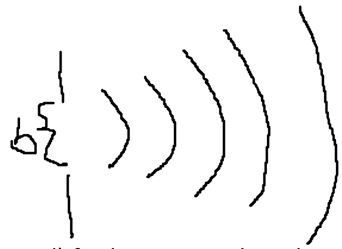
Coherence

Thursday, June 16, 2016 9:48 AM

$$\Delta s_c \approx c\Delta t = \frac{c}{\Delta\omega} = \frac{c}{2\pi\Delta v}$$

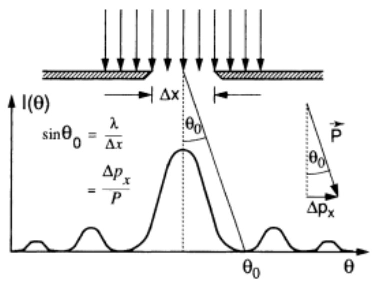
If the path difference gets to large, the wave will no longer be coherent temporally at detection
Lower Spectral widths results in larger coherent lengths or times

$$\theta \cong \frac{\lambda}{b} \approx \frac{d}{r}$$
$$d\Omega = \frac{\lambda^2}{b^2} \approx \theta^2$$



Larger divergence angle (spread) for larger wavelengths

$$\Delta p_x \Delta x \geq \hbar$$
$$\Delta p_y \Delta y \geq \hbar$$
$$\Delta p_z \Delta z \geq \hbar$$
$$\Delta p_x \Delta x \Delta p_y \Delta y \Delta p_z \Delta z \geq \hbar^3 = V_{ph}$$
$$V_{ph}\ isthe\ volume\ of\ the\ elemetary\ cell\ in\ phase\ space$$



The minimum uncertainty of the x and y
direction with z as the propagation axis

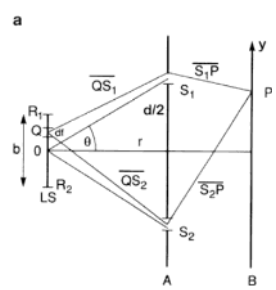
$$\Delta p_x = \Delta p_y = \frac{|p|\lambda}{2\pi b} = \frac{\hbar\omega}{c}\lambda\frac{1}{2\pi b} = \frac{\hbar\omega}{c}\frac{d}{2\pi r}$$

Natural broadening due to quantum mechanics

$$\Delta p_z = \frac{\hbar}{c}\Delta\omega, spread\ mostly\ caused\ by\ the\ uncertainty\ in\ the\ frequency$$

Quantitative description of Coherence
Coherence Function and the Degree of Coherence
Temporal and Spatial coherence, concered wit hthe correlation between optical fields at P_o at various times or at the same t iem at difference points

$$E(r,t) = A_0 e^{i(\omega t - \vec{k}\cdot\vec{r})} + c.c.$$



$$E(P,t) = k_1 E_1\left(S_1, t - \frac{r_1}{c}\right) + k_2 E_2\left(S_2, t - \frac{r_2}{c}\right)$$
$$\frac{r_1}{c} = t_1, \qquad \frac{r_2}{c} = t_2$$

$k_1\ and\ k_2$ are compelx and depend on the distance from the source to the location P

$$I_p = \epsilon_o c\langle E\cdot E^*\rangle = |k_1|^2\langle E_1 E_1^*\rangle + |k_2|^2\langle E_2 E_2^*\rangle + 2Re\{k_1 k_2^*\langle E_1(t+\tau)E_2^*(t)\rangle\}$$
$$I_p = \epsilon_o c[|k_1|^2 I_{S1} + |k_2|^2 I_{S2} + 2|k_1||k_2|Re\{\Gamma_{12}(\tau)\}]$$

