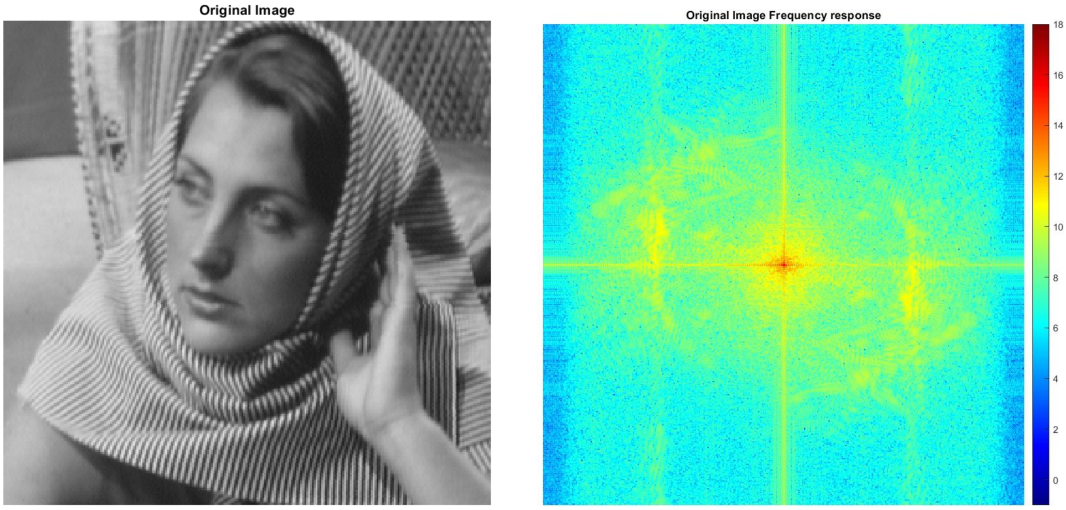
# DIP
# Assignment 3
# Question 1 to 7
# Report

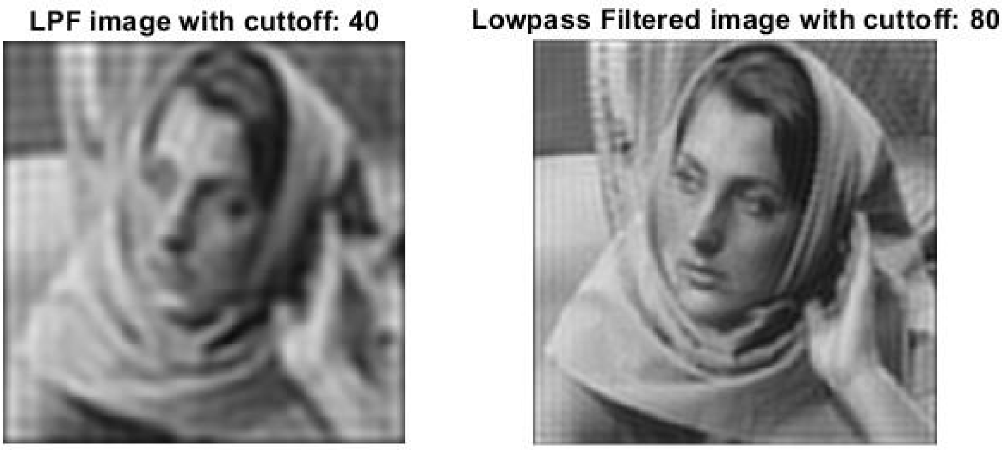
Ques 1
sol
As we increase the hs and hr the images become more smoother.

