Pattern and Speech Recognition WS2015-16
Exercise 3

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Dimensionality Reduction

1. Linear classification - Iris dataset.
   The data is linearly separable and the equation of the line is :
   $ax+b$ where $a=0.5$ *and* $b=-0.2$.

   This classification has no error since there are no misclassifications in the result.

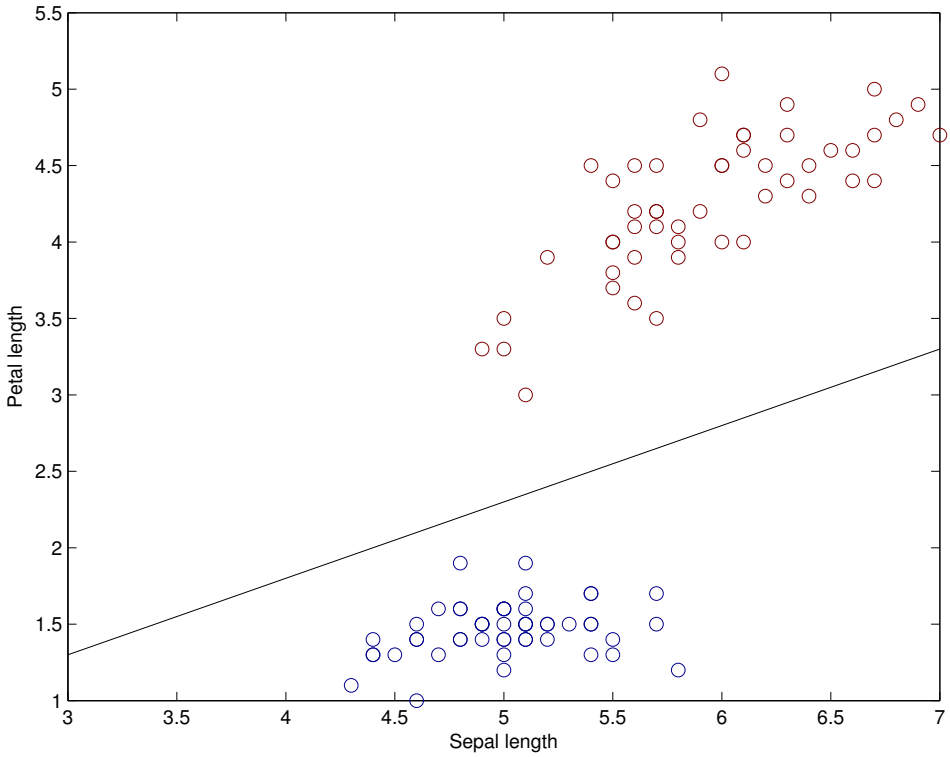

Figure 1: linear classification

2. The dimensions of W must be *n by 1*.

3. **Grid search:**

   $$\mathrm{W} = \begin{bmatrix} 0.91 \\ 0.51 \end{bmatrix}$$

   See "ex.m"

