1. Sum of the gaussian distribution for the smile face:

A smile face consists of three part, the outline, mouse and eyes.

Firstly, I sampled 128*128 points from [-5,5]*[-5,5].

The code for the outline is in listing 1.

```
%the outline of the face
sigma1=[0.05 0;0 0.05];%covariance of the outline and the mouse of face
n1=100;%number of total gaussian distribution for the outline
dtheta1=2*pi/n1;
theta1=0:dtheta1:2*pi-dtheta1;
r=4.5;
mu1=zeros(100,2);
mu1(:,1)=cos(theta1)*r;
mu1(:,2)=sin(theta1)*r;
for j=1:n1
    ft=1.5*mvnpdf([X(:) Y(:)],mu1(j,:),sigma1);
    f=f+reshape(ft,size(X));
end
```

listing 1 code to get the outline

The outline part is just a circle, which should not be so wide. Hence, I set the covariance for the gaussian distribution of outline part is Σ = [0.05 0; 0 0.05]. Then I made a set of $\mu_i = (\mu_{i1}, \mu_{i2})$, which is defined by

$$\mu_{i1} = r\cos\theta_i \ , \ \mu_{i2} = r\sin\theta_i$$

Where r is the radius of the circle, which is 4.5 in this case, and theta here is defined by

theta1=0:dtheta1:2*pi-dtheta1;

which means theta ranges from [0,2pi] and divided by n evenly.

Then we can got the outline by got the sum of each gaussian distribution with mean μ_i and covariance Σ, as we can see in figure 1.

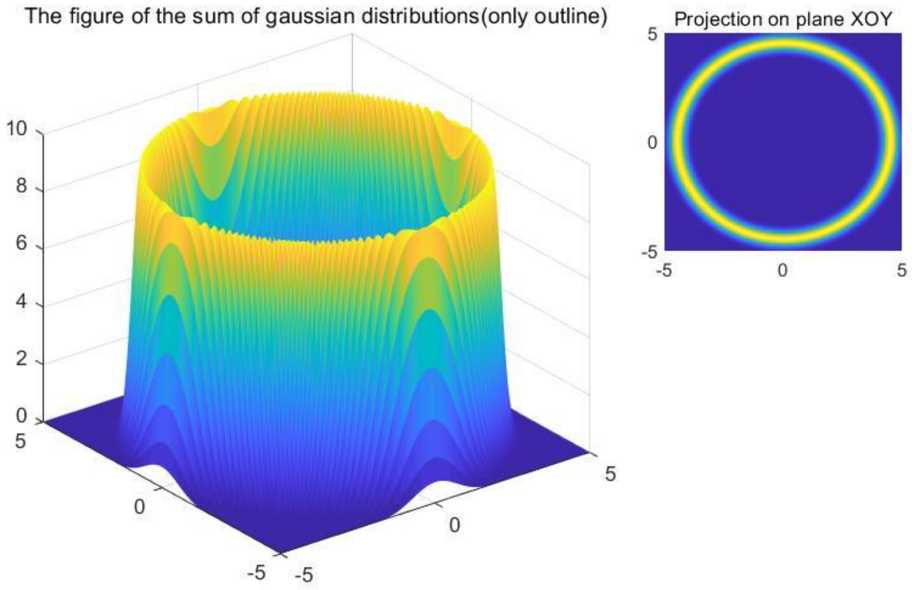

figure 1 the plot of the outline

As for the mouse, I think it will be fine if it is as wide as the outline, so I used the same Σ. What's different is the $\mu_i = (\mu_{i1}, \mu_{i2})$.

I think we can define the mouse an arc. So, we just need to reduce r to 3 and change the range of theta to $[\frac{7}{6}\pi, \frac{11}{6}\pi]$. Other step is just the same as what has been done to get the outline.

```
%the mouse
n2=20;
dtheta2=(11*pi/6-7*pi/6)/(n2-1);
theta=7*pi/6:dtheta2:11*pi/6;
r=3;
mu2=zeros(n2,2);
mu2(:,1)=cos(theta)*r;
mu2(:,2)=sin(theta)*r;
for j=1:n2
    ft=1.5*mvnpdf([X(:) Y(:)],mu2(j,:),sigma1);
    f=f+reshape(ft,size(X));
end
```

