Angular dispersion boost of high order laser harmonics with dense plasma clusters

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***Abstract*—Periodic surface gratings or photonic crystals are excellent tools for diffracting light and to collect information about the spectral intensity, if the target structure is known, or about the diffracting object, if the light source is well defined. However, this method is less effective in the case of extreme ultraviolet (XUV) light due to the high absorption coefficient of any material in this frequency range. Here we propose a nanosphere array target in the plasma phase as an efficient dispersive medium for the intense XUV light which is originated from laser-plasma interactions where various high harmonic generation processes take place. The scattering process is studied with the help of numerical simulations and we show that the angular distribution of different harmonics after scattering can be perfectly described by a simple interference theory.**

I. Introduction

Limited size targets interacting with high-intensity coherent radiation is well-studied phenomenon of linear excited surface plasmonic oscillations. Absorption and scattering of incident light in this case good described with Mie theory predicting exist of resonance corresponding to multipole oscillations of part of the target free electrons relative to positive charged ions. In resonance mode efficient exciting of surface plasmons can lead to significant boost internal and external field on fundamental cluster frequency (eigenfrequency). In turn, this can cause enhancement of field scattered on large angles relative to the direction of incident wave.

In micrometer wavelengths photon crystals and lattices can be used for direction or diffraction electromagnetic waves [1], while for x-ray radiation it is possible to use real crystals with regularly placed scattering centers (atoms) with distance of few nanometers [2]. At the same time, large interval between these wavelength orders named XUV (extreme-ultraviolet) is hard to manipulate.

Within the present work we consider the possibility of directed scattering of short wavelength radiation in the XUV range by scattering on suitable spherical clusters. Similar case with cylindrical symmetry (arrays of nanocylinders as scatters) was researched earlier [3]. Of course, nanocylinders are more suitable regarding the control of size an distance parameters at the target manufacturing stage, but arrays of spherical clusters can make possible to manipulate with light direction in three-dimensional space and give a more optimal spatial configuration.

II. Base model

A. Dielectric function

Firstly we consider a single cluster with radius a irradiated by short femtosecond pulse with intensity about $I_h \approx 10^{14}$ W/cm^2. The Drude model yields the dielectric function of the plasma:

$$\varepsilon(\omega) = 1 - \left(\frac{\omega_{pe}}{\omega}\right)^2 \frac{1}{1 + i\beta_e}, \qquad \omega_{pe} = \sqrt{\frac{4\pi e^2 n_e}{m_e}}, \tag{1}$$

where ω — harmonic (angular) frequency under consideration; ω_{pe} — the electron plasma frequency; e, m_e — electron charge and mass; $n_e = Z n_i$ — the electron number density, where Z — average ionization degree, n_i — ion density. $\beta_e = \nu_e/\omega$ and ν_e — electron-ion collision rate in Spitzer approximation. As we are going to consider scattering of harmonic radiation, the cluster should have a density above the critical one for this harmonic: $n_c = \omega^2 m_e / 4\pi e^2$. For 10-th Ti:Sa laser harmonic with wavelength $\lambda_L = 830$ nm one obtains condition $n_e > 1.3 \cdot 10^{23}$ cm^{-3}.

B. Mie Theory

In the case of linear interaction Mie theory can be used for the description of elastic electromagnetic waves scattering by arbitrary sized particles and beyond that, it allows the description of the electric and magnetic field distribution inside and outside the scatter [4]. A main step is to solve the scalar Helmholtz Equation in suitable coordinate system and gain the vector solutions. For spherical cluster the solution of corresponding equation can be written in a spherical Bessel function of the l-th kind and n-th order the spherical harmonic including the associated Legendre polynomial [4].

Assume an incident plane wave propagating along z axis of cartesian coordinate system and polarized along x axis:

$$\mathbf{E}_i = E_0\, e^{i\omega t - ikz}\, \mathbf{e}_x, \tag{2}$$

where $k = \omega/c$ — wavenumber, $\mathbf{e}_x$ — the unit vector of x axis direction and polarization vector.

