# On Compositional safety verification with Max-SMT

Author: Fabian Böller

Supervision: David Korzeniewski

RWTH Aachen

fabian.boeller@rwth-aachen.de

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#### Overview

Example execution



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- 14: return the result of a call to CheckSafe with a narrowed version

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- 10: return None



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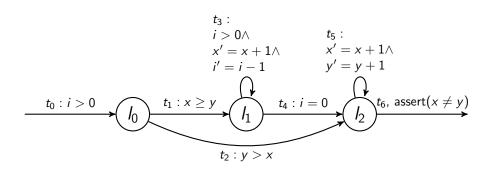
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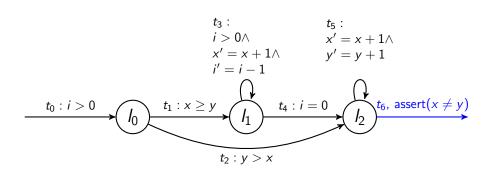
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# Example program

#### Program

$$V = \{x, y, i\}, \ \mathcal{L} = \{\ell_0, \ell_1, \ell_2\}, \ \mathcal{T} = \{t_i \mid i \in \{1, \dots, 6\}\}$$

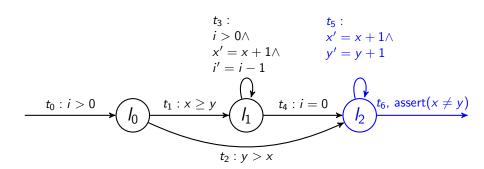




#### Task

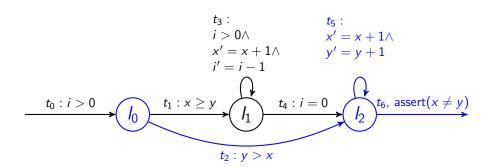
Prove that the program is safe for  $x \neq y$  at  $t_6$ 

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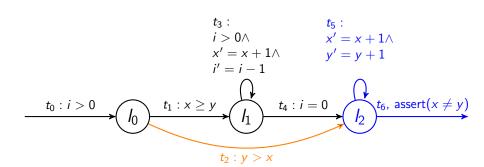
# CheckSafe on $\{\ell_2\}$ for $x \neq y$

 $t_6$  does not already imply  $x \neq y$   $t_6$  is not an initial transition Call CondSafe, get x > y as precondition



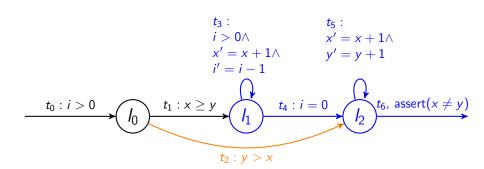
## CheckSafe on $\{\ell_0\}$ for x > y

 $t_2$  does not already imply x>y  $t_2$  is not an initial transition Call CondSafe



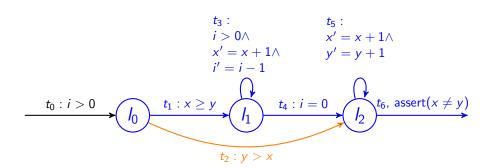
# CheckSafe on $\{\ell_0\}$ for x > y

No precondition, since y > x contradicts x > yPath is maybe safe, but not for x > y



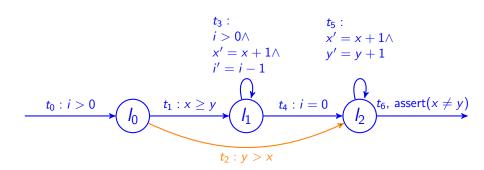
# CheckSafe on $\{\ell_1\}$ for x>y

 $t_4$  does not already imply x > y  $t_4$  is not an initial transition Call CondSafe, get  $i > 0 \land x > y$  as precondition



# CheckSafe on $\{\ell_0\}$ for i > 0

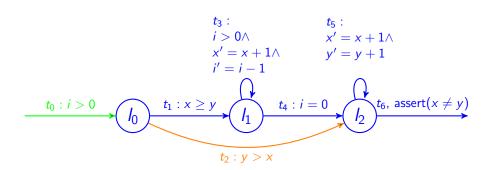
 $t_1$  does not already imply i>0  $t_1$  is not an initial transition Call CondSafe, get i>0 as precondition



#### CheckSafe on initial SCC for i > 0

 $t_0$  does already imply i > 0

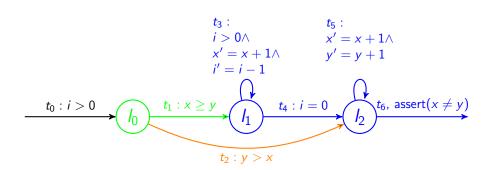
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#### CheckSafe on initial SCC for i > 0

Path is safe for i > 0

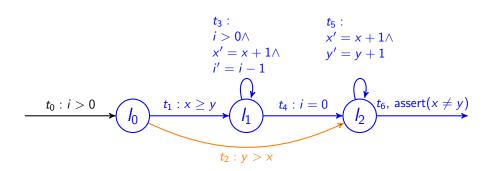
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## CheckSafe on $\{\ell_0\}$ for i > 0

