

CP Violation In and Beyond The Standard Model: Two Higgs Doublet Model Type II Corrections to Flavour Observables

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ABSTRACT: In this preliminary report, we test the Two Higgs Doublet Model (2HDM) of Type II as an extension of the Standard Model (SM) using indicative flavour observables, with particular focus on leptonic decays of B and D mesons, $B\bar{B}$ mixing, and the $b \rightarrow s\gamma$ radiative decay. Testing the 2HDM parameter space m_{H^\pm} , $\tan\beta$ to find alignment between theoretical calculations and experiment, constraints on the parameters were found for the above flavour phenomena both individually and then as a global fit. Strongly dominated by the $b \rightarrow s\gamma$ branching ratio, the mass of a charged Higgs particle would be expected to have a minimum value of 524 GeV. Discussions on the goals of the study from here are ongoing, likely a further study into the 2HDM and its validity as a SM extension.

Contents

1	Introduction	1
1.1	The Standard Model	2
1.2	Flavour Physics and CP Violation	7
1.3	The Two Higgs Doublet Model of Type II	9
2	Probing the 2HDM Type II	10
2.1	Leptonic Decays	10
2.2	Mass mixing	11
2.3	Radiative decay	12
2.4	Input and Fits	12
3	New Observables	15
4	Statistical Fitting	15
5	Results With James	15
6	CKM Element Modification	15
7	SM4x2HDM	15
8	Next Steps	15

1 Introduction

The Standard Model is one of the most successful theories ever developed, describing the fundamental forces currently observed, excluding gravity, through a quantum field theory Lagrangian,

$$\begin{aligned}\mathcal{L} = & -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} && \rightarrow \text{gauge term} \\ & +i\bar{\Psi}\not{D}\Psi && \rightarrow \text{Fermion term} \\ & +(D_\mu\Phi)^\dagger(D^\mu\Phi) - V(\phi) && \rightarrow \text{Higgs term} \\ & -Y_{ij}\bar{\Psi}_i\Phi\Psi_j + h.c. && \rightarrow \text{Yukawa term}\end{aligned}\tag{1.1}$$

Throughout the latter half of the 20th century and through the 21st, the Standard Model has found strong agreement with many observed phenomena, while also predicting many other observables that took longer to be observed (most notably, the Higgs boson). However, there are still many observations that do not align with the Standard Model: some more large scale such as unification with gravity or a description of dark matter, some more specific such as some leptonic and semi-leptonic meson decays. The modifications and additions to the Standard Model needed for these smaller scale problems could open the door to new phenomena that could bring us closer a complete theory of nature.

1.1 The Standard Model

The Standard Model is a quantum field theory, describing all constituent particles as quantum fields permeating through space-time. There are twenty-five fundamental particles currently described in the Standard Model: twelve fermions (quarks and leptons), four gauge bosons of the electroweak theory (W^\pm, Z, γ), eight gluons of the strong force, and the Higgs boson.

The bases for the Standard Model are the quantum theories of electroweak and strong interactions, and then the symmetry-breaking Higgs mechanism which gives masses to these theories. Both the electroweak and strong theories are non-Abelian gauge theories: the electroweak theory has a gauge symmetry of $SU(2)_L \otimes U(1)_Y$, and the strong theory of $SU(3)_c$ [14].¹ To formulate the Standard Model Lagrangian, we start with the free Lagrangian of a massless Dirac fermion coupled to a gauge field tensor,

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}. \quad (1.2)$$

The Lagrangian must be formed such that it is invariant for a local gauge transformation of the fermion field, ψ :

$$\psi(x) \rightarrow U(x)\psi(x), \quad \bar{\psi}(x) \rightarrow \bar{\psi}(x)U^\dagger(x), \quad (1.3)$$

where $U(x)$ is the gauge transformation. For $U(1)$ and $SU(N)$ symmetries, we have

$$U_{U(1)} = \exp[i\alpha], \quad U_{SU(N)} = \exp\left[i\alpha^a \frac{T^a}{2}\right]. \quad (1.4)$$

For $SU(N)$ symmetries, the index a is summed from 1 to $N^2 - 1$: this is the number of gauge fields of the group. T^a are the generators of these fields, which are represented by the Pauli matrices for $SU(2)$ and the Gell-Mann matrices for $SU(3)$. For $SU(2)$, there are three gauge fields, which will lead to the three associated bosons (W^\pm, Z) when mixed with $U(1)$; for $SU(3)$, there are eight gauge field, leading to the eight gluons. Clearly for $U(1)$ there is only a single gauge field, which will lead to the photon when mixed with the $SU(2)$ gauge fields. In addition to the definitions in Eq.(1.4), to make Eq.(1.2) gauge invariant, it is necessary to transform the gauge field, as well as introduce the *covariant derivative*, D_μ :

$$A_\mu^a \rightarrow A_\mu^a + \frac{1}{g}\partial_\mu\alpha^a + gf^{abc}\alpha^b A_\mu^c, \quad (1.5)$$

$$\partial_\mu \rightarrow D_\mu = \partial_\mu - igT^a A_\mu^a, \quad (1.6)$$

where g is the relevant gauge coupling parameter. Note that Eqs.(1.5) and (1.6) are the general forms for the $SU(N)$ theories (if considering $U(1)$, these are simplified). The full covariant derivative of the Standard Model is

$$D_\mu = \partial_\mu - ig_1 \frac{Y}{2} B_\mu - ig_2 \frac{\sigma^i}{2} W_\mu^i - ig_3 \frac{\lambda^a}{2} G_\mu^a, \quad (1.7)$$

