f(n1) -> t1 t2=f(n2)f(n1)t1=kt1

f(n2) -> t2?

Activity 1. Bubble algorithm

TABLE1 BUBBLE ALGORITHM (times in milliseconds and WITHOUT OPTIMIZATION):

n	t ordered	t reverse	t random
10000	870	1923	1316
2*10000	2270	8635	6554
2^2*10000	12622	30528	23658
2^3*10000	39271	OoT	OoT
2^4*10000	OoT	OoT	OoT

For the bubble algorithm, the complexity is always $O(n^2)$ for the best, worst and average complexity:

When ordered:

$t_{2,1} = \frac{(2 \cdot 10000)^2}{10000^2} \cdot 870 = 4 \cdot 870 = 3480$ The real is: 2270

$t_{2,2} = 4 \cdot 2270 = 9080$ The real is: 12622

$t_{2,3} = 4 \cdot 12622 = 50488$ The real is: 39271

When reversed:

$t_{2,1} = 4 \cdot 1923 = 7692$ The real is: 8635

$t_{2,2} = 4 \cdot 8635 = 34540$ The real is: 30528

When random:

$t_{2,1} = 4 \cdot 1316 = 5264$ The real is: 6554

$t_{2,2} = 4 \cdot 6554 = 26216$ The real is: 23658

Activity 2. Selection algorithm

TABLE 2 SELECTION ALGORITHM (times in milliseconds and WITHOUT OPTIMIZATION):

n	t ordered	t reverse	t random
10000	769	631	487
2*10000	2450	2273	2448
2^2*10000	9461	12027	8431
2^3*10000	36304	36625	33919
2^4*10000	OoT	OoT	OoT

For the selection algorithm, the complexity is always $O(n^2)$ for the best, worst and average complexity:

When ordered:

$t_{2,1} = \frac{(2 \cdot 10000)^2}{10000^2} \cdot 769 = 4 \cdot 769 = 3076$ The real is: 2450

$t_{2,2} = 4 \cdot 2450 = 9800$ The real is: 9461

$t_{2,3} = 4 \cdot 9461 = 37844$ The real is: 36304

When reversed:

$t_{2,1} = 4 \cdot 631 = 2524$ The real is: 2273

$t_{2,2} = 4 \cdot 2273 = 9092$ The real is: 12027

$t_{2,3} = 4 \cdot 12027 = 48108$ The real is: 36625

When random:

$t_{2,1} = 4 \cdot 487 = 1948$ The real is: 2448

$t_{2,2} = 4 \cdot 2448 = 9792$ The real is: 8431

$t_{2,3} = 4 \cdot 8431 = 33724$ The real is: 33919

Activity 3. Insertion algorithm

TABLE 3 INSERTION ALGORITHM (times in milliseconds and WITHOUT OPTIMIZATION):

n	t ordered	t reverse	t random
10000	LoR	733	373
2*10000	LoR	2956	1463
2^2*10000	LoR	12212	6071
2^3*10000	LoR	52504	30696
2^4*10000	LoR	OoT	OoT
2^5*10000	LoR	OoT	OoT
2^6*10000	LoR	OoT	OoT
2^7*10000	61	OoT	OoT
2^8*10000	88	OoT	OoT
2^9*10000	120	OoT	OoT
2^{10}*10000	235	OoT	OoT
2^{11}*10000	474	OoT	OoT
2^{12}*10000	942	OoT	OoT

2^{13}*10000	1872	OoT	OoT

For the insertion algorithm, the complexity is O(n) at the best case (when the vector is ordered), but has complexity $O(n^2)$ for the worst and average case

When ordered:

$t_{2,1} = \frac{(2^9 \cdot 10000)}{(2^8 \cdot 10000)} \cdot 88 = 2 \cdot 88 = 176$ The real is: 120

$t_{2,2} = 2 \cdot 120 = 240$ The real is: 235

$t_{2,3} = 2 \cdot 235 = 470$ The real is: 474

$t_{2,4} = 2 \cdot 474 = 948$ The real is: 942

$t_{2,5} = 2 \cdot 942 = 1884$ The real is: 1872

When reversed:

$t_{2,1} = \frac{(2 \cdot 10000)^2}{10000^2} \cdot 733 = 4 \cdot 733 = 2932$ The real is: 2956

$t_{2,2} = 4 \cdot 2956 = 11824$ The real is: 12212

$t_{2,3} = 4 \cdot 12212 = 48848$ The real is: 52504

When random:

$t_{2,1} = 4 \cdot 373 = 1492$ The real is: 1463

$t_{2,2} = 4 \cdot 1463 = 5852$ The real is: 6071

$t_{2,3} = 4 \cdot 6071 = 24284$ The real is: 30696

Activity 4. Quicksort algorithm

TABLE 4 QUICKSORT ALGORITHM (times in milliseconds and WITHOUT OPTIMIZATION):

Algorithmics	Student information	Date	Number of session
	UO: 294067	21/02/24	2
	Surname: Díaz Álvarez		
	Name: Paula		

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University of Oviedo

n	t ordered	t reverse	t random
250000	49	54	123
2*250000	102	111	254
2^2*250000	211	228	955
2^3*250000	436	478	1222
2^4*250000	909	998	2677
2^5*250000	2412	2034	8454
2^6*250000	5448	4312	18286

For the quicksort algorithm the complexity is O(n log(n)) for the best, average and worst, as we pick the pivot as median-3. But it would have O(n^2) if we picked a bad pivot.

