DS 623 PE02

Consider a system of two equations:

$$\begin{cases} a_{11}\, x_1 + a_{12} x_2 = b_1 \\ a_{21}\, x_1 + a_{22} x_2 = b_2 \end{cases}$$

It can be represented in the following matrix form Ax=b where:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, and\ x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

For PE02, your code in the Jupyter Notebook should have the following properties:

1) Input: 2x2 matrix A
2) Output:
    a. reduced row echelon form (rref) of the matrix A
    b. rank of the matrix A
    c. whether column vectors of matrix A form a basis for R^2
    d. (optional) kernel of A

For this assignment, you cannot use any special function other than basic NumPy operations.

Example 1)

Enter the matrix A: np.array([[3, 2], [2, 3]])

Output:
Reduced row echelon form is: np.array([[1, 0], [0, 1]])
Rank of the matrix is: 2
Column vectors form a basis for R^2

Example 2)

Enter the matrix A: np.array([[0, 2], [3, 2]])

Output:
Reduced row echelon form is: np.array([[1, 0], [0, 1]])
Rank of the matrix is: 2
Column vectors form a basis for R^2

Example 3)

Enter the matrix A: np.array([[3, 2], [-6, -4]])

Output:
Reduced row echelon form is: np.array([[1, 0.66666667
], [0, 0]])
Rank of the matrix is: 1
Column vectors do not form a basis for R^2

How to find out what kernels are in example 1, 2, and 3?

Kernels are $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ that makes $Ax = 0$. They are {0}, {0}, and any vector spanned by np.array([-0.66666667], [1])) in these examples.

In example 1, $x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ is the only solution that makes $\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

In example 2, $x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ is the only solution that makes $\begin{bmatrix} 0 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$

In example 3, $x = c \begin{bmatrix} -\frac{2}{3} \\ 1 \end{bmatrix}$ (where c is any coefficient) can make $\begin{bmatrix} 3 & 2 \\ -6 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$. In other words, any multiple of $\begin{bmatrix} -\frac{2}{3} \\ 1 \end{bmatrix}$ solves $Ax = 0$.