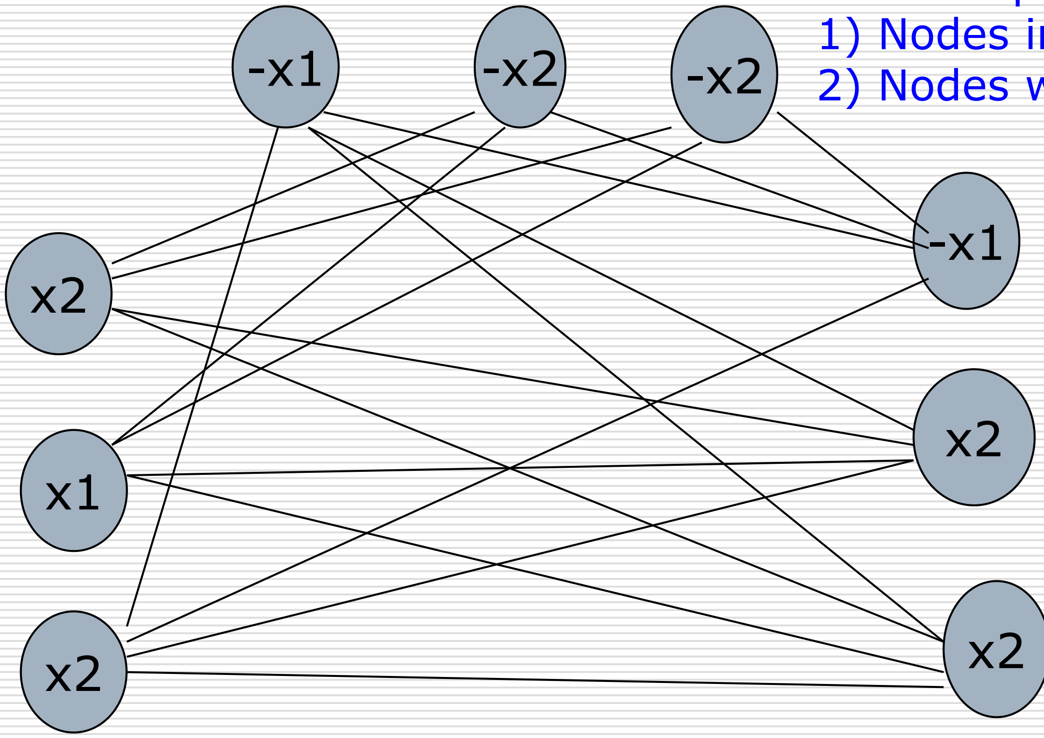
CLIQUE is NP-complete

Two exceptions:
1) Nodes in the same triples
2) Nodes with contradictory labels



$$\phi = (x_1 \vee x_1 \vee x_2) \wedge (\overline{x_1} \vee \overline{x_2} \vee \overline{x_2}) \wedge (\overline{x_1} \vee x_2 \vee x_2)$$

The formula is satisfiable iff the constructed graph has a k-clique.

