Fish Dispersal Model Post Hatching Based On Density Dependent Diffusion

Shrishti Mishra (22110246) Vedant Chichmalkar (22110282)
Shambhavi Agrawal (22110240) Sujal Patel (22110261) Mayank Gulati

October 3, 2023

Abstract

This report aims to explore the post-hatching migration of fry that results from the huge egg spawns during mating seasons. This includes time-dependent analysis of young fish post hatching and their subsequent path taken to maximise their chances for survival and food.

1 Problem Statement

Most fish reproduce by external fertilization. Both eggs and sperms are released into the water. After the sperm reaches egg, fertilization takes place, Fish populations exhibit a fascinating phenomenon where adults lay a multitude of eggs in a specific region. Following hatching, the young fish embark on a journey, dispersing in search of food resources essential for their growth and survival. These organisms are different from others in their way of reproduction. The motions, migrations, and re-distribution of these large number of fish from a specified location are hence of great interest. The population dispersal model based on density-dependent diffusion is studied to study the dispersion flow of newborn fish. Since most fish reproduce by spawning, it is an intriguing phenomenon to model.

The dispersal equations employed in our research elucidate the intricate movements of the newborn fish, shedding light on how individual fish navigate their habitats. For the study, we conduct methodical research to analyze the dispersal patterns of newly hatched fish, taking into account the effects of aeration and temperature on egg hatching, alongside the influence of variables like food availability. The effect of Population through flux analysis introduces a novel perspective, quantifying the flow of young fish across habitat boundaries.

By utilizing appropriate numerical methods, we gain the ability to capture the dynamic interplay between local population growth and migration, providing a comprehensive view of fish population dynamics.

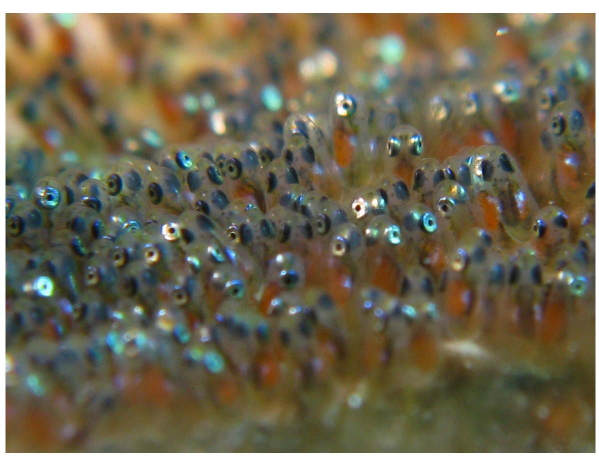

Figure 1: An Egg Spawn

2 Objectives

1. Create a scientific model to illustrate how newly born fishes disperse immediately after hatching from their eggs once deposited.

2. Establish relevant assumptions to simplify and make the problem more approachable.

3. Formulate the initial conditions, representing the hatching of eggs right after spawning, and boundary conditions to account for optimum hatching conditions, etc.

4. Select appropriate ecological parameters and properties to represent environmental conditions affecting fish development.

5. Employ effective numerical methods to perform computational simulations and analyze population dispersion.

6. Develop a computer program to solve the equations and generate numerical solutions.

7. Perform systematic investigations to examine the dispersion flow of newborn fishes considering the impacts of aeration and temperature on the hatching of eggs, as well as the influence of factors such as food availability on the mortality rate of the eggs.

3 Physical Model

Most fish lay thousands to millions of eggs at a time in a particular place during the spawning season. The eggs are considered to be laid in a specific region described by two dimensions. Initially, the eggs are assumed to be distributed in a two-dimensional Gaussian distribution. The hatching of the eggs depends on the temperature of the water and air conditions. These two factors are interdependent on each other and have a direct relation with the hatchability of the eggs. Other factors, such as the intensity of light and depth of water, seem to have no significant effect on the hatching of eggs[1]. Once the eggs start hatching, the newly born fish will disperse and hence diffuse out of the area initially defined. This population of fish is dependent on a number of factors, such as the number of eggs hatched and the mortality rate. Other factors such as food availability and optimal living conditions have not been considered keeping in mind the complexity of the equation. To make this model as realistic as possible, a non-constant rate of dispersal is considered that increases with overcrowding. Hence, a modified diffusion flux could be defined. As the population density of the fish increases, so will the diffusion coefficient.

4 Assumptions

It is difficult to consider all the parameters affecting the population dispersal of fish. In reality, there will be a number of factors affecting the hatching rate, mortality rate, and the dispersal rate of fish:

1. The problem is modelled in a two-dimensional setting, not considering differences in depth, and the dispersion is also considered to be dependent in a depth-independent manner.