Question 2

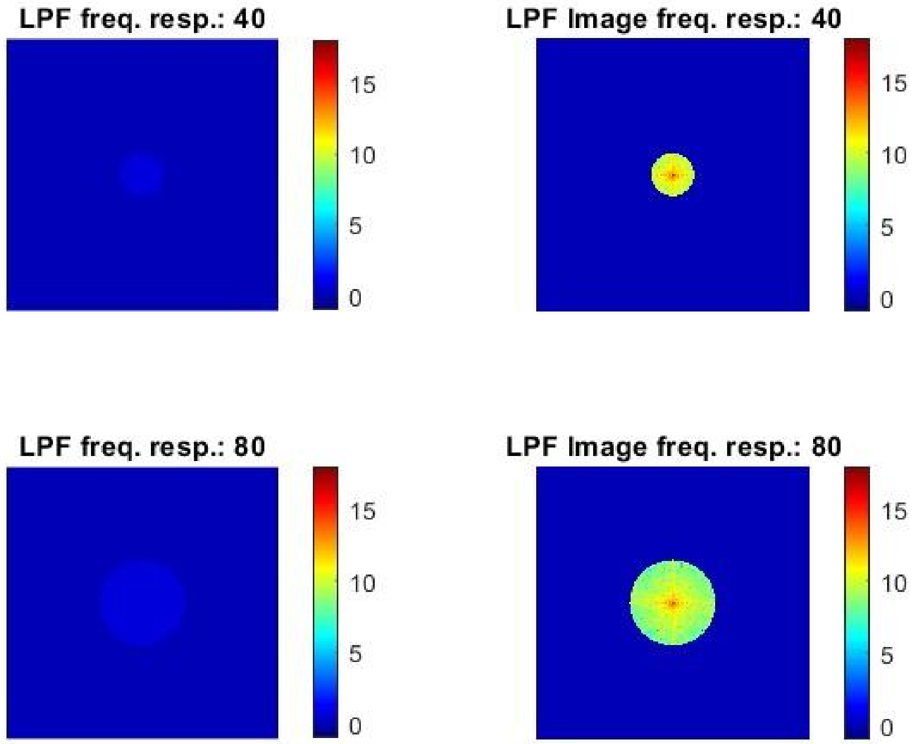
## Original Image and Its Log Absolute Frequency Response



