Massachusetts Institute of Technology
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Fall 2010)

Recitation 4
September 21, 2010

1. Problem 1.50, page 67 in the text.

   The birthday problem. Consider n people who are attending a party. We assume that every person has an equal probability of being born on any day during the year, independently of everyone else, and ignore the additional complication presented by leap years (i.e., nobody is born on February 29). What is the probability that each person has a distinct birthday?

2. Imagine that 8 rooks are randomly placed on a chessboard. Find the probability that all the rooks will be safe from one another, i.e. that there is no row or column with more than one rook.

3. Problem 1.61, page 69 in the text.

   Hypergeometric probabilities. An urn contains n balls, out of which exactly m are red. We select k of the balls at random, without replacement (i.e., selected balls are not put back into the urn before the next selection). What is the probability that i of the selected balls are red?

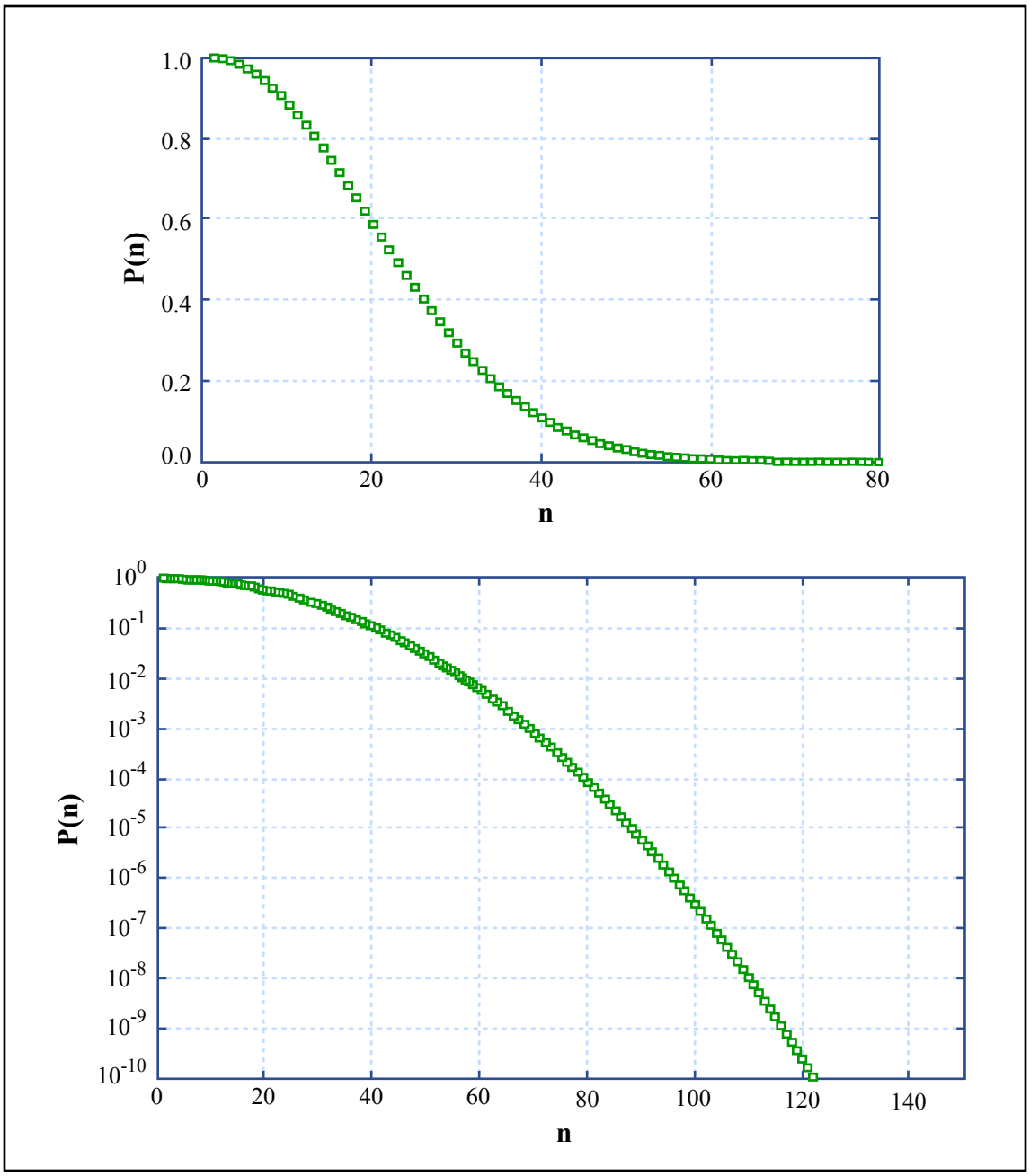
Q4 and Q5 are discussed in Rec and not in Lec hence not solvnig for time being

4. **Multinomial coefficient.** Derive the multinomial coefficient (the number of partitions of n distinct items into groups of $n_1, \ldots, n_r$) using a different argument than the one in class. Consider n items which can be placed into n slots and divide the group of n slots into segments of length $n_1, \ldots, n_r$ slots. Derive the multinomial coefficient by showing how many different ways can the n items be arranged into the r segments.

5. **Multinomial probabilities.** At each draw, there is a probability p_i $(i = 1, \ldots, r)$ of getting a ball of color i. Draw n objects. What is the probability of obtaining exactly n_i of each color i?

Recitation 4: Extra Handout
September 21, 2010

1. As part of the solution to problem 1, plotted below are the probabilities of each person having a distinct birthday versus n the number of people present.



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