2. The domain is infinite in both directions.

3. The eggs are initially normally distributed on the sea floor, and the velocity/dispersion rate depends upon the population at that point.

4. We have considered two major factors that affect the hatching of eggs which are Temperature and Aeration. Other factors like light, pressure, etc. have significantly less impact and thus are neglected.

5. Temperature and aeration factors are combined in a single skewed Gaussian distribution, which is taken to an effective amalgamation of the factors.

6. We have taken a numerically suitable value of k that can be fine-tuned to model real-life migration habits using other numerical methods suited to fitting coefficients.

5 Governing Equations

To develop a mathematical model for the said problem, we referred to literature to derive the fundamental equations of Temperature and Aeration, And then derive the relation between the density coefficient and the current population in a particular region.

5.1 Diffusion Flux

To make the model realistic assumptions about dispersal include a non-constant rate of dispersal that increases when overcrowded conditions prevail.

$$J = -D(p)\nabla p \tag{1}$$

$$D(p) = \text{Dispersion Coefficient} \tag{2}$$

Modelling the diffusion flux in two dimensions involves solving the two-dimensional diffusion equation, also known as Fick's second law of diffusion. Hence, this is the equation which describes the diffusion in two dimensions:

$$J = -D(p)\left(\frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}\right) \tag{3}$$

5.2 Diffusion Coefficient

$$D(p) = k_1 p \tag{4}$$

$$\begin{aligned} p &= \text{Population density at a given point} \\ k &= \text{A constant with the appropriate dimensions} \end{aligned} \tag{5}$$

k is a positive constant. Hence, an increase in the population causes the dispersal rate to increase.

5.3 Temperature and Aeration

Optimal temperature is necessary for hatching. Very high or low temperatures can have negative effects on the hatchability. When the area under consideration is not aerated, the there is a significant difference at 24 °C. The graph of hatchability vs temperature can be approximated as an exponentially decreasing function. We have considered the area to be aerated, and in this case the hatchability is maximum at around 27 °C. In aerated cases, the graph of hatchability vs temperature is approximated to be a skewed normal distribution.

$$f(x;\alpha,\mu,\sigma) = \frac{2}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \Phi\left(\alpha\frac{x-\mu}{\sigma}\right) \tag{6}$$

where:

α is the skewness parameter,

μ is the mean,

σ is the standard deviation,

$\Phi(x)$ is the cumulative distribution function of the standard normal distribution.

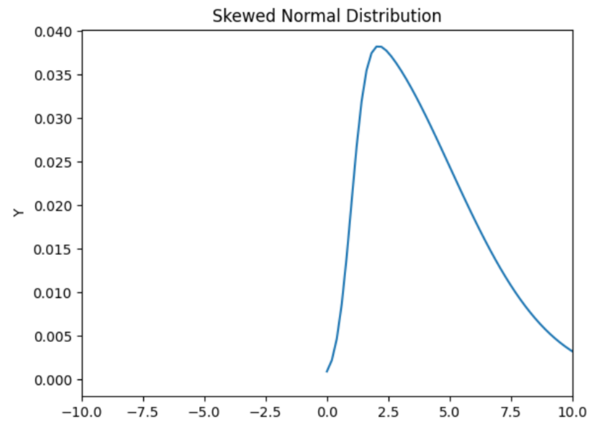

Figure 2: Hatching Rate v/s Time

5.4 Population change over time in 2D plane

By using Divergence Theorem we arrive at the following equation,

$$\frac{\partial P}{\partial t}(x,y,t) = K\left[\frac{\partial^2}{\partial x^2}\left[P^2(x,y,t)\right] + \frac{\partial^2}{\partial y^2}\left[P^2(x,y,t)\right]\right] + F(p) \tag{7}$$

6 Boundary Conditions

1. At time $t = 0$, the distribution of population density is considered to as a Gaussian distribution in two dimensions. The following plot displays the distribution in two dimensions:

2. The diffusion is maximum at an optimal temperature at which the hatching of eggs is most habitable for a given population density.