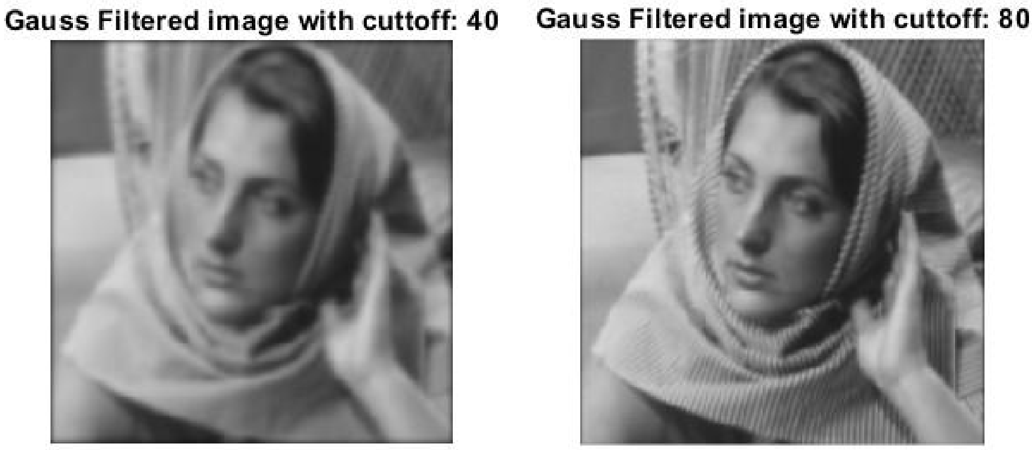

## Low pass Filtered Images



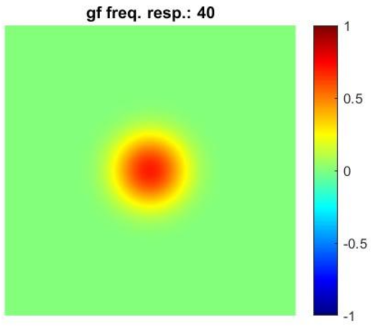

## Low Pass filter and Low pass filtered image Log absolute Frequency Response

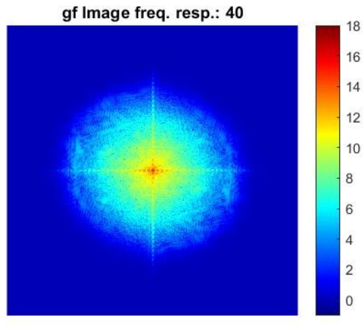


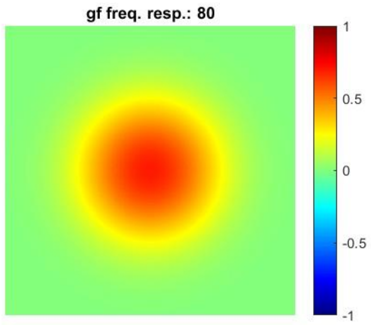

## Gaussian Filtered Images

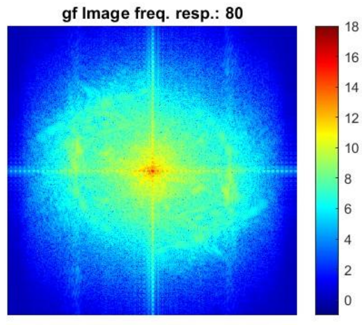



## Gaussian Filter and Gaussain Filtered Image Log absolute Frequency Response




## Observation

1. In low pass filter image horizontal and vertical blur black lines start to appear while blur is not so smooth because all frequencies become zero after cutoff frequency, but in case of gaussian filter there is very smooth blurring because frequencies start to decrease exponentially after cutoff frequency.

3

$$y(x,y) = f(x,y) * h(x,y)$$

$$\Rightarrow y(x,y) = \sum_p \sum_q f(p,q)\, h(x-p, y-q)$$

$\Rightarrow$ we know DFT of $y(x,y)$

$$Y(u,v) = \sum_x \sum_y y(x,y)\, e^{-j2\pi(ux/M + vy/N)}$$

$\Rightarrow$ Putting value of $y(x,y)$

$$Y(u,v) = \sum_x \sum_y \left[ \sum_p \sum_q f(p,q)\, h((x-p),(y-q)) \right] \times$$

$$e^{-j2\pi(ux/M + vy/N)} \times e^{-j2\pi(up/M + vq/N)} \times$$

$$e^{+j2\pi(up/M + vq/N)}$$

$$\Rightarrow Y(u,v) = \sum_x \sum_y \sum_p \sum_q f(p,q)\, e^{-j2\pi(up/M + vq/N)}$$

$$\times\, h(x-p, y-q)\, e^{-j2\pi\left(\frac{u(x-p)}{M} + \frac{v(y-q)}{N}\right)}$$

$$= F(u,v) \times \sum \sum h(x-p, y-q)\, e^{-j2\pi\left[\frac{u(x-p)}{M} + \frac{v(y-q)}{N}\right]}$$

$$\Rightarrow Y(u,v) = F(u,v)\, H(u,v)$$

Question 4.

Analytical Proof

4

$$f(x,y) = \begin{cases} 0 & x \neq 101 \\ 255 & x = 101 \end{cases}$$

$$\therefore f(x,y) = 255\,\delta(x-101)$$

$$F(u,v) = \iint f(x,y)\, e^{-j2\pi(ux+vy)}\, dx\, dy$$

$$= \iint 255\,\delta(x-101)\, e^{-j2\pi(ux+vy)}\, dx\, dy$$

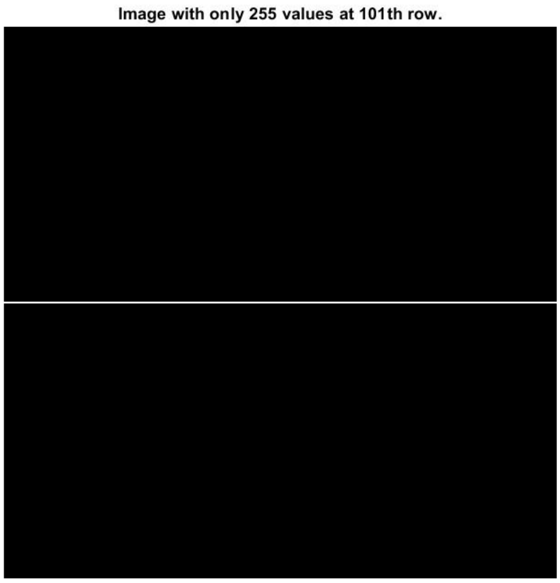
$$= \int 255\,\delta(x-101)\, e^{-2\pi j u x}\, dx \int e^{-j2\pi v y}\, dy$$

