Sr. No. 1 Date :

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Roll No.- 2020PHY1092
Subject- GE-3
Linear Programming and Game Theory
Course- B.Sc. (H) Physics

Assignment- 7

Simplex, Big M and Two Phase Method.

1. a) Maximize $Z = 2x_1 + 5x_2$
subject to
$$3x_1 + 2x_2 \geq 6$$
$$2x_1 + x_2 \leq 2$$
$$x_1, x_2 \geq 0$$

Sol- Writing the LPP in standard form,

$$\text{Min. } Z = -2x_1 - 5x_2 + MA_1$$
$$Z + 2x_1 + 5x_2 - MA_1 = 0$$

Constraints-

$$3x_1 + 2x_2 - S_1 + A_1 = 6$$
$$2x_1 + x_2 + S_2 = 2$$
$$x_1, x_2, A_1, S_1, S_2 \geq 0$$

$n - m = 5 - 2 = 3$

$3 \rightarrow$ NBV $\quad 2 \rightarrow$ BV
Let A_1, S_2 be initial basic variables.

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	C_j	2	5	0	0	$-M$	RHS	Ratio
x_B	C_B	x_1	x_2	S_1	S_2	A_1		
A_1	$-M$	3	2	-1	0	1	6	$\frac{6}{3}=2$
S_2	0	2	1	0	1	0	2	$\frac{2}{2}=1 \rightarrow$
z_j-C_j		$-3M-2$	$-2M-5$	M	0	0		
		$\uparrow$						

x_1 will enter the basis and S_2 will leave

$R_2 \rightarrow \frac{R_2}{2}$ $\quad$ $R_1 \rightarrow R_1 - 3R_2^{new}$

	C_j	2	5	0	0	$-M$	RHS	Ratio
x_B	C_B	x_1	x_2	S_1	S_2	A_1		
A_1	$-M$	0	$\frac{1}{2}$	-1	$-\frac{3}{2}$	1	3	$\frac{3}{\frac{1}{2}}=6$
x_1	2	1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	1	$\frac{1}{\frac{1}{2}}=2 \rightarrow$
z_j-C_j		0	$-\frac{M}{2}-4$	M	$\frac{3M}{2}+1$	0		
			$\uparrow$					

x_2 will enter the basis and x_1 will leave.

$R_2 \rightarrow 2R_2$ $\quad$ $R_1 \rightarrow R_1 - \frac{1}{2}R_2^{new}$

	C_j	2	5	0	0	$-M$	RHS	Ratio
x_B	C_B	x_1	x_2	S_1	S_2	A_1		
A_1	$-M$	-1	0	-1	-2	1	2	
x_2	5	2	1	0	1	0	2	
		$M+8$	0	M	$2M+5$	0		

Since $z_j - C_j \geq 0$ for all j, therefore optimal solution has been obtained. But the artificial variable A_1 is still there in the basis at a positive level, so the original problem has no feasible solution.

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b) Maximize $z = 4x_1 + 12x_2 + 8x_3$

subject to
$$3x_1 + 2x_2 - 6x_3 \leq 20$$
$$3x_1 + 6x_2 + 4x_3 \leq 30$$
$$x_1, x_2, x_3 \geq 0$$

Sol. Writing the LPP in standard form,

Min. $z = -4x_1 - 12x_2 - 8x_3$

$$z + 4x_1 + 12x_2 + 8x_3 = 0$$

Constraints are -

$$3x_1 + 2x_2 - 6x_3 + S_1 = 20$$
$$3x_1 + 6x_2 + 4x_3 + S_2 = 30$$
$$x_1, x_2, x_3, S_1, S_2 \geq 0$$

$n - m = 5 - 2 = 3$

$3 \rightarrow$ NBV $2 \rightarrow$ BV

Let S_1, S_2 be initial basic variables.

	c_j	4	12	8	0	0		
x_B	c_B	x_1	x_2	x_3	S_1	S_2	RHS	Ratio
S_1	0	3	2	-6	1	0	20	$\frac{20}{2} = 10$
S_2	0	3	6	4	0	1	30	$\frac{30}{6} = 5$ →
		-4	-12	-8	0	0		
			↑					

x_2 will enter the basis and S_2 will leave.

$R_2 \rightarrow \frac{R_2}{6}$ $R_1 \rightarrow R_1 - 2R_2^{new}$

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	C_j	4	12	8	0	0	RHS	Ratio
x_B	C_B	x_1	x_2	x_3	S_1	S_2		
S_1	0	2	0	$-\frac{22}{3}$	1	$-\frac{1}{3}$	10	
x_2	12	$\frac{1}{2}$	1	$\frac{2}{3}$	0	$\frac{1}{6}$	5	
$z_j - C_j$		2	0	0	0	2		

$z_j - C_j \geq 0$ for all j

so, optimal solution is arrived.

Solution is $\rightarrow$ $(0, 5, 0)$ { Basic feasible }

Max $Z = 4(0) + 12(5) + 8(0)$

$= 60$

for $j = 3$, $z_j - C_j = 0$ but x_3 is not there in the basis.

So, we can find one more optimal solution by entering x_3 in the basis.

	C_j	4	12	8	0	0	RHS	Ratio
x_B	C_B	x_1	x_2	x_3	S_1	S_2		
S_1	0	2	0	$-22/3$	1	$-1/3$	10.	—
x_2	12	$1/2$	1	$2/3$	0	$1/6$	5	$\frac{15}{2}$ →
$z_j - C_j$		2	0	0 ↑	0	2		

x_3 is entering the basis and corresponding to it, x_2 will leave the basis.

$R_2 \rightarrow \frac{3}{2} R_2$ $R_1 \rightarrow R_1 + \frac{22}{3} R_2^{new}$

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x_B	C_j / C_B	4 / x_1	12 / x_2	8 / x_3	0 / S_1	0 / S_2	RHS	Ratio
S_1	0	$\frac{15}{2}$	11	0	1	$\frac{3}{2}$	65	
x_3	8	$\frac{3}{4}$	$\frac{3}{2}$	1	0	$\frac{1}{4}$	$\frac{15}{2}$	
$z_j - c_j$		2	0	0	0	2		

$z_j - c_j \geq 0$,

So, the solution obtained here is optimal.

Basis feasible solution is – $(0, 0, \frac{15}{2})$

$$\text{Max. } z = 0 + 0 + 8\left(\frac{15}{2}\right) = 60$$

So, two basis feasible solutions are-

I $(0, 5, 0)$, Max. $z = 60$

II $(0, 0, \frac{15}{2})$, Max. $z = 60$

c) Minimize $z = x_1 - 3x_2 - 6x_3$

subject to
$$2x_1 - x_2 + x_3 + x_4 \leq 60$$
$$3x_1 + 4x_2 + 2x_3 - 2x_4 \leq 150$$

Sol Writing LPP in standard form,

Min $z - x_1 + 3x_2 + 6x_3 = 0$

Constraints-
$$2x_1 - x_2 + x_3 + x_4 + S_1 = 60$$
$$3x_1 + 4x_2 + 2x_3 - 2x_4 + S_2 = 150$$
$$x_1, x_2, x_3, x_4, S_1, S_2 \geq 0$$

$n - m = 6 - 2 = 4$

$4 \to NBV \quad 2 \to BV$

Let S_1, S_2 be initial basic variables.

Table 1

	C_j	-1	3	6	0	0	0	RHS	Ratio
X_B	C_B	x_1	x_2	x_3	x_4	S_1	S_2		
S_1	0	2	-1	1	1	-1	0	60	60 $\to$
S_2	0	3	4	2	-2	0	1	150	$\frac{150}{2} = 75$
$Z_j - C_j$		1	-3	-6 $\uparrow$	0	0	0		

x_3 will enter the basis and S_1 will leave.

$R_2 \to R_2 - 2R_1$

Table 2

	C_j	-1	3	6	0	0	0	RHS	Ratio
X_B	C_B	x_1	x_2	x_3	x_4	S_1	S_2		
x_3	6	2	-1	1	1	1	0	60	—
S_2	0	-1	6	0	-4	-2	1	30	5 $\to$
$Z_j - C_j$		13	-9 $\uparrow$	0	6	6	0		

x_2 will enter the basis, S_2 will leave.

$R_2 \to \frac{R_2}{6} \qquad R_1 \to R_1 + R_2^{new}$

Table 3

	C_j	-1	3	6	0	0	0	RHS	Ratio
X_B	C_B	x_1	x_2	x_3	x_4	S_1	S_2		
x_3	6	$\frac{11}{6}$	0	1	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{6}$	65	
x_2	3	$-\frac{1}{6}$	1	0	$-\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{6}$	5	
$Z_j - C_j$		$\frac{23}{2}$	0	0	0	3	$\frac{3}{2}$		

$z_j - c_j \geq 0$ for all j.
So, the optimal solution is obtained.

The basic feasible solution is.

$(0, 5, 65, 0)$

$$\text{Min } z = 0 - 3(5) - 6(65) + 0$$
$$= -405$$

for $j = 4$, $z_j - c_j = 0$
But x_4 is not there in the basis.
So, we can find second optimal solution by entering x_4 in basis

Corresponding to x_4 entering, ratio is

x_B	x_4	RHS	Ratio
x_3	$1/3$	65	195 →
x_2	$-2/3$	5	—

So, x_4 will enter the basis and x_3 will leave.

Operating $R_1 \to 3R_1$ $R_2 \to R_2 + \frac{2}{3}R_1^{new}$ on table 3

	c_j	-1	3	6	0	0	0	RHS	Ratio
x_B	c_B	x_1	x_2	x_3	x_4	S_1	S_2		
x_4	0	$\frac{11}{2}$	0	3	1	2	$\frac{1}{2}$	195	
x_2	3	$\frac{7}{2}$	1	2	0	1	$\frac{1}{2}$	135	
		$\frac{23}{2}$	0	0	0	3	$\frac{3}{2}$		

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Since, $z_j - c_j \geq 0$ for all j.