Now we can expand the plane wave into series using generalized Fourier expantions. Assuming our media is isotropic we obtain following form of scattered field [4]:

$$\mathbf{E}_s = \sum_{n=1}^{\infty} E_n \left[i a_n(ka, m) \mathbf{N}^{(3)}_{e1n} - b_n(ka, m) \mathbf{M}^{(3)}_{o1n} \right] \quad (3)$$

$$E_n = i^n E_0 \frac{2n+1}{n(n+1)}$$

n — vector harmonic number after cartesian-spherical coordinate system transformation, $m = \sqrt{\varepsilon(\omega)}$ — refractive index of the target.

Coefficients in Eq. 3 have the following form [4]:

$$a_n(x, m) = \frac{m\psi'_n(x)\psi_n(mx) - \psi'_n(mx)\psi_n(x)}{m\xi'_n(x)\psi_n(mx) - \psi'_n(mx)\xi_n(x)}, \quad (4)$$

$$b_n(x, m) = \frac{\psi'_n(x)\psi_n(mx) - m\psi'_n(mx)\psi_n(x)}{\xi'_n(x)\psi_n(mx) - m\psi'_n(mx)\xi_n(x)}, \quad (5)$$

where $\psi_n(z) = zj_n(z)$, $\xi_n(z) = zh_n(z)$ — Riccati-Bessel functions, $h_n = j_n + i\gamma_n$ — spherical Hankel functions of the first kind.

C. Resonance conditions

To investigate the conditions under which resonant field enhancement occur the determination of the desired coefficients (Eq. 4, 5) is necessary in general. Since we are only interested in particle sizes which are smaller than the incident wavelength we use the limiting forms of the respective Bessel functions. In this asymptotic limit the coefficients are of a much simpler form:

$$a_n(x \to 0, m) = \left(1 + iC_n \frac{\left(m^2 + \frac{n+1}{n}\right)}{(m^2 - 1)} \frac{1}{x^{2n+1}}\right)^{-1}, \quad (6)$$

$$b_n(x \to 0, m) = 0,$$

$$C_n = \frac{1}{2^{2n-1}} \frac{(2n-1)!(2n+1)!}{n!(n+1)!}$$

In this case amplitude of the scattered field is maximum for $m^2 = -(n+1)/n$ when $ka \ll 1$, that gain corresponding set of resonance densities in collision-less case:

$$n_e = \frac{2n+1}{n} n_c.$$

Eq. 6 can be used instead of Eq. 4, 5 for scatters with quite small radius, but for $ka \sim 1$ the approximation ceases to be reasonable already, particularly for large n. In this case the first-order approximation, that can be obtained including first term in polynomial expansion of Bessel functions, is better suited [5]. Moreover, in first-order approximation dependency of coefficients on the scatter size occurs, which lead to corresponding dependency of resonance electron density (Fig. 1).

Such approximations allow us to estimate the resonance cases for a material with pre-defined refractive index m

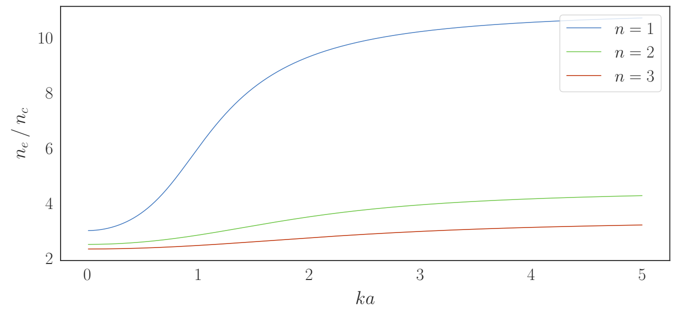

Figure 1. Resonance electron density depending on radius in collision-less case.

as well as estimate refractive index corresponding to the required wavelength. As we consider XUV range radiation (20-120 nm), radiuses of spherical scatters should be about few nanometers, that causes $ka \sim 1$. Obviously, for such ka the resonance values of the electron density can be large in considering $n = 1$ as term with the largest contribution to the scattered field. Staying within high-temperature plasma we should use only $n_e < 10^{24}$ cm^{-3}.

In first-order approximation with wavelength $\lambda_{10} = \lambda_L/10 = 83$ nm, we have $n_e \approx 5.7 \cdot 10^{23}$ cm^{-3} for $ka = 0.7$ ($a \approx 8.91$ nm) to reach efficient scattering.

