On Compositional safety verification with Max-SMT

Fabian Böller with David Korzeniewski

RWTH Aachen

fabian.boeller@rwth-aachen.de

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Overview

Terms

Safety verification

Prove that an assertion is *always* true at a location

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Non-compositional safety verification

Safety verification where the whole program is analyzed in one step

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Prove that an assertion is *always* true at a location

Non-compositional safety verification

Safety verification where the whole program is analyzed in one step

Compositional safety verification

Safety verification where program parts are analyzed semi-independently and composed

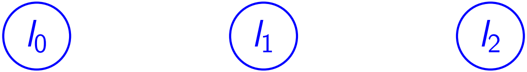
Motivation

Scalability $\longleftrightarrow$ Loss in precision

Programs

Program

$\mathcal{L} = \{\ell_0, \ell_1, \ell_2\}$



Program

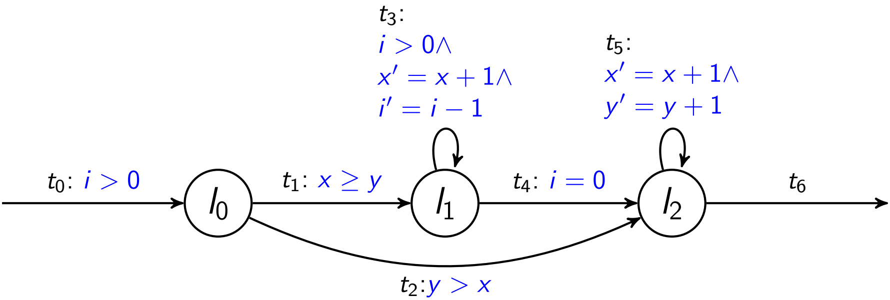
$\mathcal{L} = \{\ell_0, \ell_1, \ell_2\}$, $\mathcal{T} = \{t_i \mid i \in \{1, \ldots, 6\}\}$

$$
\begin{tikzcd}
{} \arrow[r, "t_0"] & l_0 \arrow[r, "t_1"] \arrow[rr, bend right, "t_2"'] & l_1 \arrow[loop above, "t_3"] \arrow[r, "t_4"] & l_2 \arrow[loop above, "t_5"] \arrow[r, "t_6"] & {}
\end{tikzcd}
$$

Programs

Program

$\mathcal{L} = \{\ell_0, \ell_1, \ell_2\}$, $\mathcal{T} = \{t_i \mid i \in \{1, \dots, 6\}\}$, $\mathcal{V} = \{x, y, i\}$, $\mathcal{V}' = \{x', y', i'\}$



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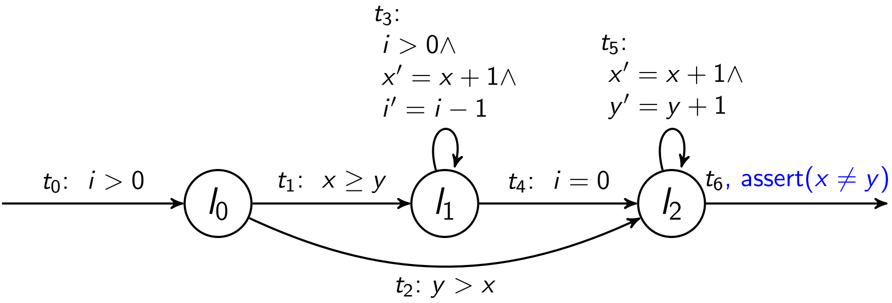

Programs

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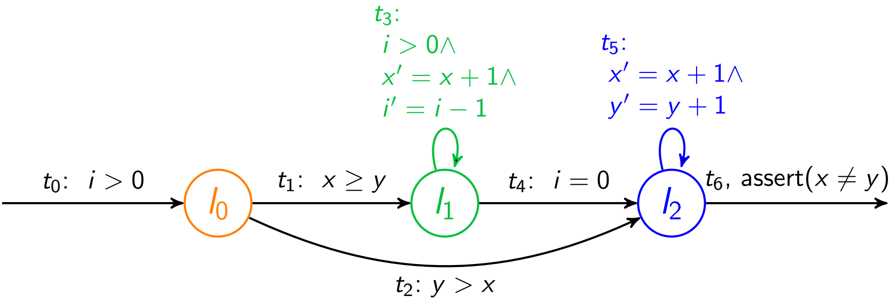