¹The subscripts of the symmetries describe the "charge" under which particles interact with each gauge theory: c represents the color charge of the strong force, L states that the weak force interacts only with left-handed doublets (through which the weak isospin will be defined analogous to spin), and Y represents the weak hypercharge of $U(1)$ before the spontaneous symmetry breaking of the Higgs mechanism to introduce the electric charge of the electromagnetic force.[15]

combining all constituent gauge groups, with $g_{1,2,3}$ the gauge coupling of $U(1)_Y$, $SU(2)_L$, and $SU(3)_c$ respectively where the gauge field(s) and generator(s) of each group are in the term of their coupling [13]. From these definitions, the gauge invariance of Eq.(1.2) for $U(1)$ symmetry is trivial, and for $SU(N)$ symmetries, it will simply follow after defining a commutation relation between generators:

$$[T^a, T^b] = if^{abc}T^c, \quad (1.8)$$

where f^{abc} are the structure constants which define each $SU(N)$ group [14]. It can be useful to define the fermion field ψ into an upper and lower part, which can be done in many ways, but it is important in the formulation of the Standard Model to separate the fermion field into left- and right-handed components. To this end, we define the operators

$$P_L = \frac{1 - \gamma^5}{2}, \quad P_R = \frac{1 + \gamma^5}{2}, \quad (1.9)$$

where γ^5 is defined in the conventional representation. The left-handed projection of the fermion field ψ would then be

$$\psi_L = P_L \psi. \quad (1.10)$$

Now when forming the $SU(2)$ symmetry of the weak force, there is a difference between the interaction of right- and left-handed fermions: that is, left-handed fermions transform under the $SU(2)$ space, whereas right-handed do not. Considering both quarks and leptons in the traditional three generations, then under the electroweak $SU(2)$ space, each generation forms a left-handed doublet and a right-handed singlet, e.g.

$$\Psi_L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \quad e_R^-, \quad \Psi_{L\alpha} = \begin{pmatrix} u_\alpha \\ d_\alpha \end{pmatrix}_L, \quad u_{R\alpha}, d_{R\alpha}, \quad (1.11)$$

where α represents the colour charge of the quarks under the strong interaction [13]. We can define the weak isospin quantum number T analogous to spin; in the left-handed doublet: up-type quarks and neutrinos have $T_3 = +\frac{1}{2}$, and down-type quarks and charged leptons have $T_3 = -\frac{1}{2}$. Neutrinos only appear in the left-handed doublets and do not form right-handed singlets, as there is currently no experimental evidence for their existence. So right- and left-handed fermions are in different $SU(2)$ multiplets; this represents a parity violation in the Standard Model which is also found in nature. This is an important criterion of a theory of nature, as will be discussed more in section 1.2.

Gauge symmetry has thus far yielded a Lagrangian to describe fermions and forces and their interactions, but only for massless particles. This however is not observed in nature. For massive fermions and bosons, the Lagrangian would require terms of the form $m\bar{\psi}\psi$ and $m_A^2 A_\mu A^\mu$ respectively. The fermion mass term would initially seem invariant, but under the electroweak symmetry, it is rewritten as

$$m\bar{\psi}\psi = m(\bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R), \quad (1.12)$$

and so is no longer invariant, due to the mixing of right-handed and left-handed spinors.

To solve this problem, the Higgs mechanism can be introduced as a way to spontaneously break the symmetry of the electroweak force to generate massive bosons. The Lagrangian of the scalar Higgs reads

$$\mathcal{L} = (D^\mu \phi)^\dagger (D_\mu \phi) - V(\phi). \quad (1.13)$$

Consider a complex scalar field under the SU(2) left-handed doublet representation,

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_0 + i\phi_3 \end{pmatrix} = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad (1.14)$$

which has a U(1) hypercharge $Y = +\frac{1}{2}$ [12]. The covariant derivative in Eq.(1.13) is that defined for SU(2)⊗U(1):

$$D_\mu \phi = \left(\partial_\mu + igT^i W_\mu^i + \frac{i}{2}g'B_\mu \right) \phi, \quad (1.15)$$

with W_μ^i and B_μ the gauge fields of SU(2)_L and U(1)_Y, and g and g' the SU(2) and U(1) gauge couplings respectively. The Higgs potential $V(\phi)$ in Eq.(1.13) is defined as

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2. \quad (1.16)$$

If $\mu^2 > 0$, then the scalar field will acquire a non-zero vacuum expectation value (VEV), which will spontaneously break the symmetry. The VEV can be defined as

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}. \quad (1.17)$$

To ensure that the symmetry of electromagnetism is not broken, the scalar field ϕ^0 is taken to have charge $Q = 0$. Under this scheme, both the hypercharge Y and the weak isospin T_3 are not conserved, but the specific combination of them which is defined as electric charge, $Q = T_3 + \frac{1}{2}Y$ is conserved, so the electroweak symmetry is spontaneously broken to form a U(1) symmetry of electric charge Q ,

$$SU(2)_L \otimes U(1)_Y \rightarrow U(1)_Q. \quad (1.18)$$

The W^\pm and Z masses can now be generated from this mechanism. In the unitary gauge, the scalar doublet can be written

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}. \quad (1.19)$$

Through substituting the above into Eq.(1.13), the gauge fields of the real electroweak bosons W^\pm , Z , γ can be found as linear combinations of W_μ^i and B_μ with their mass terms now generated in a gauge invariant form:

$$W_\mu^\pm \equiv \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2), \quad m_W = \frac{gv}{2}, \quad (1.20)$$

$$Z_\mu \equiv \frac{1}{\sqrt{g^2 + g'^2}}(gW_\mu^3 - g'B_\mu), \quad m_Z = \frac{v}{2}\sqrt{g^2 + g'^2}, \quad (1.21)$$

$$A_\mu \equiv \frac{1}{\sqrt{g^2 + g'^2}}(g'W_\mu^3 + gB_\mu), \quad m_A = 0. \quad (1.22)$$

