Part B

The given matrix is an diagonal matrix. So, the eigen-values are simply the elements on the main diagonal. The corresponding eigenvectors are the standard basis vectors.

$\therefore$ Eigenvalues and Eigenvectors are:

$\lambda_1 = 2, \; v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$\lambda_4 = -1, \; v_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

$\lambda_2 = 1.5, \; v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$\lambda_5 = 0.5, \; v_5 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

$\lambda_3 = -3, \; v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$\lambda_6 = -0.5, \; v_6 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

work for one Eigenpair.

we need to solve, $Av = \lambda v$

when $\lambda = 2$

$$\begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix} = 2 \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix}$$

this gives us the system of following equations:

$2v_1 = 2v_1$

$1.5v_2 = 2v_2 \Rightarrow 0.5v_2 = 0 \Rightarrow v_2 = 0$

$-3v_3 = 2v_3 \Rightarrow 5v_3 = 0 \Rightarrow v_3 = 0$

$-1v_4 = 2v_2 \Rightarrow 3v_4 = 0 \Rightarrow v_4 = 0$

$0.5v_5 = 2v_5 \Rightarrow 1.5v_5 = 0 \Rightarrow v_5 = 0$

$-0.5v_6 = 2v_6 \Rightarrow 2.5v_6 = 0 \Rightarrow v_6 = 0$

Thus, v_1 can be any non-zero value, and $v_2 = v_3 = v_4 = v_5 = v_6 = 0$

$\therefore$ the eigenvector when Eigenvalue (λ) is 2

is $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$