$$= 255\, e^{-j2\pi \times 101 u} \times \delta(v)$$

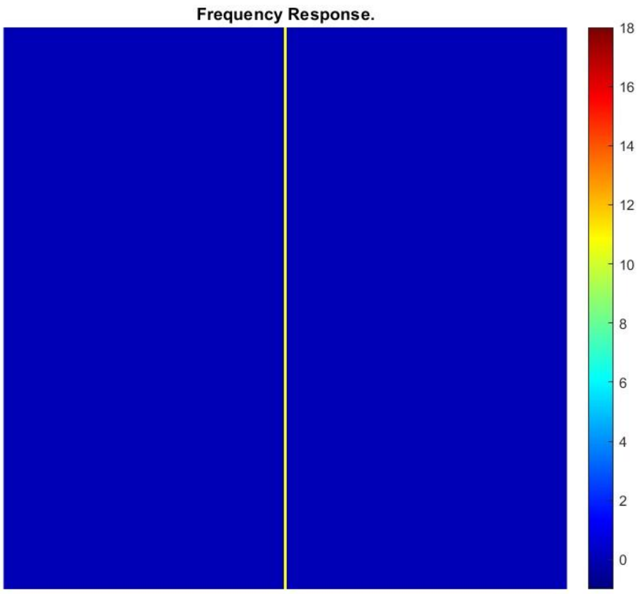
$$= 255\, e^{-j202\pi u}\, \delta(v)$$

Result from Matlab

1. Image




2. Frequency response



5 $F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)}$

(a) $F^*(u,v) = \left( \sum_0^{M-1} \sum_0^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)} \right)^*$

$= \sum_0^{M-1} \sum_0^{N-1} f^*(x,y)\, e^{j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)}$

$\Rightarrow$ as $f(x,y)$ is real $\therefore$ $f^*(x,y) = f(x,y)$

$= \sum_0^{M-1} \sum_0^{N-1} f(x,y)\, e^{-j2\pi\left(-\frac{ux}{M} - \frac{vy}{N}\right)}$

$= \sum_0^{M-1} \sum_0^{N-1} f(x,y)\, e^{-j2\pi\left(+\frac{(-u)x}{M} + \frac{(-v)y}{N}\right)}$

$= F(-u,-v)$

(b) If $f(x,y)$ is real & even.

$\therefore$ $f(x,y) = f(-x,-y)$

Taking DFT both side

$\sum_0^{M-1} \sum_0^{N-1} f(x,y)\, e^{-j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)} = \sum_0^{M-1} \sum_0^{N-1} f(-x,-y)\, e^{-j2\pi\left(\frac{ux}{M} + \frac{vy}{N}\right)}$

$\Rightarrow F(u,v) = \sum_{0}^{M-1}\sum_{0}^{N-1} f(-x,-y)\, e^{-j2\pi\left(\frac{(-u)(-x)}{M} + \frac{(-v)(-y)}{N}\right)}$

$\Rightarrow F(u,v) = F(-u,-v)$

$\therefore F(u,v) =$ even

$\Rightarrow$ from part (a) & $f(x,y)$ is real

$F^*(u,v) = F(-u,-v)$

$F^*(u,v) = F(u,v)$

$\therefore F(u,v) =$ real.

$\Rightarrow F(u,v) =$ real & even if $f(x,y)$ is real & even

6

$$\mathcal{F}(g(t)) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi ft} dt = F(f)$$

$$g(t) = \int_{-\infty}^{\infty} F(f) e^{+j2\pi ft} df.$$

$$g(-t) = \int_{-\infty}^{\infty} F(f) e^{-j2\pi ft} df.$$

$$g(-t) = \mathcal{F}(F(f)) = \mathcal{F}\mathcal{F}(g(t))$$

similarly $\mathcal{F}\mathcal{F}(g(-t)) = g(t)$

$\therefore \mathcal{F}\mathcal{F}(\mathcal{F}\mathcal{F}(g(t)) = \mathcal{F}\mathcal{F}(g(-t)) = g(t)$

Hence proved

7. All filters are high pass.

∴ Transfer function of Butterworth high pass filter

$\Rightarrow$ $H_{HP}(u,v) = \dfrac{1}{1+\left(D_0/D(u,v)\right)^{2n}}$

$\Rightarrow$ $D_0$ = cut off frequency

$\Rightarrow$ Spatial kernel, $h_{HP} = \mathcal{F}^{-1}(H_{HP}(u,v))$

$= \mathcal{F}^{-1}(1 - H_{LP}(u,v))$

$= \delta(x,y) - h_{LP}(x,y)$

$\Rightarrow$ due to presence of delta function there is spike at center.