It can be said that three of component scalar fields in Eq.(1.14) were "eaten" to form the gauge boson masses, and the fourth field goes on to become the Higgs phenomenon [15]. Now the final piece of the puzzle is fermion masses. We consider the final term of Eq.(1.1), the Yukawa Lagrangian. It is split into two terms: the one written and "h.c.", representing the hermitian conjugate of the former. The hermitian conjugate term is required so both up-type quarks, and down-type quarks and charged leptons gain mass [13]. In the unitary gauge, the down-type quark Yukawa term reads

$$-\mathcal{L}_{Yuk}^d = \sum_{a,b} Y_{ab}^{(d)} (\bar{u}_{aL} \bar{d}_{aL}) \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} d_{bR} = \sum_{a,b} Y_{ab}^{(d)} \phi_0 \bar{d}_{aL} d_{bR}, \quad (1.23)$$

where ϕ^0 is defined as $\frac{v}{\sqrt{2}}$ through the transformation

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix} \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad (1.24)$$

from the spontaneous symmetry breaking. So now down-type quarks and charged leptons have gained masses. To generate the masses of the up-type quarks, the hermitian conjugate term is used, where now a second Higgs doublet (not independent of the first, but its charge conjugate) is introduced as

$$\tilde{\Phi} = i\sigma_2 \Phi^* = \begin{pmatrix} \phi^{0*} \\ -\phi^{+*} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} v+h \\ 0 \end{pmatrix}. \quad (1.25)$$

The up-type quark Yukawa term is then

$$-\mathcal{L}_{Yuk}^u = \sum_{a,b} Y_{ab}^{(u)} (\bar{u}_{aL} \bar{d}_{aL}) \begin{pmatrix} \frac{v}{\sqrt{2}} \\ 0 \end{pmatrix} u_{bR} = \sum_{a,b} Y_{ab}^{(u)} \phi_0^0 \bar{u}_{aL} u_{bR}. \quad (1.26)$$

The charged lepton masses are generated similarly, and the expressions of masses read

$$m_i = -\frac{Y_i v}{\sqrt{2}}, \quad (1.27)$$

where i will represent the fermions [15]. The question of neutrino mass is at the forefront of some research, and so won't be mentioned in this study, other than that it is now known that neutrinos do have mass, however small.

In Eqs.(1.23) and (1.26), we have formed non-diagonal mass matrices. These can be diagonalised in which the constituent states are the mass eigenstates, e.g. for the quarks, there would be

$$\frac{v}{\sqrt{2}} V_1^\dagger Y_u V_1 = \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix}, \quad \frac{v}{\sqrt{2}} V_2^\dagger Y_d V_2 = \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix}. \quad (1.28)$$

However, generally, these cannot both be diagonalised simultaneously; this could be the case, but we do not start with this in case it is not so. By convention, it is chosen to diagonalise the up-type quarks, and then rotate the down-type quarks between their

weak eigenstates and their mass eigenstates [12]. The famous CKM matrix, defined $V_{CKM} \equiv V_1^\dagger V_2$, is the matrix connecting to mass and weak eigenstates,

$$\begin{pmatrix} d \\ s \\ b \end{pmatrix} = V_{CKM} \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}, \quad (1.29)$$

as well as describing quark mixing between the up- and down-type quarks. The CKM matrix in its full form is a unitary matrix; it is currently believed to be a 3×3 unitary matrix. This however could be a section of a 4×4 unitary matrix if a fourth generation of fermions existed, meaning the current 3×3 matrix would not necessarily be unitary. The strengths of quark coupling are stored in elements of the CKM matrix as

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \quad (1.30)$$

These individual elements can be found through quark coupling processes. For better understanding of the CKM matrix however, it can be easier to parameterise it. There are many parameterisations that can be used for this, though the two most common are the *standard parameterisation*, which is an exact parameterisation, and the *Wolfenstein parameterisation*, which is an approximation [12]. The standard parameterisation follows from extension of the Cabibbo angle for mixing between the first and second generations, where we now use three Cabibbo-like matrices for interactions of two quark generations, and multiply them for the CKM matrix, including a phase required by the formation of a 3×3 unitary matrix:

$$\begin{aligned} V_{CKM} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{23} & \sin \theta_{23} \\ 0 & -\sin \theta_{23} & \cos \theta_{23} \end{pmatrix} \begin{pmatrix} \cos \theta_{13} & 0 & \sin \theta_{13} e^{i\delta} \\ 0 & 1 & 0 \\ -\sin \theta_{13} e^{i\delta} & 0 & \cos \theta_{13} \end{pmatrix} \begin{pmatrix} \cos \theta_{12} & \sin \theta_{12} & 0 \\ -\sin \theta_{12} & \cos \theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \end{aligned} \quad (1.31)$$

The Wolfenstein parameterisation approximates the standard parameterisation as $\lambda = s_{12} \approx 0.22543$, $A\lambda^2 = s_{23}$, and $A\lambda^3(\rho - i\eta) = s_{13}e^{-i\delta}$, and performs a Taylor expansion traditionally to order λ^3 to yield

$$V_{CKM} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4). \quad (1.32)$$

This parameterisation makes it very easy to see the relative strengths of the coupling.

An important element of the CKM matrix is its complex phase.² The presence of a complex phase allows for CP-violating processes to take place in the Standard Model:

²If its real form were a 4×4 matrix, there would be three phases, implying significantly more CP violation.

the CKM matrix's hermitian conjugate will give the complex conjugated phase, yielding different physics over CP conjugation for complex coupling elements. In the Wolfenstein parameterisation, all CP violation is held with the $\rho - i\eta$ terms. Plotting measurements in the (ρ, η) plane can prove useful in understanding these terms, to which end the unitarity triangle can be formed:

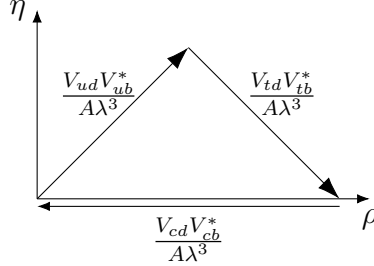


Figure 1. The unitarity triangle from the Wolfenstein parameterisation in (ρ, η) space.

