Chapter 11

Random Variable

A random variable assigns a unique numerical value to the outcome of a random experiment.

For instance, if we want to know the number of tails when flipping a coin three times then the random variable X could be zero (for HHH), one (for THH, HTH, HHT), two (for TTH, HTT, THT), or three (for TTT). This is an example of a discrete random variable because it can only take values 0, 1, 2, and 3.

If we want to know how long it takes to eat a whole pizza we could assign random variable X to be time. This is an example of a continuous random variable because it can take any value. It could take someone 1.283464527 hours to eat a whole pizza. We can discretize our random variable by saying that we want time rounded to the nearest second (1.283464527 hours $\sim$ 4621 seconds).

Discrete variables can be found by answering the question, – how many?
Continuous variables can be found by answering the question, – how much?

Probability distributions must satisfy:

$0 \leq P(X = x) \leq 1$

$\sum_{x} P(X = x) = 1$

The mean of a discrete random variable, or the expected value is

$$\mu_x = x_1 p_1 + x_2 p_2 + \cdots + x_n p_n = \sum_{i=1}^{n} x_i p_i$$

The variance of a discrete random variable is

$$\sigma_x^2 = (x_1 - \mu_x)^2 p_1 + (x_2 - \mu_x)^2 p_2 + \cdots + (x_n - \mu_x)^2 p_n = \sum_{i=1}^{n} (x_{i-}\mu_X)^2 p_i$$

The standard deviation of a discrete random variable is

$$\sigma_x = \sqrt{\sigma_x^2}$$

Binomial Random Variable

A binomial random variable comes from a binomial experiment. A binomial experiment has a fixed number of trials, two possible outcomes: either success or failure, a fixed success rate in each trial, and all trials in a binomial experiment are independent from one another.

Lets say you are flipping a coin 3 times and are calling getting tails a success.

$$The\ number\ of\ possible\ outcomes\ with\ k\ successes\ out\ of\ n\ trials = \frac{n!}{k!(n-k)!}$$

$$\frac{n!}{k!\,(n-k)!} = \frac{3!}{2!\,(3-2)!} = \frac{3*2*1}{2*1\ (1)} = \frac{6}{2} = 3$$

The three possible outcomes: TTH HTT THT

$$The\ probability\ of\ getting\ k\ successes\ out\ of\ n\ trials\ with\ probability\ p\ for\ success$$

$$P(X=k) = \frac{n!}{k!\,(n-k)!}p^k(1-p)^{(n-k)} \qquad k = 0\,,1\,,\ldots.n$$

Here, p is the probability of success, n is the total number of trials, k is the number of successes

In Excel =FACT(n)/((FACT(k)*FACT(n-k)))*(p^k)*(1-p)^(n-k)
=BINOM.DIST(k,n,p,0) for exactly k
=BINOM.DIST(k,n,p,1) for less than or equal to
=1-BINOM.DIST(k,n,p,1) for greater than

The mean of a binomial random variable, or the expected value is

$$\mu_x = np$$

The variance of a binomial random variable is

$$\sigma_x^2 = np\ (1-p)$$

The standard deviation of a binomial random variable is

$$\sigma_x = \sqrt{\sigma_x^2}$$

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