So, optimal solution is obtained.

Basic feasible solution,

$$(0, 135, 0, 195)$$

$$\text{Min. } z = 0 - 3(135) - 0$$
$$= -405$$

So, two basic feasible solutions are -

I $(0, 5, 65, 0)$, Min $z = -405$

II $(0, 135, 0, 195)$, Min $z = -405$

2. a) Maximize $z = -2x_1 - x_2$

subject to

$$3x_1 + x_2 = 3$$
$$4x_1 + 3x_2 \geq 6$$
$$x_1 + 2x_2 \leq 3$$
$$x_1, x_2 \geq 0$$

Sol- Writing the LPP in standard form,

Min. $z = 2x_1 + x_2 + MA_1 + MA_2$

$$z - 2x_1 - x_2 - MA_1 - MA_2 = 0$$

Constraints are -

$$3x_1 + x_2 + A_1 = 3$$
$$4x_1 + 3x_2 - S_1 + A_2 = 6$$
$$x_1 + 2x_2 + S_2 = 3$$

$$x_1, x_2, A_1, A_2, S_1, S_2 \geq 0$$

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$n-m = 6-3 = 3$

$3 \rightarrow$ NBV $3 \rightarrow$ BV

let A_1 A_2, S_2 be initial basis variables.

X_B	C_B \ C_j	-2 x_1	-1 x_2	0 S_1	0 S_2	-M A_1	-M A_2	RHS	Ratio
A_1	-M	3	1	0	0	1	0	3	1 →
A_2	-M	4	3	-1	0	0	1	6	$\frac{3}{2}$
S_2	0	1	2	0	1	0	0	3	3
$Z_j - C_j$		$-7M+2$	$-4M+1$	M	0	0	0		
		↑							

x_1 will enter the basis and A_1 will leave

$R_1 \rightarrow \frac{R_1}{3}$; $R_2 \rightarrow R_2 - 4R_1^{new}$; $R_3 \rightarrow R_3 - R_1$

X_B	C_B \ C_j	-2 x_1	-1 x_2	0 S_1	0 S_2	-M A_1	-M A_2	RHS	Ratio
x_1	-2	1	$\frac{1}{3}$	0	0	$\frac{1}{3}$	0	1	3
A_2	-M	0	$\frac{5}{3}$	-1	0	$-\frac{4}{3}$	1	2	$\frac{6}{5}$ →
S_2	0	0	$\frac{5}{3}$	0	1	$-\frac{1}{3}$	0	2	$\frac{6}{5}$
$Z_j - C_j$		0	$-\frac{5M}{3}+\frac{1}{3}$	M	0	$\frac{7M}{3}-\frac{2}{3}$	0		
			↑						

Ratio corresponding to A_2 and S_2 is same. But here A_2 will leave the basis because it is an artificial variable.

So, x_2 will enter and A_2 will leave.

$R_2 \to \frac{3}{5} R_2$

$R_1 \to R_1 - \frac{1}{3} R_2^{new}$ $\qquad$ $R_3 \to R_3 - \frac{5}{3} R_2^{new}$

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	c_j	-2	-1	0	0	-M	-M	RHS	Ratio
x_B	c_B	x_1	x_2	S_1	S_2	A_1	A_2		
x_1	-2	1	0	$\frac{1}{5}$	0	$\frac{3}{5}$	$-\frac{1}{5}$	3/5	
x_2	-1	0	1	$-\frac{3}{5}$	0	$-\frac{4}{5}$	$\frac{3}{5}$	6/5	
S_2	0	0	0	1	1	1	-1	0	
		0	0	$\frac{1}{5}$	0	$M-\frac{2}{5}$	$M-\frac{1}{5}$		

Since $z_j - c_j \geq 0$ for all j, and all artificial variables have left basis. So, optimal solution is obtained.

Solution is,

$$\left(\frac{3}{5}, \frac{6}{5} \right)$$

$$\text{Max. } Z = -2\left(\frac{3}{5}\right) - \frac{6}{5}$$

$$= -\frac{12}{5}$$

b) Min. $Z = 10x_1 + 4x_2$

Subject to
$$3x_1 + 2x_2 \geq 60$$
$$7x_1 + 2x_2 \geq 84$$
$$3x_1 + 6x_2 \geq 72$$
$$x_1, x_2 \geq 0$$

Sol LPP in standard form

Min. $Z = 10x_1 + 4x_2 + MA_1 + MA_2 + MA_3$

$Z - 10x_1 - 4x_2 - MA_1 - MA_2 - MA_3 = 0$

$$3x_1 + 2x_2 - S_1 + A_1 = 60$$
$$7x_1 + 2x_2 - S_2 + A_2 = 84$$
$$3x_1 + 6x_2 - S_3 + A_3 = 72$$
$$x_1, x_2, S_1, S_2, S_3, A_1, A_2, A_3 \geq 0$$

$n - m = 8 - 3 = 5$

$5 \rightarrow NBV, \quad 3 \rightarrow BV$

Let A_1, A_2, A_3 be initial basic variables

	c_j	-10	-4	0	0	0	$-M$	$-M$	$-M$		
x_B	C_B	x_1	x_2	S_1	S_2	S_3	A_1	A_2	A_3	RHS	Ratio
A_1	$-M$	3	2	-1	0	0	1	0	0	60	20
A_2	$-M$	7	2	0	-1	0	0	1	0	84	12 $\rightarrow$
A_3	$-M$	3	6	0	0	-1	0	0	1	72	24
$Z_j - C_j$		$-13M+10$	$-10M+4$	M	M	M	0	0	0		
		$\uparrow$									

x_1 will enter the basis and A_2 will leave

$R_2 \rightarrow \frac{R_2}{7}$; $R_1 \rightarrow R_1 - 3R_2^{new}$; $R_3 \rightarrow R_3 - 3R_2^{new}$

	c_j	-10	-4	0	0	0	$-M$	$-M$	$-M$	RHS	Ratio
x_B	C_B	x_1	x_2	S_1	S_2	S_3	A_1	A_2	A_3		
A_1	$-M$	0	$8/7$	-1	$3/7$	0	1	$-3/7$	0	24	$24\left(\frac{7}{8}\right) = 21$
x_1	-10	1	$\frac{2}{7}$	0	$-\frac{1}{7}$	0	0	$1/7$	0	12	$12 \times \frac{7}{2} = 42$
A_3	$-M$	0	$36/7$	0	$3/7$	-1	0	$-3/7$	1	36	$\frac{36 \times 7}{36} = 7 \rightarrow$
$Z_j - C_j$		0	$\frac{-44M}{7} + \frac{8}{7}$	M	$\frac{-6M}{7} + \frac{10}{7}$	M	0	$\frac{13M}{7} - \frac{10}{7}$	0		
			$\uparrow$								

x_2 will enter the basis and A_3 will leave

$R_3 \rightarrow \frac{7}{36} R_3$; $R_1 \rightarrow R_1 - \frac{8}{7} R_3^{new}$, $R_2 \rightarrow R_2 - \frac{2}{7} R_3^{new}$

	C_j	-10	-4	0	0	0	-M	-M	-M	RHS	Ratio
X_B	C_B	x_1	x_2	S_1	S_2	S_3	A_1	A_2	A_3		
A_1	-M	0	0	-1	$\frac{1}{3}$	$\frac{2}{9}$	1	$-\frac{1}{3}$	$-\frac{2}{9}$	16	48 →
x_1	-10	1	0	0	$-\frac{1}{6}$	$\frac{1}{18}$	0	$\frac{1}{6}$	$-\frac{1}{18}$	10	–
x_2	-4	0	1	0	$\frac{1}{12}$	$-\frac{7}{36}$	0	$-\frac{1}{12}$	$\frac{7}{36}$	7	84
$Z_j - C_j$		0	0	M	$-\frac{M}{3}+\frac{4}{3}$ ↑	$-\frac{2M}{9}+\frac{2}{9}$	0	$\frac{4M}{3}-\frac{4}{3}$	$\frac{11M}{9}-\frac{2}{9}$		

A_1 will leave the basis and S_2 will enter

$R_1 \to 3R_1$; $R_2 \to R_2 + \frac{1}{6} R_1^{new}$; $R_3 \to R_3 - \frac{1}{12} R_1^{new}$

	C_j	-10	-4	0	0	0	-M	-M	-M	RHS	Ratio
X_B	C_B	x_1	x_2	S_1	S_2	S_3	A_1	A_2	A_3		
S_2	0	0	0	-3	1	$\frac{2}{3}$	3	-1	$-\frac{2}{3}$	48	$\frac{48}{2/3} = 72$ →
x_1	-10	1	0	$-\frac{1}{2}$	0	$\frac{1}{6}$	$\frac{1}{2}$	0	$-\frac{1}{6}$	18	$\frac{18}{1/6} = 108$
x_2	-4	0	1	$\frac{1}{4}$	0	$-\frac{1}{4}$	$-\frac{1}{4}$	0	$\frac{1}{4}$	3	–
$Z_j - C_j$		0	0	4	0	$-\frac{2}{3}$ ↑	M − 4	M	$M+\frac{2}{3}$		

S_3 will enter the basis and S_2 will leave

$R_1 \to \frac{3}{2} R_1$; $R_2 \to R_2 - \frac{1}{6} R_1^{new}$; $R_3 \to R_3 + \frac{1}{4} R_1^{new}$