Algorithms

$$
\begin{tikzcd}
 & \text{CheckSafe} \arrow[loop above] \arrow[dl] \arrow[dr] & \\
\text{CondSafe} & & \text{Narrowing}
\end{tikzcd}
$$

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CheckSafe

Prove that an assertion is satisfied by backtracking through all entry components

Algorithms

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CheckSafe

Prove that an assertion is satisfied by backtracking through all entry components

CondSafe

Find a precondition for a component such that all runs satisfying the precondition always imply the postcondition

Algorithms

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CheckSafe

Prove that an assertion is satisfied by backtracking through all entry components

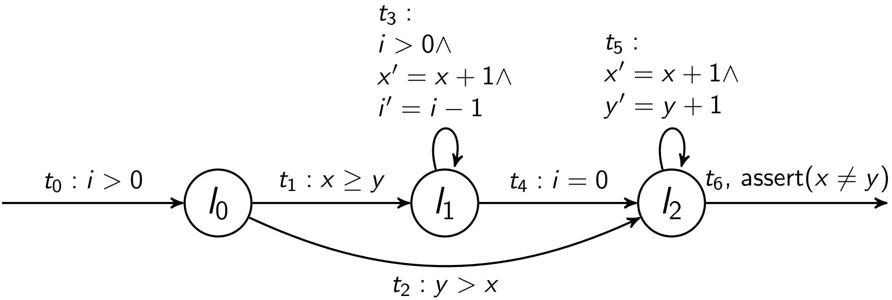
CondSafe

Find a precondition for a component such that all runs satisfying the precondition always imply the postcondition

