DS 623 PE05

For PE05, you will implement the eigendecomposition. Suppose a 2x2 matrix A has two eigenvectors. Let's form a matrix P that consists of eigenvectors of A. Then AP = PD where D is a diagonal matrix of eigenvalues. Then, A = PD^{-1}.

You should use basic NumPy operation except for calculating the inverse of a matrix.

1) Input: a 2x2 matrix A
2) Output:
    a. Eigenvalues: λ_1 and λ_2
    b. Eigenvectors: p_1 and p_2 (column eigenvectors that correspond to eigenvalues λ_1 and λ_2 respectively)
3) Verification: A – PDP^{-1} = 0. (You can use numpy.linalg.inv to calculate the inverse of matrix P)
   This is equivalent to $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} - [p_1, p_2] \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} [p_1, p_2]^{-1}$=0

Optional:

4) Compute A^{1000} and PD^{1000}P^{-1}. Compare the results and the computing time. Do you see any significant difference in computing time? Try the exercise with higher power.

Example)

Input: np.array([[-4, -3],[1, 0]])

Output:
Eigenvalues: -1, -3
Eigenvectors: np.array([[1], [-1]]), np.array([[-3], [1]])

Verification:

$$\begin{bmatrix} -4 & -3 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 & -3 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -1 & 1 \end{bmatrix}^{-1} = 0$$