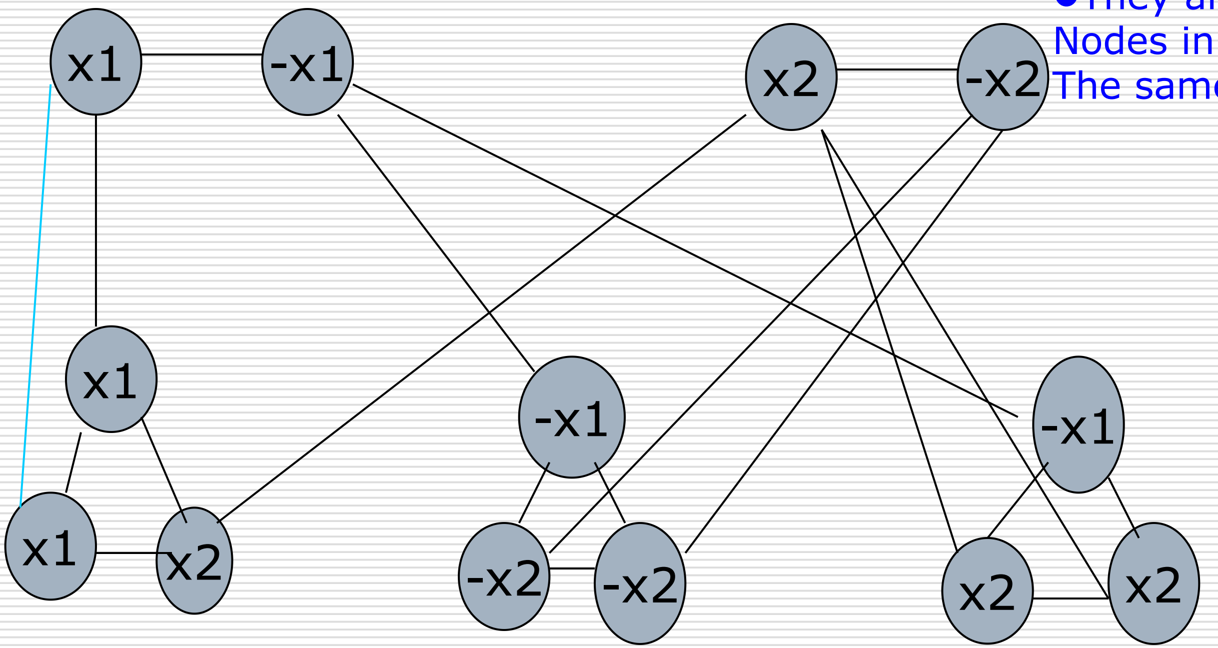
Vertex-cover problem

If G is an undirected graph, a ***vertex cover*** of G is a subset of the nodes where every edge of G touches one of those nodes. The vertex cover problem asks whether a graph contains a vertex cover of a specified size:

$$VERTEX\text{-}COVER = \{\langle G, k\rangle \mid G \text{ is an undirected graph that has a } k\text{-node vertex cover}\}.$$

