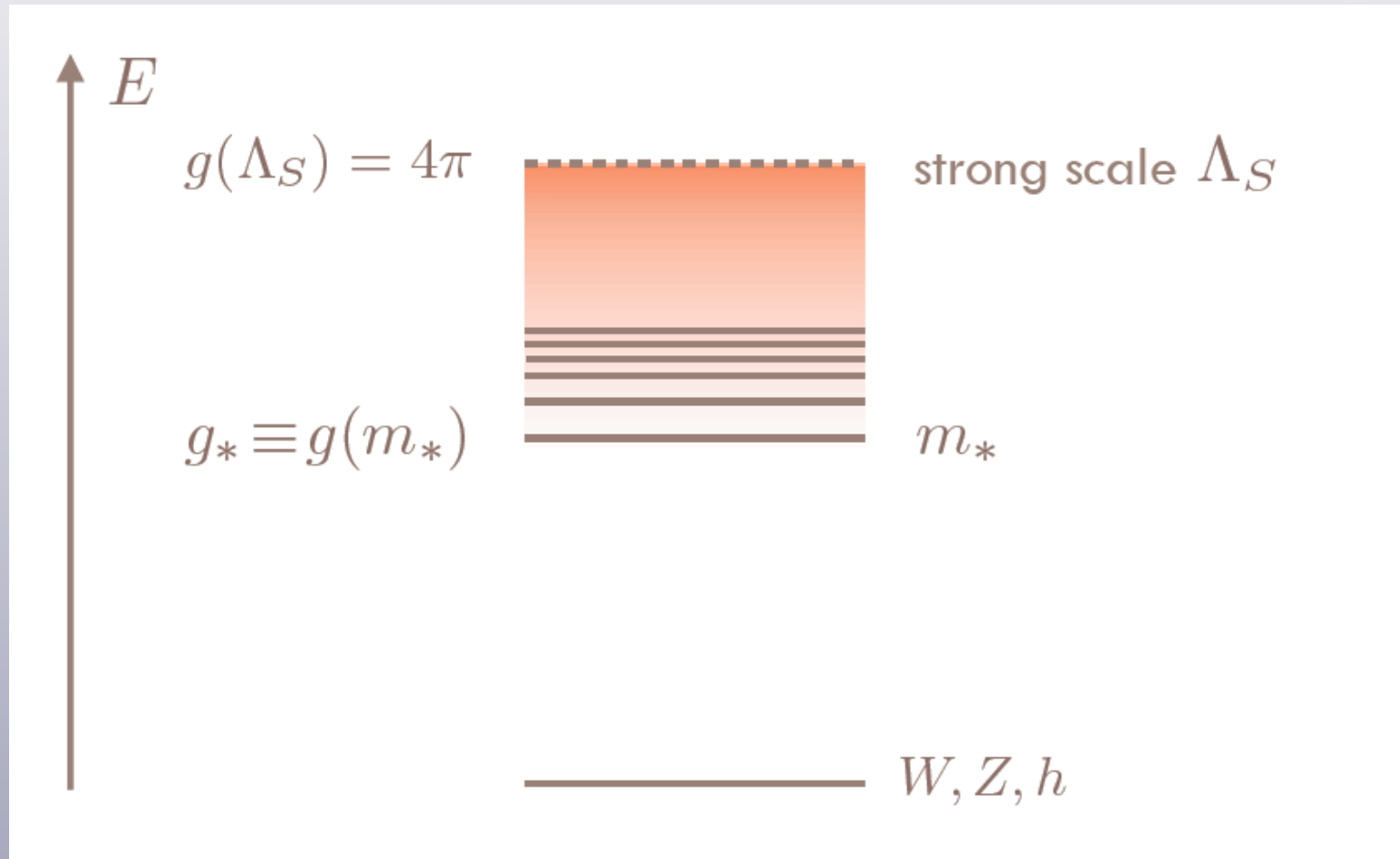
This symmetry of the QCD is known as **chiral symmetry**, and is spontaneously broken by the properties of the QCD vacuum (much as the Higgs mechanism). The **(approximately massless) pions** are the **Goldstone bosons** of the broken symmetry

This happens when the **vacuum state of the theory is not invariant** under the same symmetries as the Lagrangian. In the case of QCD it is known that

$$\langle 0 | \bar{q}q | 0 \rangle = \langle 0 | \bar{u}u + \bar{d}d | 0 \rangle \simeq (250 \text{ MeV})^3$$

Beyond QCD: Composite Higgs

Similar mechanisms to the one that break chiral symmetry in QCD have been proposed to explain the **Higgs mass lightness** in **composite Higgs** scenarios



This Higgs is the Goldstone boson of the broken symmetries of some **new strong dynamics at high scales**

These theories provide a natural explanation of the Higgs lightness

Within the **reach of the LHC** in the next years

Understanding better **QCD** is also helpful to better understand **Beyond the Standard Model** theories that are also characterized by **strong dynamics**

QCD as a non-abelian QFT

The formulation of **Quantum Chromodynamics** as a **non-abelian Quantum Field Theory** allows to:

- ✓ Describe the **hadron spectrum**
- ✓ Explain the experimentally observed symmetries of the strong interaction
- ✓ Avoid mixing between the strong and the weak interactions
- ✓ Obtain a similar field-theoretical description of the strong forces, opening the path to an unified formalism of all fundamental interactions

As we will see, the **perturbative QFT description** of the strong interactions is extremely predictive, but we understand much less the phenomena governed by **strong coupling dynamics**

- ✓ Confinement
- ✓ Hadron spectrum beyond mesons and baryons: glueballs, multi-quark hadrons
- ✓ The structure of the QCD vacuum
- ✓ The dynamical properties of quarks and gluons in the proton: energy distribution, polarization, transverse structure

In the rest of these lectures we will focus on **perturbative QCD** with the emphasis on its role at the LHC