# Solving recurrence

Tree method

1. $T(n) = T(n-1) + d, \quad n>1$

$= c, \quad n=1$

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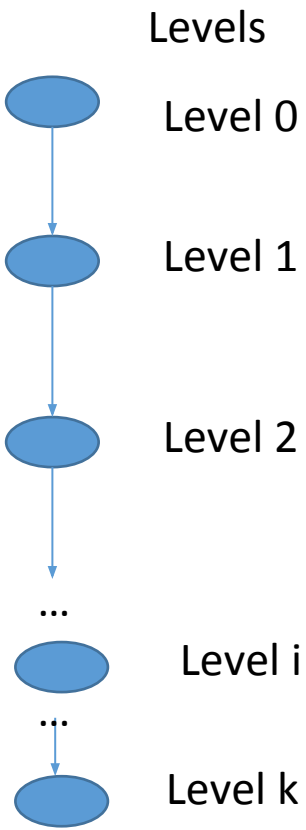

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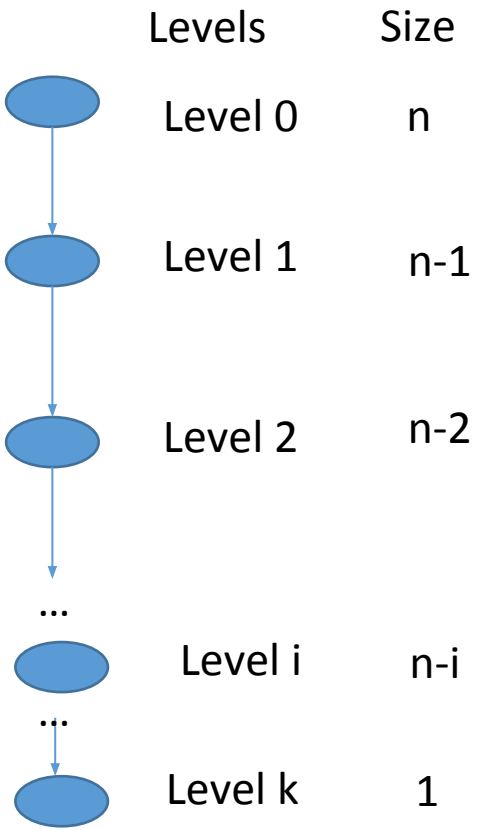

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$= c, \quad n=1$

| Levels | Size | Number of nodes |
|---|---|---|
| Level 0 | n | 1 |
| Level 1 | n-1 | 1 |
| Level 2 | n-2 | 1 |
| ... | | |
| Level i | n-i | 1 |
| ... | | |
| Level k | 1 | 1 |

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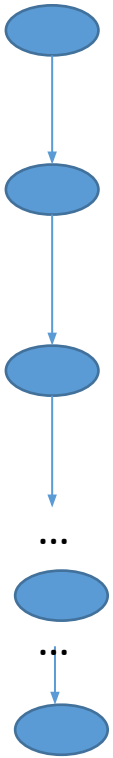

| Levels | Size | Number of nodes | Cost |
|---|---|---|---|
| Level 0 | n | 1 | d |
| Level 1 | n-1 | 1 | d |
| Level 2 | n-2 | 1 | d |
| ... | | | |
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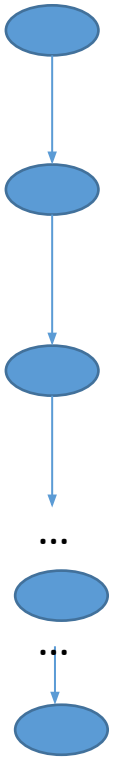
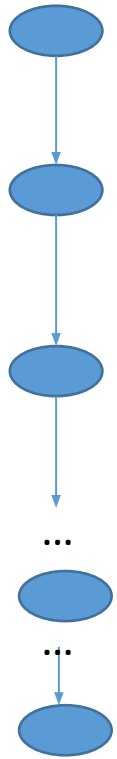


| Levels | Size | Number of nodes | Cost | Total |
|---|---|---|---|---|
| Level 0 | n | 1 | d | d |
| Level 1 | n-1 | 1 | d | d |
| Level 2 | n-2 | 1 | d | d |
| ... | | | | |
| Level i | n-i | 1 | d | d |
| ... | | | | |
| Level k | 1 | 1 | c | c |



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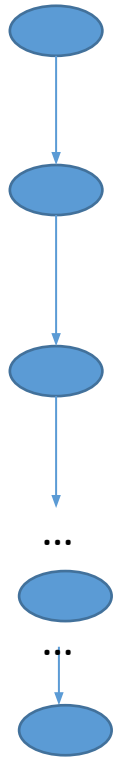