III. Simulations

A. Scattering by a single cluster

B. Scattering by an array

Using resonance conditions obtained with analytical model, we consider diffraction by arrays of plasma clusters. Simple cubic lattice with $N = 12$ edge nodes and different grating constant d is considered spatial configuration of a volume grating. As the incident field we use gaussian beam with width $w = 300$ nm, that allow to assume the lattice as cluster layers by continuing cubic structure periodically along the entrance surface vector.

Diffraction orders can be obtained with Bragg's law [6]:

$$2d \sin(\theta + \varphi) = n\lambda \quad (7)$$

where d — the fringe spacing of the grating, θ — the angle between the incident beam and the normal to the entrance surface, ϕ — the angle between the normal and the grating vector, n — the order of diffraction, λ — diffracted wavelength. As we choose cubic lattice, $\varphi = 0$ due to the symmetry, that lead to planar Wolfe–Bragg's condition.

According to Eq. 7 we obtain $\theta = \arcsin\frac{1}{4}$ for grating with $d = 2\lambda$ and $\theta = \arcsin\frac{1}{6}$ for grating with $d = 3\lambda$ in 1-st diffraction order. Using these values we numerically compute the scattered field of 10-th laser harmonic with wavelength $\lambda_{10} = 83$ nm, using CELES package, based on T-matrix calculations [7], [8].

We can see directions corresponding to the angular boost of the incident beam, in particular, for $d = 2\lambda_{10}$ there are two fairly clear directions (Fig. 2), while for $d = 3\lambda_{10}$ there are three less clear ones with wider spread of the scattered field in general (Fig. 3).

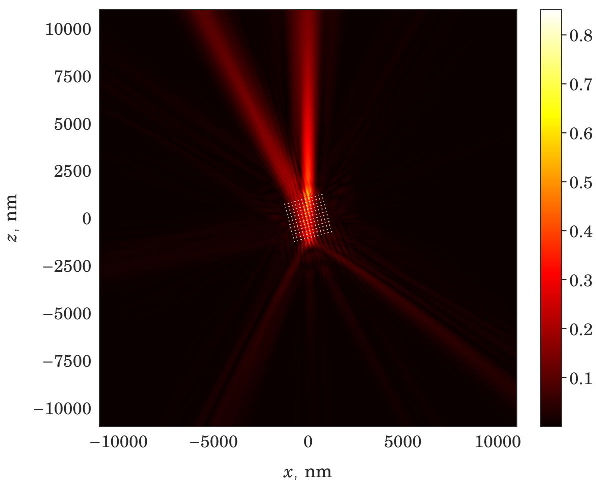

Figure 2. Scattered electric field normalized by the incident beam amplitude, the glancing angle $\theta = \arcsin \frac{1}{4}$, the grating period $d = 2\lambda_{10}$.

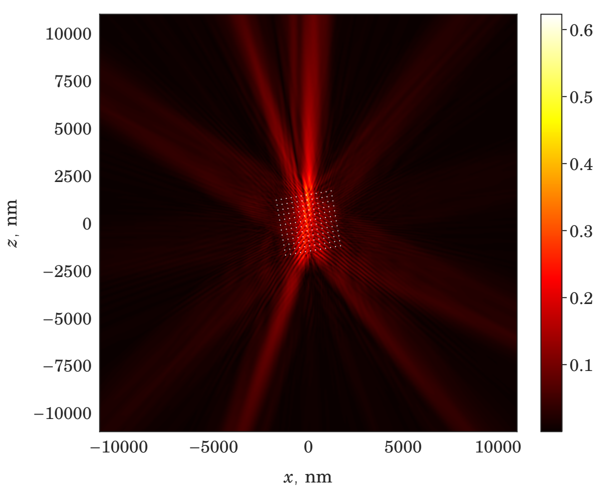

Figure 3. Scattered electric field normalized by the incident beam amplitude, the glancing angle $\theta = \arcsin \frac{1}{6}$, the grating period $d = 3\lambda_{10}$.

Conclusion

The results show the correspondence of the Bragg-Wolfe diffraction theory for planar and spatial gratings, the ability to control high harmonics of laser radiation (XUV range) using an ionized cluster gas.

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