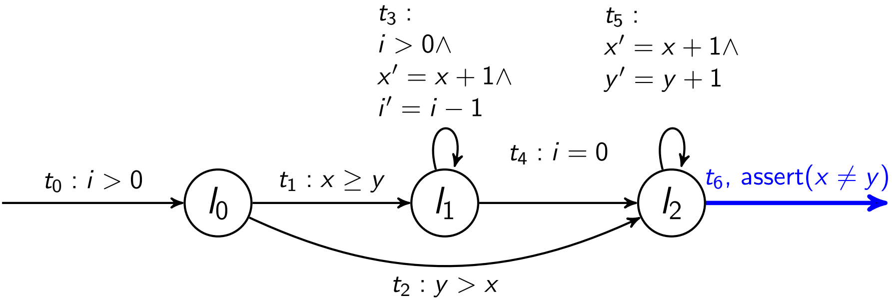
Narrowing

Manipulate the program such that new preconditions can be found

Example program



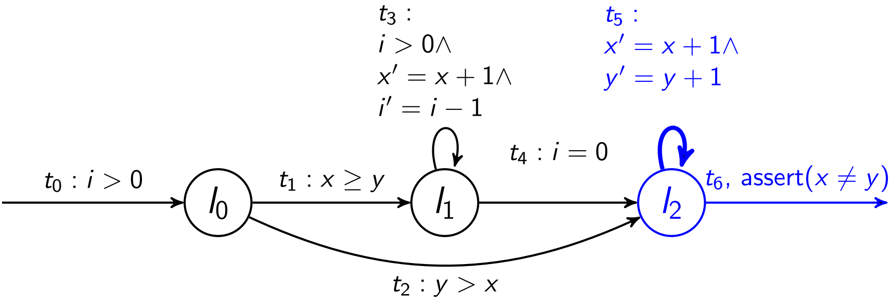
Example execution



Task

Prove that the program is safe for $x \neq y$ at t_6

Example execution



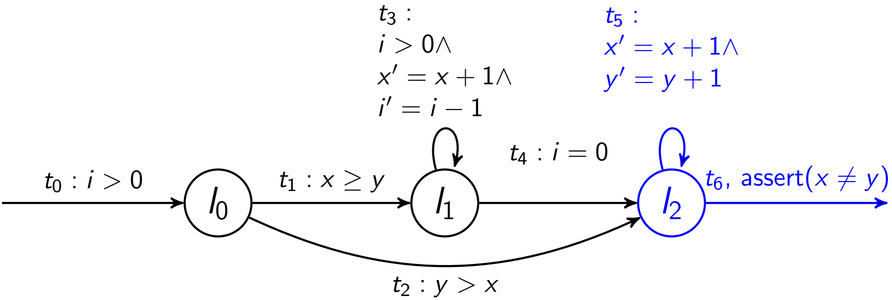
CheckSafe on $\{\ell_2\}$ for $x \neq y$

t_6 does not already imply $x \neq y$

t_6 is not an initial transition

Call CondSafe

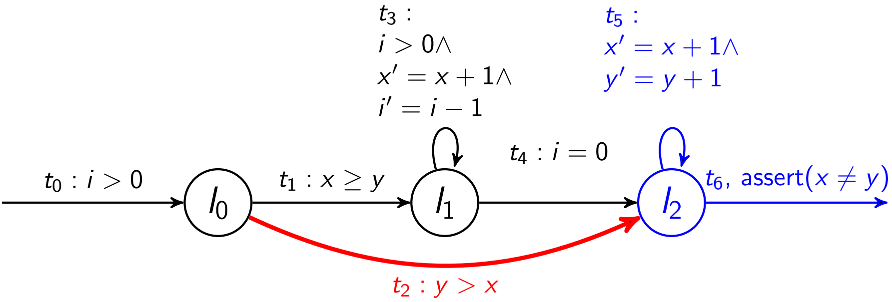
Example execution



Max-SMT on $\{\ell_2\}$ for $x \neq y$

$$I_{\ell_2,1}(\{x, y, i\}) \equiv i_{\ell_2,1} + i_{\ell_2,1,x} * x + i_{\ell_2,1,y} * y + i_{\ell_2,1,i} * i \leq 0$$

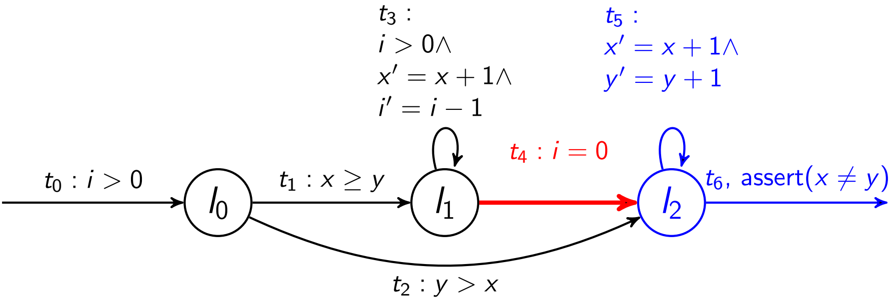
Example execution



Max-SMT on $\{\ell_2\}$ for $x \neq y$

$\mathbb{I}_{t_2,1,1} \equiv y > x \wedge i' = i \wedge x' = x \wedge y' = y \Rightarrow I'_{\ell_2,1,1}$

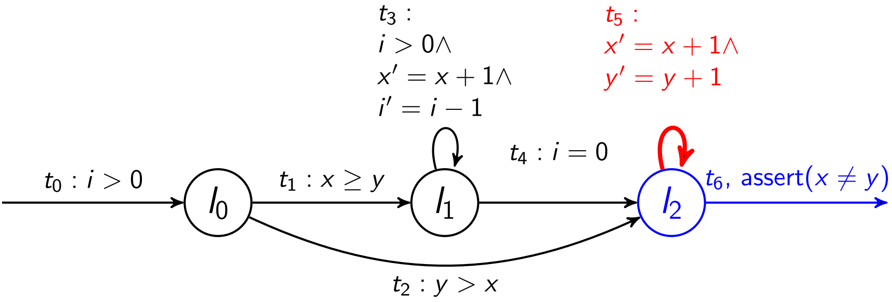
Example execution



Max-SMT on $\{\ell_2\}$ for $x \neq y$

$$\mathbb{I}_{t_4,1,1} \equiv i = 0 \wedge i' = i \wedge x' = x \wedge y' = y \Rightarrow I'_{\ell_2,1,1}$$

