Systems & Control Engg.
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SC 624, Spring 2018

Quiz 6

Date: 11 April 2018

Time: 90 min

Throughout the paper, X is an open subset of $\mathbb{R}^n$, and f, g denote vector fields on X, and γ_t^f and γ_t^g denote their respective flows.

1. (10 points) Let $\phi : X \mapsto X$ be a diffeomorphism, and f a vector field on X. We define $Ad_\phi(f)$ to be the vector field such that $Ad_\phi(f)(p) = D\phi(\phi^{-1}(p))f(\phi^{-1}(p))$. Let σ_t denote the flow of $Ad_\phi(f)$. Show that $\sigma_t = \phi \circ \gamma_t^f \circ \phi^{-1}$.

2. (5 points) Show that
$$Ad_{\gamma_t^f} f = f.$$

3. (5 points) Show that
$$[Ad_\phi f, Ad_\phi g] = Ad_\phi [f, g]$$

4. (5 points) Show that
$$\frac{\partial}{\partial t} Ad_{\gamma_t^f} g \bigg|_{t=s} = -[f, Ad_{\gamma_s^f} g] = -Ad_{\gamma_s^f}[f, g]$$
(Hint : Use the fact that $[f, g] = \frac{\partial}{\partial t} Ad_{\gamma_t^f} g \Big|_{t=0}$ and the results from Q2 and Q3.)

5. (10 points) Use the above results to show that if $[f, g] = 0$, then $\gamma_t^f \circ \gamma_s^g = \gamma_s^g \circ \gamma_t^f, \ \forall s, t \in \mathbb{R}$. (Hint : Rewrite the above relation as $\gamma_{-s}^g \circ \gamma_t^f \circ \gamma_s^g = \gamma_t^f$. Does the expression on the LHS look familiar ?)