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| Level 2 | n-2 | 1 | d | d |
| ... | | | | |
| Level i | n-i | 1 | d | d |
| ... | | | | |
| Level k | 1 | 1 | c | c |

$$c + d(k) = c + (n-1)d$$
$$= dn + c - d$$
$$= O(n)$$



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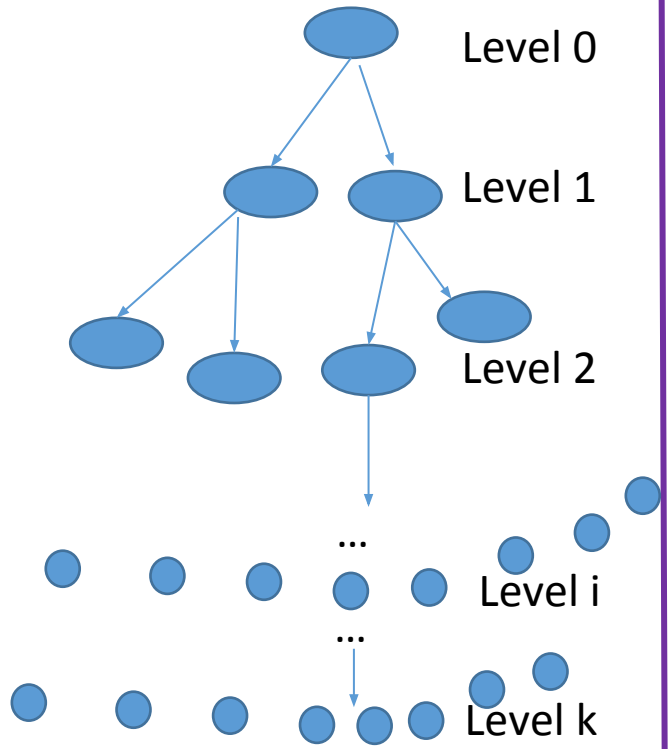
2. $T(n) = 2T(n/2) + d, \quad n>1$

$= c , \quad n=1$



| Levels | Size | Number of nodes | Cost | Total |
|---|---|---|---|---|
| Level 0 | n | 1 | d | d |
| Level 1 | n/2 | 2 | d | d |
| Level 2 | n/4 | 4 | d | d |
| ... | | | | |
| Level i | $n/2^i$ | $2^i$ | d | d |
| ... | | | | |
| Level k | $n/2^k$ | $2^k$ | c | c |





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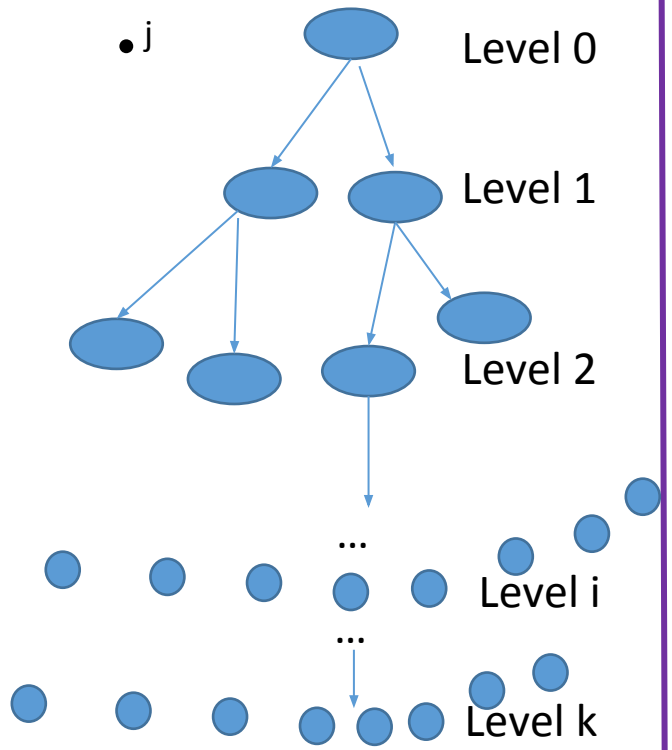


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| Level 0 | n | 1 | d | d |
| Level 1 | n/2 | 2 | d | d |
| Level 2 | n/4 | 4 | d | d |
| Level i | $n/2^i$ | $2^i$ | d | d |
| Level k | $n/2^k$ | $2^k$ | c | c |