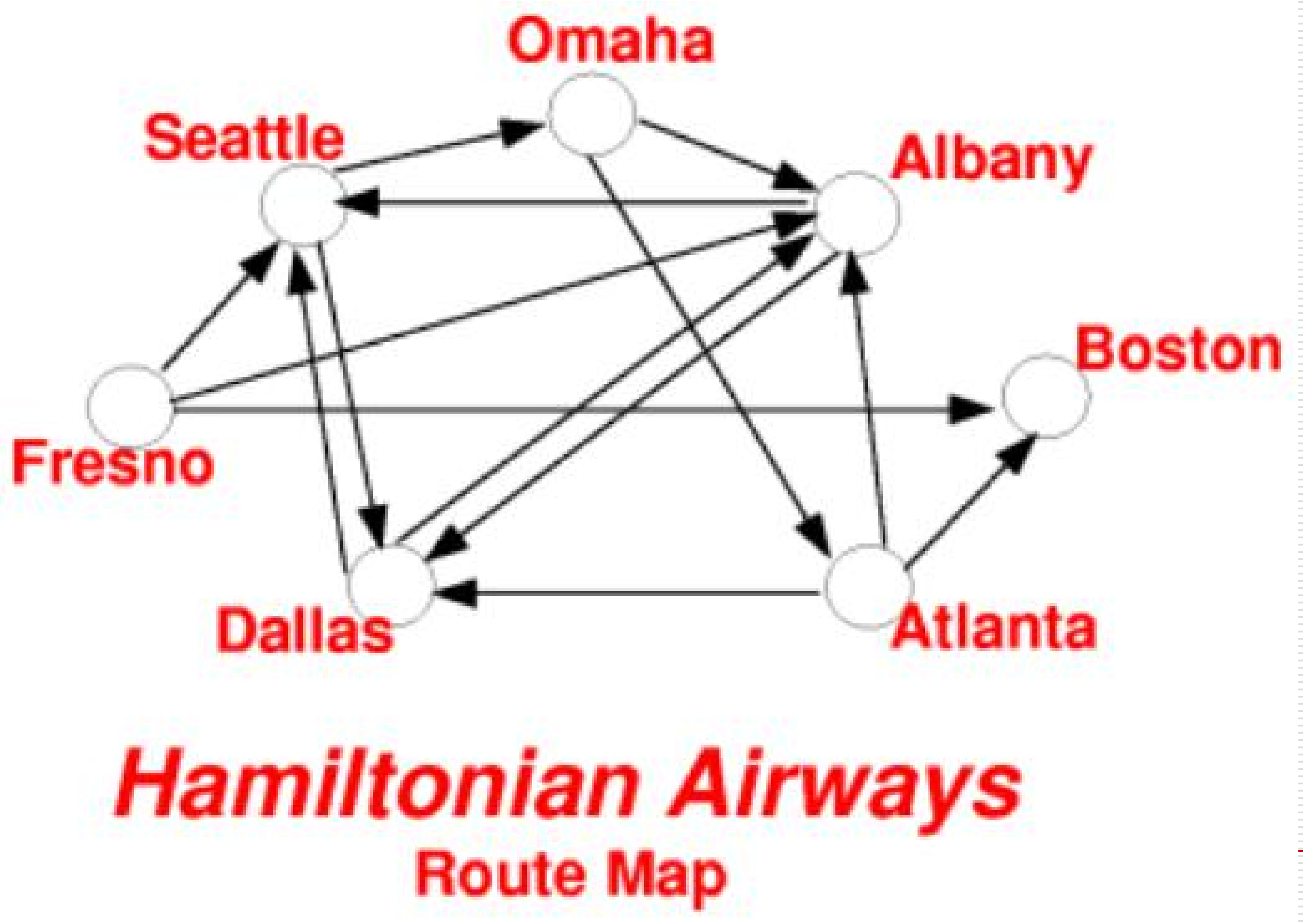
VERTEX-COVER is NP-complete

- The nodes in each gadget
Are connected to each other
- They are also connected to the
Nodes in the variable gadgets wit
The same labels.



$$\phi = (x_1 \vee x_1 \vee x_2) \wedge (\overline{x_1} \vee \overline{x_2} \vee \overline{x_2}) \wedge (\overline{x_1} \vee x_2 \vee x_2)$$