	C_j	-10	-4	0	0	0	-M	-M	-M	RHS	Ratio
X_B	C_B	x_1	x_2	S_1	S_2	S_3	A_1	A_2	A_3		
S_3	0	0	0	$-\frac{9}{2}$	$\frac{3}{2}$	1	$\frac{9}{2}$	$-\frac{3}{2}$	-1	72	
x_1	-10	1	0	$\frac{1}{4}$	$-\frac{1}{4}$	0	$-\frac{1}{4}$	$\frac{1}{4}$	0	6	
x_2	-4	0	1	$-\frac{7}{8}$	$\frac{3}{8}$	0	$\frac{7}{8}$	$-\frac{3}{8}$	0	21	
$Z_j - C_j$		0	0	1	1	0	M − 1	M − 1	M		

Since $z_j - c_j \geq 0$ for all j. So, optimal solution is obtained

Solution is $(6, 21)$

$$\text{Min } z = 10(6) + 4(21)$$
$$= 144$$

c) Min. $z = 2x_1 + 2x_2 - 5x_3$

S.T.

$$3x_1 + 2x_2 - 4x_3 = 7$$
$$x_1 - x_2 + 3x_3 = 2$$
$$x_1, x_2, x_3 \geq 0$$

Sol- Writing the LPP in standard form.

$$\text{Min. } z = 2x_1 + 2x_2 - 5x_3 + MA_1 + MA_2$$
$$z - 2x_1 - 2x_2 + 5x_3 - MA_1 - MA_2 = 0$$

S.T. $3x_1 + 2x_2 - 4x_3 + A_1 = 7$

$x_1 - x_2 + 3x_3 + A_2 = 2$

$$x_1, x_2, x_3, A_1, A_2 \geq 0$$

$n - m = 5 - 2 = 3$

3 NBV , 2 BV

Let A_1, A_2 be initial basic variables.

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	C_j	-2	-2	5	-M	-M	RHS	Ratio
x_B	C_B	x_1	x_2	x_3	A_1	A_2		
A_1	-M	3	2	-4	1	0	7	$\frac{7}{3}$
A_2	-M	1	-1	3	0	1	2	2 →
$Z_j - C_j$		-4M+2	-M+2	M-5	0	0		
		↑						

x_1 will enter the basis and A_2 will leave

$R_1 \to R_1 - 3R_2$

	C_j	-2	-2	5	-M	-M	RHS	Ratio
x_B	C_B	x_1	x_2	x_3	A_1	A_2		
A_1	-M	0	5	-13	1	-3	1	5 →
x_1	-2	1	-1	3	0	1	2	–
$Z_j - C_j$		0	-5M+4	13M-11	0	4M-2		
			↑					

x_2 will enter and A_1 will leave

$R_1 \to \frac{R_1}{5}$; $R_2 \to R_2 + R_1^{new}$

	C_j	-2	-2	5	-M	-M	RHS	Ratio
x_B	C_B	x_1	x_2	x_3	A_1	A_2		
x_2	-2	0	-1	$-\frac{13}{5}$	$\frac{1}{5}$	$-\frac{3}{5}$	$\frac{1}{5}$	–
x_1	-2	1	0	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{11}{5}$	$\frac{11/5}{2/5} = \frac{11}{2}$ →
		0	0	$-\frac{3}{5}$	$M-\frac{4}{5}$	$M+\frac{2}{5}$		
				↑				

x_3 will enter and x_1 will leave the basis

$R_2 \to \frac{5}{2} R_2$ $\quad R_1 \to R_1 + \frac{13}{5} R_2^{new}$

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	c_j	-2	-2	5	-M	-M		
x_B	c_B	x_1	x_2	x_3	A_1	A_2	RHS	Ratio
x_2	-2	$\frac{13}{2}$	1	0	$\frac{3}{2}$	2	$\frac{29}{2}$	
x_3	5	$\frac{5}{2}$	0	1	$\frac{1}{2}$	1	$\frac{11}{2}$	
		$\frac{3}{2}$	0	0	$M-\frac{1}{2}$	$M+1$		

Since $z_j - c_j \geq 0$ for all j. So, optimal solution is obtained.

Solution is -

$$\left(0, \frac{29}{2}, \frac{11}{2}\right)$$

$$\text{Min. } z = 0 + 2\left(\frac{29}{2}\right) - 5\left(\frac{11}{2}\right)$$

$$= \frac{3}{2}$$

d) Min. $z = 8x_1 - 2x_2 - x_3 - 6x_4$

S.T.

$$x_1 + x_2 - x_3 + x_4 = 12$$
$$-2x_1 + 3x_2 + 2x_4 = 42$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Sol- Writing the LPP in standard form.

Min $z - 8x_1 + 2x_2 + x_3 + 6x_4 - MA_1 - MA_2 = 0$

S.T.

$$x_1 + x_2 - x_3 + x_4 + A_1 = 12$$
$$-2x_1 + 3x_2 + 2x_4 + A_2 = 42$$

$$x_1, x_2, x_3, x_4, A_1, A_2 \geq 0$$

Sr. No. 16 Date :

$n - m = 6 - 2 = 4$

4 → NBV, 2 BV.

So, let A_1, A_2 be initial basic variables.

x_B	C_j / C_B	-8 x_1	2 x_2	1 x_3	6 x_4	$-M$ A_1	$-M$ A_2	RHS	Ratio
A_1	$-M$	1	1	-1	1	1	0	12	12 →
A_2	$-M$	-2	3	0	2	0	1	42	14
$z_j - C_j$		$M+8$	$-4M-2$ ↑	$M-1$	$-3M-6$	0	0		

x_2 will enter the basis and A_1 will leave

$R_2 \to R_2 - 3R_1$

x_B	C_j / C_B	-8 x_1	2 x_2	1 x_3	6 x_4	$-M$ A_1	$-M$ A_2	RHS	Ratio
x_2	2	1	1	-1	1	1	0	12	–
A_2	$-M$	-5	0	3	-1	-3	1	6	2 →
$z_j - C_j$		$5M+10$	0	$-3M-3$ ↑	$M-4$	$4M+2$	0		

x_3 is entering the basis and A_2 is leaving.

$R_2 \to \frac{1}{3} R_2$; $R_1 \to R_1 + R_2^{new}$

x_B	C_j / C_B	-8 x_1	2 x_2	1 x_3	6 x_4	$-M$ A_1	$-M$ A_2	RHS	Ratio
x_2	2	$-\frac{2}{3}$	1	0	$\frac{2}{3}$	0	$\frac{1}{3}$	14	$14 \times \frac{3}{2} = 21$ →
x_3	1	$-5/3$	0	1	$-\frac{1}{3}$	-1	$\frac{1}{3}$	2	–
		5	0	0	-5 ↑	$M-1$	$M+1$		

Sr. No. ___ Date : ___

x_4 will enter the basis and x_2 will leave.

$R_1 \to \frac{3}{2} R_1$

$R_2 \to R_2 + \frac{1}{3} R_1^{new}$

	C_j	-8	2	1	6	$-M$	$-M$	RHS	Ratio
x_B	C_B	x_1	x_2	x_3	x_4	A_1	A_2		
x_4	6	-1	$\frac{3}{2}$	0	1	0	$\frac{1}{2}$	21	
x_3	1	-2	$\frac{1}{2}$	1	0	-1	$\frac{1}{2}$	9	
$z_j - c_j$		0	$\frac{15}{2}$	0	0	$M-1$	$M+\frac{7}{2}$		

$z_j - c_j \geq 0$ for all j. So, optimal solution is obtained.

Solution is - $(0, 0, 9, 21)$

Min. $Z = 0 + 0 - 9 - 6(21)$

$= -135$

e) Min. $Z = x_1 - 3x_3$

S.T. $x_1 + 2x_2 - x_3 \leq 6$

$x_1 - x_2 + 3x_3 = 3$

$x_1, x_2, x_3 \geq 0$

Sol Writing the LPP in standard form

Min $Z = x_1 - 3x_3 + MA_1$

$Z - x_1 + 3x_3 - MA_1 = 0$

Constraints are $x_1 + 2x_2 - x_3 + S_1 = 6$

$x_1 - x_2 + 3x_3 + A_1 = 3$

$x_1, x_2, x_3, S_1, A_1 \geq 0$

Sr. No. 18 Date :

$n - m = 5 - 2 = 3$

3 NBV , 2 BV

Let S_1, A_1 be initial basic variables

	c_j	-1	0	3	0	-M	RHS	Ratio
x_B	c_B	x_1	x_2	x_3	S_1	A_1		
S_1	0	1	2	-1	1	0	6	—
A_1	-M	1	-1	3	0	1	3	1 →
$z_j - c_j$		-M+1	M	-3M-3	0	0		

↑

x_3 will enter the basis and A_1 will leave

$R_2 \to \frac{R_2}{3}$; $R_1 \to R_1 + R_2^{new}$

	c_j	-1	0	3	0	-M	RHS	Ratio
x_B	c_B	x_1	x_2	x_3	S_1	A_1		
S_1	0	$\frac{4}{3}$	$\frac{5}{3}$	0	1	$\frac{1}{3}$	7	$\frac{7}{5/3} = \frac{21}{5}$ →
x_3	3	$\frac{1}{3}$	$-\frac{1}{3}$	1	0	$\frac{1}{3}$	1	—
$z_j - c_j$		2	-1	0	0	M+1		

↑

x_2 will enter the basis and S_1 will leave

$R_1 \to \frac{3}{5} R_1$; $R_2 \to R_2 + \frac{1}{3} R_1^{new}$

	c_j	-1	0	3	0	-M	RHS	Ratio
x_B	c_B	x_1	x_2	x_3	S_1	A_1		
x_2	0	$\frac{4}{5}$	1	0	$\frac{3}{5}$	$\frac{1}{5}$	$\frac{21}{5}$	
x_3	3	$\frac{3}{5}$	0	1	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{12}{5}$	
$z_j - c_j$		$\frac{14}{5}$	0	0	$\frac{3}{5}$	$M+\frac{6}{5}$		

Since $z_j - c_j \geq 0$ for all j. So, optimal solution has been obtained.

