Mouvement d'une particule d'un champ $\vec{E}$ et $\vec{B}$

Données
- particule chargée q
- $\vec{E} = E\,\vec{e_x}$
- $\vec{B} = B\,\vec{e_z}$
- $\vec{v_0} = \vec{0}$

① Mvt de la particule dans Ⅰ et Ⅱ.

② Calculer le temps que passe la particule dans Ⅰ

③ Mq la particule fait un cercle dans Ⅱ

Résolution

① : On a
$$x(t) = x\,\vec{e_x} + y\,\vec{e_y} + z\,\vec{e_z}$$
$$\dot{x}(t) = \dot{x}\,\vec{e_x} + \dot{y}\,\vec{e_y} + \dot{z}\,\vec{e_z}$$
$$\ddot{x}(t) = \ddot{x}\,\vec{e_x} + \ddot{y}\,\vec{e_y} + \ddot{z}\,\vec{e_z}$$

On a aussi un champ $\vec{E} = E\,\vec{e_x}$

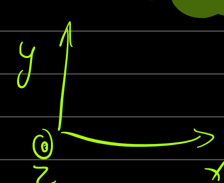
Par Newton : $\vec{F} = m\ddot{x}$ $\Rightarrow$
$$\begin{array}{l} \vec{e_x} \\ \vec{e_y} \\ \vec{e_z} \end{array} \begin{cases} qE = m\ddot{x} \\ 0 = m\ddot{y} \\ 0 = m\ddot{z} \end{cases}$$

$$\begin{cases} \dot{x}(t) = \frac{qE}{m}\cdot t + \overbrace{C_1}^{=0} \\ \dot{y}(t) = C_2 = 0 \\ \dot{z}(t) = C_3 = 0 \end{cases} \Rightarrow \begin{cases} x(t) = \frac{qE}{2m}\cdot t^2 \\ y(t) = 0 \\ z(t) = 0 \end{cases}$$

Donc la particule atteint la région Ⅱ en $x = l \Rightarrow x(t_l) = \frac{qE}{2m}\cdot t_l^2 = l$

$$\Rightarrow t_l = \sqrt{\frac{2ml}{qE}}$$

Ⅱ



On a $\vec{F_B} = q\vec{v} \wedge \vec{B} = Bq\,\vec{v} \wedge \vec{e_z}$

$= qB\cdot\left((\dot{x}\,\vec{e_x} + \dot{y}\,\vec{e_y} + \dot{z}\,\vec{e_z}) \wedge \vec{e_z}\right)$

$= qB\,(\dot{x}\,\vec{e_z} - \dot{z}\,\vec{e_x})$

Donc, par Newton : $\vec{F_B} = m\ddot{x}$

$$\begin{array}{l} \vec{e_x} \\ \vec{e_y} \\ \vec{e_z} \end{array} \begin{cases} -qB\dot{z} = m\ddot{x} & (a) \\ 0 = m\ddot{y} & \rightarrow \dot{y} = c \rightarrow y(t) = 0 \\ qB\dot{x} = m\ddot{z} & (b) \end{cases}$$

On trouve :

(a) $\ddot{x} = -\frac{qB\dot{z}}{m}$

$\dot{x} = -\frac{qB}{m} z + C_1 \quad \rightarrow z = 0$ et $x = l \Rightarrow \dot{x} = \frac{qE}{m} \cdot t_l$

$= \frac{qE}{m} \cdot \sqrt{\frac{2ml}{qE}}$

$= \sqrt{\frac{2qE \cdot l}{m}}$

Donc $C_1 = \sqrt{\frac{2qEl}{m}}$

et on trouve $\dot{x} = -\frac{qB}{m} \cdot z + \sqrt{\frac{2qEl}{m}}$ (1)