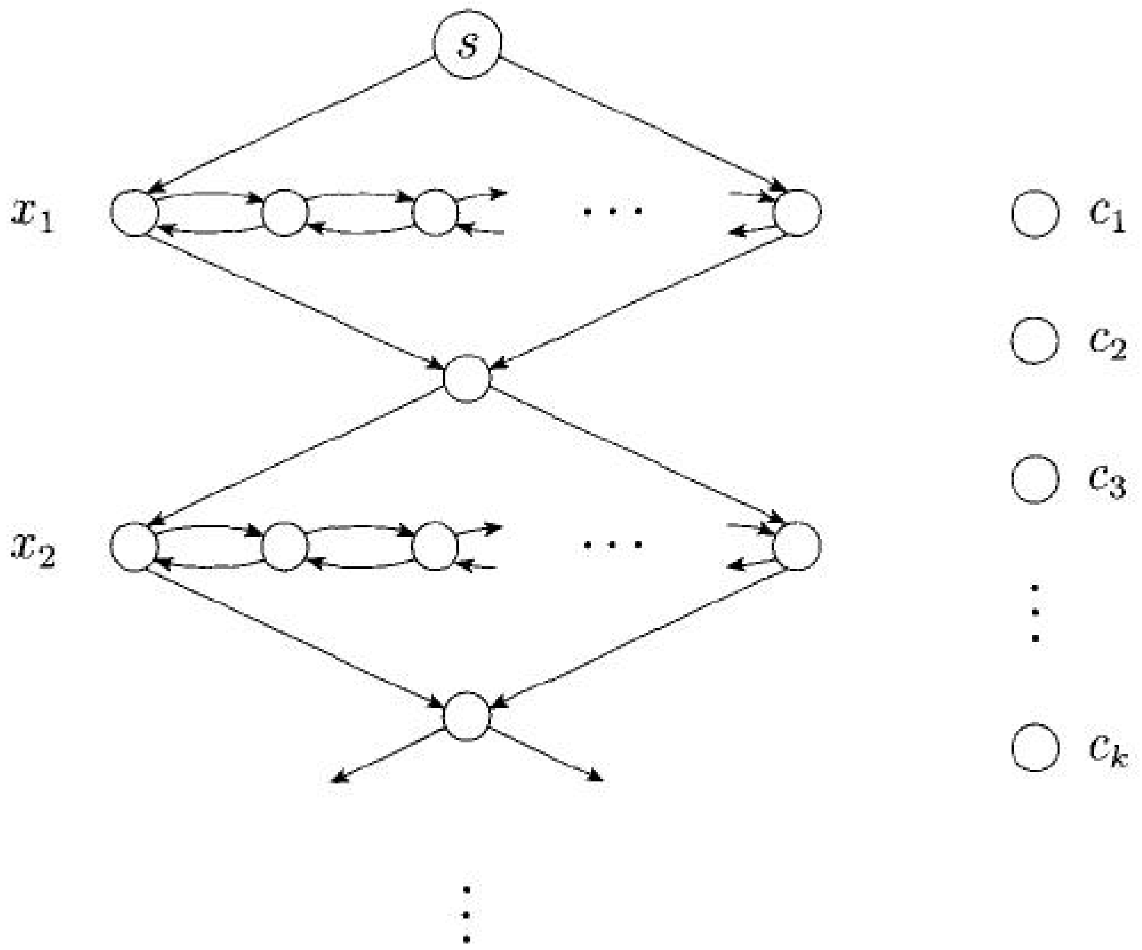
Hamiltonian Path Problem



HAMPATH is NP-complete

- HAMPATH={<G,s,t>: G is a directed graph containing a path from s to t that goes through every node exactly once}
- Proof idea: The variable gadgets are diamond structures that can be traversed in either of 2 ways and the clause gadgets are simple nodes.

Polynomial time reduction



Connecting diamonds to nodes

