Part D

Given,

$t = 22$

$\theta_{22}|Y_{1:22} \sim N(m_{22} = 3.539,\ C_{22} = 3.048)$

State equation: $\theta_j = G\theta_{j-1} + w_j$

Since observations $Y_{23}, Y_{24}, \ldots, Y_{29}$ are missing, we will skip NAs. we need to make an 8 step predictions from $t=22$ to $t=30$.

Predictive Mean $(\alpha_{30}) = G^{(30-22)} \cdot m_{22} = G^{8} \cdot m_{22}$

$$\alpha_{30} = (0.8)^8 \cdot (3.539)$$

$$= 0.16777 \times 3.539$$

$$= 0.5937$$

$$\approx 0.594$$

Predictive variance $(R_{30}) = G^{2 \cdot 8} C_{22} + \sigma_w^2 \sum_{i=0}^{7} (G^2)^i$

$$= (0.8)^{16} \cdot (3.048) + 4 \cdot (1 + 0.8^2 + 0.8^4 + \cdots + (0.8)^{14})$$

$$= 0.08579 + 10.7986 \text{ (using Calculator)}$$

$$= 10.884$$

Therefore, The predictive distribution of $\theta_{30}|Y_{1:29}$ is indeed $N(\alpha_{30} = 0.594,\ R_{30} = 10.884)$