Quarkonium

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Introduction

- Project will solve Schrodinger equation of a quarkonium system
- Study the bound states found given by solutions
- Expand from this to explore further physics
- Quantum Chromodynamics (QCD) is the study the strong force, quarks and gluons
- Gluons are the force carriers for the strong force
- Quarks cannot exist as free particles

- QCD gets split into two regimes - one can be studied similarly to QED
- Charm and Bottom quark systems have high enough masses to be studied using standard methods
- A Bottom or Charm quark-antiquark bound pair can be modeled similarly to the hydrogen atom
- Apply QCD similarly to QED for hydrogen
- The non-relativistic 3D Schrodinger equation is satisfied by these systems

$$-\frac{\hbar^2}{2\mu}\nabla^2\psi + \left[V(r) - E_{nl}\right]\psi = 0 \tag{1}$$

$$V(r) = -\frac{4\alpha_s}{3r} + \beta r \tag{2}$$

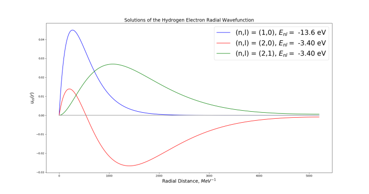
Numerical Methods for Solving the Wavefunction

- Radial wavefunction, u_{nl}, found from solving a set of ODEs

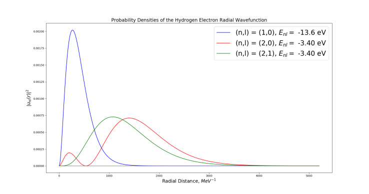
$$\frac{du_{nl}}{dr} = v_{nl} \tag{3}$$

$$\frac{dv_{nl}}{dr} = \frac{l(l+1)}{r^2}u_{nl} - 2\mu(E_{nl} - V(r))u_{nl} \tag{4}$$

- Solve iteratively to find correct energy
  - Guess three initial energies/betas and solve equation
  - Compare nodes and turning points of solution and theory
  - Narrow down energies/betas until a solution is found
  - Guess new inital energies if no solutions found
  - Use Simpsons method, or similar numerical integrator, to normalise solution



- $E_{1,0} = -13.6\,\text{eV}$
- $E_{2,0} = -3.39\,\text{eV}$
- $E_{2,1} = -3.25\,\text{eV}$



- Program for Quarkonium applied to Hydrogen
- $\frac{4\alpha_s}{3} \to \frac{1}{137}, \beta \to 0, \mu \to m_e$
- Iterate over energies

Further Extensions

- Will apply program to charmonium and bottomonium
- Possible ventures:
  - Consider other models of the potential
  - Use program to solve and study other quark systems
  - Time dependence of states and lifetime
  - Hyperfine splitting and transition between spin states
  - Charmonium melting into quark-gluon plasma

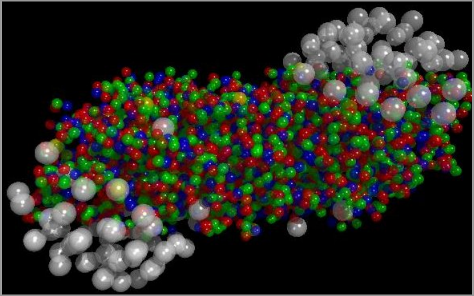


Figure: Quark-Gluon Plasma