The unitarity triangle can help describe many things in quark mixing, such as CP violation, through the length of its side and its angles. It can also be useful to define a unitarity circle through comparison of experiment and theory in CP-violating processes, and from these and others, many regions in the (ρ, η) plane can be defined to interpret various constants and effects in quark mixing.

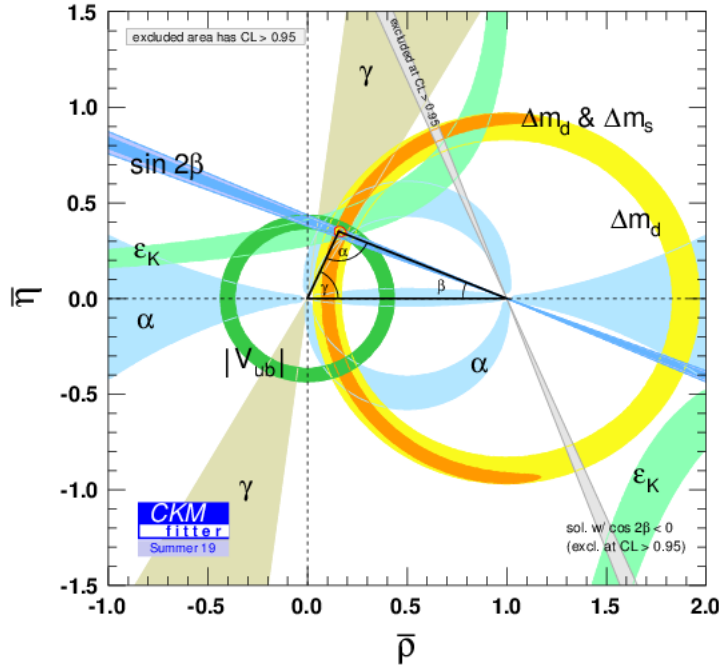


Figure 2. The global CKM fit in the (ρ, η) plane, as of 13/01/20 [7].

1.2 Flavour Physics and CP Violation

A significant focus of flavour physics is the study of CP violation - the violation of CP symmetry (the product of Charge and Parity symmetries) in the Standard Model. CP

symmetry was introduced as a candidate for the fundamental symmetry of Standard Model interactions after Parity violation was discovered in the 1950s. One of the big questions about the formulation of our universe is why is there an asymmetry between matter and anti-matter. One of the conditions Sakharov found in 1967 to answer this question is a necessity of CP violation [8]: if left-handed baryons interact differently to right-handed anti-baryons, then one of these can be more prominent than the other. Both the electromagnetic and strong forces appear to conserve CP, the weak force, however, through the chiral nature of its couplings, does not. There are two ways for CP-violating interactions to occur through the weak force: through complex Yukawa couplings in the CKM matrix; and through complex Higgs parameters, such as those in the 2 Higgs Doublet Model to be discussed in section 1.3. There is also a question of whether CP violation can be found in the strong sector, but this will not be discussed here [13].

Currently, there are two cruxes to the study of Sakharov’s condition of CP violation: experiment finds larger scattering amplitudes than theory for many CP-violating processes, and the amount of CP violation currently observed would not be enough to account for the large baryon asymmetry of the universe. Both these cruxes point towards physics beyond the Standard Model to add to the CP violation predicted and perhaps predict where to observe the CP violation needed in nature [12]. There are many proposals for Standard Model extensions which could bridge the gap between theory and experiment, although these extensions would require new particles to be observed to explain their phenomena, none of which have so far been observed.

The Z' boson is a common extension to the Standard Model, which in its simplest application behaves akin to the Z^0 boson but with a higher mass. There are many descriptions of the Z' boson ranging from the introduction of a new U(1) gauge symmetry to the inclusion of string theory [16]. The consideration of a Z' boson to flavour physics is to add extra coupling paths to scattering amplitudes which could better explain flavour observables. An example of how Z' could modify Standard Model decays is given in Figure 3.

$$\Gamma_{\text{Exp}} = B^- \left\{ \begin{array}{c} b \rightarrow s \\ \bar{u} \leftarrow \bar{u} \end{array} \right\} K^- + B^- \left\{ \begin{array}{c} b \rightarrow s \\ \bar{u} \leftarrow \bar{u} \end{array} \right\} K^- \quad (1.33)$$

Figure 3. Proposed Z' extension to $\Gamma[B^- \rightarrow K^- \mu^+ \mu^-]_{SM}$ (without higher order corrections).

It could also be possible that there is a fourth generation of fermions, perfectly replicating the first generation to higher masses as the second and third generations do. The presence of the fourth generation could add extra Feynman diagrams to decays, increasing the resulting decay amplitude. Another interesting implication of a fourth generation would be that the CKM matrix would now be a 4x4 matrix, which could see some alterations to flavour-changing interactions, even some additional complex terms where CP violation would arise. A key indicator on this extension is in the Higgs mechanism.

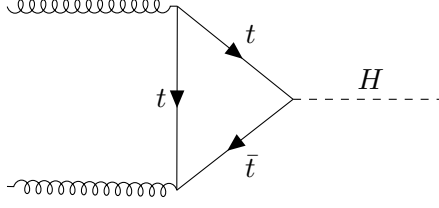


Figure 4. Higgs boson production through the top quark loop.

The Higgs couples more strongly to heavier particles, so the top-Higgs vertex seen in figure 4 should be replicated with the theoretical t' and b' quarks with increased amplitudes. However, when just considering the SM4 model, the new prediction of the Higgs scattering amplitude is far greater than that observed [18]. It could be possible that the SM4 model is physical, but would also need another extension to resolve its issues, such as the Two Higgs Doublet Model discussed below [17].