listing 2　code to get the mouse

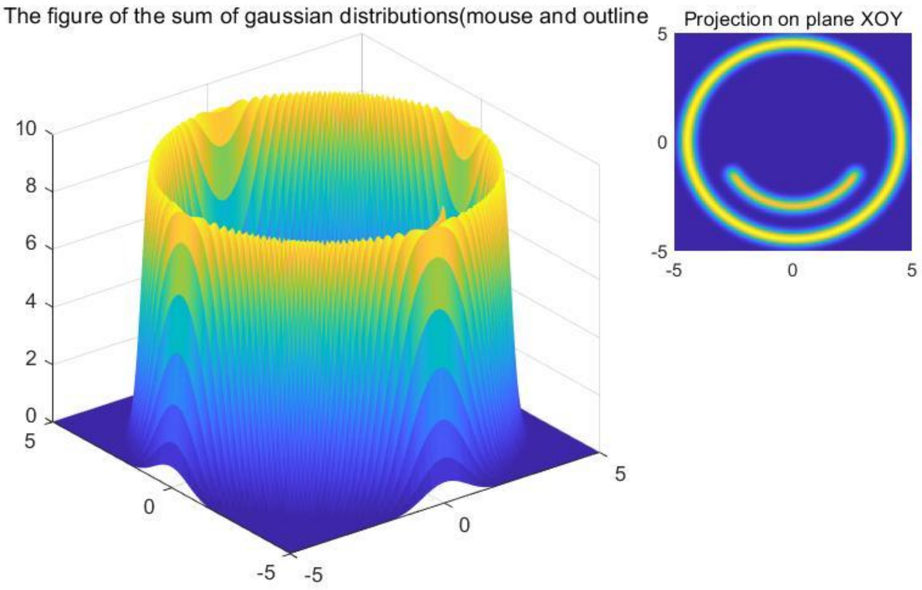

figure 2 the plot of the outline and mouse

When it comes to the eyes, I use one gaussian distribution with larger covariance for each eyes in order to simplify the plot.

The $\boldsymbol{\mu} = (\mu_1, \mu_2)$ for each eye are [-1.8,1] and [1.8,1] and the covariance is Σ = [0.05 0; 0 0.05]*10. It has to be mentioned that to prevent the eyes from looking too light, I times 30 to the value of the gaussian distribution to makes it look darker.

```
%the eyes
sigma2=sigma1*10;%covariance of the eyes
eyeleft=[-1.8,1];
eyeright=[1.8,1];
ft=30*mvnpdf([X(:) Y(:)],eyeleft,sigma2);
f=f+reshape(ft,size(X));
ft=30*mvnpdf([X(:) Y(:)],eyeright,sigma2);
f=f+reshape(ft,size(X));
```

listing 3　code to get the eyes

The final smile face is in figure 3, it looks very cute.

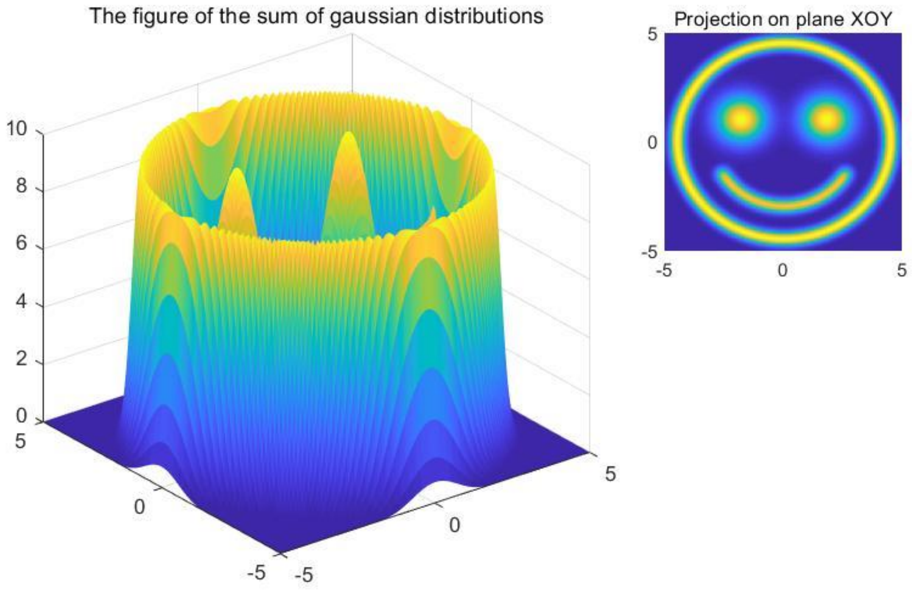

figure 3 the plot of the smile face

2. The analytical result of the Fourier transform:

Since the linearity applies on 2-D FT transform,

if $x_1(n_1,\ldots,n_M) \overset{\mathrm{FT}}{\longleftrightarrow} X_1(\omega_1,\ldots,\omega_M)$, and $x_2(n_1,\ldots,n_M) \overset{\mathrm{FT}}{\longleftrightarrow} X_2(\omega_1,\ldots,\omega_M)$ then,

$$ax_1(n_1,\ldots,n_M) + bx_2(n_1,\ldots,n_M) \overset{\mathrm{FT}}{\longleftrightarrow} aX_1(\omega_1,\ldots,\omega_M) + bX_2(\omega_1,\ldots,\omega_M)$$

We can calculate analytical FT transform of each Gaussian distribution first, then add them together. The sum will be the analytical result of the FT transform for the smile distribution.

