

Physics Problem Solving Computing Project

Quarkonium

Matthew
Rossetter

Quarkonium

Matthew Rossetter

Durham University

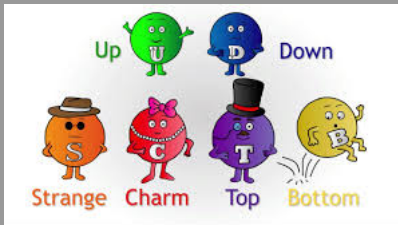
November 21, 2018

Introduction

Quarkonium

Matthew
Rossetter

- Project will solve Schrodinger equation of a quarkonium system
- Study the bound states found given by solutions
- Expand from this to explore further physics
- Quantum Chromodynamics (QCD) is the study the strong force, quarks and gluons
- Gluons are the force carriers for the strong force
- Quarks cannot exist as free particles



Introduction

Background Physics

Quarkonium

Matthew
Rossetter

- QCD gets split into two regimes - one can be studied similarly to QED
- Charm and Bottom quark systems have high enough masses to be studied using standard methods
- A Bottom or Charm quark-antiquark bound pair can be modeled similarly to the hydrogen atom
- Apply QCD similarly to QED for hydrogen
- The non-relativistic 3D Schrodinger equation is satisfied by these systems

$$-\frac{\hbar^2}{2\mu}\nabla^2\psi + [V(r) - E_{nl}]\psi = 0 \quad (1)$$

$$V(r) = -\frac{4\alpha_s}{3r} + \beta r \quad (2)$$

Numerical Methods for Solving the Wavefunction

Quarkonium

Matthew
Rossetter

- Radial wavefunction, u_{nl} , found from solving a set of ODEs

$$\frac{du_{nl}}{dr} = v_{nl} \quad (3)$$

$$\frac{dv_{nl}}{dr} = \frac{l(l+1)}{r^2}u_{nl} - 2\mu(E_{nl} - V(r))u_{nl} \quad (4)$$

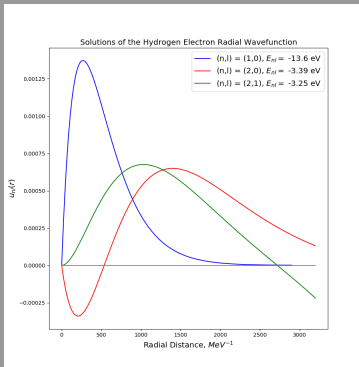
- Solve iteratively to find correct energy
 - Guess three initial energies/betas and solve equation
 - Compare nodes and turning points of solution and theory
 - Narrow down energies/betas until a solution is found
 - Guess new initial energies if no solutions found
 - Use Simpsons method, or similar numerical integrator, to normalise solution

The Milestone Project

Solving the Hydrogen Wavefunction

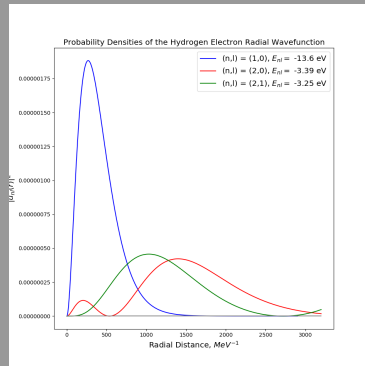
Quarkonium

Matthew
Rossetter



- Program for Quarkonium applied to Hydrogen
- $\frac{4\alpha_s}{3} \rightarrow \frac{1}{137}, \beta \rightarrow 0, \mu \rightarrow m_e$
- Iterate over energies

- $E_{1,0} = -13.6 \text{ eV}$
- $E_{2,0} = -3.39 \text{ eV}$
- $E_{2,1} = -3.25 \text{ eV}$



Further Extensions

Quarkonium

Matthew
Rossetter

- Will apply program to charmonium and bottomonium
- Possible ventures:
 - Consider other models of the potential
 - Use program to solve and study other quark systems
 - Time dependence of states and lifetime
 - Hyperfine splitting and transition between spin states
 - Charmonium melting into quark-gluon plasma

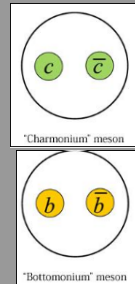
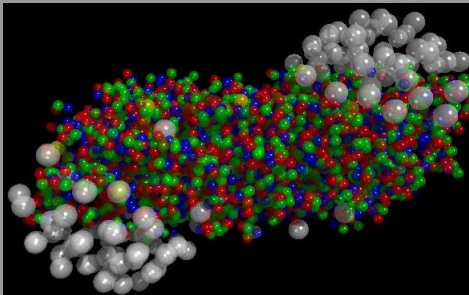


Figure: Quark-Gluon Plasma