1.3 The Two Higgs Doublet Model of Type II

Previously in the Standard Model Higgs mechanism, to generate the masses of the weak gauge bosons and fermions, a single Higgs SU(2) doublet of complex scalar fields was introduced, as in Eq.(1.14). To form masses for both up- and down-type quarks and charged leptons, we introduced the hermitian conjugate doublet in Eq.(1.25), otherwise the Higgs field would only give mass to the down-type quarks. Now two Higgs doublets can be postulated, of the general form

$$\Phi_1 = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_0 + i\phi_3 \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_5 + i\phi_6 \\ \phi_4 + i\phi_7 \end{pmatrix}. \quad (1.34)$$

These two doublets would have opposite hypercharge so one would couple to up-type quarks, and the other to down-type [19]. Each doublet would have its own VEV, with the quadrature addition of these two VEVs creating the value measured in experiment:

$$\Phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \Phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} v_2 \\ 0 \end{pmatrix}, \quad v_{SM}^2 = v_1^2 + v_2^2. \quad (1.35)$$

Under this model, the masses of the three weak gauge bosons can still be generated in the same way, each eating one of the scalar fields. This time, however, there would be 5 residual scalar fields, and so, five Higgs fields should be found: three neutral Higgs - two scalar H and h (where h is the SM Higgs particle) and one pseudoscalar A . There will also be two charged Higgs H^\pm . The introduction of these new particles requires searching for additional parameters to describe them: the masses of H , A , and H^\pm ; the mixing angle between h and H ; and the VEV ratio $\tan \beta = v_2/v_1$ [1].

The two Higgs doublet model (2HDM) described above is the type II version, where Φ_1 couples to down-type quarks and Φ_2 couples to up-type quarks and charged leptons. There are other types of 2HDM where the two Higgs doublets couple in different ways, but the promising sign from Type II is that it yields a similar expression for the Yukawa Lagrangian to the SM. This in turn yields a similar CKM matrix through which all flavour-changing interactions can be described. The addition of 4 new Higgs fields introduces

new interactions, where the charged Higgs fields can replace W fields in flavour-changing charged interactions, as well as additional neutral interactions between the neutral Higgs scalar and pseudoscalar fields [19].

The presence of charged Higgs fields presents the opportunity to bridge the gap between theory and experiment. The charged Higgs particles can mediate weak charged currents in the same way as W^\pm , and the additional contribution of H^\pm to many weak-mediated decays could correct the Standard Model to predict these decay paths [19] more accurately. The important factors to determine whether these charged Higgs could be the answer to the inconsistencies are the relevant parameters from the 2HDM: the charged Higgs mass m_{H^\pm} , and the VEV ratio $\tan\beta$ [1]. Values of these parameters which align theory and observation can indicate where LHC experiments could probe for these theorised new particles. This study considers where regions of the 2HDM Type II parameters can align SM predictions with experiment, giving indication of where they could be found in nature if this model is to be a real extension of the Standard Model.

2 Probing the 2HDM Type II

Below will follow [1] closely in describing the SM theory for the decays relevant to charged Higgs searches and extending these into the 2HDM. The measurements and parameters used for these expressions is given in Tables 1 and 2.

2.1 Leptonic Decays

In the Standard Model, the leptonic decay $M \rightarrow l\nu_l$, where M is a charged meson, has a branching ratio of

$$\mathcal{B}[M \rightarrow l\nu]_{\text{SM}} = \frac{G_F^2 m_M m_l^2}{8\pi} \left(1 - \frac{m_l^2}{m_M^2}\right)^2 |V_{quqd}|^2 f_M^2 \tau_M (1 + \delta_{EM}^{Ml2}), \quad (2.1)$$

where q_u and q_d represent the up- and down-like quarks of the meson, V_{quqd} the CKM element, and f_M the M meson's decay constant. δ_{EM}^{Ml2} is a corrective factor for electromagnetic radiative corrections. For π and K mesons, the effect is around 2-3% and around 1% for D mesons. For B meson phenomena, the effect is approximated to 0.

For the light mesons, kaons and pions, it is easier to determine the ratio of their decay constants f_K/f_π than the individual values. Thus, it is then easier to consider the ratio of their branching fractions as well. In the Standard Model,

$$\frac{\Gamma[K \rightarrow \mu\nu]_{\text{SM}}}{\Gamma[\pi \rightarrow \mu\nu]_{\text{SM}}} = \frac{m_K}{m_\pi} \left(\frac{1 - m_l^2/m_K^2}{1 - m_l^2/m_\pi^2}\right)^2 \left|\frac{V_{us}}{V_{ud}}\right|^2 \left(\frac{f_K}{f_\pi}\right)^2 (1 + \delta_{EM}^{Kl2/\pi l2}). \quad (2.2)$$

It is also worth considering the ratio of tau decays of kaons to pions:

$$\frac{\Gamma[\tau \rightarrow K\nu]_{\text{SM}}}{\Gamma[\tau \rightarrow \pi\nu]_{\text{SM}}} = \left(\frac{1 - m_K^2/m_\tau^2}{1 - m_\pi^2/m_\tau^2}\right)^2 \left|\frac{V_{us}}{V_{ud}}\right|^2 \left(\frac{f_K}{f_\pi}\right)^2 (1 + \delta_{EM}^{\tau K2/\tau\pi2}). \quad (2.3)$$

These SM branching values will take alterations from the two Higgs doublet model of the form

$$\mathcal{B}[M \rightarrow l\nu] = \mathcal{B}[M \rightarrow l\nu]_{\text{SM}}(1 + r_H)^2, \quad (2.4)$$

where r_H is the corrective factor of the two Higgs doublet model:

$$r_H = \left(\frac{m_{q_u} - m_{q_d} \tan^2 \beta}{m_{q_u} + m_{q_d}} \right) \left(\frac{m_M}{m_{H^+}} \right)^2. \quad (2.5)$$

It can still be possible through this method for some leptonic decays to have agreement between the SM predictions and experiment, where $r_H = 0, -2$. $r_H = 0$ is known as the decoupling solution which can be found as m_{H^+} approaches infinity; $r_H = -2$, the fine-tuned solution, is obtained from a linear relationship between m_{H^+} and $\tan \beta$, which will be dependent of the masses of the meson and its constituent quarks, so will vary between mesons.

