Algorithmics	Student information	Date	Number of session
	UO: 294067	14/02/24	1.2
	Surname: Díaz Álvarez		Escuela de
	Name: Paula	Ingenierí Informáti	
			Universidad de Oviedo



Activity 1. Times for iterative methods

$$f(n_1) \rightarrow t_1$$
 $t_2 = \frac{f(n_2)}{f(n_1)} \cdot t_1 = k \cdot t_1$ $f(n_2) \rightarrow t_2$?

TABLE1 (times in milliseconds and WITHOUT OPTIMIZATION):

n	T Loop 1	T Loop 2	T Loop 3	T Loop 4
100	0.0062	0.21	1.9	1.5
200	0.0114	0.80	7.8	6.4
400	0.0288	3.71	28.9	49.8
800	0.0638	17.65	89.2	405.3
1600	0.1394	71.41	302.2	4047.8
3200	0.2943	466.36	1457.3	ОоТ
6400	0.6318	ОоТ	ОоТ	ОоТ
12800	1.4708	ОоТ	ОоТ	ОоТ
25600	3.2846	ОоТ	ОоТ	ОоТ
51200	5.8693	ОоТ	ОоТ	ОоТ

Loop1:

Complexity: O(n log(n))

 We see if the complexity is the one expected with the more representative values (the biggest times obtained):

$$t_{2,1} = \frac{2^*1600\log{(2^*1600)}}{1600\log{(1600)}} \cdot 0.1394 = 0.305$$
 Real=0.2943
$$t_{2,2} = 2 \cdot \frac{\log{(2^*3200)}}{\log{(3200)}} \cdot 0.2943 = 0.6392$$
 Real=0.6318
$$t_{2,3} = 2 \cdot \frac{\log{(2^*6400)}}{\log{(6400)}} \cdot 0.6318 = 1.3638$$
 Real=1.4708

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$$t_{2,4} = 2 \cdot \frac{\log(2*12800)}{\log(12800)} \cdot 1.4708 = 3.1572$$

Real=3.2846

$$t_{2,5} = 2 \cdot \frac{\log{(2*25600)}}{\log{(25600)}} \cdot 3.2846 = 7.0178$$

Real=5.8693

• Loop2:

o Complexity: O(n² log(n))

$$t_{2,1} = \frac{(2 \cdot 100)^2 \log (2 \cdot 100)}{100^2 \log (100)} \cdot 0.21 = 4 \cdot \frac{\log (2 \cdot 100)}{\log (100)} \cdot 0.21 = 0.9664$$
 Real = 0.80
$$t_{2,2} = 4 \cdot \frac{\log (2 \cdot 200)}{\log (200)} \cdot 0.80 = 3.6186$$
 Real = 3.71

$$t_{2,3} = 4 \cdot \frac{\log(2 \cdot 400)}{\log(400)} \cdot 3.71 = 16.56$$
 Real=17.65

$$t_{2,4} = 4 \cdot \frac{\log{(2 \cdot 800)}}{\log{(800)}} \cdot 17.65 = 77.92$$
 Real=71.41

$$t_{2,5} = 4 \cdot \frac{\log(2 \cdot 1600)}{\log(1600)} \cdot 71.41 = 312.48$$
 Real=466.36

• Loop3:

o Complexity: O(n² log(n))

$$t_{2,1} = 4 \cdot \frac{\log(2 \cdot 100)}{\log(100)} \cdot 1.9 = 8.7439$$
 Real = 7.8

$$t_{2,2} = 4 \cdot \frac{\log(2 \cdot 200)}{\log(200)} \cdot 7.8 = 35.2817$$
 Real=28.9

$$t_{2,3} = 4 \cdot \frac{\log{(2 \cdot 400)}}{\log{(400)}} \cdot 28.9 = 128.97$$
 Real=89.2

$$t_{2,4} = 4 \cdot \frac{\log(2 \cdot 800)}{\log(800)} \cdot 89.2 = 393.8$$
 Real=302.2

$$t_{2,5} = 4 \cdot \frac{\log{(2 \cdot 1600)}}{\log{(1600)}} \cdot 302.2 = 1322.37$$
 Real=1457.3

Loop4:

o Complexity: O(n³)

$$t_{2,1} = \frac{(2 \cdot 100)^3}{100^3} \cdot 1.5 = 8 \cdot 1.5 = 12$$
 Real=6.4

$$t_{22} = 8 \cdot 6.4 = 51.2$$
 Real=49.8

$$t_{23} = 8 \cdot 49.8 = 398.4$$
 Real=405.3

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$$t_{2.4} = 8 \cdot 405.3 = 3242.4$$

Real=4047.8

Activity 2. Times for iterative methods given a complexity

TABLE2 (times in milliseconds and WITHOUT OPTIMIZATION):

n	T Loop 5	T Loop 6	T Loop 7
100	1.3	8.7	65
200	5.8	85.7	832
400	30.5	808.4	14134
800	153.2	ОоТ	OoT
1600	726.1	ОоТ	OoT
3200	4656.5	ОоТ	OoT
6400	ОоТ	ОоТ	OoT