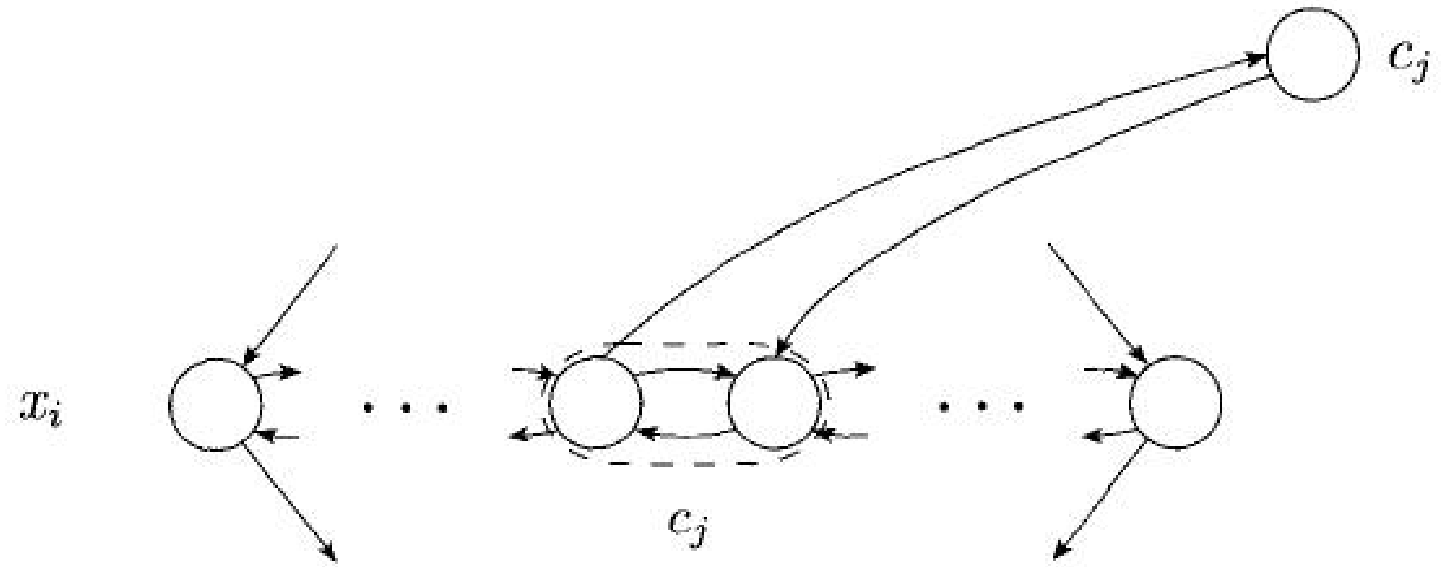


FIGURE 7.51
The additional edges when clause c_j contains x_i

UHAMPATH is NP-complete

- Proof idea: The reduction takes a directed graph G with nodes s and t, and construct a undirected graph G′ with nodes s′ and t′. A Hamiltonian path P in G

$$s \rightarrow u_1 \rightarrow u_2 \rightarrow \cdots \rightarrow u_k \rightarrow t$$

has a corresponding Hamiltonian path P′ in G′.