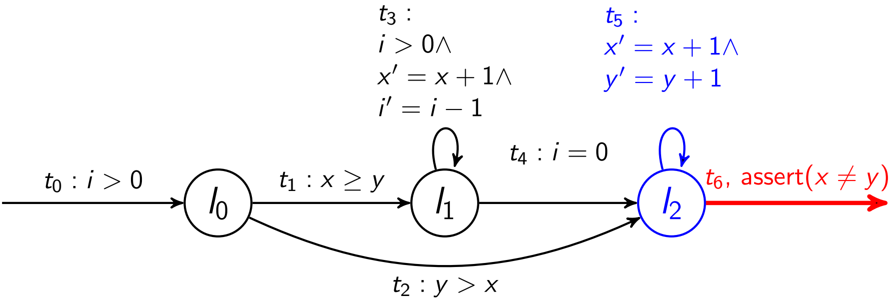
Example execution



Max-SMT on $\{\ell_2\}$ for $x \neq y$

$\mathbb{C}_{t_5,1} \equiv I_{\ell_2,1} \wedge \; x' = x + 1 \wedge y' = y + 1 \wedge i' = i \Rightarrow I'_{\ell_2,1}$

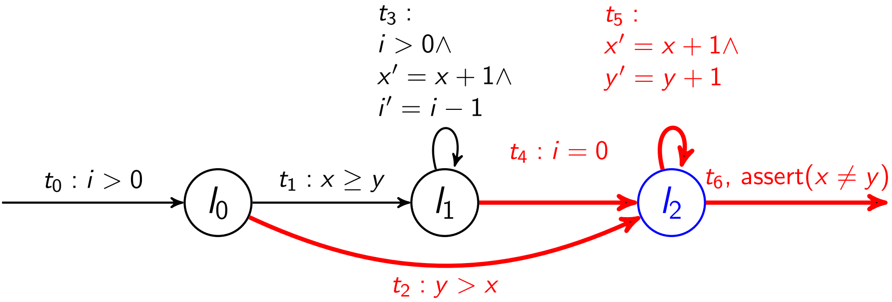
Example execution



Max-SMT on $\{\ell_2\}$ for $x \neq y$

$\mathbb{S}_1 \equiv I_{\ell_2,1} \wedge\ i' = i \wedge x' = x \wedge y' = y \Rightarrow x' \neq y'$

Example execution

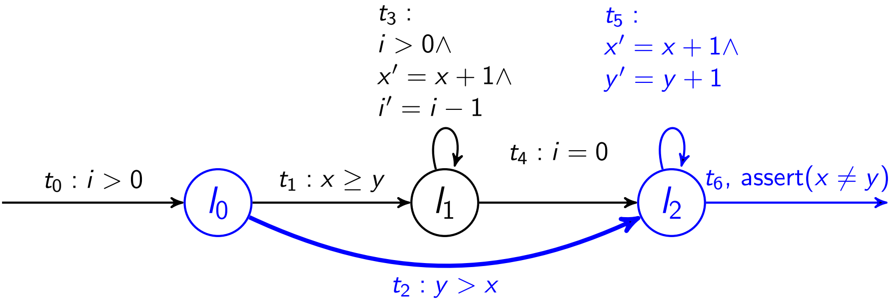


Max-SMT on $\{\ell_2\}$ for $x \neq y$

$\mathbb{F}_1 \equiv \mathbb{C}_{t_5,1} \wedge \mathbb{S}_1 \wedge (\mathbb{I}_{t_2,1,1} \vee \neg p_{t_2}) \wedge (\mathbb{I}_{t_4,1,1} \vee \neg p_{t_4}) \wedge [p_{t_2}, 1] \wedge [p_{t_4}, 1]$