Loop5:

Complexity: O(n² log²(n))

 We see if the complexity is the one expected with the more representative values (the biggest times obtained):

$$\begin{split} t_{2,1} &= \frac{(2*100)^2 \log^2(2*100)}{100^2 \log^2(100)} \cdot \ 1. \ 3 = 4 \cdot \frac{\log^2(2*100)}{\log^2(100)} \cdot \ 1. \ 3 = 6.88 & \text{Real=5.8} \\ t_{2,2} &= 4 \cdot \frac{\log^2(2*200)}{\log^2(200)} \cdot \ 5. \ 8 = 29.667 & \text{Real=30.5} \\ t_{2,3} &= 4 \cdot \frac{\log^2(2*400)}{\log^2(400)} \cdot \ 30.5 = 151.861 & \text{Real=153.2} \\ t_{2,4} &= 4 \cdot \frac{\log^2(2*800)}{\log^2(800)} \cdot \ 153. \ 2 = 746.475 & \text{Real=726.1} \end{split}$$

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$$t_{2,5} = 4 \cdot \frac{\log^2(2*1600)}{\log^2(1600)} \cdot 726.1 = 3475.779$$

Real=4656.5

• Loop6:

o Complexity: O(n³ log(n))

$$t_{2,1} = \frac{\left(2^*100\right)^3 \log\left(2^*100\right)}{100^3 \log\left(100\right)} \cdot 8.7 = 8 \cdot \frac{\log\left(2^*100\right)}{\log\left(100\right)} \cdot 8.7 = 80.076$$
 Real=85.7
$$t_{2,2} = 8 \cdot \frac{\log\left(2^*200\right)}{\log\left(200\right)} \cdot 85.7 = 775.292$$
 Real=808.4

• Loop7:

o Complexity: O(n⁴)

$$t_{2,1} = \frac{(2*100)^4}{100^4} \cdot 65 = 16 \cdot 65 = 1040$$
 Real=832
$$t_{2,2} = 16 \cdot 832 = 13312$$
 Real=14134

Activity 3. Comparison of two algorithms with different complexity

TABLE3 (times in milliseconds and WITHOUT OPTIMIZATION):

n	T Loop 1	T Loop 2	t1 / t2
100	0.0062	0.21	0.0295
200	0.0114	0.80	0.0143
400	0.0288	3.71	0.0078
800	0.0638	1.765	0.0361
1600	0.1394	7.141	0.0195
3200	0.2943	46.636	0.0063
6400	0.6318	ОоТ	~ 0

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12800	1.4708	ОоТ	~ 0
25600	5.2693	ОоТ	~ 0
51200	ОоТ	ОоТ	~ 0

Loop 1 has $O(n \log(n))$ complexity, and Loop 2 has $O(n^2 \log(n))$

As both algorithms have different complexities, when the size grows, the ratio tends to 0, so Loop 1 has a lower complexity compared with Loop 2 (As predicted before)

Activity 4. Comparison of two algorithms with same complexity

TABLE4 (times in milliseconds and WITHOUT OPTIMIZATION):

n	T Loop 3	T Loop 2	t3 / t2
100	1.9	0.21	9.0476
200	7.8	0.80	9.75
400	28.9	3.71	7.7898
800	89.2	1.765	50.5382
1600	302.2	7.141	42.3190
3200	1457.3	46.636	31.2463
6400	OoT	ОоТ	-
12800	OoT	ОоТ	-
25600	ОоТ	ОоТ	-
51200	ОоТ	ОоТ	-

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Loop 3 and 2 have $O(n^2 \log(n))$ complexity.

As both algorithms have the same complexity, the ratio tends to a constant, and as it's greater than 1; Loop 2 has a better complexity

Activity 5. Comparison of two algorithms in different development environments

TABLE5 (times in milliseconds):

n	T Loop 4: t41 (Python)	T Loop 4: t42 (Java, no optimization)	T Loop 4: t43 (Java with optimization)	t42/t41	t43/t42
100	0	1.5	0.048	inf	0.032
200	38	6.4	0.152	0.1684	0.0236
400	208	49.8	0.753	0.2394	0.0151
800	1898	405.3	4.683	0.2135	0.0116
1600	18427	4047.8	37.959	0.2197	0.0094
3200	ОоТ	ОоТ	ОоТ	-	-
6400	ОоТ	ОоТ	ОоТ	-	-

They are the same algorithm, so they have the same complexity, so the ratio tends to a constant:

- For t42/t41: the ratio tends to < 1, so the Java version with no optimization is better than the Python version
- For t43/t42: the ratio tends to <1, so the Java version with optimization is better than the Java version with no optimization