Theory of Mixing and CP violation

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Abstract

We review the current status of B-mixing observables and point out the crucial importance of a control of the hadronic uncertainties for ruling out or confirming hints of BSM physics. In addition we introduce a rating system for theory predictions for lifetimes and mixing observables, that classifies the quality of the corresponding SM values ranging from no star to ****.

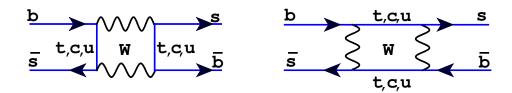


Figure 1: Standard Model diagrams for the transition between B_s and B_s mesons.

1 Introduction

In the Standard Model (SM) mixing of neutral B_q -mesons is governed by the famous box-diagrams, with internal W-bosons and internal up-, charm- and top-quarks, see Fig. 1 for the case of B_s -mesons - for a more detailed introduction into B-mixing, see e.g. [1]. The contribution of internal on-shell particles (only the charm- and the up-quark can contribute) is denoted by Γ_{12}^q ; the contribution of internal off-shell particles (all depicted particles can contribute) is denoted by M_{12}^q . In the B-system there are simple relations between Γ_{12}^q , M_{12}^q and the physical observables mass difference ΔM_q , the decay rate difference $\Delta \Gamma_q$ and the semi-leptonic asymmetries a_{sl}^q :

$$\Delta M_q \approx 2 |M_{12}^q| , \qquad \Delta \Gamma_q \approx 2 |\Gamma_{12}^q| \cos \phi_{12}^q , \qquad a_{sl}^q \approx \left| \frac{\Gamma_{12}^q}{M_{12}^q} \right| \sin \phi_{12}^q .$$
 (1)

The calculation of M_{12}^q gives

$$M_{12}^q = \frac{G_F^2}{12\pi^2} \lambda_t^2 M_W^2 S_0(x_t) B f_{B_q}^2 M_{B_q} \hat{\eta}_B , \qquad (2)$$

where λ_t denotes the CKM elements $V_{tq}^*V_{tb}$ and the Inami-Lim function S_0 [5] contains the result of the 1-loop box diagram in the SM. The bag parameter B and the decay constant f_{B_q} quantify the hadronic contribution to B-mixing, the uncertainties of their numerical values make up the by far biggest uncertainty in the SM prediction of the mass difference. Perturbative 2-loop QCD corrections have been calculated by [6] and they are compressed in the factor $\hat{\eta}_B$. The calculation of Γ_{12}^q is more involved and is based on the Heavy Quark Expansion (HQE) (see [7] for a review and the original

¹This holds not for *D*-mixing, see e.g. [2, 3, 4].

references). According to the HQE the total decay rate of a heavy hadron can be expanded in the inverse of the heavy quark mass as

$$\frac{1}{\tau} = \Gamma = \Gamma_0 + \frac{\Lambda^2}{m_b^2} \Gamma_2 + \frac{\Lambda^3}{m_b^3} \Gamma_3 + \frac{\Lambda^4}{m_b^4} \Gamma_4 + \dots$$
 (3)

The hadronic scale Λ is of order Λ^{QCD} , its numerical value has to be determined by direct computation. For hadron lifetimes it turns out that the dominant correction to Γ_0 is the third term Γ_3 . Each of the Γ_i 's can be split up in a perturbative part and non-perturbative matrix elements - it can be formally written as

$$\Gamma_i = \left[\Gamma_i^{(0)} + \frac{\alpha_S}{4\pi} \Gamma_i^{(1)} + \frac{\alpha_S^2}{(4\pi)^2} \Gamma_i^{(2)} + \dots, \right] \langle O^{d=i+3} \rangle$$
 (4)

where $\Gamma_i^{(0)}$ denotes the perturbative LO-contribution, $\Gamma_i^{(1)}$ the NLO one and so on; $\langle O^{d=i+3} \rangle$ is the non-perturbative matrix element of $\Delta B=0$ operators of dimension i+3. The mixing quantity Γ_{12}^q obeys a very similar HQE, but now the operators change the b-quantum number by two units, $\Delta B=2$:

$$\Gamma_{12} = \frac{\Lambda^3}{m_b^3} \Gamma_3 + \frac{\Lambda^4}{m_b^4} \Gamma_4 + \dots$$
 (5)