Assume $x > y$ does satisfy $\mathbb{F}_1$

Example execution



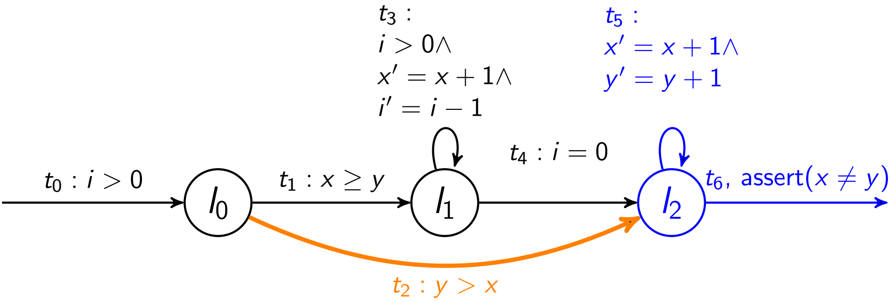
CheckSafe on $\{\ell_0\}$ for $x > y$

t_2 does not already imply $x > y$

t_2 is not an initial transition

Call CondSafe

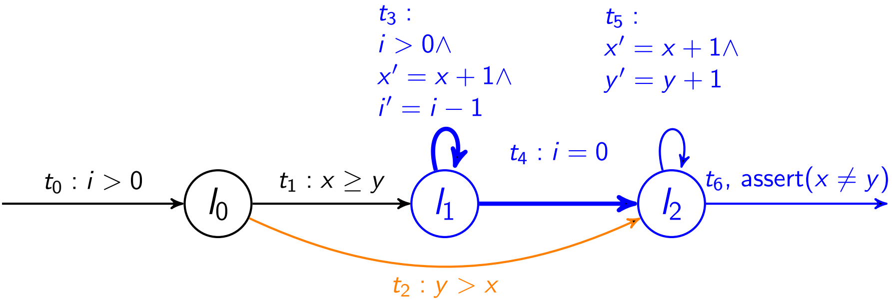
Example execution



CheckSafe on $\{\ell_0\}$ for $x > y$

No precondition, since $y > x$ contradicts $x > y$
Path is maybe safe, but not for $x > y$

