The Little Crash Course of Particle Physics

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Meeting 10/10

QED:

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{1}$$

$$\partial\!\!\!/ \rightarrow D\!\!\!\!/ : D_\mu = \partial_\mu - ieA_\mu \tag{2}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{3}$$

Symmetry:

$$\psi \rightarrow e^{i\alpha}\psi \tag{4}$$

$$A^\mu \rightarrow A'^\mu = A^\mu + \partial^\mu \alpha(x) \tag{5}$$

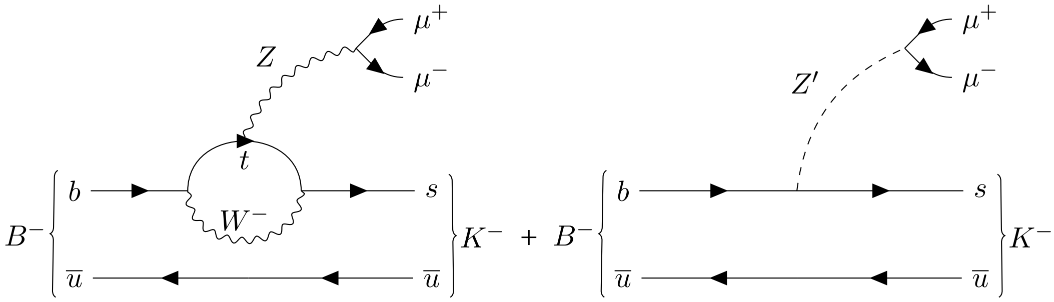
Known as a $U(1)$ local gauge symmetry, for EM.
Most accurate measurement in modern science is $(g-2)$ for the electron. Gone on to 5-loop now, > 10000 diagrams to calculate that.

gauge	SU(N)	N^2-1	N charges
strong	SU(3)	8 gluons	3 colours
weak	SU(2)	$W^{+/-}, Z^0$	2 charges
(gravity)			

$$\psi \rightarrow \begin{cases} \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} & \text{weak} \\ \begin{pmatrix} \psi_i \\ \psi_j \\ \psi_k \end{pmatrix} & \text{strong} \end{cases} \qquad e^{i\alpha} \rightarrow \begin{cases} e^{i\alpha_i \tau_i} & \text{weak, Pauli matrices, } i = 1,2,3 \\ e^{i\alpha_i \lambda_i} & \text{strong, Gell-Mann matrices, } i = 1,\ldots,8 \end{cases} \tag{6}$$

The theoretical scattering rate of $B^- \rightarrow K^-\mu^+\mu^-$ does not match that of experiment close enough that it can be simply down to innacuracies in experiment, so we posit a new boson, the Z', that could explain the difference between theory and experiment, and then we must find if such a boson can exist in harmony with the rest of the Standard Model.

Exp =



(7)

Eq (7) shows the suggested sum, with the second diagram showing the new *Z'* that may account for the difference.
For this *Z'* to be the solution to the inconsistencies, it must also hold for all other couplings it could affect in the Standard Model.

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Taken by Aleksey this week, just a few maths notes.

$$F_{\mu\nu} = D_\mu A_\nu - D_\nu A_\mu \tag{8}$$
$$= (\partial_\mu - igA_\mu)A_\nu - (\partial_\nu - igA_\nu)A_\mu \tag{9}$$
$$= \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu] \tag{10}$$
$$A_\mu = \sum_{a=1}^{n^2-1} A_\mu^a t^a,\ SU(n) \tag{11}$$
$$U(1) \implies [A_\mu, A_\nu] = 0 \tag{12}$$

Of course, for the other symmetry groups used in SM, this commutator will not be zero, as Eq (11) implies, which leads to the more complicated Lagrangian for these theories. The basic formulation for proving gauge invariance of each group is the same, and follows below, but for the SU(2) and SU(3) groups, Eq (11) must be remembered and applied in addition to the usual methods.

$$\psi(x) \to \psi'(x) = e^{ig\alpha(x)} \tag{13}$$
$$A_\mu \to A'_\mu = A_\mu + \partial_\mu \alpha(x) \tag{14}$$
$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\partial_\mu\gamma^\mu - m)\psi \tag{15}$$
$$\partial_\mu \to D_\mu = \partial_\mu - igA_\mu \tag{16}$$

