Systems & Control Engg.
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SC 624, Spring 2018
Quiz 1
Date: 13 Jan 2018
Time: 30 min

All questions are of equal weightage.

No collaboration allowed, No notes or books allowed, No electronic devices allowed.

1. Let $f : X \to Y$ be a function. Show that $f^{-1}(f(A)) = A, \forall A \subset X$ iff f is injective.

2. Let $f : X \to Y$ be a function. Show that $f(f^{-1}(A)) = A, \forall A \subset Y$ only if f is surjective.
Is it true that $f(f^{-1}(A)) = A, \forall A \subset Y$ if f is surjective ? If it is true, prove the statement. Else, give a counterexample.

3. Show that the function

$$\mathbb{R}^2 \ni (x_1, x_2) \mapsto f(x_1, x_2) := \begin{cases} 0 & \text{if } (x_1, x_2) = (0, 0) \\ \frac{{x_1}^3}{{x_1}^2 + {x_2}^2} & \text{otherwise,} \end{cases}$$

has directional derivatives at $(0, 0)$ along the x_1- and x_2-axes. Is the function f continuous at $(0, 0)$? Justify.