$d+ 2d+4d+8d+\ldots+ 2^{k-1}d + 2^kc$

$= d \sum_{j=0}^{k-1} 2^j + 2^kc$

$= c2^k + d(2^k - 1)$

$=2^k(c+d)-d$

$=2^{\log n}(c+d)-d$

$=n(c+d)-d$

$<= n(c+d)=n*\text{Constant}=O(n)$



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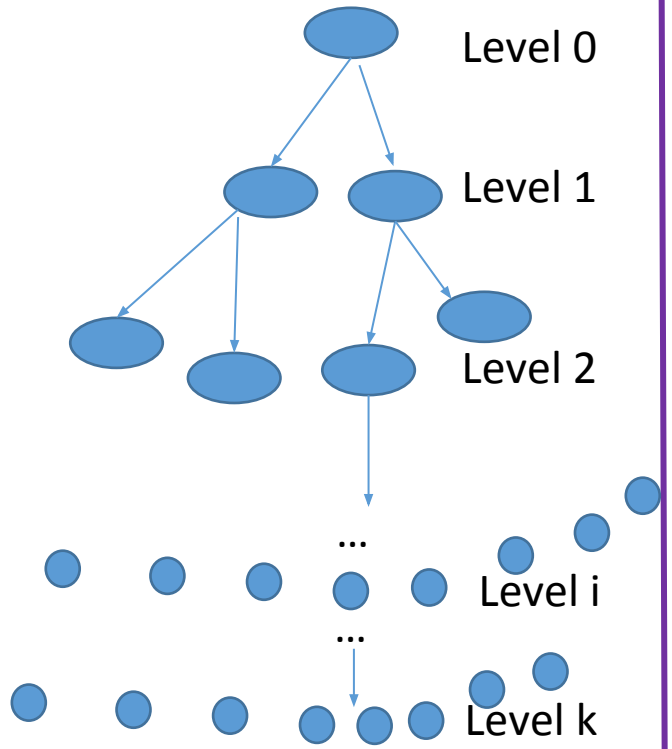
$$3.\ T(n) = 2T(n/2) + n, \quad n>1$$
$$= c, \qquad n=1$$



| Levels | Size | Number of nodes | Cost | Total |
|---|---|---|---|---|
| Level 0 | n | 1 | n | n |
| Level 1 | n/2 | 2 | n/2 | n |
| Level 2 | n/4 | 4 | n/4 | n |
| ... | | | | |
| Level i | $n/2^i$ | $2^i$ | $n/2^i$ | n |
| ... | | | | |
| Level k | $n/2^k$ | $2^k$ | c | c |





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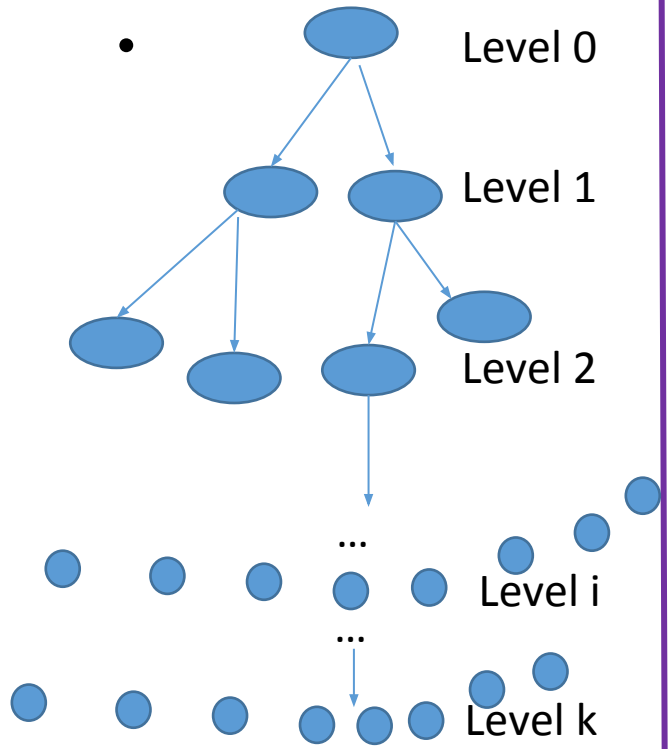


| Levels | Size | Number of nodes | Cost | Total |
|---|---|---|---|---|
| Level 0 | n | 1 | n | n |
| Level 1 | n/2 | 2 | n/2 | n |
| Level 2 | n/4 | 4 | n/4 | n |
| Level i | $n/2^i$ | $2^i$ | $n/2^i$ | n |
| Level k | $n/2^k$ | $2^k$ | c | c |

$n+ n+n+n+...+ n + c2^k$

$= \sum_{j=0}^{k-1} n + c2^k$

$= c2^k + n(k)$

$= 2^{\log n} c + n \log n$

$= nc + n\log n = O(n\log n)$

# Home Work

4. $T(n) = T(n/3) + 5n, \quad n>1$
   $= 6, \qquad n=1$


5. $T(n) = 4T(n/3) + 6n^2, \quad n>1$
   $= 11, \qquad n=1$