All of this is then mashed together into the SM Lagrangian,

$$\mathcal{L}^{SM} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \ \to \text{gauge term} \tag{17}$$
$$+ i\bar{\psi}\not{D}\psi \ \to \text{Fermion term} \tag{18}$$
$$+ (D_\mu\phi)^\dagger(D^\mu)\phi - V(\phi) \ \to \text{Higgs term} \tag{19}$$
$$- Y_{ij}\bar{\psi}_i\phi\psi_j + h.c. \ \to \text{Yukawa term} \tag{20}$$
$$SU_c(3) \otimes SU(2)_L \otimes U(1) \tag{21}$$

Very reduced form, most terms can be largely expanded, i.e.

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \underbrace{-\frac{1}{4}G_{\mu\nu}G^{\mu\nu}}_{SU_c(3)} \underbrace{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{SU(2)_L} \underbrace{-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}}_{U(1)} \tag{22}$$
$$G_{\mu\nu} = D_\mu A_\nu - D_\nu A_\mu \tag{23}$$
$$D_\mu = \partial_\mu - igA_\mu,\ A_\mu = \frac{1}{2}A_\mu^a\lambda^a,\ a = 1, \ldots, 8 \tag{24}$$
$$[A_\mu^a\lambda^a, A_\nu^a\lambda^a] \neq 0 \tag{25}$$

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Now that we're comprehending the gauge invariance of SU(N) Lagrangians, moving on.

$$\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{26}$$

Gauge symmetry thus far has been relatively straightforward, but the mass terms cause some problems.

1. Spontaneous Symmetry Breaking: We start off by discussing the issues with the mass terms in these theories, how they initially to symmetry breaking and parity violation of the weak force.
   - $m_A^2 A_\mu A^\mu$ - bosons
   - $m\bar{\psi}\psi$ - fermions

   Introducing the concept of right- and left-handed particles can resolve some of these issues within electroweak theory, where left-handed particles transform under SU(2) space, and right-handed particles transform as a number, so do not transform under the weak interaction.

$$\psi = \underbrace{\psi_L}_{SU(2)_L} + \underbrace{\psi_R}_{\wedge} = \frac{1-\gamma_5}{2}\psi + \frac{1+\gamma_5}{2}\psi \tag{27}$$

$$\implies m\bar{\psi}\psi = m\left(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L\right) \tag{28}$$

   So including right-handed fermions in the mass term leads to no gauge invariance(?). Including spontaneous symmetry breaking into the theory can account for this (?) - but the SM still has SU(2)$_L$ space.

2. Quantum Field Theory: A possible explanation used to recapture mass terms into gauge invariant lagrangian was from something called *renormalisation.* Every interaction is not a single Feynman diagram, but an infinite sum of all possible valid interactions, even for just $\psi \to \psi$.
   doodle diagrams
   Attempting to calculate for a single diagram would result in an infinity, or even a single vertex, but taking the entire sum will lead to a finite result. People proved this in the 70s as true for any gauge field theory(?) which is why like them so much and still use them.

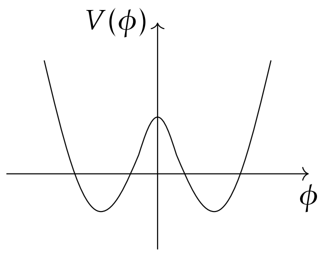
Higgs mechanism:

- sorts out $m\psi, mA$ issues
- gauge symmetry and field theory - renormalisable and stuff
- Only really issue with it was that it predicted a particle to explain it that hadn't been seen before, until 2012, then everyone was happy, *doodle mechanism*

Higgs - $\phi \in$ SU(2)$_L$:

$$(D_\mu\phi)(D^\mu\phi)^\dagger - V(\phi,\phi^\dagger) \tag{29}$$

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \tag{30}$$



In the early universe, so high energy that the Higgs mechanism resembled a simple x^2 potential around $\phi = 0$, but as the universe cooled, the symmmetry broke into the double-welled potential shown above. This is developed using Goldstone's theorem, akin to symmetry breaking in ferromagnets.