$$\Gamma_{11}(\tau) = \langle E_1(t+\tau)E_1^*(t)\rangle$$
$$\Gamma_{22}(\tau) = \langle E_2(t+\tau)E_2^*(t)\rangle$$
$$\tau = 0\ is\ the\ self\ coherence\ functions$$
$$\Gamma_{11}(\tau=0) = \frac{I_1}{\epsilon_o c}, \qquad \Gamma_{22}(\tau=0) = \frac{I_2}{\epsilon_o c}$$

$$\gamma_{12}(\tau) = \frac{\Gamma_{12}(\tau)}{\sqrt{\Gamma_{11}(0)\Gamma_{22}(0)}} = \frac{\langle E_1(t+\tau)E_2(t)\rangle}{\sqrt{\langle |E_1(t)|^2 |E_2(t)|^2\rangle}}$$

$$I_p = I_1 + I_2 + 2\sqrt{I_1 I_2}\Re\{\gamma_{12}(\tau)\}$$

$$\gamma_{12}(\tau) = |\gamma_{12}(\tau)|e^{i\phi_{12}(\tau)}$$

$\phi_{12}(\tau) = \phi_1(\tau) - \phi_2(\tau)$ *is the phase of the fields* E_1 *and* E_2

$\gamma_{12}(\tau)$ *is a measure of degree of coherence*

- $|\gamma_{12}(\tau)| = 1$ describes interference of two completely coherent waves
- $0 < |\gamma_{12}(\tau)| < 1$ *describes partial coherence and thus*

For γ_{11}

$$\gamma_{11}(\tau) = \frac{\langle E(t+\tau)E^*(t)\rangle}{|E(t)|^2} = \left\langle e^{i(\phi(t+\tau)e^{-i\phi(t)})}\right\rangle$$

$$\gamma_{11}(\tau) = \lim_{T\to\infty}\frac{1}{\mathrm{T}}\int_0^T \cos(\Delta\phi) + i\sin\Delta\phi \;\; dt$$

Monochromatic waves have infinite coherent length, $\Delta\omega = 0 \Rightarrow \Delta s_c = \infty$

$\phi(t) = \omega t - \vec{k}\cdot\vec{r}\,, \Delta\phi = \phi(t+\tau) - \phi(t) = \omega\tau$

For a wave with large spectral width, $\Delta\omega \Rightarrow \tau > \frac{\Delta s_c}{c} = \frac{1}{\Delta\omega}$, *the phase fluctuate betwen* 0 *and* 2π

Resulting in an average of 0

$\Delta s_c = c\Delta t = \frac{c}{\Delta\omega}$

$\Delta\phi(t) \approx \Delta\omega\tau$, *for large* $\Delta\omega$, this leads to $\Delta\phi$ ranging from 0 to 2π

For γ_{12}

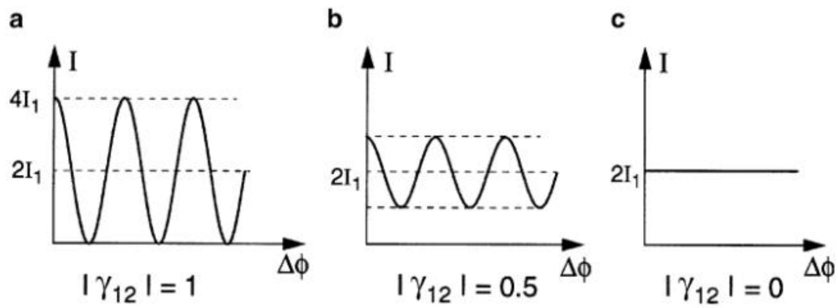


Figure 2.35 Interference pattern $I(\Delta\phi)$ of two-beam interference for different degrees of coherence

For quasi monochromatic planewaves of different paths, *both same frequency*

$\phi_{12}(\tau) = \vec{k}\cdot(\vec{r}_2 - \vec{r}_1)$

$I_p = I_1 + I_2 + 2\sqrt{I_1 I_2}|\gamma_{12}(\tau)|\cos\phi_{12}(\tau)$