$$s^{out}$$

$$u_1^{in} - u_1^{mid} - u_1^{out}$$

$$u_2^{in} - u_2^{mid} - u_2^{out}$$

$$\cdots$$

$$t^{in}$$

Traveling salesman problem (TSP)

- Input: A graph with integer weights on the edges and a weight limit W.
- Question: does the graph have a Hamilton circuit of total wieght at most W.

Traveling salesman problem (TSP)

- TSP is NP-complete.
- Proof idea: Given a graph, construct a weighted graph G′ whose nodes and edges are the same as the nodes and edges of G, with a weight of 1 on each edge, and a limit k that is equal to the number of nodes n of G.
- A Hamiltonian circuit of weight n exists in G′ iff there is a Hamilton circuit in G.
  - A Hamilton circuit is a set of edges that connect the nodes into a single cycle, with each node appearing exactly once.

Subset-sum problem

- Input: given a collection of numbers togther with a target number t
- Ask: does the collection contain a subcollection that adds up to t.

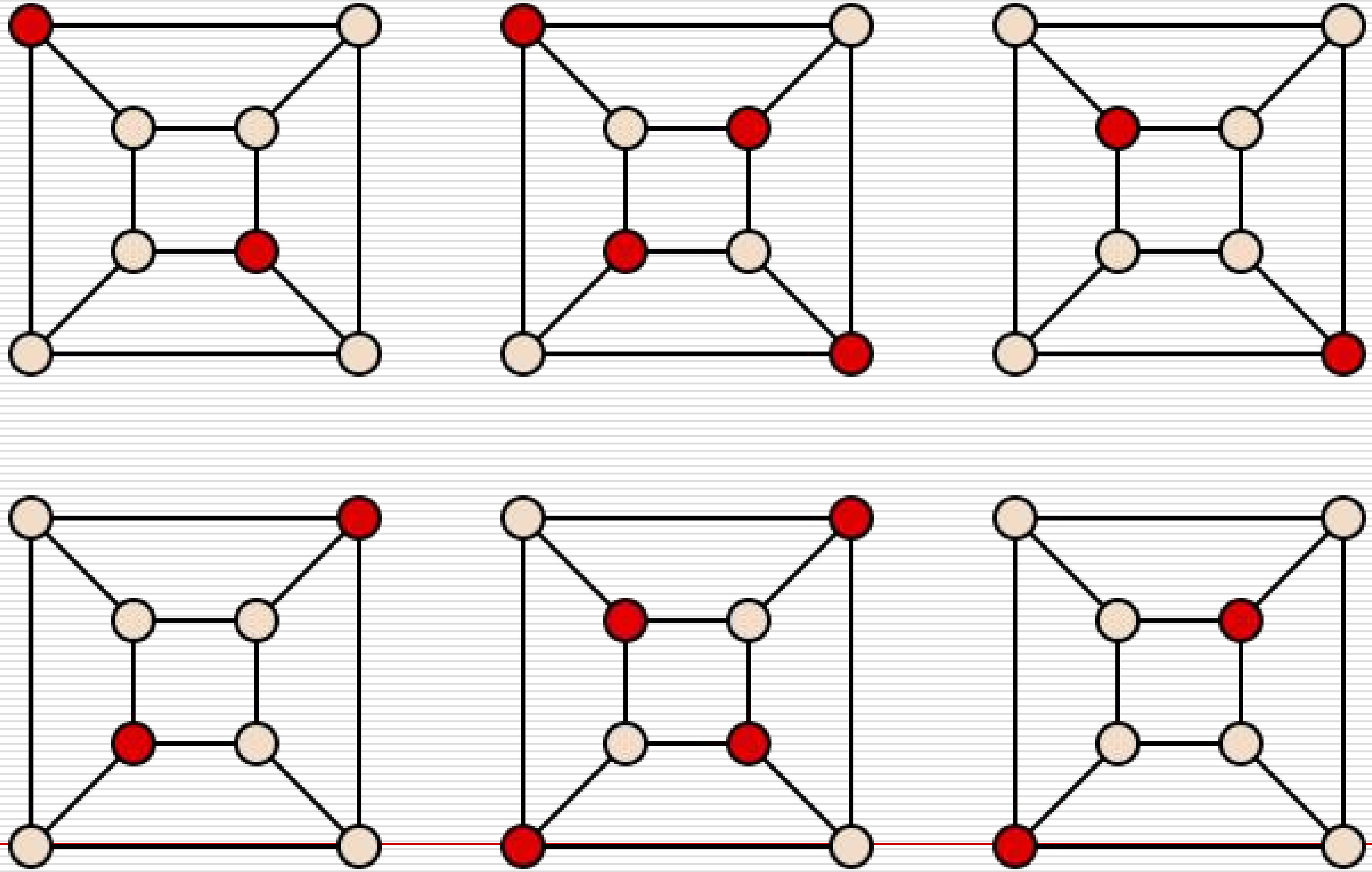
	1	2	3	4	$\cdots$	l	c_1	c_2	$\cdots$	c_k
y_1	1	0	0	0	$\cdots$	0	1	0	$\cdots$	0
z_1	1	0	0	0	$\cdots$	0	0	0	$\cdots$	0
y_2		1	0	0	$\cdots$	0	0	1	$\cdots$	0
z_2		1	0	0	$\cdots$	0	1	0	$\cdots$	0
y_3			1	0	$\cdots$	0	1	1	$\cdots$	0
z_3			1	0	$\cdots$	0	0	0	$\cdots$	1
$\vdots$					$\ddots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
y_l						1	0	0	$\cdots$	0
z_l						1	0	0	$\cdots$	0
g_1							1	0	$\cdots$	0
h_1							1	0	$\cdots$	0
g_2								1	$\cdots$	0
h_2								1	$\cdots$	0
$\vdots$									$\ddots$	$\vdots$
g_k										1
h_k										1
t	1	1	1	1	$\cdots$	1	3	3	$\cdots$	3

