5 $F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)}$

(a) $F^*(u,v) = \left(\sum_{0}^{M-1} \sum_{0}^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)} \right)^*$

$= \sum_{0}^{M-1} \sum_{0}^{N-1} f^*(x,y)\, e^{j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)}$

$\Rightarrow$ as $f(x,y)$ is real $\therefore$ $f^*(x,y) = f(x,y)$

$= \sum_{0}^{M-1} \sum_{0}^{N-1} f(x,y)\, e^{-j2\pi\left(-\frac{ux}{M} - \frac{vy}{N}\right)}$

$= \sum_{0}^{M-1} \sum_{0}^{N-1} f(x,y)\, e^{-j2\pi\left(+\frac{(-u)x}{M} + \frac{(-v)y}{N}\right)}$

$= F(-u,-v)$

(b) If $f(x,y)$ is real & even.

$\therefore$ $f(x,y) = f(-x,-y)$

Taking DFT both side

$\sum_{0}^{M-1} \sum_{0}^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)} = \sum_{0}^{M-1} \sum_{0}^{N-1} f(-x,-y)\, e^{-j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)}$

$\Rightarrow F(u,v) = \sum_{0}^{M-1} \sum_{0}^{N-1} f(-x,-y)\, e^{-j2\pi\left(\frac{(-u)(-x)}{M} + \frac{(-v)(-y)}{N}\right)}$

$\Rightarrow F(u,v) = F(-u,-v)$

$\therefore F(u,v) =$ even

$\Rightarrow$ from part (a) & $f(x,y)$ is real

$F^*(u,v) = F(-u,-v)$

$F^*(u,v) = F(u,v)$

$\therefore F(u,v) =$ real.

$\Rightarrow F(u,v) =$ real & even if $f(x,y)$ is real & even