Solution is - $\left(0, \frac{21}{5}, \frac{12}{5}\right)$

Min. $Z = 0 - 3\left(\frac{12}{5}\right)$

$= \frac{-36}{5}$

f) Min. $Z = 3x_1 - x_2 + 2x_3 + 5x_4 + 6x_5$

S.T.

$$12x_1 - 3x_2 + 5x_3 - 2x_4 + 4x_5 = 12$$
$$8x_1 - 2x_2 - 4x_3 + 5x_5 = 150$$
$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Sol. Writing LPP in standard form.

Min. $Z - 3x_1 + x_2 - 2x_3 - 5x_4 - 6x_5 - MA_1 - MA_2 = 0$

Constraints -

$$12x_1 - 3x_2 + 5x_3 - 2x_4 + 4x_5 + A_1 = 12$$
$$8x_1 - 2x_2 - 4x_3 + 5x_5 + A_2 = 150$$

$$x_1, x_2, x_3, x_4, x_6, A_1, A_2 \geq 0$$

$n - m = 7 - 2 = 5$

5 NBV, 2BV

Let A_1, A_2 be initial basic variables

C_j		-3	1	-2	-5	-6	-M	-M	RHS	Ratio
x_B	C_B	x_1	x_2	x_3	x_4	x_5	A_1	A_2		
A_1	$-M$	12	-3	6	-2	4	1	0	12	$1 \rightarrow$
A_2	$-M$	8	-2	-4	0	5	0	1	150	$\frac{150}{8}$
$z_j - c_j$		$-20M+3$	$5M-1$	$-M+2$	$2M+5$	$-9M+6$	0	0		

$\uparrow$

x_1 will enter the basis and A_1 will leave

$R_1 \rightarrow \frac{R_1}{12}$ $\quad R_2 \rightarrow R_2 - 8R_1^{new}$

C_j		-3	1	-2	-5	-6	-M	-M	RHS	Ratio
x_B	C_B	x_1	x_2	x_3	x_4	x_5	A_1	A_2		
x_1	-3	1	$-\frac{1}{4}$	$\frac{5}{12}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{12}$	0	1	$\frac{1}{\frac{1}{3}} = 3 \rightarrow$
A_2	$-M$	0	0	$-\frac{22}{3}$	$\frac{4}{3}$	$\frac{7}{3}$	$-\frac{2}{3}$	1	142	$\frac{142}{\frac{7}{3}} = \frac{426}{7}$
$z_j - c_j$		0	$-\frac{1}{4}$	$\frac{22M}{3}+\frac{3}{4}$	$-\frac{4M}{3}+\frac{11}{2}$	$-\frac{7M}{3}+5$	$\frac{5M}{3}-\frac{1}{4}$	0		

$\uparrow$

x_5 will enter the basis and x_1 will leave

$R_1 \rightarrow 3R_1$ $\quad R_2 \rightarrow R_2 - \frac{7}{3}R_1^{new}$

C_j		-3	1	-2	-5	-6	-M	-M	RHS	Ratio
x_B	C_B	x_1	x_2	x_3	x_4	x_5	A_1	A_2		
x_5	-6	3	$-\frac{3}{4}$	$\frac{5}{4}$	$-\frac{1}{2}$	1	$\frac{1}{4}$	0	3	—
A_2	$-M$	-7	$\frac{7}{4}$	$-\frac{41}{4}$	$\frac{5}{2}$	0	$-\frac{5}{4}$	1	135	$\frac{135}{\frac{5}{2}} = 54 \rightarrow$
$z_j - c_j$		$7M+15$	$-\frac{7M}{4}+\frac{7}{2}$	$\frac{41M}{4}-\frac{11}{2}$	$-\frac{5M}{2}+8$	0	$\frac{9M}{4}-\frac{3}{2}$	0		

$\uparrow$

x_4 will enter the basis and A_2 will leave.

$R_2 \rightarrow \frac{2}{5}R_2$ $\quad R_1 \rightarrow R_1 + \frac{1}{2}R_2^{new}$

Sr. No. 21 Date :

	C_j	-3	1	-2	-5	-6	-M	-M		
x_B	C_B	x_1	x_2	x_3	x_4	x_5	A_1	A_2	RHS	Ratio
x_5	-6	$\frac{8}{5}$	$-\frac{2}{3}$	$-\frac{4}{3}$	0	1	0	$\frac{1}{5}$	30	—
x_4	-6	$\frac{-14}{5}$	$\frac{7}{10}$	$-\frac{41}{10}$	1	0	$-\frac{1}{2}$	$\frac{2}{5}$	54	$\frac{54}{\frac{7}{10}} = \frac{540}{7}$ →
$z_j - c_j$		$\frac{37}{5}$	$-\frac{21}{10}$	$\frac{273}{10}$	0	0	$M+\frac{5}{2}$	$M-\frac{16}{6}$		

↑

x_2 will enter the basis and x_4 will leave.

$R_2 \to \frac{10}{7} R_2$; $R_1 \to R_1 + \frac{2}{3} R_2^{new}$

	C_j	-3	1	-2	-5	-6	-M	-M		
x_B	C_B	x_1	x_2	x_3	x_4	x_5	A_1	A_2	RHS	Ratio
x_5	-6	0	0	$-\frac{22}{7}$	$\frac{4}{7}$	1	$-\frac{2}{7}$	$\frac{3}{7}$	$\frac{426}{7}$	—
x_2	1	-4	1	$-\frac{41}{7}$	$\frac{10}{7}$	0	$-\frac{5}{7}$	$\frac{4}{7}$	$\frac{540}{7}$	—
$z_j - c_j$		-1	0	15	3	0	M+1	M-2		

↑

Here x_1 should enter the basis but all the coefficients of x_1 in the columns are either zero or negative. So, we cannot take the ratio.

This means the solution is unbounded in the direction of x_1

3. a) Minimize $z = 2x_1 + 5x_2$

S.T. $3x_1 + 2x_2 \geq 12$

$2x_1 + x_2 \leq 4$

$x_1, x_2 \geq 0$

Sr. No. 22 Date :

Sol- Standard form of given LPP:

$$\text{Min } Z = 2x_1 + 5x_2 + MA_1$$
$$Z - 2x_1 - 5x_2 - MA_1 = 0$$

Constraints -
$$3x_1 + 2x_2 - S_1 + A_1 = 12$$
$$2x_1 + x_2 + S_2 = 4$$
$$x_1, x_2, S_1, S_2, A_1 \geq 0$$

Phase I

$Z^* - A_1 = 0$ (Obj function)

S.T. $3x_1 + 2x_2 - S_1 + A_1 = 12$

$2x_1 + x_2 + S_2 = 4$ $x_1, x_2, S_1, S_2, A_1 \geq 0$

$n - m = 5 - 2 = 3$

3 NBV , 2 BV

let A_1, S_2 be initial basic variables

X_B	C_B	x_1	x_2	S_1	S_2	A_1	RHS	Ratio
	C_j	0	0	0	0	-1		
A_1	-1	3	2	-1	0	1	12	$\frac{12}{3} = 4$
S_2	0	2	1	0	1	0	4	$\frac{4}{2} = 2 \rightarrow$
$Z_j - C_j$		-3	-2	1	0	0		
		$\uparrow$						

x_1 will the basis and S_2 will leave

$R_2 \rightarrow \frac{R_2}{2}$ $R_1 \rightarrow R_1 - 3R_2^{new}$

	C_j	0	0	0	0	-1	RHS	Ratio
x_B	C_B	x_1	x_2	S_1	S_2	A_1		
A_1	-1	0	$\frac{1}{2}$	-1	$-\frac{3}{2}$	1	6	$\frac{6}{\frac{1}{2}} = 12$
x_1	0	1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	2	$\frac{2}{\frac{1}{2}} = 4 \rightarrow$
$z_j - c_j$		0	$-\frac{1}{2}$	1	$\frac{3}{2}$	0		
			$\uparrow$					

x_2 will enter the basis and x_1 will leave

$R_2 \rightarrow 2R_2$ $\quad\quad$ $R_1 \rightarrow R_1 - \frac{1}{2} R_2^{new}$

	C_j	0	0	0	0	-1	RHS	Ratio
x_B	C_B	x_1	x_2	S_1	S_2	A_1		
A_1	-1	-1	0	-1	-2	1	4	
x_2	0	2	1	0	1	0	4	
$z_j - c_j$		1	0	1	2	0		

$z_j - c_j \geq 0$ for all j. But artificial variable A_1 is still there in the basis at a positive level.

So, the original problem does not have any feasible solution.

b) Min. $z = x_1 - 3x_3$

S.T. $x_1 + 2x_2 - x_3 \leq 7$

$x_1 - x_2 + 3x_3 = 3$

$x_1, x_2, x_3 \geq 0$

Sol- Standard form of LPP -

Min. $Z = x_1 - 3x_3 + MA_1$

$Z - x_1 + 3x_3 - MA_1 = 0$

S.T. $x_1 + 2x_2 - x_1 + S_1 = 7$

$x_1 - x_2 + 3x_3 + A_1 = 3$

$x_1, x_2, x_3, S_1, A_1 \geq 0$

Phase I

$Z^* - A_1 = 0$ (obj. fun)

$n - m = 5 - 2 = 3$

3 NBV , 2 BV

Let S_1, A_1 be initial basic variables.

x_B	C_B \ C_j	0	0	0	0	-1	RHS	Ratio
		x_1	x_2	x_3	S_1	A_1		
S_1	0	1	2	-1	1	0	7	—
A_1	-1	1	-1	3	0	1	3	1 →
$Z_j - C_j$		-1	1	-3 ↑	0	0		

x_3 is entering and A_1 is leaving

$R_2 \to \frac{R_2}{3}$ $R_1 \to R_1 + R_2^{new}$

x_B	C_B \ C_j	0	0	0	0	-1	RHS	Ratio
		x_1	x_2	x_3	S_1	A_1		
S_1	0	$\frac{4}{3}$	$\frac{5}{3}$	0	1	$\frac{1}{3}$	8	
x_3	0	$\frac{1}{3}$	$-\frac{1}{3}$	1	0	$\frac{1}{3}$	1	
$Z_j - C_j$		0	0	0	0	1		

Since all $z_j - c_j \geq 0$.