3. The population density will be zero at infinity.

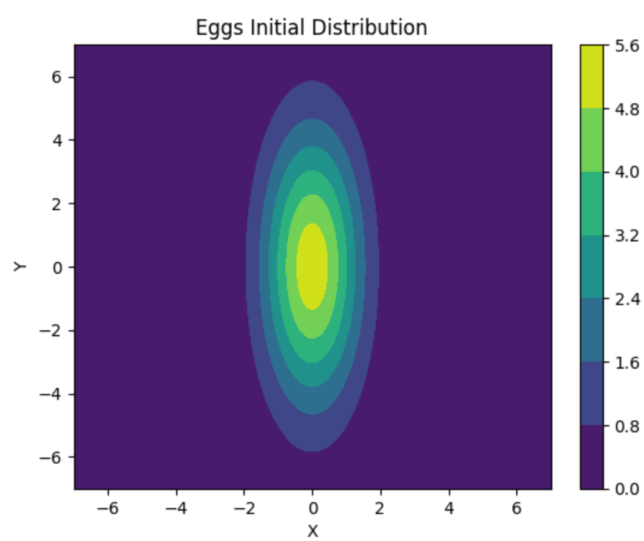

Figure 3: Initial Distribution of the Eggs

7 Numerical Solution

To solve equation 7, the finite difference method to solve PDEs is used. First the equation is discretized in both the x and y dimensions and in time (t). Let $P_{i,j}^n$ represent the numerical approximation of $P(x_i, y_i, t_n)$, where $x_i = i \cdot \Delta x, y_j = j \cdot \Delta y$ and $t_n = n \cdot \Delta t$. A short overview of the steps used to solve equation 4, using this method is given below:

1. Discretize the spatial and temporal domains:
   - Spatial grid: $x_i = i \cdot \Delta x$ and $y_j = j \cdot \Delta y$
   - Temporal grid: $t_n = n \cdot \Delta t$
2. Initialize the initial condition: Set $P_{i,j}^0$ to the initial condition $P(x_i, y_j, 0)$ for all i and j.
3. Time -stepping loop: The following finite difference equation is used to update $P_{i,j}^{n+1}$ from $P_{i,j}^n$:

$$P_{i,j}^{n+1} = P_{i,j}^n + \frac{\Delta t}{\Delta x^2}(P_{i+1,j}^n - 2P_{i,j}^n + P_{i-1,j}^n) + \frac{\Delta t}{\Delta y^2}(P_{i,j+1}^n - 2P_{i,j}^n + P_{i,j-1}^n)$$

   where i = 1, 2, ...,N$x - 2$, j = 1, 2, ...,N$y - 2$ and n = 0, 1, 2, ..., Nt-1.
4. Boundary conditions: Apply the appropriate boundary conditions.
5. Continue the time-stepping loop until the desired final time T is reached.
6. The final solution will be the values of $P_{i,j}^{n+1}$ at the final time step T.