The Problem of Independent sets

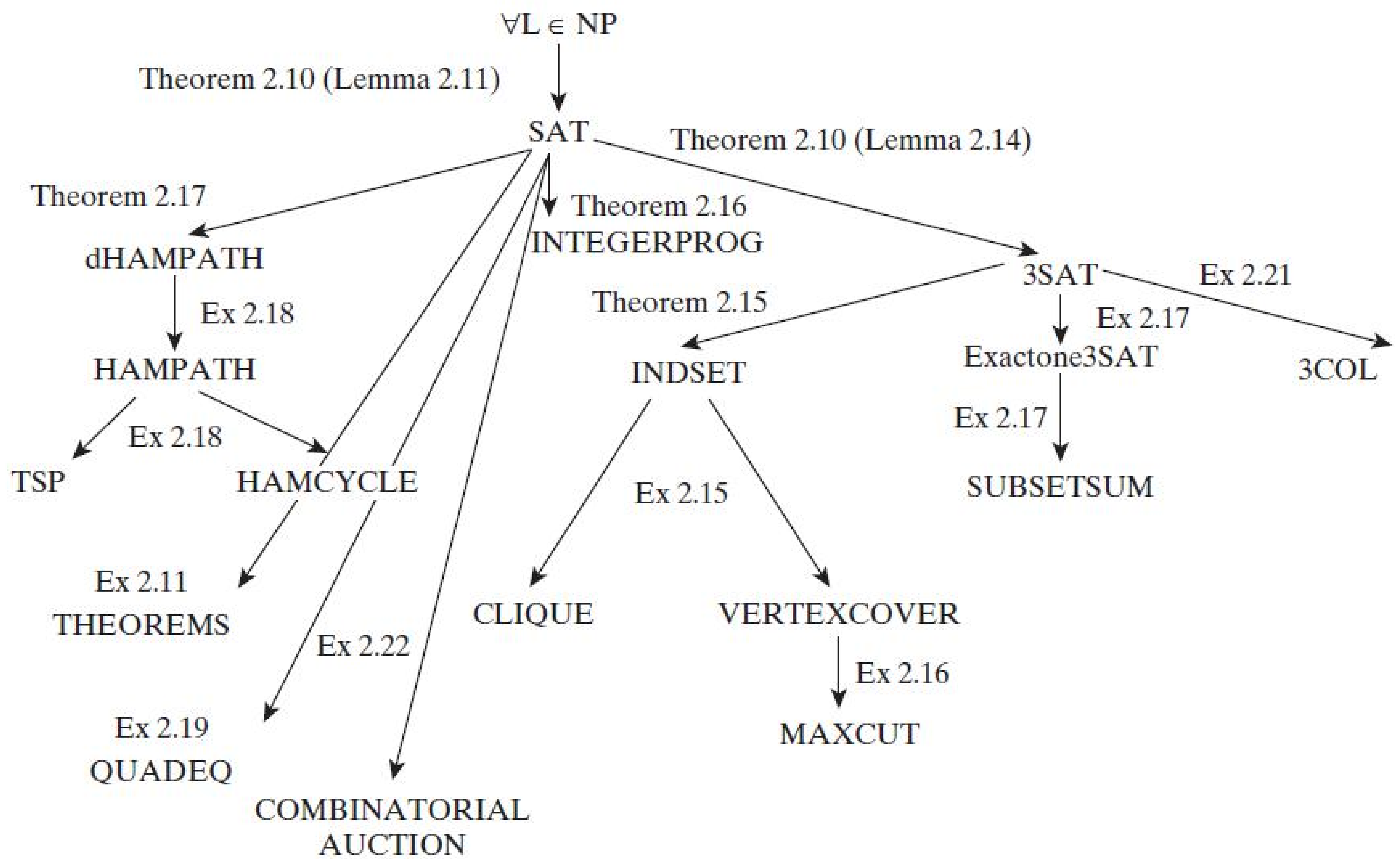
For an undirected graph, a subset I of the nodes of G is an independent set if no two nodes are connected by an edge of G