4. Plotting projection line

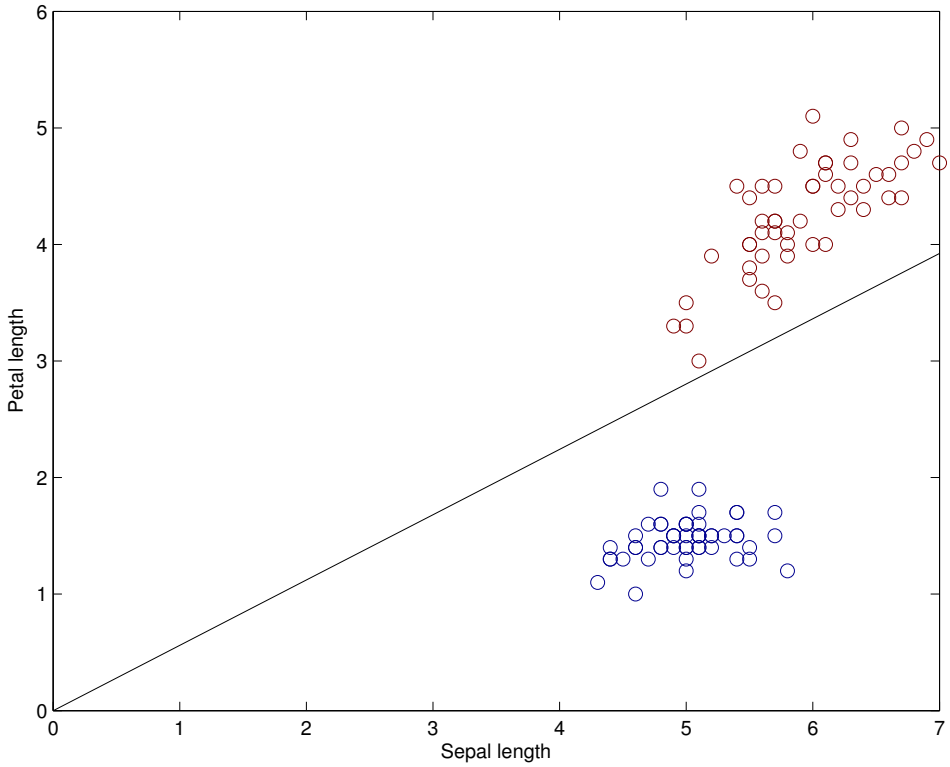

Figure 2: Projection line

The line linearly separates the data and the equation of the line is :
ax+b where *a=0.91 and b=0.51.*
This is the line which reconstructs the data with least reconstruction error.

5. Projected data

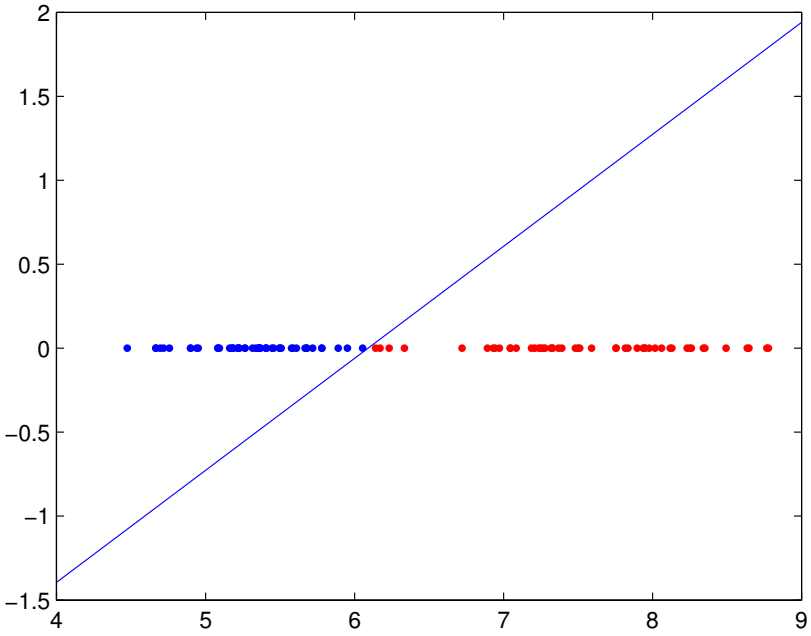

Figure 3: Projected data

Yes the data is still linearly separable. And the equation of the line is :
$ax+b$ where $a=0.666$ *and* $b=-4.06$.
And the misclassification(error) is zero.