So, optimal solution is obtained as-
$(0, 0, 1)$
Min. $Z = 0$

Phase II

Min. $Z - x_1 + 3x_3 = 0$ [obj. fun.]

	c_j	-1	0	3	0	RHS	Ratio
x_B	C_B	x_1	x_2	x_3	s_1		
s_1	0	$\frac{4}{3}$	$\frac{5}{3}$	0	1	8	$\frac{8}{5/3} = \frac{24}{5}$ →
x_3	3	$\frac{1}{3}$	$\frac{-1}{3}$	1	0	1	—
		2	-1 ↑	0	0		

x_2 will enter the basis and s_1 will leave.

$R_1 \to \frac{3}{5} R_1$ $\quad R_2 \to R_2 + \frac{1}{3} R_1^{new}$.

	c_j	-1	0	3	0	RHS	Ratio
x_B	C_B	x_1	x_2	x_3	s_1		
x_2	0	$4/5$	1	0	$3/5$	$24/5$	
x_3	3	$3/5$	0	1	$1/5$	$13/5$	
$z_j - c_j$		$\frac{14}{5}$	0	0	$\frac{3}{5}$		

Since all $z_j - c_j \geq 0$. So, optimal solution is obtained as:

$\left(0, \frac{24}{5}, \frac{13}{5}\right)$

Min. $z = 0 - 3\left(\frac{13}{5}\right) = \frac{-39}{5}$

c) Min. $z = x_1 + x_2 - x_4$

S.T. $4x_1 + x_2 + x_3 + 4x_4 = 8$

$x_1 - 3x_2 + x_3 + 2x_4 = 16$

$x_1, x_2, x_3, x_4 \geq 0$.

Sol- Standard form of LPP

Min. $Z = x_1 + x_2 - x_4 + MA_1 + MA_2$

$Z - x_1 - x_2 + x_4 - MA_1 - MA_2 = 0$

Constraints-

$4x_1 + x_2 + x_3 + 4x_4 + A_1 = 8$

$x_1 - 3x_2 + x_3 + 2x_4 + A_2 = 16$

$x_1, x_2, x_3, x_4, A_1, A_2 \geq 0$

Phase I

$Z^* - A_1 - A_2 = 0$ (obj function)

$n - m$ $6 - 2 = 4$

4 NBV, 2 BV

Let A_1, A_2 be initial basic variables.

	C_j	0	0	0	0	-1	-1		
Y_B	C_B	x_1	x_2	x_3	x_4	A_1	A_2	RHS	Ratio
A_1	-1	4	1	1	4	1	0	8	$\frac{8}{4} = 2 \rightarrow$
A_2	-1	1	-3	1	2	0	1	16	$\frac{16}{2} = 8$
$Z_j - C_j$		-5	2	-2	-6 ↑	0	0		

$x_4 \rightarrow$ entering variable, $A_1 \rightarrow$ leaving variable.

$R_1 \rightarrow \frac{R_1}{4}$, $R_2 \rightarrow R_2 - 2R_1^{new}$

C_j		0	0	0	0	-1	-1	RHS	Ratio
x_B	C_B	x_1	x_2	x_3	x_4	A_1	A_2		
x_4	0	1	$\frac{1}{4}$	$\frac{1}{4}$	1	$\frac{1}{4}$	0	2	$\frac{2}{1/4} = 8 \rightarrow$
A_2	-1	-1	$-\frac{7}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	1	12	$\frac{12}{1/2} = 24$
$Z_j - C_j$		1	$\frac{7}{2}$	$-\frac{1}{2}$ ↑	0	$\frac{3}{2}$	0		

x_3 is entering and x_4 is leaving.

$R_1 \rightarrow 4R_1$, $R_2 \rightarrow R_2 - \frac{1}{2} R_1^{new}$

C_j		0	0	0	0	-1	-1	RHS	Ratio
x_B	C_B	x_1	x_2	x_3	x_4	A_1	A_2		
x_3	0	4	1	1	4	1	0	8	
A_2	-1	-3	-4	0	-2	-1	1	8	
$Z_j - C_j$		3	4	0	2	2	0		

All $Z_j - C_j \geq 0$, so optimality has been achieved.
But artificial variable is still there in the basis at a positive level.
So, original problem has no feasible solution.

d) Max. $Z = 3x_1 - x_2$

S.T. $x_1 - x_2 \leq 3$

$2x_1 \leq x_2$

$x_1 + x_2 \geq 12$

$x_1, x_2 \geq 0$

Sr. No. 28   Date :

Sol- Standard form of LPP :

Min. $Z = -3x_1 + x_2 + MA_1$

Constraints

$$x_1 - x_2 + S_1 = 3$$
$$2x_1 - x_2 + S_2 = 0$$
$$x_1 + x_2 - S_3 + A_1 = 12$$
$$x_1, x_2, S_1, S_2, S_3, A_1 \geq 0$$

Phase I

$Z^* - A_1 = 0$ (obj. fun.)

$n - m = 6 - 3 = 4$

4 NBV , 3 BV

Let S_1, S_2, A_1 be initial basic variables.

	c_j	0	0	0	0	0	-1	RHS	Ratio
x_B	C_B	x_1	x_2	S_1	S_2	S_3	A_1		
S_1	0	1	-1	1	0	0	0	3	3
S_2	0	2	-1	0	1	0	0	0	0 →
A_1	-1	1	1	0	0	-1	1	12	12
$Z_j - C_j$		-1 ↑	-1	0	0	1	0		

(x with lower index)

x_1 (having lower index among x_1 & x_2) will enter and S_2 will leave the basis.

$R_2 \to \frac{R_2}{2}$, $R_1 \to R_1 - R_2^{new}$, $R_3 \to R_3 - R_2^{new}$

	C_j	0	0	0	0	0	-1	RHS	Ratio
x_B	C_B	x_1	x_2	S_1	S_2	S_3	A_1		
S_1	0	0	$-\frac{1}{2}$	1	$-\frac{1}{2}$	0	0	3	—
x_1	0	1	$-1/2$	0	$\frac{1}{2}$	0	0	0	—
A_1	-1	0	$3/2$	0	$-1/2$	-1	1	12	$\frac{12}{2/3} = 8 \rightarrow$
$Z_j - C_j$		0	$-\frac{3}{2}$	0	$\frac{1}{2}$	1	0		

$\uparrow$

x_2 will enter and A_1 will leave.

$R_3 \rightarrow \frac{2}{3} R_3$ $\quad$ $R_1 \rightarrow R_1 + \frac{1}{2} R_3^{new}$, $R_2 \rightarrow R_2 + \frac{1}{2} R_2^{new}$

	C_j	0	0	0	0	0	-1	RHS	Ratio
x_B	C_B	x_1	x_2	S_1	S_2	S_3	A_1		
S_1	0	0	0	1	$-\frac{2}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	7	
x_1	0	1	0	0	$\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	4	
x_2	0	0	1	0	$-\frac{1}{3}$	$-\frac{2}{3}$	$\frac{2}{3}$	8	
$Z_j - C_j$		0	0	0	0	0	1		

$Z_j - C_j \geq 0$ for all j.
So, optimal solution has been obtained as:
$(4, 8)$
Min. $Z = 0$

Phase II

$Z + 3x_1 - x_2 = 0$ (obj function).

	c_j	+3	-1	0	0	0	RHS	Ratio
x_B	C_B	x_1	x_2	S_1	S_2	S_3		
S_1	0	0	0	1	$-\frac{2}{3}$	$-\frac{1}{3}$	7	—
x_1	+3	1	0	0	$\frac{1}{3}$	$-\frac{1}{3}$	4	—
x_2	-1	0	1	0	$-\frac{1}{3}$	$-\frac{2}{3}$	8	—
$z_j - c_j$		0	0	0	$+\frac{4}{3}$	$-\frac{1}{3}$		

S_3 should enter the basis, but all the coefficients of S_3 are negative. So, we can't take the ratio.

Thus, the solution is unbounded in the direction of x_2.

e) Max. $Z = x_1 + 2x_2 + 3x_3 + 4x_4$

S.T.
$$x_1 + x_3 - 4x_4 = 2$$
$$x_2 - x_3 + 3x_4 = 9$$
$$x_1 + x_2 - 2x_3 - 3x_4 = 21$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Sl- Standard form of LPP,

Min $Z = -x_1 - 2x_2 - 3x_3 - 4x_4 + MA_1 + MA_2 + MA_3$

$Z + x_1 + 2x_2 + 3x_3 + 4x_4 - MA_1 - MA_2 - MA_3 = 0$

Constraints,
$$x_1 + x_3 - 4x_4 + A_1 = 2$$
$$x_2 - x_3 + 3x_4 + A_2 = 9$$
$$x_1 + x_2 - 2x_3 - 3x_4 + A_3 = 21$$

$$x_1, x_2, x_3, x_4, A_1, A_2, A_3 \geq 0$$

Sr. No. 31 Date :

Phase I

Obj. function, $z^* - A_1 - A_2 - A_3 = 0$

$n - m = 7 - 3 = 4$

4 NBV, 3 BV

Let A_1, A_2, A_3 be initial basic variables.

c_j		0	0	0	0	-1	-1	-1	RHS	Ratio
x_B	C_B	x_1	x_2	x_3	x_4	A_1	A_2	A_3		
A_1	-1	1	0	1	-4	1	0	0	2	$2/1 = 2 \rightarrow$
A_2	-1	0	1	-1	3	0	1	0	9	—
A_3	-1	1	1	-2	-3	0	0	1	21	$21/1 = 21$
$z_j - c_j$		-2 ↑	-2	2	4	0	0	0		

x_1 (with lower index among x_1 & x_2) will enter the basis and A_1 will leave.

