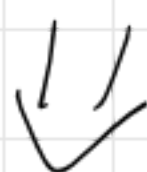


NP and Computational Intractability

Polynomial time Reduction

→ Relative difficulty of Problems

" Problem X is at least as hard as Problem Y "



\exists a black box which can solve X
then we could solve Y

Can arbitrary instance of Problem Y be solved using a polynomial # steps, plus a polynomial # of calls to a black box that solves the problem X ?

if yes

$$Y \leq_p X$$

Y is polynomial time reducible to X .

(X is at least as hard as Y .)

Suppose $Y \leq_p X$. If X can be solved in polynomial time, then Y can be solved in polynomial time. \checkmark

Suppose $Y \leq_p X$. If Y cannot be solved in polynomial time, then X cannot be solved in polynomial time.

(Y is hard $\rightarrow X$ is hard)

(Independent Set to Vertex Cover)

Decision version

Given G and a no. k , does G contain an IS of size at least k ?

Vertex Cover

Given G and a number k , does G contain a VC of size at most k ?

we don't want to solve both the problems, but want to know the relative difficulty.

Ind Set \leq_p Vertex Cover

Let G be a graph. Then S is an IS
iff $V-S$ is a VC.

\Rightarrow S is ind set $\rightarrow V-S$ is VC

$\Rightarrow V-S$ is VC

Explained in class