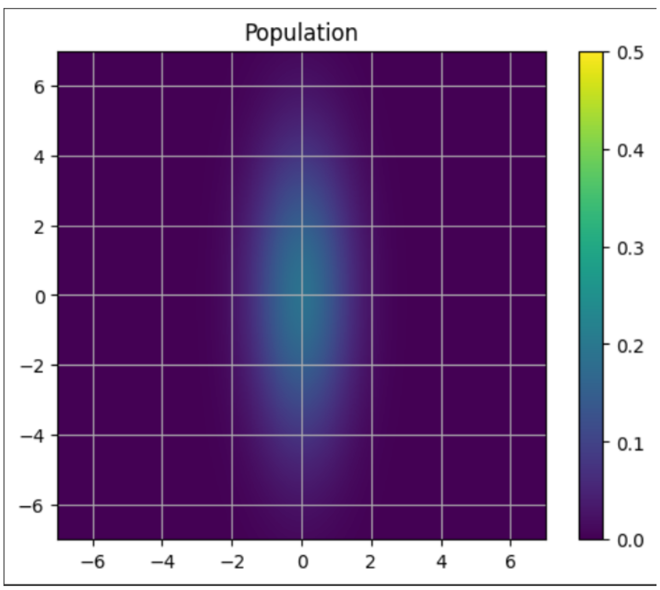

Figure 4: Population Density

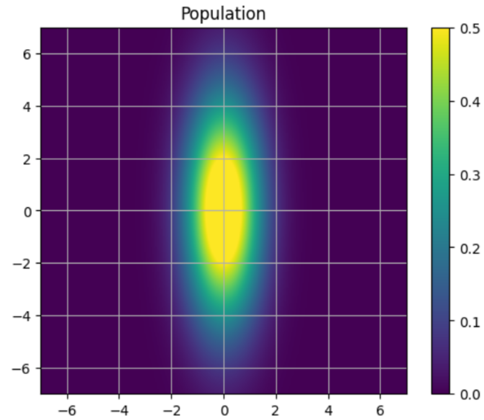

Figure 5

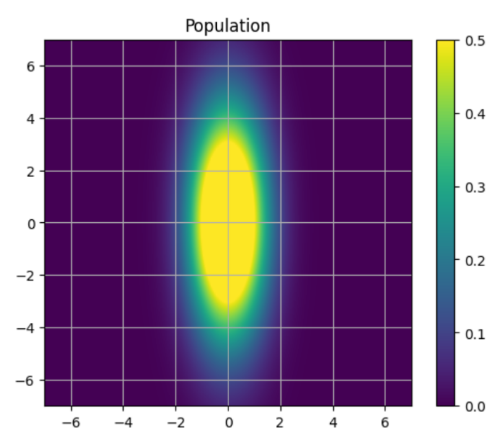

Figure 6

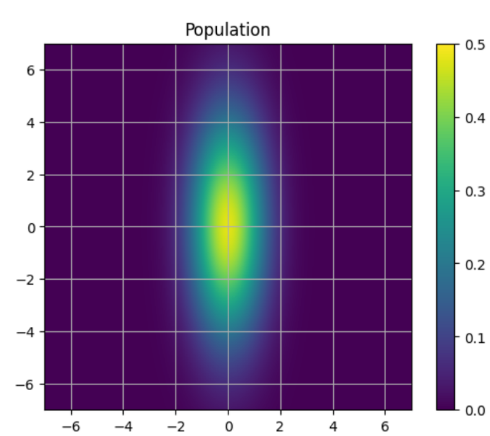

Figure 7

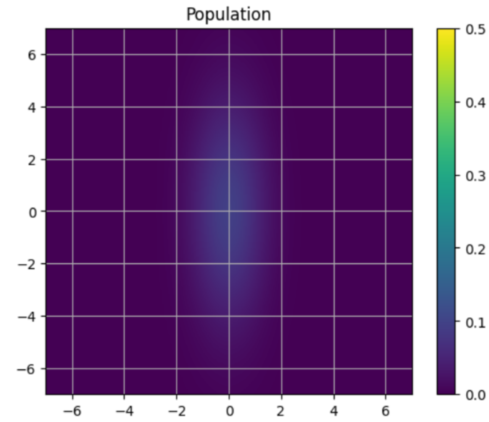

Figure 8

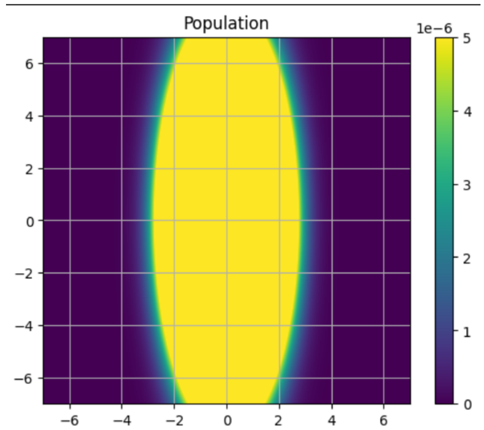

Figure 9: Population density with the scale lowered

8 Results and Conclusions

Using the above-mentioned techniques, we plotted the population density at several points in time at regular intervals, thus allowing us to see the movement at large of the fish. The results show convincingly that the population increases in the spawn region till a certain point, after which it reduces as the fish begin to spread out. To show the spread at later times, we lowered the scale.
The graphs show that young fry disperse in an efficient way to maximise their chances for survival and finding food. The fast dispersal also allows them to be less susceptible to large predator attacks and is most likely an important evolutionary trait that helps the survival of the species.

All the code required to recreate the above images and plots can be found at Link to the Colab Notebook

Acknowledgements

The authors would like to thank the institute for offering the course - MA 203, which provided the opportunity to apply the concept of Numerical Methods to ecological phenomena. This course has been instrumental in bridging the gap between theory and real-world applications, equipping the students with valuable tools to explore and analyze ecological dynamics. The authors would also like to extend their special gratitude to Prof. Akshaa Vatwani and Prof. Dilip Srinivas Sundaram for giving them the opportunity to get comfortable with the theory taught in the lectures.

During the literature review conducted for the purpose of this project, the authors found [1], [2], and [3] sources extremely insightful and helpful.

References

[1] Qihua Huang, Laura Parshotam, Hao Wang, Caroline Bampfylde, and Mark Lewis. A model for the impact of contaminants on fish population dynamics. *Journal of theoretical biology*, 334, 05 2013.

[2] Cassius Aruho, Bridget Kimera, Richard Ddungu, Rose Komugisha Basiita, Fred Bugenyi, and Justus Rutaisire. Optimal factors for egg hatchability and larvae development of barbus altianalis under captivity. *Journal of the World Aquaculture Society*, 54(4):828–843, 2023.

[3] *10. Partial Differential Equation Models in Biology*, pages 436–495.