Chapter 10

Conditional Probability

Conditional probability considers the probability of one event, given another event.

You need to be able to distinguish if you are being asked for a conditional probability with the words describing the situation.

One example they show you in OLI:

They consider different probabilities of being male or female and having pierced ears or not having pierced ears. They call being female "not M" and they call not having pierced ears "not E."

They ask which of the following describes the conditional probability:

$$P(not\ E|not\ M)$$

This conditional probability describes the probability that a person does not have pierced ears, given that they are a female. The options they give are:

(a) a randomly chosen male does not have pierced ears.
(b) a randomly chosen female does not have pierced ears.
(c) a randomly chosen student is a non-pierced female.
(d) a randomly chosen non-pierced student is a female.

The correct answer is: (b) randomly chosen female does not have pierced ears.

We know that they are talking about females and about not having pierced ears, so we can eliminate option (a), which is about males. The reason (c) is not correct, even though it describes choosing a non-pierced female, is because option (c) describes selecting a student. The conditional probability for option (c) would look something like,

$$P(not\ E\ and\ not\ M|student)$$

This statement is describing selecting someone who is both a female and does not have pierced ears, given that they are a student.

We are now left with options (b) and (d).

(b) a randomly chosen female does not have pierced ears.

(d) a randomly chosen non-pierced student is a female.

Because (b) starts by saying that we randomly chose a female we know that this is the pool of students out of which we find the probability that they do not have pierced ears. In other words, (b) describes the probability that someone does not have pierced ears, given that they are a female. Although (d) also describes a conditional probability, it describes the probability of a student being a female, given that they do not have pierced ears and so it is the other way around.

This is why the correct answer is

(b) a randomly chosen female does not have pierced ears.

$$P(not\ E|not\ M)$$

Bayes Rule

$$P(A|B) = \frac{P(B|A) * P(A)}{P(B)} = \frac{P(A\ and\ B)}{P(B)}$$

If we do not know whether some events A and B are independent, then we cannot conclude that

$$P(A\ and\ B) = P(A) * P(B) \qquad only\ for\ independent\ events$$

$$P(A\ and\ B) = P(B|A) * P(A) \qquad otherwise$$

If A and B are independent, then the probability of B, given A is just the probability of B and the situation reduced to the formula for independent events.

You can check if two events are independent by testing whether the probability of the event, given some other event is equal to the probability of that event in general.

Lets say that we test the probability that a stranger accepts an offer to go on a date with someone, who pulls up to the stranger in a car. We perform this test in different cars.

There will be some probability that the stranger will accept the date. Lets say the probability is 5%. We can then test the probability that the stranger accepts the offer, given that the person offering them a date is driving a beamer.

If pulling up in a beamer has no effect on the probability of accepting the date then,

$$P(accept\ date|beamer) = P(accept\ date)$$

We can then conclude that these are independent events.

If pulling up in a beamer changes the likelihood that they accept the date, then

$$P(accept\ date|beamer) \neq P(accept\ date)$$

We can then conclude that these are not independent events.

We can also check independence by comparing the probability that a stranger accepts a date from someone driving one of two cars: a purple 2011 BMW 7 Series vs a 9 Fiat 600 Multipla Marinella.

Purple 2011 BMW 7 Series
http://www.thesupercars.org/bmw/purple-bmw-cars/

9 Fiat 600 Multipla Marinella
https://www.driving.co.uk/news/10-ugliest-cars-ever-made/

If pulling up in a beamer changes the likelihood that they accept the date, then

$$P(accept\ date|2011\ BMW\ 7\ Series) \neq P(accept\ date|9\ Fiat\ 600\ Multipla\ Marinella)$$

We can then conclude that these are not independent events.

Four ways to test independence is to ask whether

1) $P(A|B) = P(A)$ 2) $P(A|B) = P(A|not\ B)$

3) $P(B|A) = P(B)$ 4) $P(A\ and\ B) = P(A) * P(B)$

If 1 or 2 are true, then we can conclude that A and B are independent events. Otherwise, they are not independent events.

We mentioned that,

$$P(A\ and\ B) = P(A) * P(B) \qquad only\ for\ independent\ events$$

$$P(A\ and\ B) = P(A) * P(B|A) \qquad otherwise$$

If we know that two events are not independent, then we use the bottom equation to find P(A and B).

Lets say we are picking marbles out of a bag. There are 5 red marbles, 3 blue marbles, and 2 yellow marbles. If we pick one marble and put it back into the bag, then the second time we pick out a marble the total number of marbles of each color is the same. This means that the two events are independent. On the contrary, if we do not replace the marble we selected, then the second time we pick out a marble the number of marbles is different than it was originally. Then, the two events are not independent.

What is the probability that we pick out one red and then one blue marble?

If the marble is replaced into the bag, then the events are independent. Then,

$$P(red\ and\ blue) = P(red) * P(blue) = \frac{5}{10} * \frac{3}{10}$$

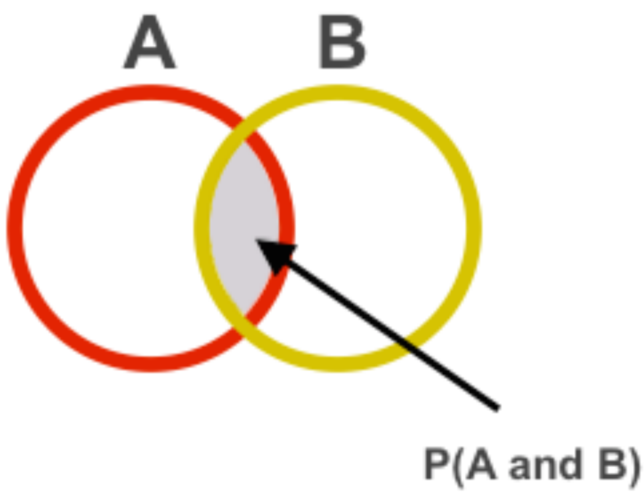
If the marble is not replaced into the bag, then the events are not independent. Then,

$$P(red\ and\ blue) = P(red) * P(blue|red) = \frac{5}{10} * \frac{3}{9}$$

We know that we picked the red marble out the first time. So, we now have only 9 marbles left in the bag.

Venn Diagrams and Conditional Probability

One way to think about conditional probability is to make a Venn diagram of the events.



$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

Checkpoint Hint

In one of the questions they ask you to calculate conditional probability, but they do not give you P(A and B). Instead, they give you P(A or B). You then need to find P(A and B).

We know that,

$$P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$$

Then, we can deduce that

$$P(A\ and\ B) = P(A) + P(B) - P(A\ or\ B)$$

This lecture was prepared by Elizaveta Latash