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \left(\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4\right) \tag{31}$$

In this Lagrangian, setting $\mu^2 > 0$ corresponds to a potential well around $\phi = 0$ as would be seen in the early universe, but there is no reason for $\mu^2 > 0$, so must also look at $\mu^2 < 0$, where you get the double well above and a Goldstone boson can be found.

$$(\partial_\mu + igA_\mu)\begin{pmatrix} 0 \\ v \end{pmatrix}(\partial^\mu + igA^\nu)\begin{pmatrix} v \\ 0 \end{pmatrix} = \cdots + \underbrace{g^2v^2}_{m_A^2} A_\mu A^\nu \tag{32}$$

$$g\bar{\psi}_L\phi\psi_R \to g\bar{\psi}_L\begin{pmatrix} 0 \\ v \end{pmatrix}\psi_R = gv\bar{\psi}_L\psi_R \tag{33}$$

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The Higgs model symmetry breaking leaves you with

$$\phi = \begin{pmatrix} \phi_1(x) \\ \phi_2(x) \end{pmatrix} \rightarrow \begin{pmatrix} 0 + \phi_1(x) \\ v + \phi_2(x) \end{pmatrix} \tag{34}$$

You should go down three steps when working through the Higgs model:

1. Test it out in U(1) symmetry
2. Try out SU(2) symmetry breaking
3. Make use of the unitary gauge, to see that we get left with 1 physical Higgs, and 3 unphysical fields that get "eaten" by the W,Z bosons to give them mass.

$$\phi = \begin{pmatrix} \phi_1(x) + i\phi_2(x) \\ \phi_3(x) + i\phi_4(x) \end{pmatrix} \tag{35}$$

If we define our ϕ as

$$\phi(x) = \left(\frac{v + \sigma(x)}{\sqrt{2}} \right) e^{i\pi(x)/F_\pi} \tag{36}$$

where F_π is just a number, $\sigma(x)$ and $\pi(x)$ are two real fields, and v is the minimum around which we expand, then the **unitary gauge** sets $\pi(x)$ to zero in the gauge symmetry,

The next big thing to look at is the CKM matrix, which on basic terms transforms quark states and allows quark mixing as

$$\psi'_d = U_d \psi_d \tag{37}$$
$$\psi'_u = U_u \psi^*_d \tag{38}$$

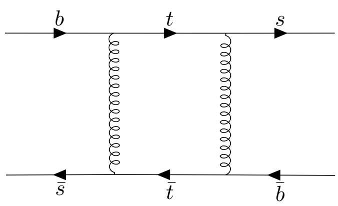
The U_d and U_u we want as unitary matrix elements to transform between the eigenstates. This leads to the CKM matrix, for all quark flavours and their mixing potentials - crucial for CP violation.
A good analogy to start with is how neutrinos have mass oscillations. In reality, neutrinos are a linear combination of electron-, and muon-neutrinos (and tau if you want to add the third family later). At one point in time, the neutrino may be fully the electron-neutrino, and then at a later time, may be the muon-neutrino, and then could be part muon-, part electron-.

$$|\nu\rangle = \alpha e^{-im_1 t}|\nu_1\rangle + \beta e^{-im_2 t}|\nu_2\rangle \tag{39}$$

7/11

We see lots of mixing in particle physics:

- Neutrino mass oscillations, *see last week for brief overview, or look at Alex's Flavour notes.*
- We see mixing of the neutral mesons, K^0, D^0, B^0_d, B^0_s.



- In $U(1) \otimes SU(2)$ symmetry, we find mixing of the gauge fields to get our physical gauge bosons.

$$W^+_\mu = \frac{1}{\sqrt{2}}(-W^1 + iW^2) \qquad W^-_\mu = \frac{1}{\sqrt{2}}(-W^1 - iW^2) \tag{40}$$

$$A_\mu = \frac{g}{\sqrt{g^2+g'^2}}B_\mu + \frac{g'}{\sqrt{g^2+g'^2}}W^3_\mu \qquad Z_\mu = -\frac{g'}{\sqrt{g^2+g'^2}}B_\mu + \frac{g}{\sqrt{g^2+g'^2}}W^3_\mu \tag{41}$$

Where g and g' are the U(1) and SU(2) coupling constants respectively, $W_\mu^{1,2,3}, B_\mu$ are the SU(2) and U(1) gauge fields, and $W_\mu^{+,-}, Z_\mu, A_\mu$ are the $W^{+,-}$, Z, and photon fields in $U(1)\otimes SU(2)$.

