EE774 - Lab 6

For all the problems below, please adhere to the following:

- Put each C++ source file in a separate directory, such as `prob1`, `prob2` etc.
- Add a `Makefile` in each directory to build the source code
- Try to accept the file input as a command line argument. Examples have been provided in class.
- Submit the assignment as a single `zip` or `tar.gz` file with all the subdirectories.

1. (10 points) In this question, you will use overdetermined fitting to fit to a non-linear model. The model you will fit to is

$$y(x) = a + bx + cexp(-x)$$

Unfortunately, you are given noisy samples. There are three files that fit to the same model, viz. `data_1.txt, data_2.txt, data_3.txt`. Perform the least squares fit to the model and plot them on the same plot. What is your prediction on the noise variance in each case? (Hint: just find the sum squared error and make a prediction). The data in the files has the form (x_i, y_i), where x_i and y_i are provided in two columns.

2. (10 points) One interesting application of underdetermined least squares is to perform deconvolution. Consider the discrete time sequence $h[n] = \delta[n] + \delta[n-1] - \delta[n-2] - \delta[n-3]$, alternately represented as $\{1, 1, -1, -1\}$. The convolution relationship is given by:

$$y[n] = x[n] + x[n-1] - x[n-2] - x[n-3]$$

In all the following questions, we find the "least energy" causal sequence, i.e. we find the values $x[0], x[1], ...$ with the minimum value of $\sum_{k=0}^{\infty} |x[k]|^2$ that satisfy the required property.

   - Find the minimum energy causal sequence $x[n]$ that yields the precise output $y[n] = \{1, 2, -1, -4, -1, 2, 1\}$. Note that the length of this sequence indicates that there should be a unique solution.
   - Find the minimum energy causal sequence $x[n]$ that gives you $y[2] = -1$ when we **don't care** about the values of $y[n]$ for $n \neq 2$. This is definitely underdetermined.

- Now, we wish to impose the following simultaneous constraints: $y[2] = -1, y[3] = -4, y[4] = -1$. Find the minimum energy causal $x[n]$ that achives this.

- Finally, we wish to obtain the least energy causal $x[n]$ that yields $y[4] \neq 0$ and $y[0] = y[1] = y[2] = y[3] = y[5] = y[6] = 0$. What is the complete convolution of this $x[n]$ with $h[n]$ for all n? Did this achieve your goal? Why or why not?