2 Current Status

We introduce in this section a rating system for the robustness of lifetime and mixing predictions. Any calculation of a perturbative term $(\Gamma_i^{(j)})$ or a non-perturbative matrix element $(\langle O^{d=k} \rangle)$ gets a "+"; if the calculation is confirmed by an independent collaboration it gets a "++". In the case of non-perturbative matrix elements one can even gain a "+++" for two independent lattice evaluations and one sum rule evaluation. A missing non-perturbative matrix element of dimension 6 is punished by a "--" contribution. Non-perturbative estimates different from lattice or sum rules (like quark models) will be valued by a "0". Partial perturbative calculations will be rated with a "+/2". The possible number of 15 "+" will be classified in 5 categories: **** (at least 12 "+"), *** (at least 8 "+"), ** (at least 4 "+"), * (at least 2 "+") and no star for 1 or less "+".

For the lifetimes of heavy hadrons we get the following overview:

Obs.	$\Gamma_3^{(0)}$	$\Gamma_3^{(1)}$	$\Gamma_3^{(2)}$	$\langle O^{d=6} \rangle$	$\Gamma_4^{(0)}$	$\Gamma_4^{(1)}$	$\langle O^{d=7} \rangle$	Σ	
$\tau(B^+)/\tau(B_d)$	++	++	0	+	++	0	0	**	(7+)
$\tau(B_s)/\tau(B_d)$	++	++	0	$\frac{\pm}{2}$ \pm	++	0	0	**	(6.5+)
$\tau(\Lambda_b)/\tau(B_d)$	++	$\frac{\pm}{2}$	0	$\frac{\pm}{2}$	+	0	0	**	(4+)
$\tau(b-baryon)/\tau(B_d)$	++	0	0	0	+	0	0	*	(3+)
$ au(B_c)$	+	0	0	+	0	0	0	*	(2+)
$\tau(D^+)/\tau(D^0)$	++	++	0	+	++	0	0	**	(7+)
$\tau(D_s^+)/\tau(D^0)$	++	++	0	$\frac{\pm}{2}$	++	0	0	**	(6.5+)
$\tau(c-baryon)/\tau(D^0)$	++	0	0	0	+	0	0	*	(3+)

The LO-QCD part $\Gamma_3^{(0)}$ was first done with the full charm quark mass dependence in 1996 by Uraltsev [8] and Neubert and Sachrajda [9]. For the B_c -meson one has to estimate also the leading HQE term Γ_0 - the full estimate of the lifetime was done by Beneke and Buchalla [10] - to some extent this quantity does not perfectly fit in our list. The NLO-QCD corrections $\Gamma_3^{(1)}$ to B^+ , B_d and B_s were done by [11] and the Rome group [12] - the Rome group also presented part of the NLO-QCD corrections for the Λ_b . In the charm system the NLO-QCD corrections were done by [13] for *D*-mesons. The dimension 6 matrix elements for mesons (except for small corrections arising in B_s and D_s) were recently calculated via HQET sum rules [14] here a complementary lattice evaluation would be very important, either for looking for BSM effects in the very precisely predicted ratio $\tau(B_s)/\tau(B_d)$ this could point towards new effects in hadronic tree-level decays [15] -, or for testing the convergence of the HQE in the b- and in particular in the charmsystem. For baryons we do not have a complete first principle determination of the non-perturbative matrix elements - there are sum rule determinations of the condensate contribution for the Λ_b [16] - we have, however, some estimates [7, 18] of the size of the matrix elements using spectroscopy as an input (based on [17]). LO dimension 7 contributions to B^+ , B_s , B_d and Λ_b were done in [19]. These authors also considered dimension 8 contribution, but since there are operators arising where we even cannot use vacuum insertion approximation, we did not include these corrections in our list. There are unpublished calculations of the dimension 7 terms to B^+ , B_s and B_d by Uli Nierste and myself, that agree with [19], therefore the "++" in the table. Perturbative dimension 7 contributions to D mesons were determined in [13] and to charmed baryons in [18]. So far there exists no non-perturbative de-