6. *(sepal length | sepal width)* projection

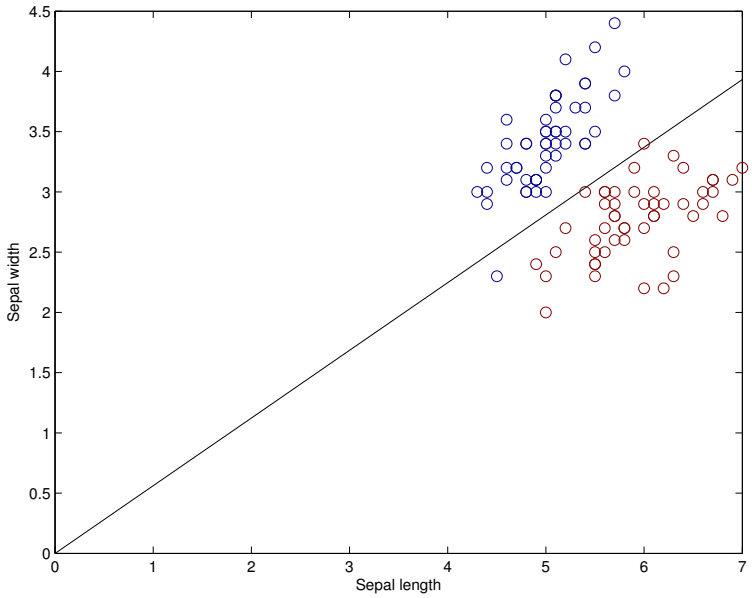

Figure 4: (sepal length | sepal width projection)

The error between the projected data and the original one, (i.e.)

$$\left\| \left(\mathbf{X} - \hat{\mathbf{X}} \right) \right\|_2^2$$

is 31.97

The projection line does not separate the classes without error. This implies that this data cannot be projected into lower dimensional space and do linear separation without errors.

Also, see *"ex6.m"*

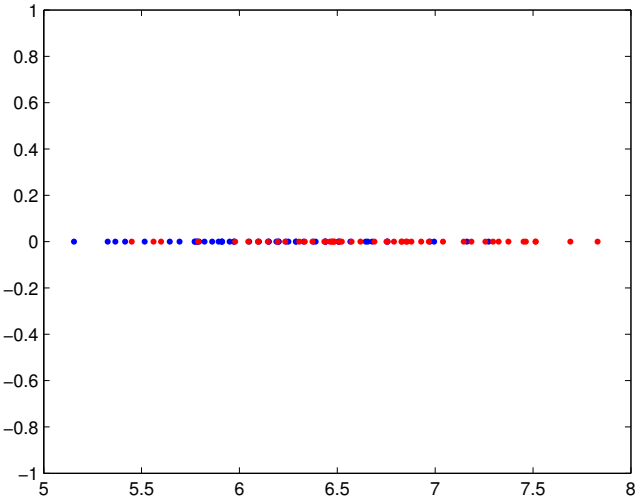

Figure 5: (sepal length | sepal width 1D-projection)

Bonus

Please see: "bonus.m"

When we multiply the data by the transformation matrix 'W' and compute the covariance matrix *(i.e. matlab subtracts the mean from each element before calculating the covariance matrix)* of the result, we get the following *4X4* matrix,

$$\begin{bmatrix} 2.767 & 6.094e-07 & -3.509e-07 & 6.855e-07 \\ 6.094e-07 & 0.228 & -7.534e-09 & -1.002e-07 \\ -3.509e-07 & -7.534e-09 & 0.051 & 8.207e-09 \\ 6.855e-07 & -1.002e-07 & 8.207e-09 & 0.010 \end{bmatrix}$$

The covariance matrix of the 3D projection is the following *3X3* matrix,

$$\begin{bmatrix} 2.767 & 6.094e-07 & -3.509e-07 \\ 6.094e-07 & 0.228 & -7.534e-09 \\ -3.509e-07 & -7.534e-09 & 0.051 \end{bmatrix}$$

The covariance matrix of the 2D projection is the following *2X2* matrix,

$$\begin{bmatrix} 2.767 & 6.094e-07 \\ 6.094e-07 & 0.228 \end{bmatrix}$$

The covariance matrix of the 1D projection is the following *1X1* matrix,

$$\begin{bmatrix} 2.767 \end{bmatrix}$$

Conclusion: Linear dimensionality reduction does not change the covariance/variance.