$R_3 \rightarrow R_3 - R_1$

c_j		0	0	0	0	-1	-1	-1	RHS	Ratio
x_B	C_B	x_1	x_2	x_3	x_4	A_1	A_2	A_3		
x_1	0	1	0	1	-4	1	0	0	2	—
A_2	-1	0	1	-1	3	0	1	0	9	$\frac{9}{3} = 3 \rightarrow$
A_3	-1	0	1	-3	1	-1	0	1	19	19
$z_j - c_j$		0	-2	4	-4 ↑	2	0	0		

x_4 will enter the basis and A_2 will leave.

$R_2 \rightarrow \frac{R_2}{3}$, $R_3 \rightarrow R_3 - R_2^{new}$, $R_4 \rightarrow R_4 + 4R_2^{new}$

	C_j	0	0	0	0	-1	-1	-1	RHS	Ratio
x_B	C_B	x_1	x_2	x_3	x_4	A_1	A_2	A_3		
x_1	0	1	$4/3$	$-1/3$	0	1	$4/3$	0	14	$\frac{14}{4/3} = \frac{21}{2}$
x_4	0	0	$1/3$	$-1/3$	1	0	$1/3$	0	3	$\frac{3}{1/3} = 9 \rightarrow$
A_3	-1	0	$2/3$	$-8/3$	0	-1	$-1/3$	1	16	$16/_{2/3} = 24$
$z_j - c_j$		0	$-\frac{2}{3}$ ↑	$\frac{8}{3}$	0	2	$\frac{4}{3}$	0		

x_2 will enter the basis and x_4 will leave the basis.

$R_2 \rightarrow 3R_2$, $R_1 \rightarrow R_1 - \frac{4}{3} R_2^{new}$, $R_3 \rightarrow R_3 - \frac{2}{3} R_2^{new}$

	C_j	0	0	0	0	-1	-1	-1	RHS	Ratio
x_B	C_B	x_1	x_2	x_3	x_4	A_1	A_2	A_3		
x_1	0	1	0	1	-4	1	0	0	2	
x_2	0	0	1	-1	3	0	1	0	9	
A_3	-1	0	0	-2	-2	-1	-1	1	10	
$z_j - c_j$		0	0	2	2	2	2	0		

Since $z_j - c_j \geq 0$, for all j.
So, optimality has been attained.

But artificial variable A_3 is still there in the basis at positive level.

So, the original problem has no feasible solution.

Sr. No. 33 Date :

f) Maximize $z = 2x_1 + 3x_2 - 5x_3$

S.T. $x_1 + x_2 + x_3 = 7$

$2x_1 - 5x_2 + x_3 = 10$

$x_1, x_2, x_3 \geq 0$

Sol- Standard form of given LPP,

Min. $z = -2x_1 - 3x_2 + 5x_3 + MA_1 + MA_2$

$z + 2x_1 + 3x_2 - 5x_3 - MA_1 - MA_2 = 0$

Constraints, $x_1 + x_2 + x_3 + A_1 = 7$

$2x_1 - 5x_2 + x_3 + A_2 = 10$

$x_1, x_2, x_3, A_1, A_2 \geq 0$

Phase I

$z^* - A_1 - A_2 = 0$ (objective function)

$n - m = 5 - 2 = 3$

3 NBV, 2 BV

Let A_1, A_2 be initial basic variables.

	c_j	0	0	0	-1	-1	RHS	Ratio
x_B	c_B	x_1	x_2	x_3	A_1	A_2		
A_1	-1	1	1	1	1	0	7	7
A_2	-1	2	-5	1	0	1	10	$\frac{10}{2} = 5 \rightarrow$
$z_j - c_j$		-3	4	-2	0	0		
		↑						

x_1 will enter the basis and A_2 will leave.

$R_2 \rightarrow \frac{R_2}{2}$ $R_1 \rightarrow R_1 - R_2^{new}$

	C_j	0	0	0	-1	-1	RHS	Ratio
x_B	C_B	x_1	x_2	x_3	A_1	A_2		
A_1	-1	0	7/2	1/2	1	-1/2	2	$\frac{2}{7/2} = \frac{4}{7}$ →
x_1	0	1	-5/2	1/2	0	1/2	5	—
$z_j - c_j$		0	-7/2 ↑	-1/2	0	3/2		

A_1 will leave the basis and x_2 will enter.

$R_1 \to +\frac{2}{7} R_1$; $R_2 \to R_2 + \frac{5}{2} R_1^{new}$

	C_j	0	0	0	-1	-1	RHS	Ratio
x_B	C_B	x_1	x_2	x_3	A_1	A_2		
x_2	0	0	1	1/7	2/7	-1/7	4/7	
x_1	0	1	0	6/7	5/7	1/7	45/7	
$z_j - c_j$		0	0	0	0	1		

$z_j - c_j \geq 0$ for all j. So, optimal solution is obtained as: $\left(\frac{45}{7}, \frac{4}{7}, 0 \right)$.

Min. $z = 0$

Phase II :

objective function: $z + 2x_1 + 3x_2 - 5x_3 = 0$

	C_j	2	3	-5		Ratio
x_B	C_B	x_1	x_2	x_3	RHS	
x_2	3	0	1	1/7	4/7	
x_1	2	1	0	6/7	45/7	
$z_j - c_j$		0	0	50/7		

$z_j - c_j \geq 0$ for all j.

So, optimal solution is obtained.
Solution is,

$$\left(\frac{45}{7}, \frac{4}{7}, 0 \right)$$

$$\text{Max. } z = 2\left(\frac{45}{7}\right) + 3\left(\frac{4}{7}\right) + 0$$

$$z = \frac{102}{7}$$

Sol: 4.

LPP in its standard form:

$$\text{Min. } z = x_1 + 4x_2 + 3x_2 + 2x_4 + MA_1 + MA_2 + MA_3$$

$$z - x_1 - 4x_2 - 3x_3 - 2x_4 - MA_1 - MA_2 - MA_3 = 0$$

Constraints:

$$x_1 + 2x_2 + x_4 + A_1 = 20$$
$$2x_1 + x_2 + x_3 + A_2 = 10$$
$$-x_1 + 4x_2 - 2x_3 + 3x_4 + A_3 = 40$$
$$x_1, x_2, x_3, x_4, A_1, A_2, A_3 \geq 0$$

Phase I

$$z^* - A_1 - A_2 - A_3 = 0 \quad \text{[objective function]}$$

$$n - m = 7 - 3 = 4$$

4 NBV, 3 BV

Let A_1, A_2, A_3 be initial basic variables.

x_B	c_j / C_B	x_1 (0)	x_2 (0)	x_3 (0)	x_4 (0)	A_1 (−1)	A_2 (−1)	A_3 (−1)	RHS	Ratio
A_1	−1	1	2	0	1	1	0	0	20	$\frac{20}{2}=10$ →
A_2	−1	2	1	1	0	0	1	0	10	10
A_3	−1	−1	4	−2	3	0	0	1	40	$\frac{40}{4}=10$
		−2	−7	1	−4	0	0	0		
			↑							

x_2 will enter the basis and A_1 will leave

$R_1 \to \frac{R_1}{2}$; $R_2 \to R_2 - R_1^{new}$, $R_3 \to R_3 - 4R_1^{new}$

x_B	c_j / C_B	x_1 (0)	x_2 (0)	x_3 (0)	x_4 (0)	A_1 (−1)	A_2 (−1)	A_3 (−1)	RHS	Ratio
x_2	0	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	10	$\frac{10}{1/2}=20$
A_2	−1	$\frac{3}{2}$	0	1	$-\frac{1}{2}$	$-\frac{1}{2}$	1	0	0	—
A_3	−1	−3	0	−2	1	−2	0	1	0	0 →
$z_j - c_j$		$\frac{3}{2}$	0	1	$-\frac{1}{2}$	$\frac{5}{2}$	0	0		
					↑					

x_4 will enter and A_3 will leave the basis.

$R_1 \to R_1 - \frac{1}{2}R_3$; $R_2 \to R_2 + \frac{1}{2}R_3$

x_B	c_j / C_B	x_1 (0)	x_2 (0)	x_3 (0)	x_4 (0)	A_1 (−1)	A_2 (−1)	A_3 (−1)	RHS	Ratio
x_2	0	2	1	1	0	$\frac{3}{2}$	0	$-\frac{1}{2}$	10	
A_2	−1	0	0	0	0	$-3/2$	1	$\frac{1}{2}$	0	
x_4	0	−3	0	−2	1	−2	0	1	0	
$z_j - c_j$		0	0	0	0	$\frac{5}{2}$	0	$\frac{1}{2}$		

$z_j - c_j \geq 0$ for all j, but one artificial variable A_2 is still there in the basis at zero level.

So, A_2 will not exit the basis, but the coefficients of x_1 and x_3 corresponding to A_2 are zero.
So, neither x_1 nor x_3 can enter the basis.

So, this means 2^{nd} constraint is redundant constraint.