(b) $\ddot{z} = \frac{qB}{m}\dot{x}$

$\dot{z} = \frac{qB}{m} x + C_2 \quad \rightarrow x = l$ et $z = 0 \rightarrow \dot{z} = 0 \Rightarrow C_2 = -\frac{qBl}{m}$

et on trouve $\dot{z} = \frac{qB}{m} x - \frac{qBl}{m}$ (2)

Voyons $\ddot{x} \cdot \dot{x} + \ddot{z} \cdot \dot{z} = -\frac{qB\dot{z}}{m} \cdot \dot{x} + \frac{qB\dot{x}}{m} \cdot \dot{z}$

$\frac{q}{2} \cdot (\ddot{x} \cdot \dot{x} + \ddot{z} \cdot \dot{z}) = 0$

$\frac{d}{dt}(\dot{x}^2 + \dot{z}^2) = 0$

$\rightarrow \dot{x}^2 + \dot{z}^2 = K_1$

On a $\dot{x} = \sqrt{\frac{2qEl}{m}}$ et $\dot{z} = 0 \rightarrow K_1 = \frac{2qEl}{m}$

On a finalement avec les valeurs de (1) et (2)

$$\left(-\frac{qB}{m} \cdot z + \sqrt{\frac{2qEl}{m}}\right)^2 + \left(\frac{qB}{m} x - \frac{qBl}{m}\right)^2 = \frac{2qEl}{m}$$

Trouvons $x(t)$ et $z(t)$ par le formalisme complexe

$Z = x + iz$

$\dot{Z} = \dot{x} + i\dot{z}$

$\ddot{z} = \ddot{x} + i\ddot{z}$

$\rightarrow$ On trouve $\dot{z} = -\frac{qB}{m} z + i\frac{qB}{m}\dot{x}$

$= i\frac{qB}{m}(\dot{x} + i\dot{z}) = i\frac{qB}{m}\dot{z}$, avec $\underbrace{\dot{x} + i\dot{z}}_{=\dot{z}}$

Donc $\ddot{z} - i\frac{qB}{m}\dot{z} = 0$

$p^2 - i\frac{qB}{m}p = 0 \rightarrow p_{1,2} = 0, \ i\frac{qB}{m}$

$e^{ix} = \underbrace{\cos(x)}_{\Re e^{ix}} + i\underbrace{\sin(x)}_{I e^{ix}}$

$C e^{i\frac{qB}{m}.t} + D = z(t)$

$C + D = z(t_0) = l$

$|\dot{z}| = i\frac{qB}{m}.C = \sqrt{\frac{2lqE}{m}} \Rightarrow C = -i\sqrt{\frac{2lmE}{qB^2}}$

$\Rightarrow x^2 + z^2 = C^2 + D^2$ en $t=0$

$\rightarrow D = l - C = l + i\sqrt{\frac{2lmE}{qB^2}}$

Donc $z(t) = -i\sqrt{\frac{2lmE}{qB^2}}.e^{i\frac{qB}{m}.t} + l + i\sqrt{\frac{2lmE}{qB^2}}$

$z(t) = l + i\sqrt{\frac{2lmE}{qB^2}}.\left(1 - e^{i\frac{qB}{m}.t}\right)$

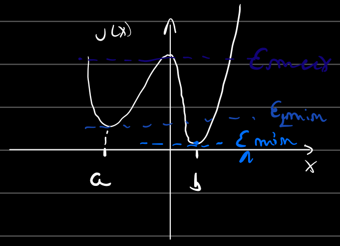
$z(t) = l + i\sqrt{\frac{2lmE}{qB^2}}.\left(1 - \cos\left(\frac{qB}{m}t\right) - i\sin\left(\frac{qB}{m}t\right)\right)$

- $x(t) = l + \sin\left(\frac{qB}{m}.t\right)\sqrt{\frac{2lmE}{qB^2}}$

$z(t) = \sqrt{\frac{2lmE}{qB^2}}.\left(1 - \cos\left(\frac{qB}{m}.t\right)\right)$

Description portrait de phase

→ potentiel bizarre :



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