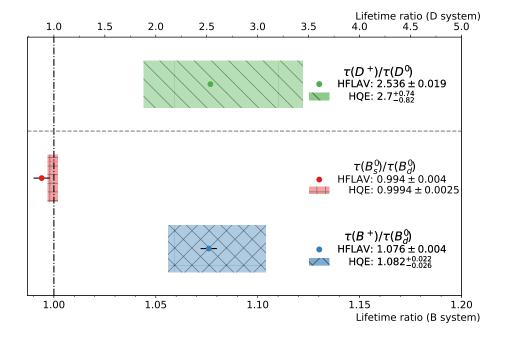


Figure 2: Comparison of the most solid SM predictions for heavy lifetimes with experiment.

termination of the matrix elements of dimension 7 operators. In Fig. 2, taken from [14], we compare the most solid SM predictions for heavy lifetimes with experiment and find an excellent agreement.

The SM prediction for the mass difference is completely dominated by the non-perturbative input for the matrix element of the dimension 6 operator with a V-A Dirac structure. Depending on this input we get the range of predictions for the mass difference in the B_s -system as indicated in Table 1, taken from [20].

For the SM predictions of the decay rate differences in the B_d and B_s -system we get the following list:

Obs.	$\Gamma_3^{(0)}$	$\Gamma_3^{(1)}$	$\Gamma_3^{(2)}$	$\langle O^{d=6} \rangle$	$\Gamma_4^{(0)}$	$\Gamma_4^{(1)}$	$\langle O^{d=7} \rangle$	\sum
Γ_{12}^s	++	++	$\frac{\pm}{2}$	++	++	0	0	8.5 + (***)
Γ_{12}^d	++	++	0	+++	++	0	0	9 + (***)

Source	$f_{B_s}\sqrt{\hat{B}}$	$\Delta M_s^{ m SM}$
HPQCD14 [21]	$(247 \pm 12) \text{ MeV}$	$(16.2 \pm 1.7) \mathrm{ps^{-1}}$
HQET-SR [14]	$(261 \pm 8) \text{ MeV}$	$(18.1 \pm 1.1) \mathrm{ps^{-1}}$
ETMC13 [22]	$(262 \pm 10) \text{ MeV}$	$(18.3 \pm 1.5) \mathrm{ps}^{-1}$
HPQCD09 [23] = FLAG13 [24]	$(266 \pm 18) \text{ MeV}$	$(18.9 \pm 2.6) \mathrm{ps^{-1}}$
FLAG17 [25]	$(274 \pm 8) \text{ MeV}$	$(20.01 \pm 1.25) \mathrm{ps}^{-1}$
Fermilab16 [26]	$(274.6 \pm 4) \text{ MeV}$	$(20.1 \pm 0.7) \mathrm{ps^{-1}}$
HPQCD06 [27]	$(281 \pm 20) \text{ MeV}$	$(21.0 \pm 3.0) \mathrm{ps^{-1}}$
RBC/UKQCD14 [28]	$(290 \pm 20) \text{ MeV}$	$(22.4 \pm 3.4) \mathrm{ps}^{-1}$
Fermilab11 [29]	$(291 \pm 18) \text{ MeV}$	$(22.6 \pm 2.8) \mathrm{ps^{-1}}$

Table 1: List of predictions for the non-perturbative parameter $f_{B_s}\sqrt{\hat{B}}$ and the corresponding SM prediction for ΔM_s . The current FLAG average is dominated by the FERMILAB/MILC value from 2016.

