Numerical Solutions to Quarkonia Wavefunctions

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Introduction

Quarks are fundamental particles of the Standard Model of particle physics, most well-known for being the constituent particles of the proton and neutron. They **cannot exist as free particles**, and so will only exist inside of composite particles with other quarks, such as the proton. They can form quark-antiquark pairs known as mesons - these particles can be studied similarly to the Hydrogen atom.

Theory

$$-\frac{\hbar^2}{2\mu}\nabla^2\psi + [V(r) - E_{nl}]\psi = 0 \tag{1}$$

$$V(r) = -\frac{4\alpha_s}{3r} + \beta r \tag{2}$$

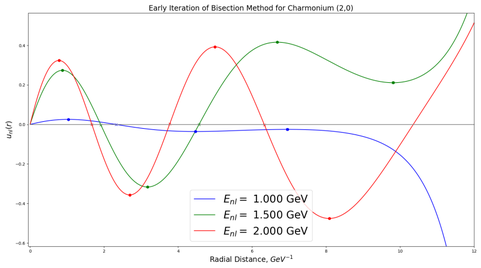
Radial wavefunction, $u_{nl}(r) = rR_{nl}(r)$

$$\frac{du_{nl}}{dr} = v_{nl} \tag{3}$$

$$\frac{dv_{nl}}{dr} = \frac{l(l+1)}{r^2}u_{nl} - 2\mu[E_{nl} - V(r)]u_{nl} \tag{4}$$

The Bisection Method

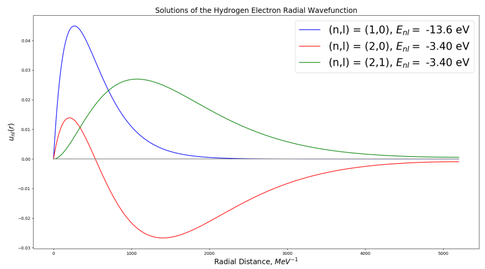
The correct u_{nl} can be found using the **bisection method**. This method guess three energies, E_1, E_3, and $E_2 = \frac{E_1+E_3}{2}$. Equations (3) and (4) can be solved for these energies using `scipy.integrate.odeint` or a similar ode solver. Count the nodes and turning points for each function - if these differ between two solutions, E_{nl} lies somewhere between those energies.



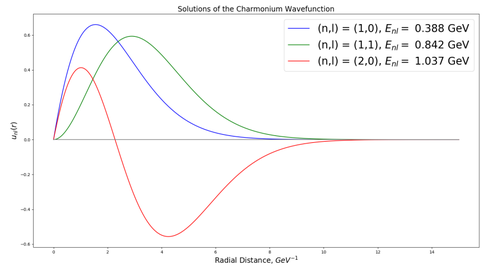
E_1 and E_3 are set to the two energies over the difference, and E_2 calculated again. A correct solution to u_{nl} will have $(n-1)$ nodes and n turning points, as energies converge on E_{nl}.

The Hydrogen Wavefunction

$$\frac{4\alpha_s}{3} \to \alpha = \frac{1}{137},\ \beta \to 0,\ \mu \to m_e \tag{5}$$

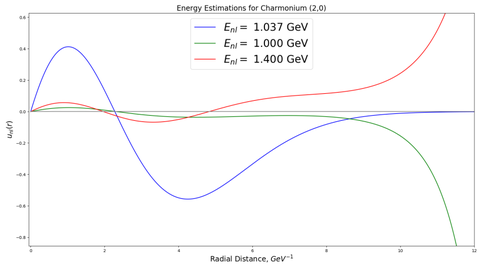


Results of Computations



Outlook

Faults of the Method



References

[1] C. Amsler et al [Particle Data Group]. Review of particle physics. http://pdg.lbl.gov/, February 2019.

[2] Alan D. Martin Francis Halzen. *Quarks and Leptons: An Introductory Course In Moden Particle Physics.* John Wiley and Sons, 1984.