- Mixing that we'll be dealing with mainly from here is the quark flavour mixing, through the CKM matrix.

CKM matrix is a 3x3 unitary matrix, holding the strength of transition for one quark to another. The strength for anti-quarks is given by the Hermitian conjugate of the matrix. There is no current theoretical method of deriving these matrix elements, instead we can simply measure them through experiment. However, there is a common parameterisation of the CKM matrix that can be used to help describe this theoretically - the Wolfenstein parameterisation, where $\lambda \approx 0.22$.

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & \lambda^3 \\ -\lambda & 1-\frac{\lambda^2}{2} & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \tag{42}$$

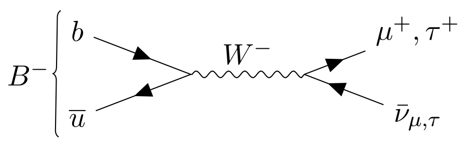
We can however write this out in full, using an extension of the Cabibbo angle. The matrix has 3 rotations, so three angles are needed - $\theta_{12}, \theta_{13}, \theta_{23}$, where θ_{12} is the Cabibbo angle, and each θ_{ij} describes couplings between the ith and the jth generation of quark. There is also 1 phase, $e^{i\delta}$ - the existence of this phase means the matrix is not Hermitian, and CP symmetry will be violated in matrix elements that contain this phase. The CKM matrix was formed using these angles and phase, as three matrices (one for each inter-generation coupling) were multiplied together.

$$V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix} \begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -\sin\theta_{13}e^{i\delta} & 0 & \cos\theta_{13} \end{pmatrix} \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{43}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{44}$$

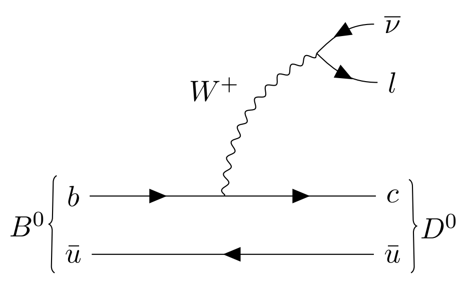
For current values of all CKM parameters, see CKMfitter. But how do we measure these values? We can use the cross sections of processes where all values are known but the matrix elements:

- $\Gamma(B^- \to \tau^+\nu_\tau) \approx f[V_{ub}, m_b, m_W, m_\tau, m_\nu, \alpha_s, \ldots]$



In this coupling, we simply have a decay constant, f_B, which is simpler to figure out but less informative *? he said something that effect at least.*

- $\Gamma(B^- \to D^0 l\nu) \approx f[V_{cb}, \ldots]$



This coupling is more complex as F_B is a form factor and requires more precision, but can be more informative.

So we know that the CKM matrix is Hermitian, i.e.

$$V_{CKM}V_{CKM}^{\dagger} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{45}$$

From the full Wolfenstein parameterisation,

$$V_{CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4) \tag{46}$$

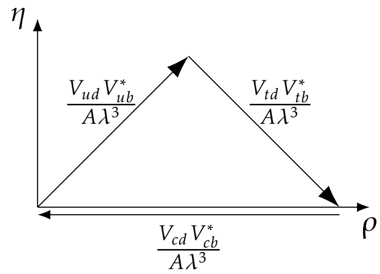
we can apply the Hermitian constraints of the CKM matrix to derive the **unitarity triangle**.
Note that we can see all CP violation is contained within the $\rho - i\eta$ terms.

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \tag{47}$$

$$\left(1-\frac{\lambda^2}{2}\right)(A\lambda^3)(\rho+i\eta) + (-\lambda)(A\lambda^2) + A\lambda^3(1-\rho-i\eta)\cdot 1 = 0 \tag{48}$$

$$A\lambda^3\left[\left(1-\frac{\lambda^2}{2}\right)(\rho+i\eta) - 1 + 1 - \rho - i\eta\right] = 0 \tag{49}$$

And so we can define the unitarity triangle in the $\rho - \eta$ plane.