2.2 Mass mixing

Neutral B meson mixing is a common study in flavour physics, due to the strong dependence on the heavy W exchange.

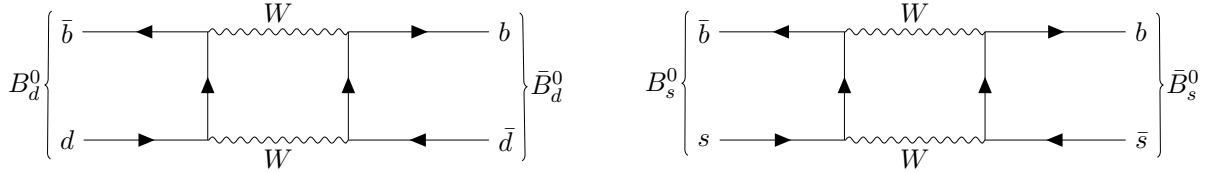


Figure 5. Examples of the box diagrams detailing B_d and B_s meson mixing.

In 2HDM, the mass difference between the two mixed states receives two additional contributions from box diagrams using the heavy charged Higgs in place of both W s or one W . For B_q , where $q = d, s$, this results in:

$$\Delta m_q = \frac{G_F^2}{24\pi^2} (V_{tq} V_{tb}^*)^2 \eta_B m_B m_t^2 f_{B_q}^2 \hat{B}_{B_q} (S_{WW} + S_{WH} + S_{HH}), \quad (2.6)$$

$$S_{WW} = \left(1 + \frac{9}{1 - x_{tW}} - \frac{6}{(1 - x_{tW})^2} - \frac{6x_{tW}^2 \ln(x_{tW})}{(1 - x_{tW})^3} \right), \quad (2.7)$$

$$S_{WH} = \frac{x_{tH}}{\tan^2 \beta} \left(\frac{(2x_{HW} - 8) \ln(x_{tH})}{(1 - x_{HW})(1 - x_{tH})^2} + \frac{6x_{HW} \ln(x_{tW})}{(1 - x_{tW})(1 - x_{tW})^2} - \frac{8 - 2x_{tW}}{(1 - x_{tW})(1 - x_{tH})} \right), \quad (2.8)$$

$$S_{HH} = \frac{x_{tH}}{\tan^4 \beta} \left(\frac{1 + x_{tH}}{(1 - x_{tH})^2} + \frac{2x_{tH} \ln(x_{tH})}{(1 - x_{tH})^3} \right), \quad (2.9)$$

where $x_{ij} = m_i^2/m_j^2$, and S_{xy} describes the internal bosonic lines for the two bosons $x, y = H^\pm, W^\pm$. The current SM predictions for the mass differences are calculated in [9] to be

$$\Delta m_d^{\text{SM}} = (0.533_{-0.036}^{+0.022}) ps^{-1} = (1.05_{-0.07}^{+0.04}) \Delta m_d^{\text{exp}}, \quad (2.10)$$

$$\Delta m_s^{\text{SM}} = (18.4_{-1.2}^{+0.7}) ps^{-1} = (1.04_{-0.07}^{+0.04}) \Delta m_s^{\text{exp}}. \quad (2.11)$$

2.3 Radiative decay

The radiative decay $b \rightarrow s\gamma$ occurs through the flavour-changing neutral currents of penguin diagrams. The Standard Model calculations of $\bar{B} \rightarrow X_s\gamma$ have been performed up to Next-to-Next Leading Order (NNLO), which yields a complicated expression, so the result is parameterised in line with [1, 2].

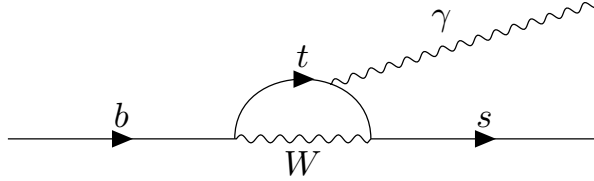


Figure 6. A one-loop penguin diagram for $b \rightarrow s\gamma$ decay.

Similarly to B meson mixing, W can be substituted for a charged Higgs in all the relevant penguin diagrams, giving extra contributions again to the theory. The 2HDM corrections given by these additional contributions are calculated to NLO in [2], so the SM parameters must be limited to NLO. The normalised branching ratio of $\bar{B} \rightarrow X_s\gamma$ is given by

$$\mathcal{R}_{b \rightarrow s\gamma} = \frac{\mathcal{B}[\bar{B} \rightarrow X_s\gamma]}{\mathcal{B}[\bar{B} \rightarrow X_c l \bar{\nu}]} = \left| \frac{V_{ts}^* V_{tb}}{V_{cb}} \right|^2 \frac{6\alpha_{\text{EM}}}{\pi C} (P + N). \quad (2.12)$$

Here, P represents the perturbative leading contribution in the heavy quark expansion, and N the non-perturbative higher-order contributions of the heavy quark expansion. P and N are parameterised in [1, 2] as

$$P + N = (C_{7,SM}^{\text{eff},(0)} + B\Delta C_{7,H^+}^{\text{eff},(0)})^2 + A, \quad (2.13)$$

where A and B are functions defined in [1], $C_{7,SM}^{\text{eff},(0)}$ is one of the Wilson coefficients of the effective Hamiltonian in the SM, and $\Delta C_{7,H^+}^{\text{eff},(0)}$ holds the charged Higgs contributions to the SM Wilson coefficient.

