Problem Sheet [1]

1. Call a sequence $X[1..n]$ of numbers bitonic if there is an index i with $1 < i < n$, such that the prefix $X[1..i]$ is increasing and the suffix $X[i..n]$ is decreasing. Describe an efficient algorithm to compute the length of the longest bitonic subsequence of an arbitrary array A of integers.

2. Call a sequence $X[1..n]$ of numbers oscillating if $X[i] < X[i+1]$ for all even i, and $X[i] > X[i+1]$ for all odd i. Describe an efficient algorithm to compute the length of the longest oscillating subsequence of an arbitrary array A of integers.

3. Describe an efficient algorithm to compute the length of the shortest oscillating supersequence of an arbitrary array A of integers.

4. Call a sequence $X[1..n]$ of numbers weakly increasing if each element is larger than the average of the two previous elements; that is, $2X[i] > X[i-1] + X[i-2]$ for all $i > 2$. Describe an efficient algorithm to compute the length of the longest weakly increasing subsequence of an arbitrary array A of integers.

5. Call a sequence $X[1..n]$ of numbers double-increasing if $X[i] > X[i-2]$ for all $i > 2$. (In other words, a double-increasing sequence is a perfect shuffle of two increasing sequences.) Describe an efficient algorithm to compute the length of the longest double-increasing subsequence of an arbitrary array A of integers.

6. Recall that a sequence $X[1..n]$ of numbers is increasing if $X[i] < X[i+1]$ for all i. Describe an efficient algorithm to compute the length of the longest common increasing subsequence of two given arrays of integers. For example, $< 1, 4, 5, 6, 7, 9 >$ is the longest common increasing subsequence of the sequences $< 3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5, 8, 9, 7, 9, 3 >$ and $< 1, 4, 1, 4, 2, 1, 3, 5, 6, 2, 3, 7, 3, 0, 9, 5 >$.

7. For each of the following problems, the input consists of two arrays $X[1 \ldots k]$ and $Y[1 \ldots n]$ where $k \leq n$.

   a Describe and analyze an algorithm to find the smallest number of symbols that can be removed from Y so that X is no longer a subsequence. For example, after removing removing two symbols from the string PENPINEAPPLEAPPLEPEN, the string PPAP is no longer a subsequence.

   b Describe and analyze an algorithm to determine whether X occurs as two disjoint subsequences of Y. For example, the string PPAP appears as two disjoint subsequences in the string PENPINEAPPLEAPPLEPEN.

[1] Prepared by Pawan K. Mishra