NP and Computational Intractability

Polynomial time Reduction

→ Relative difficulty of Problems

"Problem X is at least as hard as Problem Y"

⇓

∃ a black box which can solve X then we could solve Y

Can arbitrary instance of Problem Y be solved using a polynomial # steps, plus a polynomial # of calls to a black box that solves the problem X?

if Yes $Y \leq_p X$

Y is polynomial time reducible to X

(X is at least as hard as Y.)

Suppose $Y \leq_p X$. If X can be solved in polynomial time, then Y can be solved in polynomial time. ✓

Suppose $Y \leq_p X$. If Y cannot be solved in polynomial time, then X cannot be solved in polynomial time.

(Y is hard $\longrightarrow$ X is hard)

(Independent Set to Vertex Cover)

Decision version

Given G and a no. k, does G contain an IS of size at least k?

Vertex Cover

Given G and a number k, does G contain a VC of size at most k?

We don't want to solve both the problems, but want to know the relative difficulty.

Ind Set $\leq_p$ Vertex Cover

Let G be a graph. Then S is an IS iff $V-S$ is a VC.

$\Rightarrow$ S is ind set $\rightarrow$ $V-S$ is VC

$\Rightarrow$ $V-S$ is VC

Explained in class