# NUMERICAL METHODS

ASSIGNMENT

MISHA AKRAM
CS-18118
SUBMITTED TO:
DR. FAHIM RAESS

# Numerical Methods MT-442

**Problem definition**

You are required to modify **Four stage Runge-Kutta (R.K) method** for proper choice of weighting coefficients (ci) and the location of slope computation i.e. parameters ($\theta i \; and \; wi$).

Four stage R.K. methods are defined in the following way in terms of general coefficients.

$yi$+1 = $yi$ + 4 $i$=1 $ciki$ , Such that 4 $i$=1 $ci$ = 1.

Where, $k1 = \hbar f \; xi, yi$ And

$kj$+1 = $\hbar f \; xi + \theta j \; \hbar, yi + wj \; kj$ , ∀ $j$ = 1, 2 $and$ 3. 'h' is width of interval.

**Test Problem**

$\boldsymbol{dy/dx}$ =($\boldsymbol{ax}$+$\boldsymbol{by}$)/($\boldsymbol{bx}$+$\boldsymbol{ay}$ )

At a=18 and b=1.33

$\boldsymbol{dy/dx}$ =(**18$\boldsymbol{x}$+1.33$\boldsymbol{y}$**)/ (1.**33$\boldsymbol{x}$+18$\boldsymbol{y}$** )

domain 0 ≤ $x$ ≤ 18.

The initial condition is $y$ 0 = 18/1.33

## 1-STANDARD RK METHOD:

Four stage R.K. methods are defined in the following way in terms of general coefficients.

Here:

- X0 = 0
- Y0 = 18 / 1.33
- Xn = 18
- h = 0.5
- θ1= 0.5
- θ2= 0.5
- W1 = 0.5
- W2=0.5
- c1 = 1/6
- c2 = 2/6
- c3 = 2/6
- c4 = 1/6

At **x = 18, y = 23.1835**

**T**he value obtained using 4 stage standard runge-kutta method is considered as the exact value for the rest of the computation.

## 2- MODIFIED RK METHOD:

For modified RK method we use same function but with different coefficients that are given below

Here:

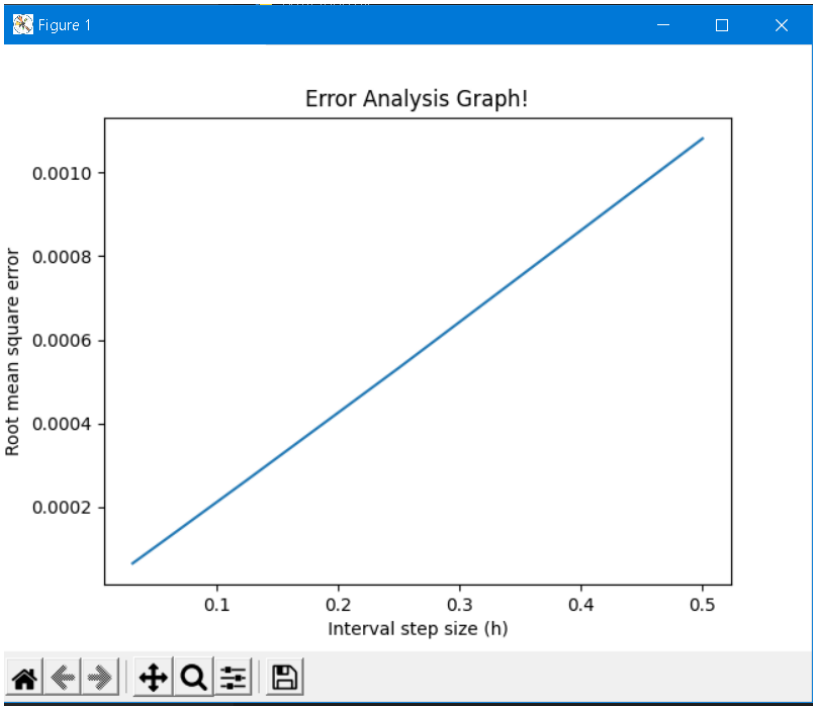
- X0 = 0
- Y0 = 62.6016
- Xn = 77
- h = 0.5
- θ1 = 0.8
- θ2=0.5
- W1 = 0.8
- W2=0.4
- c1 = 2/9
- c2 = 3/9
- c3 = 4/9
- c4 = 0/9

For the selection of w1, w2, θ1, θ2, c1, c2, c3, c4 trial and error method is used. To determine the values

## Scalability Test:

| Iterations | Interval size (h)=0.5 | Y(standard) | Y(Modified) | Root mean square error |
|---|---|---|---|---|
| 1 | h = 0.5 | 23.183527837687038 | 23.185544653707176 | 0.001081202279350705 |
| 2 | h/2 = 0.25 | 23.18352780215029 | 23.184532070795548 | 0.0005326799522002232 |
| 3 | h/4 = 0.125 | 23.18352779995445 | 23.184029118593728 | 0.00026430099265719914 |
| 4 | h/8 = 0.0625 | 23.183527799818002 | 23.183778282549444 | 0.000131632582507721 |
| 5 | h/16 = 0.03125 | 23.18352779980948 | 23.183653000441232 | 6.568574277127841e-05 |

From the above values obtained we can conclude that our provided modified RK-method is scalable, as we reduce the interval size the value of root mean square reduces.




In this graph the x-axis is the interval size (h) and y-axis represents the root mean square error between values obtained from standard and modified RK method.'

From the graph it can be concluded that as the value of interval size (h) reduces rmse also reduces, hence a linear line is obtained which indicates that the condition for super convergence has been achieved, and our proposed method is scalable.