Example execution



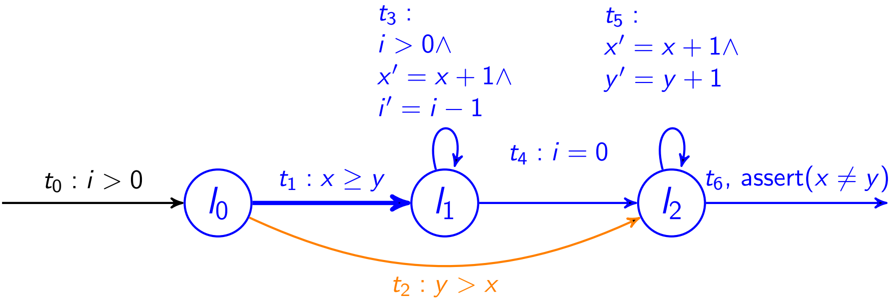
CheckSafe on $\{\ell_1\}$ for $x > y$

t_4 does not already imply $x > y$

t_4 is not an initial transition

Call CondSafe, get $i > 0 \wedge x \geq y$ as precondition

Example execution



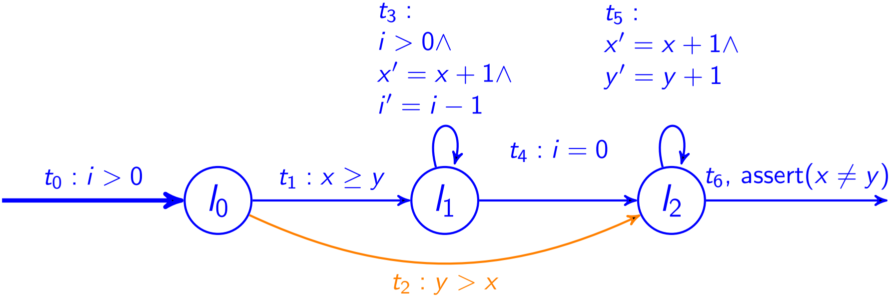
CheckSafe on $\{\ell_0\}$ for $i > 0$

t_1 does not already imply $i > 0$

t_1 is not an initial transition

Call CondSafe, get $i > 0$ as precondition

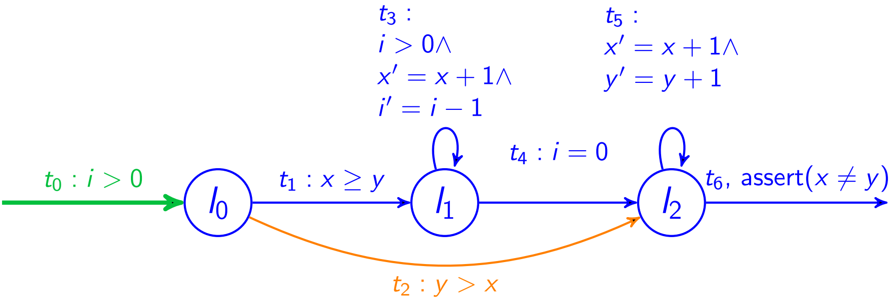
Example execution



CheckSafe on initial SCC for $i > 0$

t_0 does already imply $i > 0$

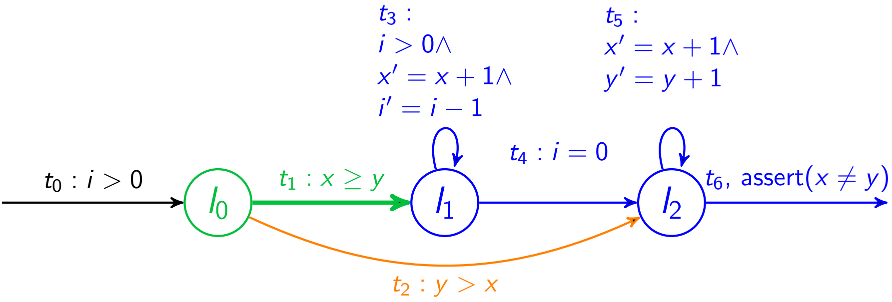
Example execution



CheckSafe on initial SCC for $i > 0$

Path is safe for $i > 0$

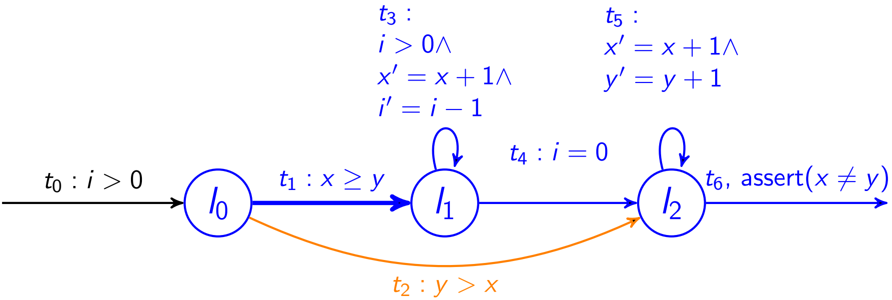
Example execution