Our solution to exercise1 last week is

$$\hat{f}\left(\omega;\mu,\sigma^2 I\right) = \exp(-\frac{\omega^T \Sigma \omega}{2} - \omega^T \mu j)$$

Based on that, we can calculate the analytical FT transform for each distribution, which is on the listing 4.

Note that in outline and mouse, I times 1.5 , while in eyes, I times 30. This is just for I times this number on the Gaussian distribution in Section 1 to make the smile clearer to see. When it comes to FT transform, we cannot miss this modification.

In the ana.m file, the matrix **f** is the sum of the Gaussian distributions, which makes the smile face. Matrix **G_2d** is the result of analytical FT transform of **f**. Matrix

G_2dm is the magnitude of **G_2d**, while **G_2dp** is the phase of **G_2d**.

```
for i=1:n
    for j=1:n
        WXY=[WX(i,j),WY(i,j)];
        for k=1:n1%for outline
            G_2d(i,j)=G_2d(i,j)+1.5*exp(-0.5*WXY*sigma1*WXY'-1i*WXY*mu1(k,:)');
        end
        for k=1:n2%for mouse
            G_2d(i,j)=G_2d(i,j)+1.5*exp(-0.5*WXY*sigma1*WXY'-1i*WXY*mu2(k,:)');
        end
        G_2d(i,j)=G_2d(i,j)+30*exp(-0.5*WXY*sigma2*WXY'-1i*WXY*eyeleft');%lefteye
        G_2d(i,j)=G_2d(i,j)+30*exp(-0.5*WXY*sigma2*WXY'-1i*WXY*eyeright');%righteye
    end
end
```

Listing 4 The analytical FT transform

I also plot the figure of magnitude—w, which can be seen in figure 4. I think it should be right. If you have any questions, please just tell me.

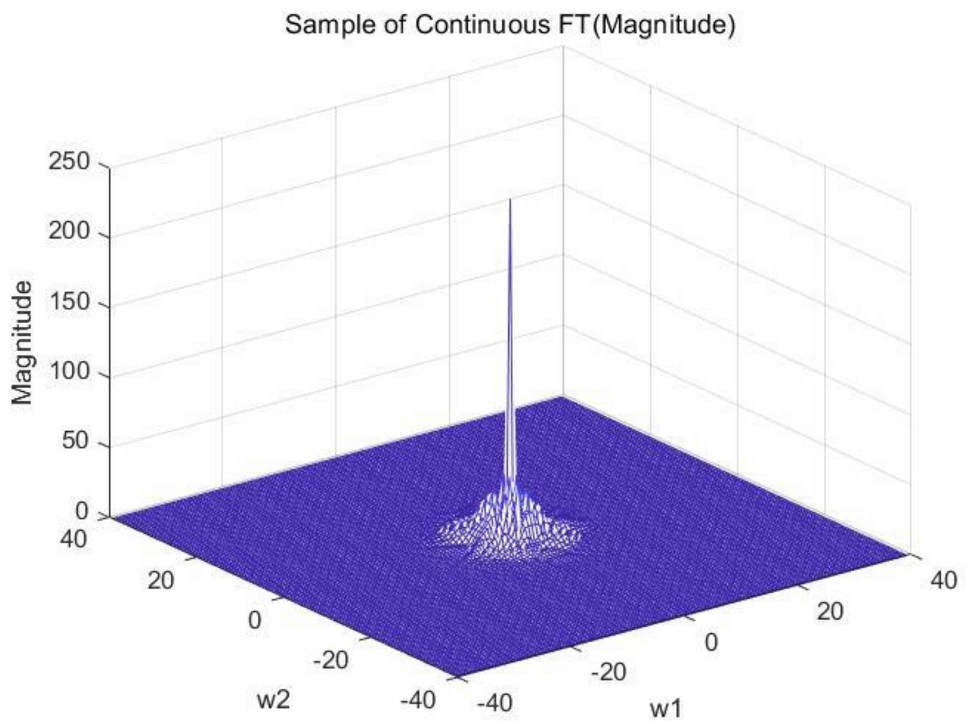

figure 4 the plot of the magnitude—w (analytical FT transform)