Problem Sheets [1]

1. Given $n = 2^k - 1$ elements, construct a binary min heap using Divide and Conquer in $O(n)$ time. *(Note that, in general, we can construct binary min heap in $O(n)$ time without using DnC.)*

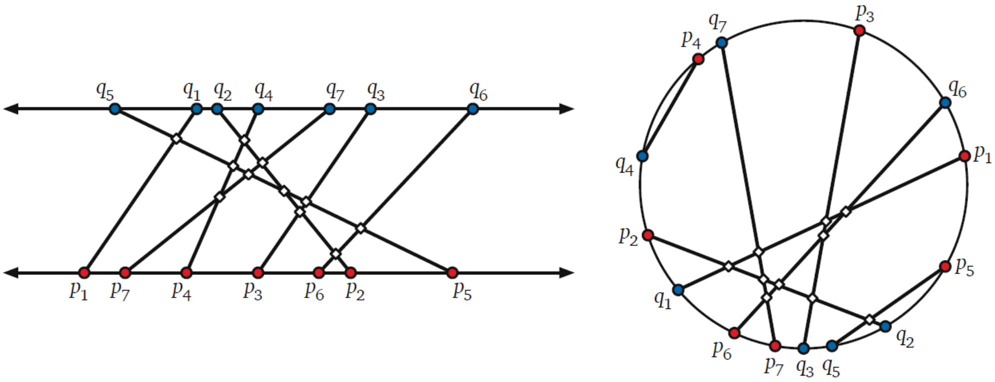
2. Given a set $P = \{p_1, p_2, \ldots p_n\}$ of n points in 2-D, and distance between two points is Manhattan distance(not the Euclidean distance). Find the closest pair of points using divide and conquer in $O(n \log n)$ time. Justify the time complexity. *(Hint: Find number of points to be checked for a point in a strip)*

Computational Geo in C expln poor than MIT lec

3. Given a set $P = \{p_1, p_2, \ldots p_n\}$ of n points, construct a convex hull using divide and conquer in in $O(n \log n)$ time. Justify the time complexity.

4. Suppose you are given two sets of n points, one set $p_1, p_2, \ldots p_n$ on the line $y = 0$ and the other set $q_1, q_2, \ldots q_n$ on the line $y = 1$. Create a set of n line segments by connect each point p_i to the corresponding point q_i. Describe and analyze a divide-and-conquer algorithm to determine how many pairs of these line segments intersect, in $O(n \log n)$ time. *(See the left image of the figure below to understand the problem).*

5. Now suppose you are given two sets $p_1, p_2, \ldots p_n$ and $q_1, q_2, \ldots q_n$ of n points on the unit circle. Connect each point p_i to the corresponding q_i. Describe and analyze a divide-and-conquer algorithm to determine how many pairs of these line segments intersect in $O(n \log^2 n)$ time. *(See the right image of the figure below to understand the problem). Hint Use sorting with respect to polar coordinate.*



6. Let S be a set of n-distinct real numbers and let $k \leq n$ be a positive integer (k may not be a constant). Design an algorithm, running in $O(n)$ time, that determines the k numbers in S that are closest to the median of S. Justify the time complexity.

7. For n distinct elements $x_1, x_2, \ldots x_n$ with positive weights $w_1, w_2, \ldots, w_n$ such that $\sum_{i=1}^{n} w_i = 1$, the weighted median is the element x_k satisfying $\sum_{x_i < x_k} w_i < 1/2$ and $\sum_{x_i > x_k} w_i \leq 1/2$. Show how to compute the weighted median of n elements in $O(n)$ time using a linear-time median algorithm discussed in the class.

8. Need more of ☕☕☕ to solve the question number 5 in $O(n \log n)$ time ? Try this after you solve all the questions. Good luck and become algo 🥷.

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