The NLO-QCD corrections $\Gamma_3^{(1)}$ have been calculated in [30, 31, 32], recently also a part of the NNLO-QCD has been determined [33]. At dimension 6 two additional operators to the one appearing in the mass difference are arising. We have currently a HQET sum rule determination for B_d mesons [34, 14] and lattice determinations from 2016 [26] $(N_f = 2 + 1)$ and 2013 [22] $(N_f = 2)$. The dimension 7 perturbative part has been determined already in 1996 by Buchalla and Beneke [35] for B_s and in [36] for B_d . For numerical values of the mixing observables see e.g. the aggressive scenario of [2]

$$\Delta\Gamma_s = (0.098 \pm 0.014) \text{ps}^{-1}, \quad a_{sl}^s = (2.27 \pm 0.25) \cdot 10^{-5}, \quad (6)$$

 $\Delta\Gamma_d = (2.99 \pm 0.52) \cdot 10^{-3} \text{ps}^{-1}, \quad a_{sl}^d = -(4.90 \pm 0.54) \cdot 10^{-4}. \quad (7)$

$$\Delta\Gamma_d = (2.99 \pm 0.52) \cdot 10^{-3} \text{ps}^{-1}, \quad a_{sl}^d = -(4.90 \pm 0.54) \cdot 10^{-4}.$$
 (7)

3 One constraint to kill them all

The importance of the precise value of SM predictions and a strict control of the corresponding uncertainties was highlighted recently in [20]. Leptoquarks and Z' models are popular explanations of the B anomalies²; these new models would also affect B-mixing - in the case of Z' models already at tree-level. In Fig. 3 (from [20]) we show the allowed parameter range

²Due to time and space restrictions I will not attempt to cite the numerous relevant papers in that field.

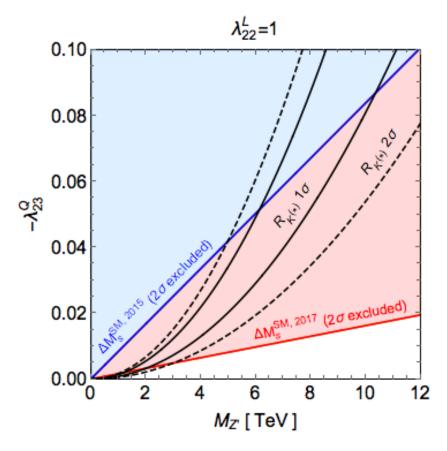


Figure 3: Allowed parameter space of Z' models that try to explain the B anomalies.

for a Z' model: in order to explain e.g. $R_{K^{(*)}}$ the mass of the Z' and the coupling to the b- and s-quark should lie within the black parabola-like shape (the 1 sigma bound is a solid line, the 2 sigma one a dotted line). Taking the FLAG inputs from 2013 for the mass difference one can exclude the blue region. Taking the new FLAG average, that is dominated by the 2016 FNAL/MILC we are left with the red exclusion region and almost all of the possible parameter space of the Z' model is excluded.

4 Conclusion

We presented an overview of the current theoretical status of lifetime and mixing predictions. $\Delta\Gamma_q$ and a_{sl}^q get the highest ranking (***). Γ_{12}^s is slightly less precise known, because the HQET sum rule calculation does not include yet m_s -effects. To improve further the reliability of these predictions one needs a non-perturbative determination of the dimension 7 matrix elements (first steps have been done in [37]) and perturbative evaluations of the α_s^2 and α_s/m_b -corrections. The next solid class of theoretical rigidness is (**) for $\tau(B^+)/\tau(B_d)$ and $\tau(D^+)/\tau(D^0)$. Here an independent lattice determination of the dimension 6 matrix elements is urgently needed. $\tau(B_s)/\tau(B_d)$ and $\tau(D_s^+)/\tau(D^0)$ is slightly less well known, because the m_s corrections to the HQET sum rule are not yet available. Finally Λ_b is considerably less well-known but still a (**) - here we need urgently a first non-perturbative determination of the dimension 6 matrix element. Finally we have the (*) class, which one should consider more an estimate than a precise SM prediction with well-defined uncertainties. We pointed out the crucial significance of a precise non-perturbative input for ΔM_q and related BSM studies - here an independent $N_f = 2+1$ or $N_f = 2+1+1$ confirmation of the FNAL/MILC result of 2016 would be desirable.

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