Path is safe for i > 0

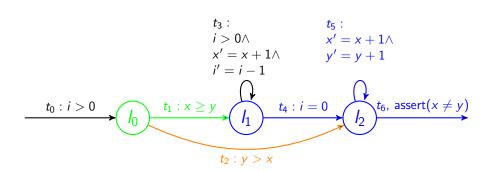
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# CheckSafe on $\{\ell_0\}$ for $x \geq y$

 $t_1$  does already imply  $x \ge y$ 

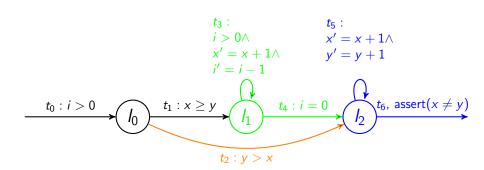
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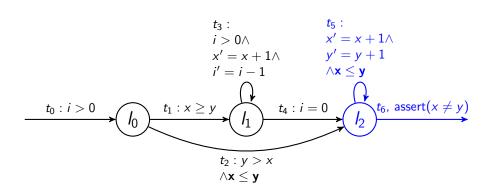
Path is safe for  $x \ge y$ 

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## CheckSafe on $\{\ell_1\}$ for x > y

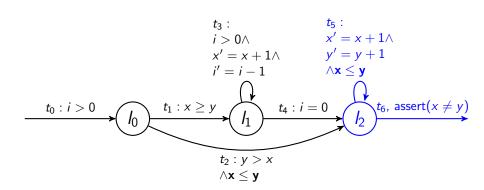
Path is safe for x > y



## Narrow on $\{\ell_2\}$

Add  $x \leq y$  to  $t_2$ 

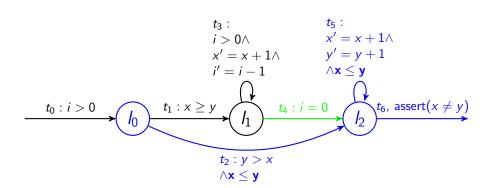
Add  $x \leq y$  to  $t_5$ 



## CheckSafe on $\{\ell_2\}$ for $x \neq y$

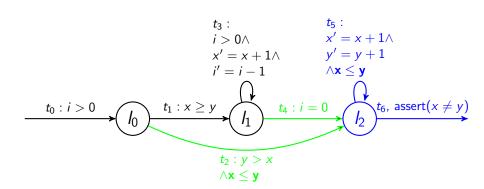
Call CondSafe, get y > x instead of x > y as precondition

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#### CheckSafe on $\{\ell_0\}$ for y > x

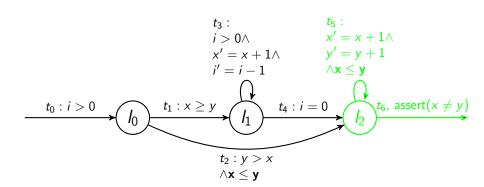
 $t_2$  does already imply y > x



#### CheckSafe on $\{\ell_0\}$ for y > x

Path is safe for y > x

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#### CheckSafe on $\{\ell_0\}$ for y > x

Program is safe for  $x \neq y$ 

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CheckSafe( $t_6$ ,  $x \neq y$ )

CondSafe(
$$x \neq y$$
)

CheckSafe( $t_2, x > y$ ) on  $C_0$ 

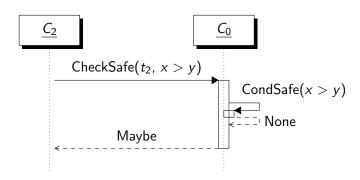
CheckSafe( $t_4, x > y$ ) on  $C_1$ 

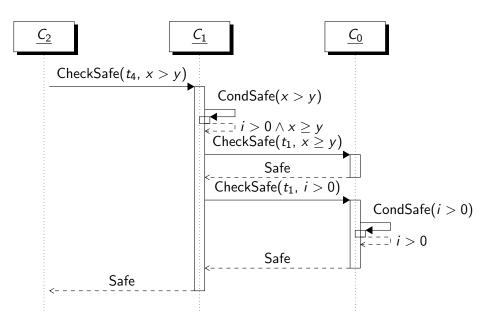
CheckSafe( $t_4, x > y$ ) on  $C_1$ 

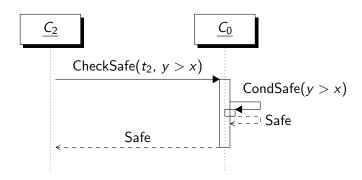
CondSafe( $t_4, x > y$ ) on  $C_1$ 

CondSafe( $t_4, x > y$ ) on  $C_1$ 

CondSafe( $t_4, x > y$ )







#### References



Brockschmidt, Marc and Larraz, Daniel and Oliveras, Albert and Rodriguez-Carbonell, Enric and Rubio, Albert (2015)

Compositional Safety Verification with Max-SMT

Proceedings of FMCAD'15

# The End