In Eq.(2.12), C is a phase space normalisation factor for the difference between the two branching ratios. From [6], it is defined as

$$C = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \frac{\Gamma(\bar{B} \rightarrow X_c e \bar{\nu})}{\Gamma(\bar{B} \rightarrow X_u e \bar{\nu})}. \quad (2.14)$$

2.4 Input and Fits

All processes mentioned above were used to test the constraints on the 2HDM parameters m_{H^+} and $\tan\beta$. The 2HDM-extended branching ratios were tested in $(m_{H^+}, \tan\beta)$ parameter space to search for regions where a charged Higgs would be allowed by nature.

The individual constraints found for leptonic decays, B meson mixing, and the radiative $b \rightarrow s\gamma$ decay are shown in figure 7 in a log-log scale. After the constraints on the 2HDM for each individual process were found, they were combined for a global fit shown in figure 8 (again in a log-log scale), yielding an overall allowed region in $(m_{H^+}, \tan\beta)$ space. In previous works, such as [1], the global fit has extended to the very top of the parameter

Branching Ratios	Value	Unit	Reference
$\mathcal{B}[B \rightarrow \tau \nu]$	1.09 ± 0.24	10^{-4}	[4]
$\mathcal{B}[D \rightarrow \mu \nu]$	3.82 ± 0.33	10^{-4}	[4]
$\mathcal{B}[D_s \rightarrow \tau \nu]$	5.48 ± 0.23	10^{-2}	[4]
$\Gamma[K \rightarrow \mu \nu]/\Gamma[\pi \rightarrow \mu \nu]$	1.337 ± 0.003		[4]
$\Gamma[\tau \rightarrow K \nu]/\Gamma[\tau \rightarrow \pi \nu]$	6.438 ± 0.094	10^{-2}	[4]
Δm_d	0.5064 ± 0.0019	ps^{-1}	[3]
Δm_s	17.757 ± 0.021	ps^{-1}	[3]
$\mathcal{B}[\bar{B} \rightarrow X_s \gamma]$	3.32 ± 0.15	10^{-4}	[3, 6]
$\mathcal{B}[\bar{B} \rightarrow X_c e \bar{\nu}]$	10.65 ± 0.16	10^{-2}	[3, 4]
$\Gamma[\bar{B} \rightarrow X_u e \bar{\nu}]$	8.41 ± 0.59	10^{-4}	[5]

Table 1. Branching ratios using in testing parameter space of 2HDM Type II. In the case of normalised ratios, values have been calculated using the constituent values taken from reference.

Input	Value	Unit	Reference
Decay Constants			
f_K/f_π	1.1932 ± 0.0019		[4]
f_B	190 ± 1.3	MeV	[10]
f_{B_s}	230.3 ± 1.3	MeV	[10]
f_D	212 ± 0.7	MeV	[10]
f_{D_s}	249.9 ± 0.5	MeV	[10]
Radiative corrections			
$\delta_{EM}^{Kl2/\pi l2}$	-0.0069 ± 0.0017		[4]
$\delta_{EM}^{\tau K2/\tau \pi 2}$	0.003		[1]
B meson mixing			
\hat{B}_{B_d}	1.268 ± 0.042		[9]
\hat{B}_{B_s}	1.290 ± 0.035		[9]
η_B	0.537856		[11]
$b \rightarrow s \gamma$ parameters			
m_t^{pole}	173.1 ± 0.9	GeV	[4]
$\alpha_S(m_Z)$	0.1179 ± 0.001		[4]

Table 2. Parameters used in the 2HDM fits for decays outlined in sections 2.1-2.3.

space, so it could not be ruled out that the allowed region did not extend higher. However, here it can be seen that the global constraint region is limited to within the region of parameter space shown, so both upper and lower limits for the 2HDM parameters can be found. Here, it is found as

$$370 < m_{H^+}(\text{GeV}) < 891, \quad 2.16 < \tan \beta < 33.0. \quad (2.15)$$