- **Input**: A graph G and a lower bound k
- **Ask**: does G have an independent set of size k.

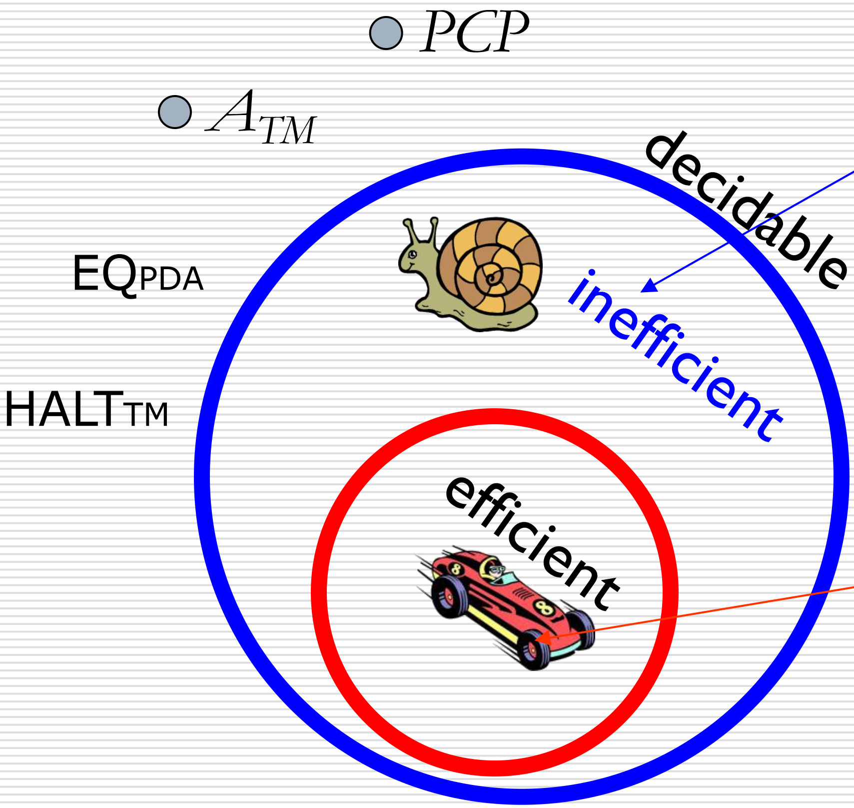
An example



Web of reductions between NP-complete problems

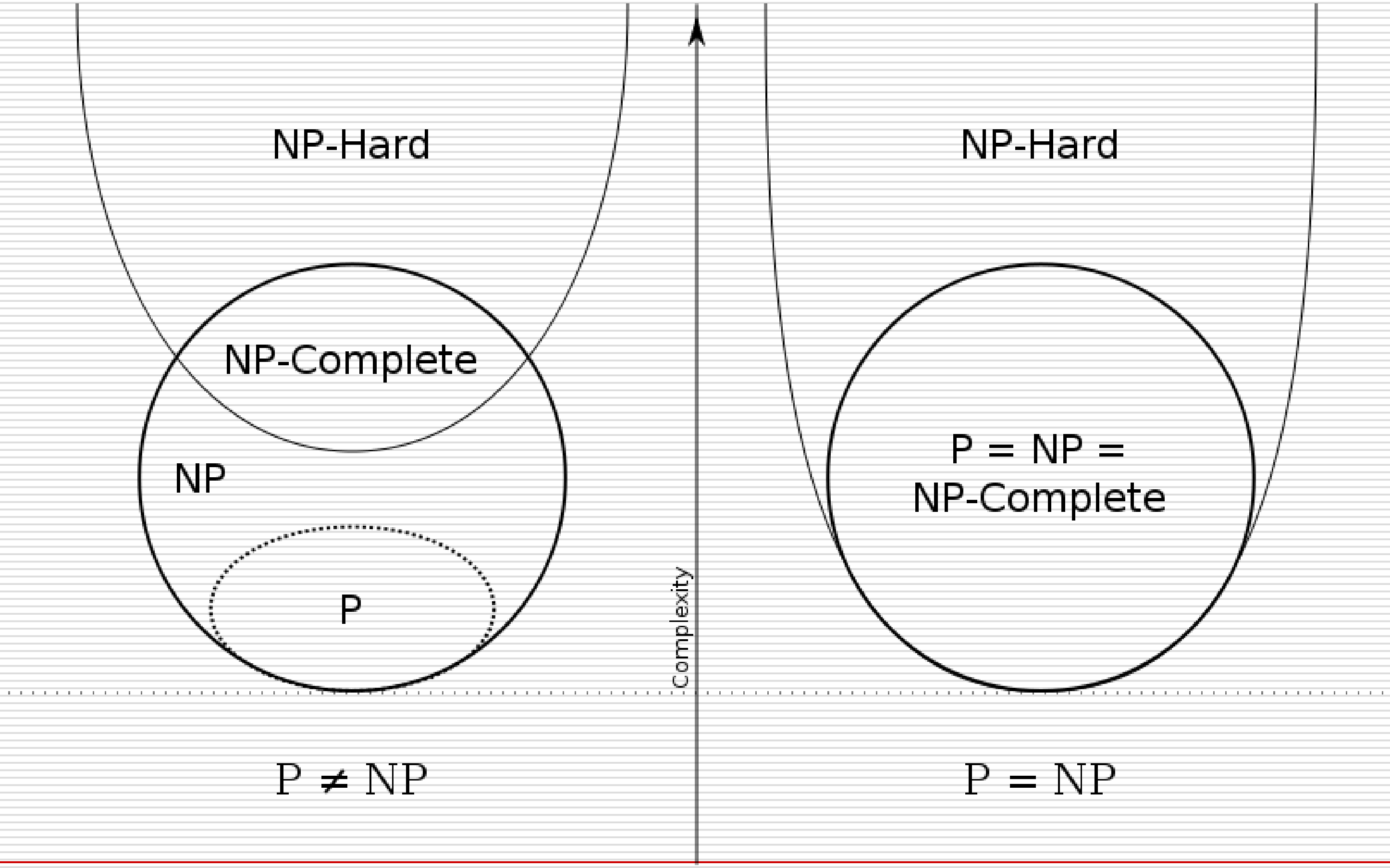


Where are we now?



- SAT
  3-SAT
  CLIQUE
  Vertex-Cover
  SUBSET-SUM
  HAMPATH
- PATH
  REGULAR (o(nlogn))
  CFL (O(n^3))
  Dynamic Programming
  Greedy algorithm

Location of NP-completeness



coNP

- A language L is in coNP if its complement is in NP.

- Example:

  TAUT = {<p>: p is a tautology which is Boolean formula satisfied by every assignment}