Redundant Constraint : $2x_1 + x_2 + x_3 = 10$

5. a) $2x_1 + 3x_2 = 11$ $\begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$
$3x_1 + 2x_2 = 9$

Sol- Variables are unrestricted in sign,
So, put
$x_1 = x_1' - x_1''$
$x_2 = x_2' - x_2''$ $\quad x_1', x_2', x_1'', x_2'' \geq 0$

So, our system becomes,

$$2x_1' - 2x_1'' + 3x_2' - 3x_2'' = 11$$
$$3x_1' - 3x_1'' + 2x_2' - 2x_2'' = 9$$

Writing in standard form,
$$2x_1' - 2x_1'' + 3x_2' - 3x_2'' + A_1 = 11$$
$$3x_1' - 3x_1'' + 2x_2' - 2x_2'' + A_2 = 9$$
$$x_1', x_1'', x_2', x_2'', A_1, A_2 \geq 0$$

Sr. No. 38 Date :

$n - m = 6 - 2 = 4$

4 NBV , 2 BV

Let A_1, A_2 be initial basic variables.

Phase I

Min. $Z^* - A_1 - A_2 = 0$

x_B	C_j / C_B	0 / x_1'	0 / x_1''	0 / x_2'	0 / x_2''	−1 / A_1	−1 / A_2	RHS	Ratio
A_1	−1	2	−2	3	−3	1	0	11	$\frac{11}{2}$
A_2	−1	3	−3	2	−2	0	1	9	$\frac{9}{3} = 3 \rightarrow$
		−5	5	−5	5				
		↑							

x_1' will enter the basis and A_2 will leave

$R_2 \rightarrow \frac{R_2}{3}$ $R_1 \rightarrow R_1 - 2R_2^{new}$

x_B	C_j / C_B	0 / x_1'	0 / x_1''	0 / x_2'	0 / x_2''	−1 / A_1	−1 / A_2	RHS	Ratio
A_1	−1	0	0	$5/3$	$-5/3$	1	$-\frac{2}{3}$	5	$\frac{5}{5/3} = 3 \rightarrow$
x_1'	0	1	−1	$\frac{2}{3}$	$-\frac{2}{3}$	0	$\frac{1}{3}$	3	$\frac{3}{2/3} = \frac{9}{2}$
		0	0	$-\frac{5}{3}$	$\frac{5}{3}$	0	$\frac{5}{3}$		
				↑					

x_2' will enter the basis and A_1 will leave

$R_1 \rightarrow \frac{3}{5} R_1$ $R_2 \rightarrow R_2 - \frac{2}{3} R_1^{new}$

x_B	C_j / C_B	0 / x_1'	0 / x_1''	0 / x_2'	0 / x_2''	−1 / A_1	−1 / A_2	RHS	Ratio
x_2'	0	0	0	1	−1	$\frac{3}{5}$	$-\frac{2}{5}$	3	—
x_1'	0	1	−1	0	0	$-\frac{2}{5}$	$\frac{3}{5}$	1	—
		0	0	0	0	1	1		

Sr. No. 39 Date

$z_j - c_j \geq 0$ for all j and all artificial variables have left the basis

Initial basic feasible solution is $(1, 0, 3, 0)$

$$x_1 = x_1' - x_1'' = 1 - 0 = 1$$
$$x_2 = x_2' - x_2'' = 3 - 0 = 3$$

Inverse of coefficient matrix is,

$$\begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} -2/5 & 3/5 \\ 3/5 & -2/5 \end{bmatrix}$$

b) $2x_1 - 3x_2 = -5$

$5x_1 + 4x_2 = 22$

$$\begin{bmatrix} -2 & 3 \\ 5 & 4 \end{bmatrix}$$

Sol. Since the variables are unrestricted in sign, so put

$$x_1 = x_1' - x_1''$$
$$x_2 = x_2' - x_2'' \qquad x_1', x_2', x_1'', x_2'' \geq 0$$

Our system becomes,

$$2x_1' - 2x_1'' - 3x_2' + 3x_2'' = -5$$
$$5x_1' - 5x_1'' + 4x_2' - 4x_2'' = 22$$

Writing in standard form,

$$-2x_1' + 2x_1'' + 3x_2' - 3x_2'' + A_1 = -5$$
$$5x_1' - 5x_1'' + 4x_2' - 4x_2'' + A_2 = 22$$

$$x_1', x_1'', x_2', x_2'', A_1, A_2 \geq 0$$

Sr. No. 40 Date :

$n - m = 6 - 2 = 4$

4 NBV , 2 BV

Let A_1, A_2 be initial basic variables.

Phase I

Min. $z^* - A_1 - A_2 = 0$

x_B	c_B \ c_j	x_1' (0)	x_1'' (0)	x_2' (0)	x_2'' (0)	A_1 (−1)	A_2 (−1)	RHS	Ratio
A_1	−1	−2	+2	+3	−3	1	0	5	—
A_2	−1	5	−5	4	−4	0	1	22	$\frac{22}{5} \rightarrow$
$z_j - c_j$		−3 ↑	3	−1	1	0	0		

x_1' will enter the basis and A_2 will leave

$R_2 \rightarrow \frac{R_2}{5}$ $R_1 \rightarrow R_1 + 2R_2^{new}$

x_B	c_B \ c_j	x_1' (0)	x_1'' (0)	x_2' (0)	x_2'' (0)	A_1 (−1)	A_2 (−1)	RHS	Ratio
A_1	−1	0	0	$\frac{23}{5}$	$-\frac{23}{5}$	1	$+\frac{2}{5}$	$69/5$	$\frac{69}{5} \times \frac{5}{23} = 3 \rightarrow$
x_1'	0	1	−1	$\frac{4}{5}$	$-\frac{4}{5}$	0	$\frac{1}{5}$	$\frac{22}{5}$	$\frac{22}{5} \times \frac{5}{4} = \frac{11}{2}$
$z_j - c_j$		0	0	$-\frac{23}{5}$ ↑	$\frac{23}{5}$	0	$\frac{3}{5}$		

x_2' will enter the basis and A_1 will leave.

$R_1 \rightarrow \frac{5}{23} R_1$

$R_2 \rightarrow R_2 - \frac{4}{5} R_1^{new}$

C_j		0	0	0	0	-1	-1		
x_B	C_B	x_1'	x_1''	x_2'	x_2''	A_1	A_2	RHS	Ratio
x_2'	0	0	0	1	-1	$\frac{5}{23}$	$\frac{2}{23}$	3	
x_1'	0	1	-1	0	0	$-\frac{4}{23}$	$\frac{3}{23}$	2	
$z_j - c_j$		0	0	0	0	1	1		

Since all $z_j - c_j \geq 0$ for all j.
~~Also~~, artificial variables have left the basis.

Initial basic feasible solution is $(2, 0, 3, 0)$.

$$x_1 = x_1' - x_1'' = 2 - 0 = 2$$
$$x_2 = x_2' - x_2'' = 3 - 0 = 3$$

Inverse of coefficient matrix is,

$$\begin{bmatrix} -2 & 3 \\ 5 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} -4/23 & 3/23 \\ 5/23 & 2/23 \end{bmatrix}$$

c) $x_1 + 3x_2 = -1$ $\quad\quad \begin{bmatrix} -1 & -3 \\ -7 & 2 \end{bmatrix}$
$7x_1 - 2x_2 = -3$

Sol- Since x_1 and x_2 are unrestricted in sign
Putting,

$$x_1 = x_1' - x_1''$$
$$x_2 = x_2' - x_2'' \quad\quad x_1', x_1'', x_2', x_2'' \geq 0$$

So, our system becomes

$$-x_1' + x_1'' - 3x_2' + 3x_2'' = 1$$
$$-7x_1' + 7x_1'' + 2x_2' - 2x_2'' = 3$$
$$x_1', x_2', x_1'', x_2'' \geq 0$$

Writing in standard form.

$$-x_1' + x_1'' - 3x_2' + 3x_2'' + A_1 = 1$$
$$-7x_1' + 7x_1'' + 2x_2' - 2x_2'' + A_2 = 3$$
$$x_1', x_1'', x_2', x_2'', A_1, A_2 \geq 0$$

$n - m = 6 - 2 = 4$

4 NBV, 2 BV

Let A_1, A_2 be initial basic variables.

Phase I

Min $Z^* - A_1 - A_2 = 0$

	c_j	0	0	0	0	-1	-1		
x_B	C_B	x_1'	x_1''	x_2'	x_2''	A_1	A_2	RHS	Ratio
A_1	-1	-1	1	-3	3	1	0	1	1
A_2	-1	-7	7	2	-2	0	1	3	$\frac{3}{7} \rightarrow$
$Z_j - C_j$		8	-8	1	-1	0	0		
			$\uparrow$						

x_1'' will enter the basis and A_2 will leave.

$R_2 \rightarrow \frac{R_2}{7}$ $R_1 \rightarrow R_1 - R_2^{new}$

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C_j		0	0	0	0	-1	-1		
x_B	C_B	x_1'	x_1''	x_2'	x_2''	A_1	A_2	RHS	Ratio
A_1	-1	0	0	$-\frac{23}{7}$	$\frac{23}{7}$	1	$-\frac{1}{7}$	$\frac{4}{7}$	$\frac{4}{23} \rightarrow$
x_1''	0	-1	1	$\frac{2}{7}$	$-\frac{2}{7}$	0	$\frac{1}{7}$	$\frac{3}{7}$	—
$z_j - c_j$		0	0	$\frac{23}{7}$	$-\frac{23}{7}$ ↑	0	$\frac{8}{7}$		

x_2'' will enter the basis and A_1 will leave.