When ordered:

$$t_{2,1} = \frac{4 \cdot 250000 \cdot \log(4 \cdot 250000)}{2 \cdot 250000 \cdot \log(2 \cdot 250000)} \cdot 102 = 217.056$$ The real is: 211

$$t_{2,2} = \frac{8 \cdot 250000 \cdot \log(8 \cdot 250000)}{4 \cdot 250000 \cdot \log(4 \cdot 250000)} \cdot 211 = 447.32$$ The real is: 436

$$t_{2,3} = \frac{16 \cdot 250000 \cdot \log(16 \cdot 250000)}{8 \cdot 250000 \cdot \log(8 \cdot 250000)} \cdot 436 = 921.704$$ The real is: 909

$$t_{2,4} = \frac{32 \cdot 250000 \cdot \log(32 \cdot 250000)}{16 \cdot 250000 \cdot \log(16 \cdot 250000)} \cdot 909 = 1916.172$$ The real is: 2412

$$t_{2,5} = \frac{64 \cdot 250000 \cdot \log(64 \cdot 250000)}{32 \cdot 250000 \cdot \log(32 \cdot 250000)} \cdot 2412 = 5070.024$$ The real is: 5448

When reversed:

$$t_{2,1} = \frac{4 \cdot 250000 \cdot \log(4 \cdot 250000)}{2 \cdot 250000 \cdot \log(2 \cdot 250000)} \cdot 111 = 236.208$$ The real is: 228

$$t_{2,2} = \frac{8 \cdot 250000 \cdot \log(8 \cdot 250000)}{4 \cdot 250000 \cdot \log(4 \cdot 250000)} \cdot 228 = 483.36$$ The real is: 478

$$t_{2,3} = \frac{16 \cdot 250000 \cdot \log(16 \cdot 250000)}{8 \cdot 250000 \cdot \log(8 \cdot 250000)} \cdot 478 = 1010.492$$ The real is: 998

$t_{2,4} = \frac{32 \cdot 250000 \cdot log\,(32 \cdot 250000)}{16 \cdot 250000 \cdot log\,(16 \cdot 250000)} \cdot 998 = 2103.784$ The real is: 2034

$t_{2,5} = \frac{64 \cdot 250000 \cdot log\,(64 \cdot 250000)}{32 \cdot 250000 \cdot log\,(32 \cdot 250000)} \cdot 2034 = 4275.468$ The real is: 4312

When random:

$t_{2,1} = \frac{4 \cdot 250000 \cdot log\,(4 \cdot 250000)}{2 \cdot 250000 \cdot log\,(2 \cdot 250000)} \cdot 254 = 540.512$ The real is: 955

$t_{2,2} = \frac{8 \cdot 250000 \cdot log\,(8 \cdot 250000)}{4 \cdot 250000 \cdot log\,(4 \cdot 250000)} \cdot 955 = 2024.6$ The real is: 1222

$t_{2,3} = \frac{16 \cdot 250000 \cdot log\,(16 \cdot 250000)}{8 \cdot 250000 \cdot log\,(8 \cdot 250000)} \cdot 1222 = 2583.308$ The real is: 2677

$t_{2,4} = \frac{32 \cdot 250000 \cdot log\,(32 \cdot 250000)}{16 \cdot 250000 \cdot log\,(16 \cdot 250000)} \cdot 2677 = 5643.116$ The real is: 8454

$t_{2,5} = \frac{64 \cdot 250000 \cdot log\,(64 \cdot 250000)}{32 \cdot 250000 \cdot log\,(32 \cdot 250000)} \cdot 8454 = 17770.308$ The real is: 18286

Time for each algorithm with n= 16000000 in random order:

- Bubble: $t_2 = \frac{(16{,}000{,}000)^2}{40{,}000^2} \cdot 23658 = 3,785,280,000\,ms = 43811\,days$
- Selection: $t_2 = \frac{(16{,}000{,}000)^2}{80{,}000^2} \cdot 33919 = 1,356,760,000\,ms = 15703\,days$
- Insertion: $t_2 = \frac{(16{,}000{,}000)^2}{80{,}000^2} \cdot 30696 = 1,227,840,000\,ms = 14211\,days$

Activity 5. Quicksort + Insertion algorithm

TABLE 4 QUICKSORT ALGORITHM (times in milliseconds and WITHOUT OPTIMIZATION):

n	t random
Quicksort	14086
Quicksort + Insertion (k=5)	15276

Quicksort + Insertion (k=10)	15494
Quicksort + Insertion (k=20)	13521
Quicksort + Insertion (k=30)	15079
Quicksort + Insertion (k=50)	14629
Quicksort + Insertion (k=100)	13328
Quicksort + Insertion (k=200)	10543
Quicksort + Insertion (k=500)	12486
Quicksort + Insertion (k=1000)	24676

With k=0, the algorithm uses only Quicksort, but as k increases. Insertion is done more each time, compared to Quicksort.

We can see this as at first time is 14086, but then with k=1000 time is 24676 as Insertion is done more than Quicksort and has worse complexity.

Between them, the time starts being worse than quicksort and after that, the time starts decreasing (they are in equilibrium) until k=500 where Insertion probably will be the one dominating so the times starts getting worse.