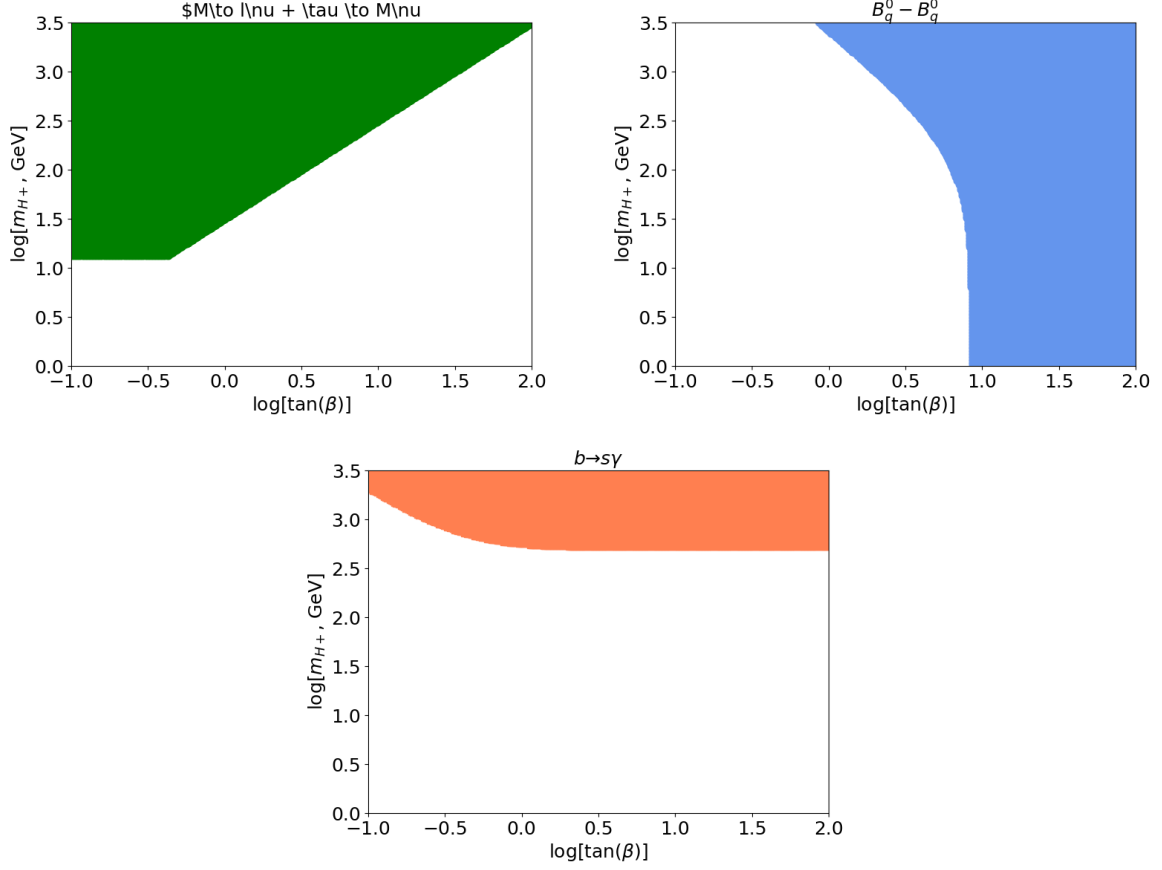


Figure 7. Constraints of $(m_{H+}, \tan \beta)$ space from leptonic decays (top left), meson mixing (top right), and the radiative $b \rightarrow s \gamma$ decay (bottom). The leptonic decays used were $B^+ \rightarrow \tau^+ \nu$, $D^+ \rightarrow \mu^+ \nu$, $D_s^+ \rightarrow \tau^+ \nu$, and the ratios $K \rightarrow \mu \nu / \pi \rightarrow \mu \nu$ and $\tau \rightarrow K \nu / \tau \rightarrow \pi \nu$. The meson mixings considered are that of B_d^0 and B_s^0 .

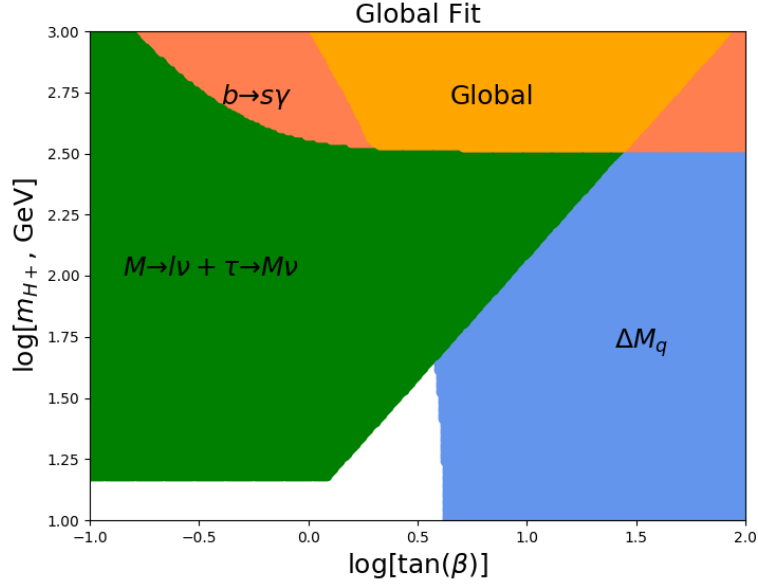


Figure 8. Global fit of constraints on m_{H+} and $\tan \beta$ from all observables considered. The green, blue, and coral regions represent constraints shown individually in figure 7, and the orange region is the global constraint to 1σ .

3 New Observables

4 Statistical Fitting

5 Results With James

6 CKM Element Modification

7 SM4x2HDM

8 Next Steps

- Move SM intro to appendix
- Add appendix for $b \rightarrow s\gamma$ maths

Model:	$\cos(\beta - \alpha) = 0.465$		$\cos(\beta - \alpha) = 0$	
	Flavour	Flavour + Oblique	Flavour	Flavour + Oblique
m_{H^+} (GeV): 95% CL	> 580	$560 \rightarrow 830$	> 500	$630 \rightarrow 870$
1σ	> 1020	$600 \rightarrow 830$	> 790	$660 \rightarrow 870$
χ^2_{min}	3200	790	3200	810
$\tan\beta$: 95% CL	< 3.7	$1.3 \rightarrow 4.1$	< 10.6	$1.4 \rightarrow 11.1$
1σ	< 3.1	$1.6 \rightarrow 3.5$	< 8.6	$1.8 \rightarrow 9.3$
χ^2_{min}	1.5	2.4	2.9	4.4
χ^2_{min}	10.3	13.9	9.9	12.1
χ^2_ν	0.94	0.99	0.90	0.86
ν	11	14	11	14
p-value	49.5%	54.0%	45.7%	40%

Table 3. Constraints and statistics for various global fits. ‘Flavour’ denotes all of the flavour observables discussed throughout, disregarding $R(D^*)$ for now; ‘Oblique’ denotes adding the three oblique parameters from James’ work, S,T,U. The fits have been done in the wrong-sign limit and the exact alignment limit as shown, and setting the additional parameters at play for now to $M = m_{H^0} = m_{A^0} = 750$ GeV. The 2HDM potential parameter M does not have much impact on the fit as it varies, although through the oblique parameters, m_{H^0} and m_{A^0} will cause our results to vary on their values as these require all new Higgs masses to be approximately equal, thus confining the space tightly near whatever values are chosen for these two masses. More thorough fits, e.g. varying the other parameters as well, to higher σ , or looking at χ^2 and p-values for a fit including $R(D^*)$ to check that validity will be incoming.

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