$R_1 \rightarrow \frac{7}{23} R_1$ $\qquad$ $R_2 \rightarrow R_2 + \frac{2}{7} R_1^{new}$

C_j		0	0	0	0	-1	-1		
x_B	C_B	x_1'	x_1''	x_2'	x_2''	A_1	A_2	RHS	Ratio
x_2''	0	0	0	-1	1	$\frac{7}{23}$	$-\frac{1}{23}$	$\frac{4}{23}$	
x_1''	0	-1	1	0	0	$\frac{2}{23}$	$\frac{3}{23}$	$\frac{11}{23}$	
$z_j - c_j$		0	0	0	0	1	1		

Since all $z_j - c_j \geq 0$ and all artificial variables have left the basis.

Initial basic feasible solution - $\left(0, \frac{11}{23}, 0, \frac{4}{23}\right)$

$$x_1 = x_1' - x_1'' = 0 - \frac{11}{23} = -\frac{11}{23}$$

$$x_2 = x_2' - x_2'' = 0 - \frac{4}{23} = -\frac{4}{23}$$

Inverse of Coefficient Matrix,

$$\begin{bmatrix} -1 & -3 \\ -7 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} -\frac{2}{23} & -\frac{3}{23} \\ -\frac{7}{23} & \frac{1}{23} \end{bmatrix}$$

d) $2x_1 + x_2 = -4$ $\begin{bmatrix} -2 & -1 \\ 5 & -3 \end{bmatrix}$

$5x_1 - 3x_2 = 1$

Sol- x_1 and x_2 are unrestricted in sign,
So, put,

$x_1 = x_1' - x_1''$

$x_2 = x_2' - x_2''$ $x_1', x_1'', x_2', x_2'' \geq 0$

So, our system becomes (In standard form)

$-2x_1' + 2x_1'' - x_2' + x_2'' + A_1 = 4$

$5x_1' - 5x_1'' - 3x_2' + 3x_2'' + A_2 = 1$

$x_1', x_1'', x_2', x_2'', A_1, A_2 \geq 0$

$n - m = 6 - 2 = 4$

4 NBV, 2 BV

Phase I:

Min. $Z^* - A_1 - A_2 = 0$

x_B	C_j / C_B	x_1' (0)	x_1'' (0)	x_2' (0)	x_2'' (0)	A_1 (-1)	A_2 (-1)	RHS	Ratio
A_1	-1	-2	+2	-1	+1	1	0	4	4
A_2	-1	5	-5	-3	+3	0	1	1	$\frac{1}{3}$ →
		-3	3	4	-4 ↑	0	0		

x_2'' will enter the basis and A_2 will leave.

$R_2 \to \frac{R_2}{3}$, $R_1 \to R_1 - R_2^{new}$

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Date :

	C_j	0	0	0	0	-1	-1	RHS	Ratio
x_B	C_B	x_1'	x_1''	x_2'	x_2''	A_1	A_2		
A_1	-1	$-\frac{11}{3}$	$\frac{11}{3}$	0	0	1	$-\frac{1}{3}$	$\frac{11}{3}$	1 →
x_2''	0	$\frac{5}{3}$	$-\frac{5}{3}$	-1	1	0	$\frac{1}{3}$	$\frac{1}{3}$	—
$z_j - c_j$		$\frac{11}{3}$	$-\frac{11}{3}$	0	0	0	$\frac{4}{3}$		
			↑						

x_1'' will enter the basis and A_1 will leave

$R_1 \to \frac{3}{11} R_1$ $\quad$ $R_2 \to R_2 + \frac{5}{3} R_1^{new}$

	C_j	0	0	0	0	-1	-1	RHS	Ratio
x_B	C_B	x_1'	x_1''	x_2'	x_2''	A_1	A_2		
x_1''	0	-1	1	0	0	$\frac{3}{11}$	$-\frac{1}{11}$	1	
x_2''	0	0	0	-1	1	$\frac{5}{11}$	$\frac{2}{11}$	2	
$z_j - c_j$		0	0	0	0	1	1		

Since $z_j - c_j \geq 0$ for all j and all artificial variables have left the basis

So, initial basic feasible solution is $(0, 1, 0, 2)$.

$x_1 = x_1' - x_1'' = 0 - 1 = -1$

$x_2 = x_2' - x_2'' = 0 - 2 = -2$

Inverse of coefficient Matrix is -

$$\begin{bmatrix} -2 & -1 \\ 5 & -3 \end{bmatrix}^{-1} = \begin{bmatrix} -\frac{3}{11} & \frac{1}{11} \\ -\frac{5}{11} & -\frac{2}{11} \end{bmatrix}$$

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Sol. 6 Given Matrix -
$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 2 \\ 1 & 3 & -1 \end{bmatrix}$$

Since there is no RHS to apply simplex method, let us introduce a dummy vector

$b = \begin{bmatrix} 2 \\ 6 \\ 8 \end{bmatrix}$ so that given matrix reduces to system of equations:

$$x_1 - x_2 + 2x_3 = 2$$
$$3x_2 + 2x_3 = 6$$
$$x_1 + 3x_2 - x_3 = 8$$

Since variables are unrestricted in sign. So, putting,

$$x_1 = x_1' - x_1''$$
$$x_2 = x_2' - x_2'' \qquad x_1', x_2', x_3' \geq 0$$
$$x_3 = x_3' - x_3'' \qquad x_1'', x_2'', x_3'' \geq 0$$

Putting x_1, x_2 and x_3 and introducing artificial variables A_1, A_2, A_3 in the constraints -

$$x_1' - x_1'' - x_2' + x_2'' + 2x_3' - 2x_3'' + A_1 = 2$$
$$3x_2' - 3x_2'' + 2x_3' - 2x_3'' + A_2 = 6$$
$$x_1' - x_1'' + 3x_2' - 3x_2'' - x_3' + x_3'' + A_3 = 8$$

$$x_1', x_1'', x_2', x_2'', x_3', x_3'', A_1, A_2, A_3 \geq 0$$

$n - m = 9 - 3 = 6$

6 NBV, 3 BV.

let A_1, A_2, A_3 be initial basic variables.

<u>Phase I</u>

Min. $z^* - A_1 - A_2 = 0$

	C_j	0	0	0	0	0	0	-1	-1	-1		
x_B	C_B	x_1'	x_1''	x_2'	x_2''	x_3'	x_3''	A_1	A_2	A_3	RHS	Ratio
A_1	-1	1	-1	-1	1	2	-2	1	0	0	2	—
A_2	-1	0	0	3	-3	2	-2	0	1	0	6	2 →
A_3	-1	1	-1	3	-3	-1	1	0	0	1	8	$\frac{8}{3}$
$z_j - c_j$		-2	2	-5	5	-3	3	0	0	0		

↑

x_2' will enter the basis and A_2 will leave

$R_2 \to \frac{R_2}{3}$ $\quad R_1 \to R_1 + R_2^{new}$ $\quad R_3 \to R_3 - R_2^{new}$

	C_j	0	0	0	0	0	0	-1	-1	-1		
x_B	C_B	x_1'	x_1''	x_2'	x_2''	x_3'	x_3''	A_1	A_2	A_3	RHS	Ratio
A_1	-1	1	-1	0	0	$\frac{8}{3}$	$-\frac{8}{3}$	1	$\frac{1}{3}$	0	4	4
x_2'	0	0	0	1	-1	$\frac{2}{3}$	$-\frac{2}{3}$	0	$\frac{1}{3}$	0	2	—
A_3	-1	1	-1	0	0	-3	3	0	-1	1	2	2 →
$z_j - c_j$		-2	2	0	0	$\frac{1}{3}$	$-\frac{1}{3}$	0	$\frac{2}{3}$	0		

↑

x_1' will enter the basis and A_3 will leave.

$R_1 \to R_1 - R_3$

c_j		0	0	0	0	0	0	-1	-1	-1		
x_B	C_B	x_1'	x_1''	x_2'	x_2''	x_3'	x_3''	A_1	A_2	A_3	RHS	Ratio
A_1	-1	0	0	0	0	17/3	-17/3	1	4/3	-1	2	$\frac{6}{17}$ →
x_2'	0	0	0	1	-1	2/3	-2/3	0	1/3	0	2	3
x_1'	0	1	-1	0	0	-3	3	0	-1	1	2	—
$z_j - c_j$		0	0	0	0	-17/3 ↑	17/3	0	$-\frac{1}{3}$	2		

x_3' will enter the basis and A_1 will leave the basis.

$R_1 \to \frac{3}{17} R_1$ $\quad R_2 \to R_2 - \frac{2}{3} R_1^{new}$ $\quad R_3 \to R_3 + 3 R_1^{new}$

c_j		0	0	0	0	0	0	-1	-1	-1		
x_B	C_B	x_1'	x_1''	x_2'	x_2''	x_3'	x_3''	A_1	A_2	A_3	RHS	Ratio
x_3'	0	0	0	0	0	1	-1	3/17	4/17	-3/17	6/17	
x_2'	0	0	0	1	-1	0	0	-2/17	3/17	2/17	2/3	
x_1'	0	1	-1	0	0	0	0	9/17	-5/17	8/17	16/17	
$z_j - c_j$		0	0	0	0	0	0	1	1	1		

$z_j - c_j \geq 0$ for all j and all artificial variables have left the basis.

So, initial basic feasible solution is : $(x_1', x_1'', x_2', x_2'', x_3', x_3'')$
$= \left(\frac{16}{17}, 0, \frac{2}{3}, 0, \frac{6}{17}, 0\right)$

$x_1 = x_1' - x_1'' = 16/17 - 0 = 16/17 > 0$

$x_2 = x_2' - x_2'' = 2/3 - 0 = 2/3 > 0$

$x_3 = x_3' - x_3'' = 6/17 - 0 = 6/17 > 0$

The inverse of given matrix is $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 2 \\ 1 & 3 & -1 \end{bmatrix}^{-1} = \begin{bmatrix} 9/17 & -5/17 & 8/17 \\ -2/17 & 3/17 & 2/17 \\ 3/17 & 4/17 & -3/17 \end{bmatrix}$