CheckSafe on $\{\ell_0\}$ for $i > 0$

Path is safe for $i > 0$

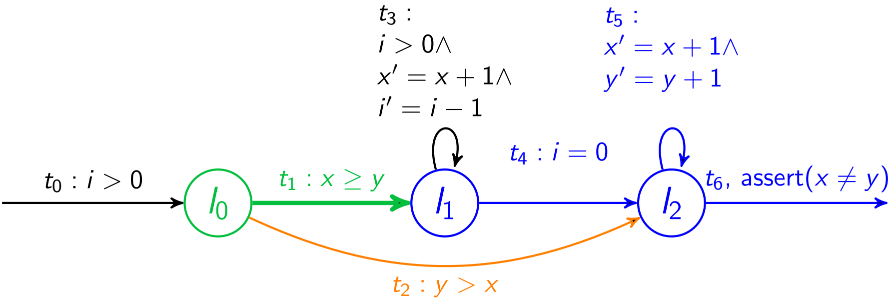
Example execution



CheckSafe on $\{\ell_0\}$ for $x \geq y$

t_1 does already imply $x \geq y$

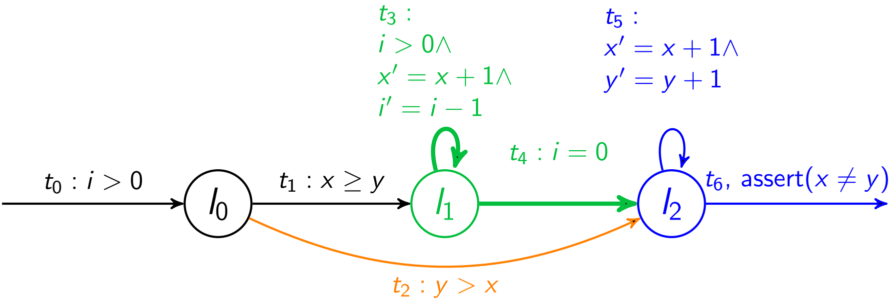
Example execution



CheckSafe on $\{\ell_0\}$ for $x \geq y$

Path is safe for $x \geq y$

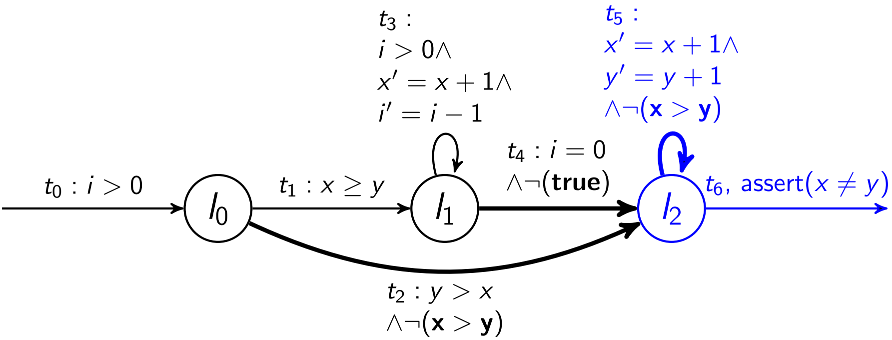
Example execution



CheckSafe on $\{\ell_1\}$ for $x > y$

Path is safe for $x > y$

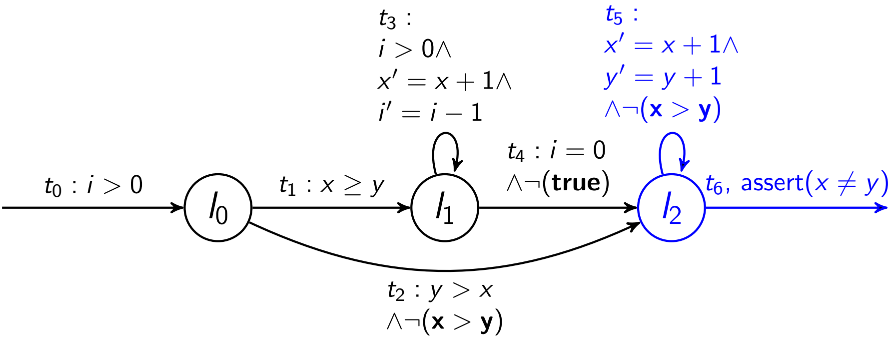
Example execution



Narrow on $\{\ell_2\}$

Add $\neg(x > y)$ to t_2
Add $\neg(x > y)$ to t_5

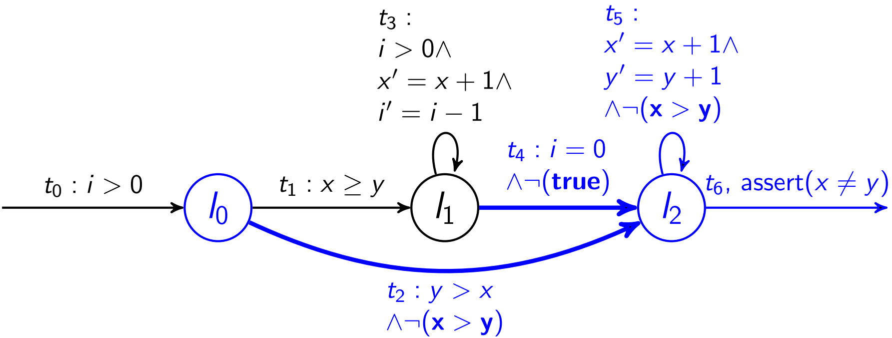
Example execution



CheckSafe on $\{\ell_2\}$ for $x \neq y$

Call CondSafe, get $y > x$ instead of $x > y$ as precondition