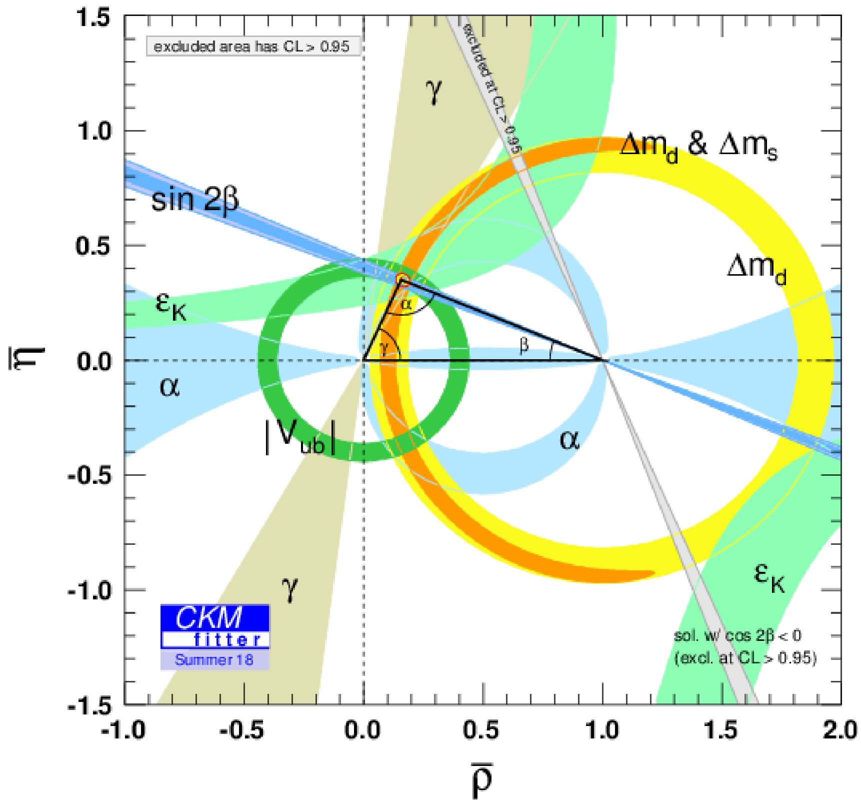
We can use the unitarity triangle to describe many things in quark mixing, such as CP violation, which will depend on one of the angles of the triangle.

$$\Gamma(B^- \to \tau\nu_\tau) = |V_{ub}|^2 f \tag{50}$$

$$\Gamma^{exp} = (\rho^2+\eta^2)A^2\lambda^6 f \tag{51}$$

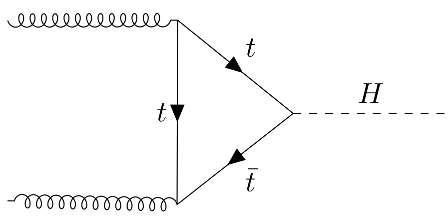
$$\rho^2+\eta^2 = \frac{\Gamma^{exp}}{A^2\lambda^6 f} \tag{52}$$

So by equating theory and experimental methods for a decay width, we can settle on a unitarity circle, that will centre on the origin of the $\rho - \eta$ plane. This will in fact be a disc around the origin due to uncertainties. We can do this with many processes and develop the $\rho - \eta$ plane to describe most of the current picture of flavour physics, using all measurements and behaviours of CP violations and rare decays. The shaded regions in this plane can then give us hints to new physics if they do not align properly with the unitarity triangle fit.
clarify what all the circles and areas mean?



Global CKM fit in $\rho - \eta$ plane from CKMfitter's current plots, as of 12/11/2019.

Could a lot of the possible signs of new physics be down to a fourth generation of particles? Well one indicator of that is through the Higgs mechanism. Only massive particles can couple well to the Higgs, so the lighter quarks essentially do not couple with it, while the top quark does:



For a fourth generation to exist, we would expect this more massive pair of the top and bottom quarkto have a coupling like above to the Higgs boson. A fourth generation would be expected to have the exact physics replicated as in the Standard Model for three generations, and this would lead to a scattering amplitude for the above Higgs production 6 times that of the current amplitude; however, other methods would suggest 9 times the amplitude *(I think I'm missing something in this explanation, but can't remember it).* This discrepancy can possibly be reconciled by using the two Higgs doublet model.