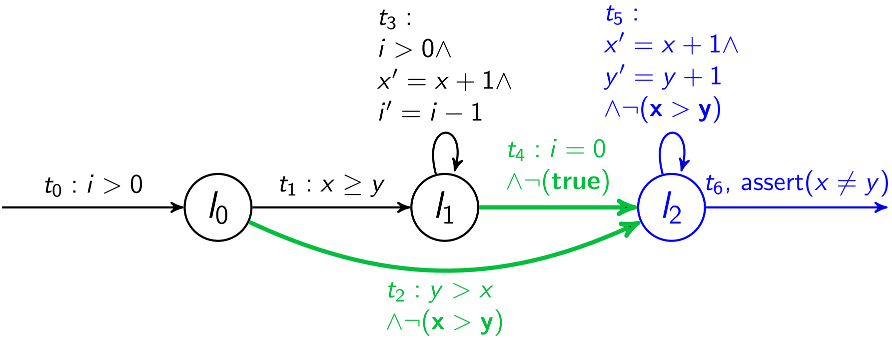
Example execution



CheckSafe on $\{\ell_0\}$ for $y > x$

t_2 does already imply $y > x$ t_4 does already imply $y > x$

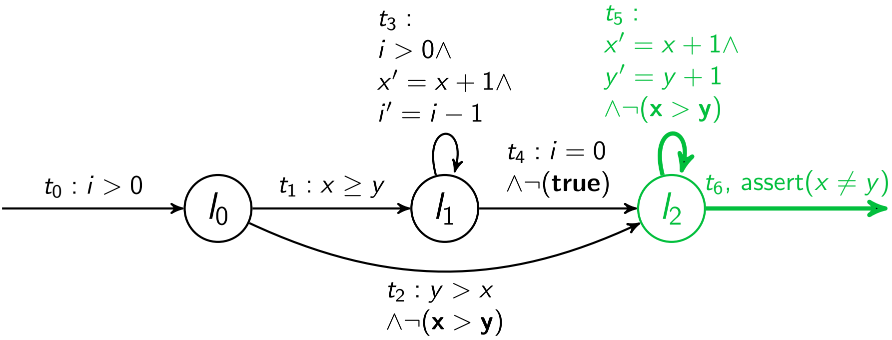
Example execution



CheckSafe on $\{\ell_0\}$ for $y > x$

Paths are safe for $y > x$

Example execution



CheckSafe on $\{\ell_0\}$ for $y > x$

Program is safe for $x \neq y$

Conclusion

We saw:

1. The exploration of multiple entry SCCs by CheckSafe
2. The effect of narrowing if a path to the SCC is proved safe and another could not be proved safe
3. The behavior of CheckSafe if a CII with multiple conjunctions is found

We didn't saw:

1. SCCs with multiple transitions (effects consecution conditions in Max-SMT and entry locations in Narrowing)